Essays in International Macroeconomics

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ESSAYS IN INTERNATIONAL MACROECONOMICS

a dissertation

by

GOHAR MINASYAN

submitted in partial fulfillment of the requirements

for the degree of

Doctor of Philosophy

August, 2015
Abstract

ESSAYS IN INTERNATIONAL MACROECONOMICS

by

GOHAR MINASYAN

Dissertation Committee:
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Professor PETER IRELAND
Professor EYAL DVIR

This thesis includes three essays. The first chapter analyzes how the implications of productivity shocks in an open economy can differ depending on the size of the economy relative to the rest of the world. It employs a stylized two-country general equilibrium model with love of variety, where economies differ in size and shows that a dynamic home market effect is present: productivity shocks that lower production and entry costs lead to deterioration of home terms of trade when home is small relative to the rest of the word but to improvement of terms of trade when home is large.

The second chapter analyzes the role of globalization in the lack of convergence of living standards within Europe, despite integration processes. Building on theoretical and empirical literature on trade and income inequality in the U.S. this chapter proposes a model that describes how globalization affects disparities between countries in Europe. To quantitatively assess this effect, a measure of exposure to globalization is constructed, using detailed trade, employment, and output data. The chapter shows that the relative performance of countries within Europe is correlated with their
exposure to globalization. In particular, countries that experienced relative declines of living standards over the past decade have been most exposed to globalization.

The third chapter explores the implications of demand side pricing complementarities and endogenous markups in open economy. It shows that endogenous markups resulting from translog preferences imply richer dynamics for international relative prices that have better chances to match the data. Further, countercyclical markups lead to endogenous procyclical movement as well as cross-country correlation of measured TPF. It also shows that in a stylized model endogenous markups may act as a transmission mechanism, leading in particular to positive GDP comovement across borders as opposed to a benchmark CES model.
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Chapter 1

Productivity Shocks and Terms of Trade:
The Dynamic Home Market Effect

1.1 Introduction

Understanding how international relative prices are related to productivity and output growth is a key question in international macroeconomics. A central idea in traditional trade and growth theory is that the benefits of improvements in domestic productivity are transmitted to the rest of the world as the country’s terms of trade deteriorate.\(^1\) The empirical evidence, however, suggests that in the U.S. terms of trade tend to improve rather than deteriorate following an increase in productivity.\(^2\) In response to this, a number of papers have introduced various modifications to benchmark international real business cycle models and shown that varying trade elasticities, persistence of shocks, degree of capital mobility, and openness to trade can help models match the data.\(^3\) The literature focusing on the extensive margin of trade has argued that when productivity improvements lead to introduction of new varieties,

\(^{1}\)Acemoglu and Ventura (2002) have argued that the association of capital accumulation with deteriorating terms of trade is an important factor contributing to stable world income distribution.

\(^{2}\)Corsetti, Dedola, Leduc (2006, 2008)

\(^{3}\)Corsetti, Dedola, Leduc (2008); Enders and Muller (2009)
the terms of trade would not deteriorate. Raffo (2010) finds that the correlation between terms of trade and output ranges from -0.18 to 0.43 in OECD countries. In a panel regression of 20 OECD countries, Corsetti, Martin, and Pesenti (2007) find that the relationship between output growth and relative export prices varies across countries.

In this paper I explore a new factor that can cause different countries to experience different responses of terms of trade to domestic productivity shocks: namely, the size of the economy relative to the rest of the world. This approach is motivated by the empirical regularity documented in the literature that for the U.S. and possibly some other relatively large economies terms of trade tend to be procyclical, while in smaller economies they tend to be countercyclical. In particular, in a VAR analysis for G7 countries Corsetti et al (2006) find that productivity shocks, which are identified by long-term restrictions, tend to improve terms of trade in case of large economies such as U.S. and Japan but worsen it in case of others.

Table 1.1 presents contemporaneous correlations between terms of trade (the ratio of export price index to import price index) and total factor productivity in a sample of industrialized economies. The data are from OECD’s Quarterly National Accounts database and Haver Analytics. The longest available quarterly time series has been used for each country, however, the correlations do not appear sensitive to changing the sample period. The statistics refer to the residual component obtained after applying HP-filter with a smoothing parameter equal to 1600 to the natural logarithm of each series. For the majority of countries the correlation is negative, implying that improvements in total factor productivity are associated with deteriorations of their

\[4\text{Corsetti, Martin, Pesenti (2007)}\]
terms of trade. The exceptions are the U.S. and some countries where commodity exports play an important role, including Canada, Norway and Brazil.

Table 1.1: Terms of Trade and Total Factor Productivity:
Concomporaneous Correlations

<table>
<thead>
<tr>
<th>Country</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>0.290</td>
</tr>
<tr>
<td>U.K.</td>
<td>-0.062</td>
</tr>
<tr>
<td>Turkey</td>
<td>-0.048</td>
</tr>
<tr>
<td>Switzerland</td>
<td>-0.303</td>
</tr>
<tr>
<td>Sweden</td>
<td>-0.156</td>
</tr>
<tr>
<td>Spain</td>
<td>-0.070</td>
</tr>
<tr>
<td>Slovenia</td>
<td>-0.155</td>
</tr>
<tr>
<td>Slovakia</td>
<td>-0.077</td>
</tr>
<tr>
<td>Portugal</td>
<td>-0.512</td>
</tr>
<tr>
<td>Norway</td>
<td>0.141</td>
</tr>
<tr>
<td>New Zealand</td>
<td>-0.064</td>
</tr>
<tr>
<td>Netherlands</td>
<td>-0.302</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>-0.158</td>
</tr>
<tr>
<td>Japan</td>
<td>0.073</td>
</tr>
<tr>
<td>Italy</td>
<td>-0.672</td>
</tr>
<tr>
<td>Ireland</td>
<td>-0.203</td>
</tr>
<tr>
<td>Iceland</td>
<td>-0.198</td>
</tr>
<tr>
<td>Hungary</td>
<td>-0.034</td>
</tr>
<tr>
<td>Germany</td>
<td>-0.003</td>
</tr>
<tr>
<td>France</td>
<td>-0.127</td>
</tr>
<tr>
<td>Estonia</td>
<td>-0.023</td>
</tr>
<tr>
<td>Denmark</td>
<td>-0.096</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>-0.493</td>
</tr>
<tr>
<td>Canada</td>
<td>0.056</td>
</tr>
<tr>
<td>Brazil</td>
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</tr>
<tr>
<td>Belgium</td>
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</tr>
<tr>
<td>Austria</td>
<td>-0.422</td>
</tr>
<tr>
<td>Australia</td>
<td>-0.162</td>
</tr>
</tbody>
</table>

From the theoretical standpoint the goal of this paper is to extend the well established notion of home market effect and explore in what ways economies that are not symmetric in steady state may respond differently to identical shocks. The home market effect in its original formulation, as discussed for example in Feenstra (2004) refers to the idea that when consumers love variety and trade frictions are present, the larger economy will produce more than proportionately more varieties. I show
that there is also a "dynamic" home market effect, which, in a model with trade fric-
tions and incomplete markets, in particular, makes terms of trade improve following
a positive productivity shock if the economy is relatively large and deteriorate when
the economy is relatively small.

I use a simple general equilibrium two-country model, where the countries differ
in economic (or market) size. This difference can come from assuming an asymmetry
in population size, steady state productivity levels, or other parameters but the exact
assumption is not relevant as long as it results in a difference between the economic
size of countries in the steady state. In this respect this paper differs from most of the
international macro literature that focuses either on small open economy models or
models where the two countries are symmetric, at least in the initial steady state. The
model features monopolistically competitive firms that produce an endogenous num-
ber of varieties. Labor is used both for production and innovation, i.e. introduction
of new varieties and is not mobile across countries. International trade is balanced
and subject to frictions as some varieties are nontradable and there are iceberg trade
costs to exporting.

This paper relates to the literature on international business cycle comovements
that seeks to explain the cyclical behavior of international relative prices. Most closely
it relates to Corsetti, Martin and Pesenti (2007) (CMP, hereafter), who introduce a
model where productivity gains that enhance manufacturing efficiency are distinct
from those that lower the cost of firms’ entry and of product differentiation. They
show that lower manufacturing costs lead to lower terms of trade while lower entry
(innovation) costs lead to improved terms of trade. The CMP model is therefore
capable of explaining the heterogeneous effect of productivity on terms of trade. One
difficulty however arises in that, as the authors acknowledge, productivity improve-
ments that lead to lower costs of production are probably very highly correlated with those that lead to lower costs of entry. This means that for the CMP model to explain why in case of some countries productivity tends to be positively associated with terms of trade while in case of others the relationship tends to be negative, it would have to assume that the nature of productivity shocks is systematically different in different countries. My model is similar to CMP, with the important distinctions that I do not impose a symmetric steady state and allow productivity improvements to reduces both the cost of producing and the cost of innovating.

In terms of exploring the economic significance of country size, this paper relates to Alesina et al. (2005), who focus on the interdependence of country size, openness and growth. An interesting parallel between this paper and Alesina et al. (2005) is that they both show how the size of countries influences their economic performance as long as trade is subject to frictions. In terms of relation to the empirical evidence, this paper relates to Corsetti et al. (2006) who document the cross-country heterogeneity of the relationship between productivity and terms of trade, as well as Hummels and Klenow (2005), who document empirically that the extensive margin of trade is more important for larger economies, which tend to export more varieties and benefit from stronger terms of trade.

The rest of the paper is structured as follows. Section 2 introduces the stylized model. Section 3 next discusses the properties of the asymmetric steady state. Section 4 analyzes the implications of productivity shocks. Section 5 concludes.
1.2 A Stylized General Equilibrium Model for Two Asymmetric Countries

The world consists of two countries, Home and Foreign, that are not necessarily symmetric in size. Foreign variables are denoted with an asterisk. Home and Foreign economies are populated by respectively \( L \) and \( L^* \) households. Households love variety and derive utility from consuming goods produced in Home and Foreign as well as leisure. Households own shares in their own country firms and get their profits as dividends. Labor is not mobile across countries. \( N \) \((N^*)\) firms operate in Home (Foreign). Firms are monopolistically competitive and each firm produces one variety\(^5\) either in traded (serving both Home and Foreign markets) or nontraded (serving only the domestic market) sectors, using domestic labor. The number of varieties produced in each country is endogenously determined in the model. There is free entry, but firms face fixed entry costs, which consist of wages paid for introducing a new variety. International trade is balanced and subject to iceberg trade costs. Prices are flexible, which allows to focus only on real variables. Because there are no state variables in the model, time subscripts can be conveniently omitted.

\(^5\)This assumption implies that a firm can be thought of as identical to a production line of a variety as, for example, in Bilbie, Ghironi, Melitz (2012). An increase in \( N \) \((N^*)\) corresponds to both introduction of new varieties and creation of new firms.
### 1.2.1 Households

The representative household maximizes the following separable utility function, increasing in the home consumption composite $C$ and decreasing in labor effort $l^s$:

$$U = \frac{C^{1 - \frac{1}{\psi}}}{1 - \frac{1}{\psi}} - l^s,$$

(1.1)

where $\psi \geq 0$.\(^6\)

$C$ is a CES aggregator of varieties available to Home consumers: all varieties produced in Home ($N$) and a fraction of varieties produced in Foreign $(1 - \phi)N^*$. $\phi$ is the fraction of varieties that are non-traded. The expression for Home consumption aggregator therefore is:

$$C = \left[ \int_0^N c(h)^{1 - \frac{1}{\sigma}} dh + \int_{N^*(1-\phi)}^{N^*} c(f)^{1 - \frac{1}{\sigma}} df \right]^\frac{\sigma}{1-\sigma},$$

(1.2)

where $c(h)$ is consumption of a Home produced variety, $c(f)$ is consumption of a Foreign produced variety and $\sigma > 1$ denotes the elasticity of substitution across varieties. A similar expression holds for the Foreign consumption aggregator.

Domestic households own the domestic firms. They finance the fixed costs of introducing new varieties and in return receive all the profits earned by the firms. In addition, they earn wages $W$ on labor supplied to the domestic firms. The budget constraint of a representative household therefore is:

$$\int_0^N p(h)c(h)dh + \int_{N^*(1-\phi)}^{N^*} p(f)c(f)df + I = l^sW + \Pi,$$

(1.3)

---

\(^6\)This formulation assumes constant marginal disutility of labor, corresponding to an infinite Frisch elasticity of labor.
where $p(h)$ is the price of a Home-produced variety and $p(f)$ is the price of a Foreign-produced variety. $I$ is the representative household’s share of the cost of introducing new varieties, which can be thought of as ”investment”, while $\Pi$ represents the equal share of profits that each household receives.

Dividing the household budget constraint by the Home consumption price index, it can be written in real terms:

$$\int_0^N \rho(h)c(h)dh + \int_0^{N(1-\phi)} \rho(f)c(f)df + i = lw + \pi,$$

where: $\rho(h) = p(h)/P$, $\rho(f) = p(f)/P$, $w = W/P$, $i = I/P$, and $\pi = \Pi/P$.

### 1.2.2 Firms

$N$ firms operate in Home, each producing a different variety. A fraction $\phi$ of all firms specializes in a nontraded variety, while $(1 - \phi)$ specializes in a traded variety. Strategic interaction between firms does not arise due to the assumption that the number of firms operating at any given time is large. Production uses only domestic labor and aggregate labor productivity $z$ is common for firms in traded and nontraded sectors. $z$ represents effectiveness of one unit of labor at production. The production function of a representative firm specializing in the nontraded sector is:

$$y^N = zl^{Nd},$$

and the production function of a representative firm specializing in the traded sector is:

$$y^T = zl^{Td},$$
where $l^{Nd}$ ($l^{Td}$) is the nontraded (traded) firm’s labor demand for production purposes.

To start the production of a variety a firm needs to bear a fixed cost of $f^N$ units of labor in case of a nontraded variety and $f^T$ in case of a traded variety. The fixed costs in the two sectors respectively are:

\[ \nu^N = f^N w, \]  
\[ \nu^T = f^T w. \]  

(1.7) \hspace{1cm} (1.8)

Optimizing firms set prices at a markup over marginal cost. Prices for varieties consumed in Home therefore are:

\[ p(h) = \mu \frac{W}{z}, \]  
\[ \rho(h) = \mu \frac{w}{z}, \]  

(1.9) \hspace{1cm} (1.10)

where $W$ is the nominal wage and $\mu = \sigma / (\sigma - 1)$ is the standard markup for a CES utility function.

In real terms, dividing both sides by the consumption price index $P$, equation (1.9) can be written as:

\[ \rho(h) = \mu \frac{w}{z}, \]

(1.10)

where $w = W/P$ is the real wage.

Prices of Home produced varieties consumed in Foreign are:

\[ \varepsilon p(h)^* = \mu \frac{W}{z} (1 + \tau), \]

(1.11)

where $\varepsilon$ is the nominal exchange rate (units of Home currency per units of Foreign)
and \( \tau \geq 0 \) is the iceberg trade cost parameter.

Dividing both sides by the Foreign price index \( P^* \), the real price of Home varieties consumed in Foreign is:

\[
\epsilon \rho(h)^* = \mu \frac{w}{z}(1 + \tau),
\]  

where \( \epsilon \) is the real exchange rate: \( \epsilon = \frac{\varepsilon P^*}{P} \), representing the relative price of a Foreign consumption basket in terms of Home.

In a similar fashion, the real price of Foreign produced varieties consumed in Foreign is:

\[
\rho(f)^* = \mu \frac{w^*}{z^*},
\]  

and the real price of Foreign produced varieties consumed in Home is:

\[
\rho(f) = \mu \frac{w^*}{z^*}(1 + \tau)\epsilon.
\]

Firms operating in the nontraded sector serve only the local market, therefore the market clearing condition for a representative Home nontraded variety is:

\[
y^N = Lc(h),
\]

where \( L \) is the population size of Home.

Firms operating in the traded sector serve both local and export markets, implying the following market clearing condition for a Home produced traded variety:

\[
y^T = Lc(h) + (1 + \tau)c^*(h)L^*,
\]

where \( L^* \) is the population size of Foreign. The presence of the iceberg cost parameter
in the equation indicates that in order to supply Foreign consumers with one unit of its output, the Home firm needs to ship \((1 + \tau)\) units to Foreign.

Operating profits of a representative Home firm equal total sales minus total production costs. Real profits for a nontraded variety are given by the following expression:

\[
d^N = (\mu - 1) \frac{w}{z} Lc(h). \tag{1.17}
\]

Real profits of a firm operating in the traded sector on the other hand are given by:

\[
d^T = (\mu - 1) \frac{w}{z} Lc(h) + (\mu - 1) \frac{w}{z} L^* c^*(h)(1 + \tau). \tag{1.18}
\]

Similar expressions hold for Foreign.

Equations (1.15)-(1.18) show that firms operating in the traded sector are larger and more profitable, since in this model profits are proportionate to sales.

### 1.2.3 Equilibrium

The first-order conditions of Household optimization give the following expressions for consumption and labor effort in terms of real wages:

\[
C = w^\psi, \tag{1.19}
\]

\[
l^* = w^{\psi - 1}. \tag{1.20}
\]

Consumption by a Home household of a Home-produced variety is given by:

\[
c(h) = \rho(h)^{-\sigma} C, \tag{1.21}
\]
and consumption of a Foreign-produced variety is given by:

\[ c(f) = \rho(f)^{-\sigma} C. \]  \hfill (1.22)

With CES preferences, the Home price index is given by:

\[ P = \left[ \int_0^N p(h)^{1-\sigma} dh + \int_0^{N^*(1-\phi)} p(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}}. \]  \hfill (1.23)

In equilibrium with free entry optimal investment into new varieties implies that entry costs must equal operating profits. Since households own equal shares in all firms and the distribution of firms across tradable vs. nontradable sectors is given exogenously, it is useful to define average entry costs and profits over all traded and nontraded varieties respectively.

\[ \nu = \phi \nu^N + (1 - \phi) \nu^T, \]  \hfill (1.24)

\[ d = \phi d^N + (1 - \phi) d^T. \]  \hfill (1.25)

In similar fashion, average firm size is:

\[ y = \phi y^N + (1 - \phi) y^T. \]  \hfill (1.26)

Defining in turn \( f = \phi f^N + (1 - \phi) f^T \), the zero profit condition for Home firms becomes:

\[ d = fw. \]  \hfill (1.27)

Combining equations for profits (1.17) and (1.18) and substituting for equations
for demand (1.15) and (1.16) average profits can be written as:

\[ d = (\mu - 1) \frac{w}{z} y, \]  

(1.28)

which, combined with (1.27) gives the following expression for the size of average firm:

\[ y = \frac{f z}{\mu - 1}. \]  

(1.29)

Using equations (1.17) and (1.18) and substituting for demand for Home-produced varieties by Home (equations (1.21) and by Foreign consumers (Foreign counterpart of equation (1.22)), the zero profit condition for Home firms can be written as:

\[ \frac{f z}{\mu - 1} = L(\mu \frac{w}{z})^{-\sigma} w^\psi + L^*(1 - \phi)(1 + \tau)^{1-\sigma}(\mu \frac{w^*}{\epsilon z})^{-\sigma} w^{*\psi}, \]  

(1.30)

where the expression for consumption (1.19) has also been used.

Similarly, the zero profit condition for Foreign firms is:

\[ \frac{f^* z^*}{\mu - 1} = L^*(\mu \frac{w^*}{z^*})^{-\sigma} w^{*\psi} + L(1 - \phi)(1 + \tau)^{1-\sigma}(\mu \frac{w^*}{\epsilon z})^{-\sigma} w^{\psi}. \]  

(1.31)

Since trade is balanced, Home imports have to equal Home exports:

\[ c(f) \rho(f) N^* L = c^*(h) \rho^*(h) \epsilon N L^*, \]  

(1.32)

which, using equations for real prices (1.12) and (1.14), the demand equation (1.22) and its foreign counterpart, as well as equation (1.19) and its Foreign counterpart,
can be rewritten as:

$$N^* L(\mu \frac{w^* \epsilon}{z^*})^{1-\sigma} w^\psi = NL^*(\mu \frac{w}{\epsilon z})^{1-\sigma} w^* \psi \epsilon. \quad (1.33)$$

Finally, labor market clearing conditions requiring that labor supplied in each country be demanded either for production or for innovation, close the model.\(^7\) For Home, the labor market clearing condition is:

$$Lt^* = \frac{y}{z} N + f N, \quad (1.34)$$

which, using equations (1.29) and (1.20) can be written as:

$$N = w^\psi \frac{L \mu - 1}{f \mu}. \quad (1.35)$$

Similarly, the labor market clearing condition for Foreign is:

$$N^* = w^* \psi \frac{L^* \mu - 1}{f^* \mu}. \quad (1.36)$$

Equilibrium then is described by the zero-profit conditions for Home and Foreign, (equations (1.30) and (1.31)), the balance trade condition (1.33) and labor market clearing conditions (1.35) and (1.36) for Home and Foreign respectively. The endogenous variables of the model are $N, N^*, w, w^*, \epsilon$ and the exogenous shocks are $z, z^*, f, f^*$. \(^7\)The nontradable goods market clearing condition is redundant.

Finally, terms of trade in this model is defined as the ratio of Home export price
to Home import price.

\[ TOT = \frac{\epsilon \rho(h)^*}{\rho(f)}. \quad (1.37) \]

Using the pricing equations (1.12) and (1.14), TOT can be rewritten as:

\[ TOT = \frac{w/z}{\epsilon w^*/z^*}. \quad (1.38) \]

which shows that since trade costs and markups are symmetric across countries, terms of trade is equivalent to the inverse of "terms of labor".

1.3 The Steady State

In this section I solve for the steady state where productivity, including both the efficiency of production as well as the cost of introducing new varieties, is at its steady state value. The key difference of this model from other two-country models in the literature is that I am not imposing symmetry in the steady state. It should be noted that the source of asymmetry is not important. I assume that Home and Foreign economies differ in population size, however similar results could be obtained by assuming, for example, that they differ in steady state level of productivity, or any other assumption that would result in the market size in the two countries being asymmetric.

I assume that \( L \neq L^* \) and this is the only source of asymmetry of the economies in steady state. In particular, I assume that \( L/L^* \epsilon(0,1) \), i.e. the largest Home can be is as large as the rest of the world (as is realistic) and the smallest it can be is as small as a small open economy. Further, to remove "scale effects" I assume that the size of the world i.e. \( L + L^* \) is fixed.
Output of an average firm (the weighted average of traded and nontraded sectors as explained in equation (1.26) is fixed in steady state and is the same in Home and Foreign:

\[ y^{ss} = y^{s*ss} = \frac{f^{ss}z^{ss}}{\mu - 1} = f^{ss}z^{ss}(\sigma - 1), \]  

(1.39)
as given by equation (1.29).

The extensive margin on the other hand is affected by the asymmetry. The model features the traditional home market effect in that the larger economy produces more varieties. When there are no trade costs and all goods are traded, the ratio of Home-to-Foreign-produced varieties equals the relative size of Home to Foreign, however in the presence of trade costs the larger economy will produce more than proportionally more varieties in steady state.

The larger economy will also have higher wages in steady state. This is because while labor demand for production does not depend on the size of the economy, labor demand for setting up new firms does. Households in the larger economy consume and work more in the steady state. Even though output of a representative firm is not dependent on the size of the economy, what becomes important is the portion of output consumed domestically versus the portion that is exported: \( y = y^d + y^x \). In steady state output consumed domestically is:

\[ y^d_{ss} = L(\frac{\sigma - 1}{\sigma})^\sigma z^{ss}w^{\phi - \sigma}, \]  

(1.40)
and as long as \( \sigma > \psi \) domestic consumption of Home-produced varieties is smaller when the economy is smaller. In other words, consistent with empirical evidence documented for example in Alesina et al. (2005), smaller economies tend to be more open.
The expression for real exchange rate in steady state is:

$$\epsilon_{ss} = \left( \frac{N_{ss} L^*}{N_{ss} L} \right)^{(2\sigma-1)(\psi-1)},$$  \hspace{1cm} (1.41)

and for terms of trade it is:

$$TOT_{ss} = \left( \frac{N_{ss} L^*}{N_{ss} L} \right)^{(2\sigma-1)(\psi-1) - 1}.$$ \hspace{1cm} (1.42)

In a symmetric steady state both $\epsilon_{ss} = 1$ and $TOT_{ss} = 1$. This also holds when $L \neq L^*$ but there are no trade costs as in this case the steady state number of varieties is proportional to the country size. However, in the presence of trade costs when $L < L^*$ then $\epsilon_{ss} < 1$ and $TOT_{ss} < 1$ as long as $\psi > 1$. These results are consistent with empirical evidence known as the "Penn effect" or the purchasing power parity puzzle as in Rogoff (1996).

### 1.4 Responses to Productivity Shocks

In this section I explore the model responses to productivity shocks and how these responses differ depending on the relative size of Home versus Foreign countries. For this goal I log-linearize the model around the steady state assuming a shock to Home productivity, affecting efficiency of production and entry cost $\hat{\varepsilon} > 0$, $\hat{f} < 0$ and no shocks to Foreign: $\hat{z}^* = \hat{f}^* = 0$. The important aspect of the log-linear solution (the full solution is described in Appendix A) is that the coefficients are functions of the relative size of the two economies in steady-state.

As is evident from equation (1.29) the response of average Home firm size (or scale of production) is generally ambiguous: $\hat{y} = \hat{\varepsilon} + \hat{f}$. It depends on whether
improvements in productivity result more in reduction of production costs or of costs to innovation. The Foreign firm size is unaffected. Note that, the response of firm size is independent of the asymmetry.

Home productivity improvement results in higher wages at Home as it increases the demand for labor both for production and innovation. While wage increase in unambiguous, exactly how much wages increase depends on the value of $\psi$. As long as $\psi > 1$ labor supply also increases, but not enough to lead to decline of wages. If $\psi < 1$ then because of a strong wealth effect, labor supply decreases, leading to even larger increase of wages.

The response of wages to productivity depends on the steady state asymmetry. In particular if we express the solution as $\dot{w} = \gamma_z \dot{z} + \gamma_f \dot{f}$, then $\gamma_z > 0$ and $\gamma_f < 0$ for all parameter values but $\partial \gamma_z / \partial (L/L^*) > 0$ and $|\partial \gamma_f / \partial (L/L^*)| > 0$, i.e. the impact of shocks is magnified when Home is large. The intuition is that the positive impact of Home productivity improvements on Foreign depends positively on the size of Home. In turn this translates into higher Foreign demand for home goods, magnifying the response of Home variables. When the size of Home approaches zero (as in the small open economy model) the impact on Foreign variables also approaches zero.

As a result of Home productivity improvement the number of varieties produced in Home increases for two reasons: lower cost of introduction of new varieties and higher demand from both Home and Foreign, provided that $\psi > 1$. Higher demand results from the fact that consumption increases more than proportionately with income as long as $\psi > 1$. On the other hand, when $\psi < 1$, the demand effect is negative and the impact on the number of produced varieties may decrease if the impact of lower entry costs is not large enough. As is expected, the response of varieties again is less pronounced when Home is small.
The log-linear equation for real exchange rate is:

$$\hat{\epsilon} = \frac{\sigma}{2\sigma - 1} (\hat{w} - \hat{w}^*) - \frac{\sigma - 1}{2\sigma - 1} \hat{z} + \frac{1}{2\sigma - 1} \hat{f},$$  \hspace{1cm} (1.43)$$

and for terms of trade it is:

$$T\hat{O}T = \frac{\sigma - 1}{2\sigma - 1} (\hat{w} - \hat{w}^*) - \frac{\sigma}{2\sigma - 1} \hat{z} - \frac{1}{2\sigma - 1} \hat{f}$$ \hspace{1cm} (1.44)$$

The response of the terms of trade is generally ambiguous. As can be seen from equation (1.44), the ambiguity, can come from two sources: (i) the nature of the productivity shock, i.e. to what extent the productivity improvements lead to reduction of production costs $\hat{z}$ versus reduction of costs of innovation $-\hat{f}$ and (ii) the impact of the productivity shock on the log wage differential $\hat{w} - \hat{w}^*$. The first one, i.e. the difference between productivity shocks enhancing the efficiency of production and productivity shocks lowering costs of entry and their impact on international relative prices is the subject of Corsetti et al. (2007). Instead I focus on the second source of ambiguity, i.e. the log wage differential and how it systematically depends on the relative size of Home compared to Foreign.

It is useful to rewrite the terms of trade equation as:

$$T\hat{O}T = \frac{\sigma - 1}{2\sigma - 1} (\hat{w} - \hat{w}^* - \hat{z}) - \frac{1}{2\sigma - 1} (\hat{f} + \hat{z}),$$ \hspace{1cm} (1.45)$$

and consider the special case when efficiency in production and in innovation are perfectly correlated $\hat{z} = -\hat{f}$. In that case $T\hat{O}T$ will increase as long as the Home to Foreign wage differential is larger than the productivity shock.

The dependence of the log wage differential on the relative country size is the
“dynamic home market effect”. It can be written as:

\[ \hat{w} - \hat{w}^* = \Gamma_1 \hat{z} - \Gamma_2 \hat{f}, \] (1.46)

where, as shown in Appendix A, both of the coefficients \( \Gamma_1 \) and \( \Gamma_2 \) are functions of \( (y^d + y^{d*}) \), i.e. the sum of steady state Home demand for Home-produced goods and Foreign demand for Foreign produced goods, which can be called world domestic demand. It is straightforward to verify that \( \partial \Gamma_1 / \partial (y^d + y^{d*}) > 0 \) and \( \partial \Gamma_2 / \partial (y^d + y^{d*}) > 0 \) as long as \( \sigma > \phi \).

When there are no trade costs and all goods are traded, world domestic demand \( (y^d + y^{d*}) \) does not depend on the relative sizes of Home and Foreign. This is not surprising as the conventional home market effect occurs only when there are trade frictions. However, in the presence of trade frictions, the larger the asymmetry, the smaller world domestic demand will be. This means that as long as there are trade frictions, there will be a dynamic home market effect in the sense that following a positive Home productivity shock the Home to Foreign wage differential will increase by more if Home is a larger economy (accounting for up to half of the world) and by less if it is small relative to Foreign. Further, the dynamic home market effect suggests that following a positive Home productivity shock terms of trade are more likely to improve if Home is large relative to the rest of the world and more likely to deteriorate when Home is small relative to the rest of the world.

Figures 1.1-1.3 illustrate the responses to a positive Home productivity shock and their dependence on the relative size of Home versus Foreign. A positive shock to productivity is assumed to make it easier to produce existing varieties (Figure 1.1), to introduce new ones (Figure 1.2), or both (Figure 1.3). The horizontal axis is the share of Home size \( L \) in the world economy, which is normalized to \( L + L^* = 2 \). The
solid lines show results with trade frictions, assuming that 50 percent of all varieties are nontraded ($\phi = 0.5$) and there are iceberg trade costs ($\tau = 0.2$). Dashed lines show results with no trade frictions. The parameter $\psi$ is calibrated at $1.2$ and the elasticity of substitution across varieties is calibrated at $\sigma = 3.8$.\(^8\)

As shown in Figure 1.3, when productivity of both production and innovation increase, the response of the terms of trade is positive when Home is large and gradually declines and becomes negative when Home is small. The bottom charts show the responses of welfare as measured by utility in the two countries, which show that welfare responses both in Home and Foreign depend negatively on the asymmetry. The welfare log differential ($\hat{U} - \hat{U}^*$) also depends negatively on the share of Home in the world, implying that relatively small economies share the benefits of their productivity improvement with the rest of the world mode than large economies do.

1.5 Conclusion

While the open economy macro literature mostly uses either small open economy models or models where the two countries are symmetric in steady state, the question of interest for this paper is precisely what happens in between these two extremes and how such asymmetry can affect the way economies react to shocks. I find that in a model with trade frictions there is a dynamic home market effect: the responses of key macroeconomic variables to Home productivity improvements depend on how large Home is relative to the rest of the world. In particular terms of trade improve following a positive productivity shock if Home is relatively large and deteriorate, when it is relatively small. These results can help explain the empirical regularity documented

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\(^8\) This calibration follows the one in Corsetti et al. (2007).
in the literature that for the U.S. productivity growth tends to be associated with improvement in the terms of trade while in most other economies it tends to be associated with deterioration of the terms of trade.
Figure 1.1: Responses to Home Productivity $z$-Shock
Figure 1.2: **Responses to Home Productivity f-Shock**
Figure 1.3: Responses to Home Productivity Combined Shock

- **Home Wage**
- **Foreign Wage**
- **Exchange Rate**
- **Terms of Trade**
- **Home Number of Varieties**
- **Foreign Number of Varieties**
- **Home Consumption**
- **Foreign Consumption**
- **Home Labor Effort**
- **Foreign Labor Effort**
- **Home Welfare**
- **Foreign Welfare**
Appendix A

The Log-Linear Solution of the Model

The log-liner system is:

\[
\hat{N} = (\psi - 1)\hat{w} - \hat{f}, \quad (A.1)
\]

\[
\hat{N}^* = (\psi - 1)\hat{w}^*, \quad (A.2)
\]

\[
((\psi - \sigma)y^d - \sigma(1 - \phi)y^x)\hat{w} + (\psi(1 - \phi)y^x)\hat{w}^* + \sigma(1 - \phi)y^x\hat{\epsilon} = ((1 - \sigma)y)\hat{z} + y\hat{f}, \quad (A.3)
\]

\[
((\psi - \sigma)y^{d*} - \sigma(1 - \phi)y^{x*})\hat{w}^* + (\psi(1 - \phi)y^{x*})\hat{w} - \sigma(1 - \phi)y^{x*}\hat{\epsilon} = 0, \quad (A.4)
\]

\[
\hat{N}^* - \hat{N} + (\psi - 1 + \sigma)\hat{w} - (\psi - 1 + \sigma)\hat{w}^* - (2\sigma - 1)\hat{\epsilon} = (\sigma - 1)\hat{z}, \quad (A.5)
\]

where \(y^d (y^{d*})\) is Home (Foreign) demand for a Home (Foreign) produced variety and \(y^x (y^{x*})\) is Home (Foreign) exports of a Home (Foreign) produced variety. The system can be simplified into two equations in two endogenous variables \(\hat{w}\) and \(\hat{w}^*\):

\[
\Lambda_1\hat{z} + \Lambda_2\hat{f} = \Lambda_3\hat{w} + \Lambda_4\hat{w}^*, \quad (A.6)
\]

\[
\Lambda_5\hat{z} + \Lambda_6\hat{f} = \Lambda_7\hat{w} + \Lambda_8\hat{w}^*, \quad (A.7)
\]

where:

\[
\Lambda_1 = -\frac{\sigma - 1}{2\sigma - 1}(f(\sigma - 1)^2 + \sigma y^d), \quad (A.8)
\]

\[
\Lambda_2 = \frac{1}{2\sigma - 1}(f(\sigma - 1)^2 + \sigma y^d), \quad (A.9)
\]
\[ \Lambda_3 = y^d(\psi - \frac{\sigma^2}{2\sigma - 1}) - \frac{\sigma(\sigma - 1)^2}{2\sigma - 1}, \quad (A.10) \]

\[ \Lambda_4 = (\psi - \frac{\sigma^2}{2\sigma - 1})(f(\sigma - 1) - y^d), \quad (A.11) \]

\[ \Lambda_5 = -\frac{\sigma(\sigma - 1)}{2\sigma - 1}(f(\sigma - 1) - y^{sd}), \quad (A.12) \]

\[ \Lambda_6 = \frac{\sigma}{2\sigma - 1}(f(\sigma - 1) - y^{sd}), \quad (A.13) \]

\[ \Lambda_7 = (\psi - \frac{\sigma^2}{2\sigma - 1})(f(\sigma - 1) - y^{sd}), \quad (A.14) \]

\[ \Lambda_8 = y^{sd}(\psi - \frac{\sigma^2}{2\sigma - 1})(\frac{f\sigma(\sigma - 1)^2}{2\sigma - 1}), \quad (A.15) \]

and it was used that \( y = y^* = f(\sigma - 1) \) and \( y^* = \frac{1}{1-\phi}(f(\sigma - 1) - y^d) \).

Note that the asymmetry of the steady state is captured by two steady state values \( y^d \) and \( y^{d*} \) that represent respectively Home demand for a Home produced variety and Foreign demand for a Foreign produced variety:

\[ \hat{w} - \hat{w}^* = \Gamma_1 \hat{\varepsilon} + \Gamma_2 \hat{f}, \quad (A.16) \]

where:

\[ \Gamma_1 = \frac{f(\sigma - 1)^2(\sigma - \phi)(\sigma(y^d + y^{d*}) - f(\sigma - 1))}{(f(\sigma - 1)\frac{\sigma(\sigma - 1)}{2\sigma - 1})^2 + f(\sigma - 1)((\frac{\sigma^2}{2\sigma - 1} - \phi)(f(\sigma - 1) + (y^d + y^{d*})(\frac{\sigma(\sigma - 1)}{2\sigma - 1} - 1))}, \quad (A.17) \]

\[ \Gamma_2 = \frac{f(\sigma - 1)^2(\sigma - \phi)(\sigma(y^d + y^{d*}) - f(\sigma - 1))}{(f(\sigma - 1)\frac{\sigma(\sigma - 1)}{2\sigma - 1})^2 + f(\sigma - 1)((\frac{\sigma^2}{2\sigma - 1} - \phi)(f(\sigma - 1) + (y^d + y^{d*})(\frac{\sigma(\sigma - 1)}{2\sigma - 1} - 1))} \quad (A.18) \]
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Chapter 2

Globalization and (Lack of) Convergence in Europe

2.1 Introduction

Convergence of living standards within Europe is a key idea behind European integration. Nevertheless, the evidence of convergence within Europe is mixed at best: as illustrated in Figure 2.1, the relative position of most European economies in early 2010s was similar to that in early 1990s. In particular, Greece, Portugal, and Spain, which started with GDP per capita significantly below the EU average, experienced some convergence towards it in the earlier period of European integration that was however reversed in the later years. In case of Italy, GDP per capita relative to EU average has been continuously declining. The opposite pattern is observed in countries that started with higher than average GDP per capita: in particular, in Austria, Belgium, Denmark, Finland and Sweden initial convergence towards the average was followed by divergence in the later period. Ireland seems to be a notable exception, as starting from below average it first grew fast to exceed it and then continued growing slower, moving towards the average.

In this paper I exclude the Eastern European transition economies from the analysis. The fast growth of most of these economies starting from a low base has been largely attributed to structural transformation related to transition into a market economy, which is outside the scope of this paper.
Literature has offered various explanations for the lack of convergence in Europe, and the topic gained more attention after the global financial crisis, especially in the context of the performance of Southern European countries. Some authors have noted that persistent trade deficits of the Southern European economies may not be as benign and natural attributes of the catching-up process as was initially believed. Others argued that if countries do not use international borrowing to build up adequate production capacity, their intertemporal budget constraint (which rationalized borrowing to finance the catching up process) may not be satisfied. As for the root causes for the lack of convergence in Europe the literature has traditionally focused on what can be referred to as the incomplete nature of integration, including the significant "reform gap" in Southern Europe, focusing in particular on problems with product and factor markets and persisting heavy regulation in several areas.

This paper aims to contribute to the discussion about lack of convergence in Europe from a new angle, by building on theoretical and empirical literature on trade and inequality. As Krugman (2008) has shown, trade with low-income countries has especially important implications for inequality in relatively high-income countries. The increase of trade between advanced and developing countries has been remarkable in the last couple of decades, largely driven by China, whose accession into the WTO in 2001 in fact marks a turning point in international trade. A number of papers have addressed the relationship between trade and inequality in the context of the U.S. In particular, Autor, Dorn and Hanson (2012) (hereafter: ADH) relate changes in labor market outcomes from 1990 to 2007 across U.S. local labor markets (commuting zones) to changes in exposure to Chinese import competition, which depends on the relative

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2 Giavazzi and Spaventa (2010)

3 E.g. see Nowotny et al (2012) for a survey.

4 Chen, Jin, and Yue (2010), Hsieh and Klenow (2009), Naughton (2007)
importance of different industries for local employment in which China has been specializing. Other papers on the impact of globalization on income distribution in the U.S. include Bernard, Jensen, and Schott (2006), Liu and Trefler (2008), Ebenstein et al. (2010) and others.

The issue of widening disparities within Europe has not been viewed in the light of the globalization and inequality debate. Yet, as far as international trade is concerned, Europe is very much a single region. Unified trade policy vis a vis non-EU countries as well as elimination of trade barriers within the EU are the areas where European integration has advanced to the fullest. Because of this, globalization has implications for convergence/divergence within Europe that are quite similar to implications for inequality in the U.S., which have been widely studied both theoretically and empirically.

In this paper I use a simple model of a small open economy that is meant to represent a "typical" economy in Southern Europe to show how such an economy is affected when integration processes within the European Union are combined with globalization. The country’s integration into the EU is modeled as lifting of trade barriers within Europe and technology transfers as well as consumption transfers from the "core" to "periphery". The small open economy is then hit by a globalization shock represented by increased trade between the European Union and the "third world" that specializes in low-skill intensive industries. I show that this shock leads to lower income and lower employment in the traded sector in the small open economy, counteracting the positive effects of the integration processes. To relate these results to the data I derive from the model a measure of exposure to globalization which depends on the country’s initial industry specialization profile and find that the countries in Europe that experienced relative declines of living standards over the
past decade have been most exposed to globalization.

This paper relates to the recent mostly empirical work on the post global financial crisis performance in Europe. In particular, Chen et al. (2013) point out that China explains a large share of the rising trade deficits of periphery European economies, as exports of Greece, Italy and Portugal have been challenged by Chinese exports. They find that at the detailed product level China has been rapidly entering product categories that these countries were traditionally active, implying intensified competition in the same products in the same markets. In total, the fraction of product codes where both exporters are active in the EU 15 market continually increased and reached more than 60 percent of all trade links at the 6-digit level by at least one partner, i.e. China has been increasingly “fishing in the same pool”.

Raising policy-relevant questions is a useful aspect of this paper’s approach to the problem of (lack of) convergence in Europe. In particular, while the questions whether a common currency or monetary policy can be optimal for all countries in the Union has been broadly studied in the literature, the question whether a single trade policy can be optimal for all countries has not. As Giavazzi and Spaventa (2010) argue “the current account has always been a neglected variable in management of the Euro area and in the assessment of its member’s performance.” Policies both at the country level and at the level of the European Union should be mindful that a unified trade policy implies asymmetric shocks to different countries stemming from globalization. To address this issue at the centralized level could mean developing mechanisms of centralized risk sharing and transfers across countries of the union to mitigate the asymmetric impact of the globalization shock. Given labor mobility between countries as well as between sectors is limited, transfers could play an important role. For example, continuing the parallel to the U.S. exposure to these shocks could be
covered by a scheme like Trade adjustment Assistance (TAA) in the U.S., which is the primary federal program that provides financial support to workers who lose their jobs as a result of foreign trade.

The rest of the paper is structured as follows. Section 2 presents the small open economy model and discusses the model responses to trade and technology shocks. Section 3 analyzes integration processes in Europe and globalization with the help of the model. It then derives model-based measures of exposure to globalization, constructed from detailed trade, employment and output data. Section 4 concludes.

2.2 The Model

In this section I describe a simple model of a small open economy that produces heterogeneous tradable goods as well as a homogeneous nontradable good. Trade is based on monopolistic competition with a "gravity" structure as in Arkolakis et al (2012). Labor is the only productive factor and labor supply is fixed. The small open economy (denoted as home economy $i$) is first subjected to integration shocks that include (i) lifting of trade barriers within the European Union; (ii) technology transfers leading to lower cost of production in the traded sector; (iii) consumption transfers, that allow the economy to run a trade deficit. Then it is subjected to a globalization shock, whereby the trade between the European Union and the third world increases.
2.2.1 Households

A representative household maximizes utility from consumption:

\[ U_i = \ln C_i, \tag{2.1} \]

where the consumption composite \( C \) is a Cobb-Douglas aggregator of nontradable and tradable consumption:

\[ C = C_{T_i}^\gamma C_{N_i}^{1-\gamma}. \tag{2.2} \]

The tradable consumption \( C_T \) in its turn is a Cobb-Douglas aggregator of sectoral composites \( C_{Tj} \), with equal expenditure share \((\gamma/J)\) for each, defined as:

\[ C_{Ti} = \prod_{j=1}^{J} C_{Tji}^\gamma. \tag{2.3} \]

Each sectoral composite \( C_{Tji} \) is a composite of symmetric product varieties indexed by \( \omega \in \Omega \) within sector \( j \) given by:

\[ C_{Tji} = (\int_{\omega \in \Omega} c_{ji}(\omega)^{\frac{\sigma_j-1}{\sigma_j}})^{\frac{\sigma_j}{\sigma_j-1}}, \tag{2.4} \]

where \( \sigma_j > 1 \) is the elasticity of substitution between any pair of varieties within sector \( j \) and can be different in different sectors.

The household supplies inelastically \( L_i \) units of labor to domestic firms at wage \( W_i \). The budget constraint of the household is:

\[ P_i C_i = W_i L_i + T_i, \tag{2.5} \]
where $P_i$ is the aggregate price index in the home economy $i$ and $T_i$ is a lump-sum consumption transfer. In the aggregate, budget constraint for the economy $i$ is isomorphic to equation (2.5), assuming that the economy is of measure 1.

### 2.2.2 Firms

Firms operate either in the nontraded or traded sectors. The nontraded sector is perfectly competitive and the production function of the homogeneous nontraded good is given by:

$$X_{Ni} = L_{Ni}^\eta,$$

(2.6)

where $L_{Ni}$ is labor employed in the nontraded sector, and $\eta \in (0, 1)$, implying that there are diminishing marginal returns to labor.

There are a total of $j = [1 : J]$ traded sectors and $M_j$ monopolistically competitive firms operate in each sector $j$. Each firm serves both the domestic and export markets. The production function can be written as:

$$l_{ij}(\omega) = \alpha_{ij} + \beta_{ij}x_{ij}(\omega),$$

(2.7)

where $l_{ij}(\omega)$ is the amount of labor required to produce $x_{ij}$ units of the traded variety $\omega$. $\alpha_{ij} > 0$ and $\beta_{ij} > 0$ are the technology parameters of producing a traded variety of sector $j$ in home economy $i$.

As it evident from (2.7) there are increasing returns to scale in the production of a traded variety, implying that the minimum cost of producing $x_{ij}(\omega)$ units of variety $\omega$ in sector $j$ is:

$$c_{ij}(W_i, x_{ij}(\omega)) = W_i(\alpha_{ij} + \beta_{ij}x_{ij}(\omega)).$$

(2.8)
\( \alpha_{ij} W_i \) is the fixed cost and \( \beta_{ij} W_i \) is the marginal cost. These are the sectoral productivity parameters of the home economy \( i \) and therefore determine comparative advantage.

As part of European integration the the country \( i \) can benefit from technology transfer from the "core" (EUC) which would lead to reduction of both fixed and marginal costs of production. In particular the positive effect of FDI from EUC can be reflected in lower fixed costs of production in \( i \), while learning from the technologically more advanced EUC would bring production processes in \( i \) closer to the technological frontier and lower marginal costs of production.

In symmetric equilibrium, each variety is produced by a single monopolistically competitive firm and the \( \omega \) can be dropped. Since \( \Omega \) is large, the price of each variety is a constant markup \( \mu = \sigma_j/(\sigma_j - 1) \) over the marginal cost. However, prices will vary across domestic and export markets due to the presence of destination-specific trade costs: to ship one unit of good \( j \) from country \( i \) to country \( k \) the firm must cover a "iceberg" trade cost of \( \tau_{ijk} \). Therefore, the price of a variety of good \( j \) produced in \( i \) and sold in \( k \) is given by:

\[
P_{ijk} = \sigma_j \frac{\beta_{ij} W_i \tau_{ijk}}{(\sigma_j - 1)},
\]

where \( \tau_{ijk} > 1 \) unless \( i = k \).

2.2.3 Trade

Demand in country \( k \) for a traded variety of \( j \) produced in country \( i \) is given by:

\[
x_{ijk} = (\frac{P_{ijk}}{\Phi_{jk}})^{-\sigma_j} C_{Tjk},
\]

where \( \Phi_{jk} \) is the production cost in country \( k \) and \( C_{Tjk} \) is the consumption of good \( j \) in country \( k \).
which is a standard expression for a CES preference structure, where \( \Phi_{jk} \) is the price index of traded sector \( j \) in market \( k \) and \( C_{Tjk} \) is the consumption of traded goods \( j \) in market \( k \). The sectoral price index is given by:

\[
\Phi_{jk} = \left[ \sum_h M_{hj} P_{hjk}^{1-\sigma_j} \right]^{\frac{1}{1-\sigma_j}}, \tag{2.11}
\]

where \( M_{hj} \) is the number of \( j \) varieties produced in region \( h \). The sectoral price is a measure of intensity of competition in market \( k \). Since the constant expenditure share property of Cobb-Douglas preferences implies that:

\[
\Phi_{jk} C_{Tjk} = \frac{\gamma E_k}{J}, \tag{2.12}
\]

where \( E_k \) is the expenditure in market \( k \): \( E_k = W_k L_i + T_k \), as in the household budget constraint. Equation (2.10) can be rewritten as:

\[
x_{ijk} = \left( \frac{P_{ijk}^{-\sigma_j}}{\Phi_{jk}^{1-\sigma_j}} \right) \frac{\gamma E_k}{J}. \tag{2.13}
\]

The total demand for output of an individual variety \( x_{ij} \) is therefore the sum of demands from all destination markets:

\[
x_{ij} = \sum_k x_{ijk} = \sum_k \frac{P_{ijk}^{-\sigma_j}}{\Phi_{jk}^{1-\sigma_j}} \frac{\gamma E_k}{J}. \tag{2.14}
\]

### 2.2.4 Equilibrium

The market clearing condition for the nontraded good is:

\[
P_{N_i} X_{N_i} = (1 - \gamma)(W_i L_i + T_i). \tag{2.15}
\]
Demand for goods is given by a Cobb-Douglas utility function with share $\gamma$ of expenditures going to traded and share $1 - \gamma$ to nontraded goods.

Wages are pinned down from profit maximization in the nontraded sector:

$$W_i = \eta P_{N_i} L_{N_i}^{\eta - 1}. \quad (2.16)$$

Note that when employment in the nontraded sector increases, wages fall. Labor employed in the nontraded sector can alternatively be interpreted as leisure and then wages would have to equal the marginal utility of leisure.

In the traded sector, because of free entry in equilibrium profits are zero. This implies that the firm size is given by the cost parameters:

$$x_{ij} = \frac{\alpha_{ij}}{\beta_{ij}} (\sigma_j - 1). \quad (2.17)$$

Adjustments to shocks other than technology therefore will occur on the extensive margin, through changes in the number of varieties in each sector: $M_{ij}$.

Combining the zero-profit condition with equation (2.14) gives the market clearing condition for traded goods:

$$\frac{\alpha_{ij}}{\beta_{ij}} (\sigma_j - 1) = \sum_k P_{ijk}^{\sigma_j} \gamma E_k / J. \quad (2.18)$$

Finally, the labor market clearing conditions is:

$$L_i = L_{N_i} + \sum_j M_{ij} l_{ij}, \quad (2.19)$$
which, using equation (2.7) and equation (??), can be rewritten as:

$$L_i = L_{N_i} + \sum_j \sigma_j M_{ij} \alpha_{ij},$$  \hspace{1cm} (2.20)

The endogenous variables of the model are: $W_i$, $L_{iN}$, $P_{iN}$ and $M_{ij}$ for $j = 1, ..., J$ and the equilibrium is described by equations (2.15), (2.16), (2.20) and (2.18), the latter specified for each $j = 1, ..., J$. All the foreign variables are exogenous, $\alpha_{ij}$ and $\beta_{ij}$ are the exogenous technology shocks while $T_i$ is an exogenous transfer shock.

### 2.2.5 Log-Linear Solution with Trade and Technology Shocks

In this section I explore log-linear deviations from steady state due to technological and foreign-trade-related shocks. Log-linearizing the wage equation gives:

$$\hat{W}_i = \hat{P}_{Ni} - (1 - \eta)\hat{L}_{Ni},$$  \hspace{1cm} (2.21)

while the market-clearing condition for the nontraded good in log-linear terms is:

$$\hat{P}_{Ni} + \eta\hat{L}_{Ni} = \rho(\hat{W}_i + \hat{L}_i) + (1 - \rho)\hat{T}_i,$$  \hspace{1cm} (2.22)

where: $\rho = L_i W_i / (L_i W_i + T_i)$, the steady-state share of labor income in total expenditure.

Log-linearizing the labor market clearing condition and using the assumption that $L_i = const$ gives:

$$\hat{L}_i = (1 - \sum_j \delta_{ij})\hat{L}_{N} + \sum_j \delta_{ij}(\hat{M}_{ij} + \alpha_{ij}) = 0,$$  \hspace{1cm} (2.23)
where $\delta_{ij} = M_{ij}l_{ij}/L_i$ is the steady-state share of $j$ sector employment in total employment and therefore $(1 - \sum_j \delta_{ij})$ is the steady-state share of nontraded employment in total.

The price of a traded variety produced in $i$ and sold in $k$ in log-linear terms is:

$$\hat{P}_{ijk} = \hat{\beta}_{ij} + \hat{W}_i + \hat{\tau}_{ijk},$$

and the price index for the traded sector $j$ in country $k$ is:

$$\hat{\Phi}_{jk} = -\frac{1}{\sigma_j - 1} \sum_h \phi_{hjk}(\hat{M}_{hj} - (1 - \sigma_j)(\hat{W}_h + \hat{\beta}_{hj} + \hat{\tau}_{hjk})), \quad (2.25)$$

where $\phi_{hjk}$ is the steady-state share of region $h$ in purchases of sector $j$ goods by market $k$:

$$\phi_{hjk} = \frac{M_{hj}P_{hjk}x_{hjk}}{\sum_i M_{ij}P_{ijk}x_{ijk}}. \quad (2.26)$$

The term in parentheses on the right-hand-side of equation (2.25) can be interpreted as change in export capability of region $h$ in market $k$. For simplicity of notation it will be denoted as $\hat{A}_{hjk}$:

$$\hat{A}_{hjk} = \hat{M}_{hj} - (1 - \sigma_j)(\hat{W}_h + \hat{\beta}_{hj} + \hat{\tau}_{hjk}). \quad (2.27)$$

Export capability of region $h$ in market $k$ increases if either region $h$ produces more varieties, wages fall in region $h$, productivity increases in region $h$ or trade costs between $h$ and $k$ fall. Note that only the latter is specific to $k$. 

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Equation (2.18) in log-linear can be written as:

\[ \sigma_j \hat{W}_i + \hat{\alpha}_{ij} - \hat{\beta}_{ij} = \sum_k \theta_{ijk} \hat{E}_k - \sum_k \theta_{ijk} \sum_h \phi_{hjk} \hat{A}_{hjk}, \quad (2.28) \]

where: \( \theta_{ijk} = x_{ijk}/\sum_l x_{ijl} \) is region \( i \)'s steady state market share in \( k \) for sector \( j \) goods; and:

\[ \hat{E}_k = \rho_k \hat{W}_k + (1 - \rho_k) \hat{T}_k \quad (2.29) \]

As discussed above, there are two sources of foreign shocks important for this model: shocks from the EU Center (EUC) and shocks from the Third World (TW). Equation (2.28) can be rewritten to show them explicitly as:

\[ \sigma_j \hat{W}_i + \hat{\alpha}_{ij} - \hat{\beta}_{ij} = \theta_{iji} \hat{E}_i - \sum_k \theta_{ijk} \phi_{ijk} \hat{A}_{ijk} + \hat{\Gamma}_{EUCj} + \hat{\Gamma}_{TWj}, \quad (2.30) \]

where: \( \hat{\Gamma}_{EUCj} \) is the combined shock originating from EUC:

\[ \hat{\Gamma}_{EUCj} = \theta_{ijEUC} \hat{E}_{EUCj} - \sum_k \theta_{ijk} \phi_{EUCjk} \hat{A}_{EUCjk}, \quad (2.31) \]

and: \( \hat{\Gamma}_{TWj} \) is the combined shock originating from TW:

\[ \hat{\Gamma}_{TWj} = \theta_{ijTW} \hat{E}_{TWj} - \sum_k \theta_{ijk} \phi_{TWjk} \hat{A}_{TWjk} \quad (2.32) \]

The first term on the right-hand-side of each equation (2.31) and (2.32) is increased demand for \( i \)'s exports: expenditure respectively in EUC and TW, weighted by \( i \)'s steady state market share for sector \( j \). The second term shows increased competition for \( i \)'s goods: export capability respectively in EUC and TW, weighted by each region’s importance for \( i \).
The log-linear system can be solved for the endogenous variables $\hat{W}_i, \hat{P}_{Ni}, \hat{L}_{Ni}$ and $\hat{M}_{ij}$ as linear functions of the exogenous shocks: the transfer form the EU Center $\hat{T}_i$, the sector-specific technology shocks $\hat{\alpha}_{ij}$ and $\hat{\beta}_{ij}$, and the sector-specific trade shocks originating from the EUC or TW, $\hat{\Gamma}_{EUCj}$ and $\hat{\Gamma}_{TWj}$. For simplicity I assume that there are 2 traded sectors, i.e. $J = 2$. This assumption is non-restrictive and similar results can be obtained without it.\(^5\) The full log-linear solution is described in Appendix B, while the signs of the coefficients are summarized in table below:

<table>
<thead>
<tr>
<th>$\partial y/\partial x$</th>
<th>$\partial(...)/\partial \Gamma_j$</th>
<th>$\partial(...)/\partial T$</th>
<th>$\partial(...)/\partial \alpha_j$</th>
<th>$\partial(...)/\partial \beta_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\partial W/\partial(...)$</td>
<td>+</td>
<td>+</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>$\partial L_N/\partial(...)$</td>
<td>−</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$\partial P_N/\partial(...)$</td>
<td>+</td>
<td>+</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>$\partial M_{ij}/\partial(...)$</td>
<td>(+) if $j = l$, ambiguous</td>
<td>(−) if $j = l$, ambiguous if $j = l$,</td>
<td>(+) otherwise</td>
<td>(+) otherwise</td>
</tr>
<tr>
<td></td>
<td>(−) otherwise</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As expected, positive trade shocks lead to higher wages and higher employment in the traded sector in the small open economy. Positive trade shocks in sector $j$ also lead to more varieties being produced in that sector and less varieties being produced in other sectors. If the small open economy gets higher transfers, it will have higher wages, lower employment in the traded sector and higher prices in the nontradable sector. Higher fixed or marginal costs in the traded sector would lead to lower wages and lower employment in the traded sector. Higher fixed or marginal costs in one sector would cause less varieties to be produced in that sector and more varieties to be produced in others.

\(^5\)E.g. Hanson and Xiang (2002) use a model with a similar trade structure with N number of traded sectors.
2.3 Integration and Globalization

In this section I employ the model described above to study the various stages of integration and globalization in Europe. The period prior to the start of "integration" corresponds to the steady state of the model, where all shocks are at their long-term levels. The home economy $i$ is relatively poor and is specializing in the relatively non-skill-intensive good, which is $j = 2$.

In the next stage the small open economy becomes integrated within the EU. In the model "integration" is captured by the following shocks. First, trade barriers are lifted within the EU and, as a result, along the lines of comparative advantage $i$'s net imports of the skill intensive good increase: $\Gamma_{EUC1} < 0$ while net imports of the non-skill-intensive good decrease: $\Gamma_{EUC2} > 0$. (Note that at this stage $\Gamma_{TWj} = 0$.) Second, the home economy $i$ benefits from technology transfer from the more advanced core. This implies that both marginal costs and fixed costs parameters decrease: $\alpha_{1,2} < 0$, $\beta_{1,2} < 0$. Note that increased FDI can also be captured by $\alpha_{1,2} < 0$, as a decline in fixed costs. Finally, the home economy $i$ receives transfers from the EU Center: $T > 0$. These transfers may capture not only direct transfers from the EU center (e.g. through the government) but also other flows that may allow the home economy $i$ to consume more than it produces, i.e. run trade deficits. While international borrowing is not part of the model, to the extent that such borrowing by the home economy $i$ could be driven by the expectations of higher future income due to integration with EU, it can be captured in $T > 0$.

In order to illustrate the impact of these shocks, the log-linear solution of the above model is summarized below. The signs for the $\gamma$ coefficients are given in the summary table in the previous section (and their expressions are given in Appendix
It is evident that "integration" leads to higher wages in the home economy $i$. The net effect of increased trade $\gamma_{T_{1W}}\Gamma_{EUC1} + \gamma_{T_{2W}}\Gamma_{EUC2}$ across the two goods is positive, in particular, because the steady-state share of employment in the non-skilled sector 2 is higher than that in the skilled sector 1 and therefore $\gamma_{T_{2}} > \gamma_{T_{1}}$. The response of nontradable employment to "integration" is ambiguous. Higher demand for the low-skill-intensive good 2 from the EU center would lead to higher employment in that sector more than compensating for the employment lost due to higher imports of the high-skill-intensive good 1. Higher technology transfers, making the tradable sector more productive, would also encourage employment in the tradable sector. However, higher consumption transfers would work in the opposite direction. Prices of nontradable goods will increase as all of the shocks work in the same direction. The impact of "integration" on the number of varieties produced in the high-skill-intensive...
and low-skill-intensive sectors ($M_1$ and $M_2$) is generally ambiguous. However, the latter is likely to increase due to both trade and technology shocks.

The following stage is "globalization", when the EU opens to trade with the Third World (TW), which largely specializes in the low-skill-intensive good. The EU’s imports of mostly the low-skill-intensive good from the TW and its exports of mostly the high-skill-intensive good to the TW increase. As far as the home economy $i$ is concerned "globalization" implies that $i$'s net imports of the low-skill-intensive goods increase: it consumes more of the low-skill-intensive imports from the TW and also exports less to the EUC (because EUC switches to importing the low-skill-intensive goods from TW). At the same time, $i$'s net imports of the high-skill-intensive good are unchanged, because the TW largely does not specialize in the skill-intensive good. Since by this stage integration processes within Europe are assumed to be over, consumption and technology transfers as well as trade within Europe do not increase further.

Model responses to the "globalization" shock are straightforward, as it is described by only the trade component: $\hat{\Gamma}_{TW2} < 0$, while $\hat{I} = 0$, $\hat{\alpha}_j = 0$ and $\hat{\beta}_j = 0$. It is important to note that the "integration" shock did not qualitatively change the steady state of the home economy $i$ and in particular, $i$ still specializes in the low-skill-intensive good 2. Wages in $i$ decrease because of lower demand for labor in traded sector 2 which accounts for a large fraction of employment. As a result of this, also employment in the tradable sector shrinks. Prices in the nontradable sector decrease due to lower incomes. The extensive margin adjusts to the shock as the number of firms operating in the low-skill-intensive sector decreases and the number of firms operating in the high-skill-intensive sector increases.
2.3.1 Empirical Analysis

In this section I aim to match the theoretical results described above to the experience of European integration and globalization. Empirically mapping many industry specific shocks into a few aggregate outcomes is a common challenge for trade and inequality literature. In case of studying cross-country inequality within Europe identifying the role of trade or globalization is even more difficult because there are many country-specific variables at play, including quality of institutions, product and labor market imperfection and others that have been shown to be important factors affecting cross country differences. At the same time, lack of detailed sub-country data make fixed effect estimations non-feasible. Nevertheless, model-consistent measures of country exposure to globalization derived from detailed trade, employment and output data give useful insights when put side by side with individual country performances.

Subject to availability of sub-national level data for Europe, econometric analysis along the lines of ADH (which uses U.S. data on commuting zone level) could potentially be a promising avenue of future work. In particular, ADH treat commuting zones as local labor markets subject to differential trade shocks according on their pattern of industry specialization and identify the impact of increased imports from China on U.S. local labor markets, which they call the "China syndrome".

Data

I use data on 14 European countries’ (which exclude the transition economies of Eastern Europe as explained above) detailed data on employment and GDP by industry and each country’s bilateral detailed trade with China. The time period covered is
1995-2013. Since in the international trade literature the year 2001, when China joined the WTO, marks the ”turning point of globalization”, I divide the the time period into pre-globalization (1995-2000) and post-globalization (2001-2013). In addition, I split the post-globalization period at 2007 to see if the global financial crisis may have altered the role of globalization in relative country performance in Europe.

A key challenge is to match detailed trade commodity classification with employment/output industry classification. Unlike for the U.S. there are no straightforward or universally accepted methodologies for ensuring consistency between various classifications of trade versus sectors of economic activity. As a result, I use multi-step transformations to match trade data in SITC (4-digit classification) to employment and national accounts in NACE classification. Data on international trade are taken from the UN COMTRADE database. Data on bilateral imports and exports of European countries from and to China for the period starting from late 1980s till 2013 in SITC Rev.3 classification are used. For employment as well as output EuroStat data Labor Force Surveys on employment by detailed economic activity and GDP by detailed sector (NACE Rev. 2) are used.

Exposure to Globalization

The sector-specific trade shock according to the model discussed in Section 2 is:

$$ \hat{\Gamma}_{Cj} = \theta_{ij}C_{\hat{E}C} - \sum_k \theta_{ijk} \hat{\phi}_{Cjk} \hat{A}_{Cjk} $$

(2.33)

6 Several methods described in particular in ”ICT sector definition: Transition from NACE Rev. 1.1. to NACE Rev. 2” and methodological manuals compiled by Eurostat have been employed.
as defined in equation (2.32), where $\theta_{ijC}\hat{E}_C$ is the change in demand for home economy $i$’s exports to China (total expenditure in China $\hat{E}_C$, weighted by $i$’s steady state market share for sector $j$ in China) and $\sum_k \theta_{ijk}\phi_{Cjk}\hat{A}_{Cjk}$ is increased Chinese competition for $i$ in sector $j$, which depends on increased export capability of China (that could come from higher production or lower trade costs as shown in equation (2.27)) and on how important Chinese competition is for home economy $i$. The term $\sum_k \theta_{ijk}\phi_{Cjk}$ will be higher if if China is an important source of supply in markets that $i$ also serves. For example, if Portugal specializes in textile and apparel industry then growing Chinese exports of textile and apparel would have important implications for Portugal in both domestic and export markets.

The impact of globalization on income in the model is given by $\sum_j \gamma_{jW}\hat{\Gamma}_{Cj}$, which, as it is shown in Appendix B can be approximated by $\sum_j \tilde{\delta}_{ij}\kappa_W\hat{\Gamma}_{Cj}$, where $\tilde{\delta}_{ij} = L_{ij}/L_{iN}$ is the ratio of employment in tradable sector $j$ to nontradable employment and $\kappa_W$ is a positive constant. Similarly, the impact of globalization on tradable employment is given by $\rho_i \sum_j \tilde{\delta}_{ij}\kappa_LT\hat{\Gamma}_{Cj}$, where $\rho_i = L_iW_i/(L_iW_i + Ti)$ is the share of trade imbalance in total expenditures of $i$.

Further, since the EU is a single trade zone, what matters for home economy $i$ is Chinese imports to the EU. Regardless of which EU country is the final destination for Chinese imports, they pose the same competition to all countries. Using this, and to facilitate mapping to data, the the globalization shock can be rewritten as:

$$\hat{\Gamma}_{Cj} = \frac{1}{x_{ij}}\Delta X_{ijC} - \frac{1}{x_j}I_{CjEU},$$

where: $\Delta X_{ijC} = x_{iJC}\hat{E}_{Cj}$ is the change in exports of good $j$ from country $i$ to China, driven by growing expenditures in China and $\Delta I_{CjEU} = I_{CjEU}\hat{A}_{Cj}$ is the change in EU imports of good $j$ from China, driven by growth in export supply capacity in
China and reduction in trade costs.

Finally, the measure of exposure to globalization (EGL) for country income would be:

\[ EGL_W = \sum_j \frac{L_{ij}}{L_{iN}} \left( \frac{1}{x_{ij}} \Delta X_{ijC} - \frac{1}{x_j} \Delta I_{CjEU} \right), \]

and the measure of exposure to globalization (EGL) for tradable employment would be:

\[ EGL_{LT} = \rho_i \sum_j \frac{L_{ij}}{L_{iT}} \left( \frac{1}{x_{ij}} \Delta X_{ijC} - \frac{1}{x_j} \Delta I_{CjEU} \right). \]

While derived from broadly the same theoretical model the measure of exposure to globalization in this paper is different from ADH’s measure of “import per worker” that they use for empirical analysis. The reason is that ADH concentrate only on Chinese imports to the U.S. and do not consider U.S. exports to China as these are seen as less relevant in view of the large aggregate trade deficit of the U.S. vis a vis China. As shown below, this assumption would not be valid for some European countries. Also, since output data are not available on U.S. commuting zone level, ADH use the approximation \( x_{ij}/x_j = L_{ij}/L_j \).

Figures 2.2 and 2.3 illustrates respectively exposure of wages and of tradable employment to globalization and their components. The solid color represents the changes between averages over 1995-2000 and 2001-2007 and the shaded color represents the changes between averages over 1995-2000 and 2001-2013. As it is evident, the global financial crisis did not appear to change the pattern of exposure to globalization. For all countries except for Belgium exposure continued to increase in the more recent period.

Figure 2.2 shows that Portugal, Italy, Greece and Spain, the countries that experienced declines in GDP per capita relative to the EU14 average (as shown in Figure
2.1) have been most exposed to globalization. It is interesting to look at exposure to exports and imports separately. While imports have been much higher, for some countries growing exports to China made a difference in terms of net exposure. For example, German wages had a large negative exposure to imports from China (third largest after Portugal and Italy), but it was compensated by the gains from higher exports to China, which were the largest for Germany compared to the other countries.

Next, it is interesting to compare the model-based measures of exposure of tradable employment to globalization (Figure 2.3) and relative developments of tradable employment in the data (Figure 2.4). Tradable employment in Portugal, Greece and Spain appear to be most exposed to globalization according to the model, and these are the countries that experienced declines in tradable employment over the past decade not just in absolute terms (as percent of population) but also relative to the European average.

2.4 Conclusion

Convergence of living standards within Europe is a key idea behind European integration. Nevertheless, the evidence of convergence within Europe is mixed at best. This paper proposes a new approach to the problem of lack of convergence in Europe by building on theoretical and empirical literature on trade and inequality in the U.S.. Using a simple model of trade with monopolistic competition it illustrates how globalization may have affected countries in southern Europe, counteracting the positive effects of integration processes. Measures of exposure to globalization depending on country’s initial industry specialization profile are derived from the model and cal-
culated using detailed trade and employment data for all European countries. Comparing these measures to individual country performances in Europe suggests that globalization may have played an important role for lack or reversal of convergence processes in Europe.
Figure 2.1: GDP Per Capita Relative to EU14 Average
Figure 2.2: Trade with China and Exposure of Wages to Globalization

- **Exposure to Globalization**
  - 1995-2007
  - 1995-2013

- **Gains through Exports**

- **Losses due to Import Competition**
Figure 2.3: Trade with China and Exposure of Tradable Employment to Globalization
Figure 2.4: Tradable Employment as a Share of Population (solid line) and Relative to EU14 Average (dotted line)
Appendix B

The Log-Linear Solution of the Model

\[ a_{ij} \hat{W}_i = b_{ij} \hat{T}_i - c_{ij} \hat{M}_{ij} + \Gamma_{EUC} + \Gamma_{TW} - \alpha_{ij} - d_{ij} \hat{\beta}_{ij}, \]  
\begin{align*}
  a_{ij} &= \sigma_j (1 - \sum_k \theta_{ijk} \phi_{ijk}) + \sum_k \theta_{ijk} \phi_{ijk} - \rho \theta_{iji} \\
  b_{ij} &= \theta_{iji} (1 - \rho) \\
  c_{ij} &= \sum_k \theta_{ijk} \phi_{ijk} \\
  d_{ij} &= 1 - (\sigma_j - 1) \sum_k \theta_{ijk} \phi_{ijk}
\end{align*}

where:

\[ \rho_i = \frac{L_{Wi}}{L_{Wi} + T_i} \] is the share of wage income in total expenditures in \( i \). \( \delta_{i1} = \frac{\delta_{i1}}{1-\delta_{i1}-\delta_{i2}} \) and \( \delta_{i2} = \frac{\delta_{i2}}{1-\delta_{i1}-\delta_{i2}} \) are respectively the initial ratios of employment in each
traded sector to employment in the nontraded sector. The rest of the terms are as defined above.

\[
\dot{W} = \gamma_T \dot{T} + \sum_j (\gamma_{\Gamma j} \hat{\Gamma}_{EUc_j} + \gamma_{\Gamma j} \hat{\Gamma}_{TW_j} + \gamma_{\alpha j} \hat{\alpha}_j + \gamma_{\beta j} \hat{\beta}_j) \tag{B.11}
\]

\[
\dot{L}_N = (1 - \rho)(1 - \gamma_T) \dot{T} - (1 - \rho) \sum_j (\gamma_{\Gamma j} \hat{\Gamma}_{EUc_j} + \gamma_{\Gamma j} \hat{\Gamma}_{TW_j} + \gamma_{\alpha j} \hat{\alpha}_j + \gamma_{\beta j} \hat{\beta}_j) \tag{B.12}
\]

\[
\dot{P}_N = (\gamma_T - (1 - \eta)(1 - \gamma_T)) \dot{T} + (1 - (1 - \eta)(1 - \rho)) \sum_j (\gamma_{\Gamma j} \hat{\Gamma}_{EUc_j} + \gamma_{\Gamma j} \hat{\Gamma}_{TW_j} + \gamma_{\alpha j} \hat{\alpha}_j + \gamma_{\beta j} \hat{\beta}_j) \tag{B.13}
\]

\[
\gamma_{\Gamma 1} = \frac{\partial \dot{w}}{\partial \hat{\Gamma}_{EUc 1}} = \frac{\partial \dot{w}}{\partial \hat{\Gamma}_{TW 1}} = \frac{\delta_1 c_2}{\delta_1 a_1 c_2 + a_2 \delta_2 c_1 + c_1 c_2 (1 - \rho)} \tag{B.14}
\]

\[
\gamma_{\Gamma 2} = \frac{\partial \dot{w}}{\partial \hat{\Gamma}_{EUc 2}} = \frac{\partial \dot{w}}{\partial \hat{\Gamma}_{TW 2}} = \frac{\delta_2 c_1}{\delta_2 a_2 c_1 + a_1 \delta_1 c_2 + c_1 c_2 (1 - \rho)} \tag{B.15}
\]

\[
\gamma_T = \frac{\partial \dot{w}}{\partial \hat{T}} = \frac{c_1 c_2 (1 - \rho) + \delta_1 b_1 c_2 + \delta_2 b_2 c_1}{c_1 c_2 (1 - \rho) + \delta_2 a_2 c_1 + \delta_1 a_1 c_2} \tag{B.16}
\]

\[
\gamma_{\alpha 1} = \frac{\partial \dot{w}}{\partial \hat{\alpha}_1} = \frac{c_2 \delta_1 (c_1 - 1)}{a_1 c_2 \delta_1 + a_2 c_1 \delta_2 + c_2 c_1 (1 - \rho)} \tag{B.17}
\]

\[
\gamma_{\alpha 2} = \frac{\partial \dot{w}}{\partial \hat{\alpha}_2} = \frac{c_1 \delta_2 (c_2 - 1)}{a_2 c_1 \delta_2 + a_1 c_2 \delta_1 + c_1 c_2 (1 - \rho)} \tag{B.18}
\]

\[
\gamma_{\beta 1} = \frac{\partial \dot{w}}{\partial \hat{\beta}_1} = -\frac{c_2 d_1 \delta_1}{a_2 \delta_2 \delta_1 + a_2 c_1 \delta_2 + c_2 c_1 (1 - \rho)} \tag{B.19}
\]

\[
\gamma_{\beta 2} = \frac{\partial \dot{w}}{\partial \hat{\beta}_2} = -\frac{c_1 d_2 \delta_2}{a_1 \delta_1 \delta_2 + a_1 c_2 \delta_1 + c_1 c_2 (1 - \rho)} \tag{B.20}
\]

\[
\frac{\partial \dot{L}_N}{\partial \hat{\Gamma}_{EUc_j}} = \frac{\partial \dot{L}_N}{\partial \hat{\Gamma}_{TW_j}} = -(1 - \rho) \gamma_{\Gamma j} \leq 0 \tag{B.21}
\]

\[
\frac{\partial \dot{L}_N}{\partial \hat{T}} = (1 - \rho)(1 - \gamma_T) \geq 0 \tag{B.22}
\]

\[
\frac{\partial \dot{L}_N}{\partial \hat{\alpha}_j} = -(1 - \rho) \gamma_{\alpha j} \geq 0 \tag{B.23}
\]
\[
\frac{\partial \hat{L}_N}{\partial \hat{\beta}_j} = -(1 - \rho)\gamma_{\beta j} \geq 0 \quad (B.24)
\]

\[
\frac{\partial \hat{P}_N}{\partial \Gamma_{EUC_j}} = \frac{\partial \hat{L}_N}{\partial \Gamma_{TW_j}} = (1 + (1 - \eta)(1 - \rho))\gamma_{\Gamma j} \geq 0 \quad (B.25)
\]

\[
\frac{\partial \hat{P}_N}{\partial \hat{T}} = (\gamma_T - (1 - \eta)(1 - \rho)(1 - \gamma_T)) \geq 0 \quad (B.26)
\]

\[
\frac{\partial \hat{P}_N}{\partial \hat{\alpha}_j} = (1 + (1 - \eta)(1 - \rho))\gamma_{\alpha j} \leq 0 \quad (B.27)
\]

\[
\frac{\partial \hat{P}_N}{\partial \hat{\beta}_j} = (1 + (1 - \eta)(1 - \rho))\gamma_{\beta j} \leq 0 \quad (B.28)
\]
Bibliography


Chapter 3

Demand Side Pricing Complementarities and Endogenous Markups in Open Economy

3.1 Introduction

There has been a recent increased attention to pricing to market and in particular, to its role in explaining the behavior of international relative prices. One strand of literature, notably Atkeson and Burstein (2008) and De Blass and Russ (2012) have adopted a game-theoretic approach to model endogenous markups that lead to large and persistent deviations from PPP. Some papers, including Davis and Huang, (2011) and Cook (2002) have shown that models with strategic interaction of domestic and foreign firms can help explain business cycle volatility and cross-country comovements. Bergin and Feenstra (2001) have shown that translog preferences can help staggered contracts generate significantly greater endogenous persistence in the real exchange rate that the standard CES specification does. Similar to Bergin and Feensra (2001) this paper also explores the role of translog preferences in an international business cycle model but assumes endogenous firm entry and flexible prices.

I introduce demand side pricing complementarities through translog preferences into a simple two-country general equilibrium model with endogenous producer entry, monopolistic competition, sunk entry costs, flexible prices and balanced trade. With
translog preferences, as the number of available varieties increases, goods become closer substitutes, causing the elasticity of substitution to increase and markups to decrease. I show that in an open economy markups depend not only on the number of available varieties but also on the relative marginal cost in the two countries and, as long as iceberg trade costs are present, markups charged in domestic market are different from those charged in export market.

I use the markup equations for translog preferences in open economy to derive expressions for international relative prices. In particular, I show that (independent of other aspects of the model and the nature of the shock) terms of trade move only half-way with relative marginal cost and half-way with relative competition in the two countries, which is an intuitive open-economy extension of the well-known result about optimal prices under translog preferences in Feenstra (2003). This contrasts the result from a benchmark CES model that terms of trade deteriorate one-for-one with improvement of relative marginal costs and implies that open economy models using translog preferences may have a better chance to match terms of trade dynamics observed in the data.

To explore the relevance of endogenous markups for the international business cycle I compare qualitatively the log-linear model responses to a positive Home productivity shock to those from a benchmark model that features CES preferences but is otherwise identical. I find that endogenous markup dynamics resulting from translog preferences acts as a potentially important transmission mechanism. In particular, it amplifies the responses of Foreign variables to a Home productivity improvement and, in contrast to a benchmark CES model, causes a positive GDP comovement across borders.

Endogenous markup dynamics also has important implications for the relation of
observed TFP to exogenous productivity shocks. I decompose the TFP measure and find that in addition to the exogenous productivity shock there are two endogenous sources for movements in TFP: first, lower monopoly power due to countercyclical markups and second, reallocation of recourses between domestic market and exports, towards the more competitive one. An important result is that due to these endogenous factors there will be a cross country correlation of measured TPFs even when there is no cross-country correlation of exogenous productivity shocks.

This paper relates closely to Feenstra (2002) and Bergin and Feenstra (2001) in exploring the implications of translog preferences. It also relates closely to Atkeson and Burstein (2008) and de Blass and Russ (2010) in which pricing to market arises from aggregate shocks and the presence of trade costs in a flexible price environment. An important difference is that both Atkeson and Burstein (2008) and de Blass and Russ (2010) use models with strategic interaction between firms. Similar to this paper, Rodriguez-Lopez (2011) uses translog preferences, however unlike this paper it also employs heterogeneous firms as a result of which choke prices become binding. In terms of using translog preferences in models with endogenous entry for the analysis of macroeconomic fluctuations this paper also relates to Bilbiiee, Ghironi and Melitz (2005) that similarly predicts procyclical variety and procyclical profits with countercyclical markups. Lewis and Stevens (2015) also study a closed economy case with endogenous firm entry and translog preferences and by using Bayesian methods estimate the impact of entry on markups and inflation.

While this paper does not perform quantitative analysis of international business cycles, in terms of its qualitative results it relates to the literature that seeks to address the comovement puzzles of standard international real business cycle (IRBC) models. In particular, as shown by Backus, Kehoe and Kydland (1995), Chari, Kehoe
and McGrattan (2002) and others, benchmark IRBC models predict negative GDP comovement across countries, while the opposite is observed in the data. Among the active research that has offered various modification to resolve these discrepancies this paper most closely relates to Davis and Huang (2011) and Cook (2002), which introduce endogenous markups into a benchmark IRBC model through strategic decisions made at the level of individual firms.\footnote{E.g. Baxter and Crucini (1995), Kehoe and Perri (2002), Heatcote and Perri (2003) show that cross country GDP correlation increases when international asset markets are restricted. Burstein et al. (2008) show that international business cycle comovement increases if intermediate inputs are added into the model.} By employing translog preferences this paper offers a much simpler approach that nevertheless yields several intuitive results.

Finally, this paper relates to Jaimovic and Floetotto (2008) where in a closed economy model countercyclical markups cause endogenous procyclical movements in measured TFP. It also relates to Holmes, Hsu and Lee (2014) who develop an index of allocative efficiency (first best allocative efficiency being when the price ratio equals the marginal cost ratio) that depends on the distribution of markups across goods. They then study how international trade affects allocative efficiency in an oligopoly model.

The rest of the paper is structured as follows. Section 2 presents the stylized model. Section 3 analyzes the model’s implications of productivity shocks and compares the results to those of from benchmark CES model. Section 4 discusses the decomposition of measured total factor productivity. Section 5 focuses on the dynamics of international relative prices. Section 6 concludes.
3.2 A Stylized General Equilibrium Model

This section describes a simple model, where the world consists of two countries, Home and Foreign, that are symmetric in size. Foreign variables are denoted with an asterisk. Households love variety and derive utility from consuming goods produced in Home and Foreign and supply labor to domestic firms. \( N (N^*) \) firms operate in Home (Foreign). Firms are monopolistically competitive and each firm produces one variety using domestic labor. The number of varieties produced in each country is endogenously determined in the model. There is free entry, but firms face fixed entry costs, which consist of wages paid for introducing a new variety. International trade is balanced and labor is not mobile across countries. Prices are flexible, which allows to focus only on real variables. The only non-standard feature of the model is that households’ preferences are defined according to the translog expenditure function.

3.2.1 Households

The representative household maximizes the following separable utility function, increasing in the home consumption composite \( C_t \) and decreasing in labor effort \( l^*_t \):

\[
U_t = \frac{C_t^{1-\frac{1}{\psi}}}{1 - \frac{1}{\psi}} - l^*_t,
\]

(3.1)

where \( \psi \geq 0. \)

With translog preferences the consumption composite \( C_t \) is associated with the

\[2\text{This formulation assumes constant marginal disutility of labor, corresponding to an infinite Frisch elasticity of labor.}\]
welfare-based price index $P_t$ given by the translog expenditure function as in Feenstra, 2002.

$$\ln P_t = \sum_{i=1}^{\hat{N}_t} \alpha_{it} \ln p_{it} + \frac{1}{2} \sum_{i=1}^{\hat{N}_t} \sum_{j=1}^{\hat{N}_t} \gamma_{ijt} \ln p_{it} \ln p_{jt},$$

(3.2)

where $p_{it}$ is the price of an individual variety and $\hat{N}_t$ is the number of varieties available for consumptions. In open economy, where Home produces $N_t$ varieties and Foreign produces $N_t^*$ varieties and all varieties are traded $\hat{N}_t = N_t + N_t^*$. $\gamma_{ijt} = \gamma_{jiti}$ and, to ensure that the expenditure function is homogenous of degree one, the following restrictions hold: $\sum_{i=1}^{\hat{N}_t} \alpha_{it} = 1$ and $\sum_{i=1}^{\hat{N}_t} \gamma_{ijt} = 0$. In addition, to ensure that all goods enter ‘symmetrically’ into the expenditure function the following restrictions are imposed:

$$\alpha_{it} = \frac{1}{\hat{N}_t},$$

(3.3)

$$\gamma_{iit} = -\gamma(\hat{N}_t - 1) / N_t,$$

(3.4)

$$\gamma_{ijt} = \gamma / N_t,$$

(3.5)

where $\gamma$ is a positive constant. It can be easily confirmed that these restrictions satisfy the conditions specified above for homogeneity of degree one.

In a symmetric equilibrium, for Home consumers all Home produced varieties will have the same price $p_t(h)$ and all Foreign produced varieties will have the same price $p_t(f)$.\(^3\) The share of each variety $i$ in expenditure can be computed by differentiating the unit expenditure function with respect to $\ln p_{it}$, which gives:

$$s(h)_t = \frac{1}{N_t} + \frac{\gamma N_t^*}{N_t} (\ln \rho_t(f) - \ln \rho_t(h))$$

(3.6)

\(^3\)Similarly, in Foreign, all Foreign produced varieties will have the same price $p_t^*(f)$ and all Home produced varieties will have the same price $p_t^*(h)$.
for Home produced varieties, and

\[ s(f)_t = \frac{1}{N_t} + \frac{\gamma N_t}{N_t} (\ln \rho_t(h) - \ln \rho_t(f)) \]  

(3.7)

for Foreign produced varieties, where \( \rho_t(h) = p_t(h)/P_t \), is the real price of Home produced varieties for Home consumers and \( \rho_t(f) = p_t(f)/P_t \) is the real price of Foreign produced varieties for Home consumers.\(^4\) As in Feenstra, 2002 the elasticity of demand is given by:

\[ \eta_{it} = 1 + \frac{\gamma}{s_{it}}, \]  

(3.8)

which in an open economy setting will be different for Home and Foreign produced varieties. Subsequently, the markup is is given by the following expression:

\[ \mu_{it} = 1 + \frac{1}{\eta_{it} - 1} = 1 + \frac{s_{it}}{\gamma}, \]  

(3.9)

implying that the markups for Home and Foreign produced varieties can be expressed respectively as:

\[ \ln \mu(h)_t = \frac{1}{\gamma N_t} + \frac{N_t^*}{N_t} (\ln \rho(f)_t - \ln \rho(h)_t), \]  

(3.10)

\[ \ln \mu(f)_t = \frac{1}{\gamma N_t} + \frac{N_t^*}{N_t} (\ln \rho(h)_t - \ln \rho(f)_t). \]  

(3.11)

It is evident from these equations that in an open economy model with translog preferences markups depend not only on the number of varieties (as in a closed economy model) but also on Home to Foreign relative prices.

Domestic households own the domestic firms. They finance the fixed costs of introducing new varieties and in return receive all the profits earned by the firms. In

\(^4\)Details of this and subsequent derivations are given in Appendix C.
addition, they earn wages $W_t$ on labor supplied to the domestic firms. The budget constraint of the representative household therefore is:

$$
\int_0^{N_t} p_t(h)c_t(h)dh + \int_0^{N^*_t} p_t(f)c_t(f)df + I_t = l_tW_t + \Pi_t, \quad (3.12)
$$

where $c_t(h)$ ($c_t(f)$) is consumption of a Home (Foreign) variety, and $p_t(h)$ ($p_t(f)$) is its respective price. $I_t$ is the representative household’s share of cost of introducing new varieties, which can be thought of as ‘investment’, while $\Pi_t$ represents the equal share of profits that the household receives. Dividing the household budget constraint by the Home consumption price index, it can be written in real terms:

$$
\int_0^{N_t} \rho_t(h)c_t(h)dh + \int_0^{N^*_t} \rho_t(f)c_t(f)df + i_t = l_tw_t + \pi_t, \quad (3.13)
$$

where: $w_t = W_t/P_t$, $i_t = I_t/P_t$, and $\pi_t = \Pi_t/P_t$.

### 3.2.2 Firms

$N_t$ firms operate in Home, each producing a different variety. Strategic interaction between firms does not arise due to the assumption that the number of firms operating at any point in time is large. Production uses only domestic labor and aggregate labor productivity $z_t$ represents effectiveness of one unit of labor. Production function for a representative firm is:

$$
y_t = z_tl^d_t, \quad (3.14)
$$

where $l^d_t$ is the firm’s labor demand for production purposes. To start the production of a variety a firm needs to bear a fixed cost of $f$ units of labor.

Optimizing firms set prices at a markup over marginal cost. I assume that market
segmentation is possible, i.e. firms are able to set different prices in the domestic and export markets. Prices for varieties consumed in Home are:

\[ p_t(h) = \mu_t(h) \frac{W_t}{z_t}, \]  

(3.15)
or, in real terms, dividing by the aggregate price index \( P_t \):

\[ \rho_t(h) = \mu_t(h) \frac{w_t}{z_t}. \]  

(3.16)

Prices for Home produced varieties consumed in Foreign are:

\[ \varepsilon p_t(h)^* = \mu_t^*(h) \frac{W_t}{z_t}(1 + \tau), \]  

(3.17)

where \( \varepsilon_t \) is the nominal exchange rate (units of Home currency per units of Foreign) and \( \tau \geq 0 \) is the iceberg trade cost parameter. Dividing both sides by the Foreign price index \( P_t^* \), the real price of Home varieties consumed in Foreign is:

\[ \varepsilon_t \rho_t(h)^* = \mu_t^*(h) \frac{w_t}{z_t}(1 + \tau), \]  

(3.18)

where \( \varepsilon \) is the real exchange rate: \( \varepsilon = \varepsilon P^* / P \) representing the relative price of a Foreign consumption basket in terms of Home. In similar fashion, the real price of Foreign produced varieties consumed in Foreign is:

\[ \rho_t(f)^* = \mu_t^*(f) \frac{w_t^*}{z_t^*}, \]  

(3.19)
and the real price of Foreign produced varieties consumed in Home is:

$$\rho_t(f) = \mu_t(f) \frac{w_t^*}{z_t} (1 + \tau) \epsilon_t.$$  \hfill (3.20)

As it is clear, pricing to market arises due to different markups in domestic and export markets.

A representative Home firm serves both the Home and Foreign markets. The goods market clearing condition is:

$$y_t = Lc_t(h) + (1 + \tau)c_t^*(h)L^*,$$  \hfill (3.21)

where \( L \) (\( L^* \)) is the population size of Home (Foreign). The first term on the right hand side of (3.21) is the domestic demand and the second term is exports of the Home produced good. The presence of the iceberg trade cost parameter in the equation indicates that in order to supply Foreign consumers with one unit of its output the Home firm needs to ship \((1 + \tau)\) units to Foreign.

Operating profits of a representative Home firm equal total sales minus total production costs and are given by the following expression, which is written in real terms:

$$d_t = (\mu_t(h) - 1) \frac{w_t}{z_t} Lc_t(h) + (\mu_t^*(h) - 1) \frac{w_t}{z_t} L^*c_t^*(h)(1 + \tau),$$  \hfill (3.22)

where the first term on the right hand side is profits from domestic sales and the second term is profits from exports.
3.2.3 Equilibrium

Since there are no state variables in the model, starting from this section time subscripts will be omitted for ease of notation. The first-order conditions of Household optimization give the following expressions for consumption and labor effort in terms of real wages:

\[ C = w^\psi, \quad (3.23) \]

\[ l^* = w^{\psi-1}. \quad (3.24) \]

Consumption of each individual variety is given by:

\[ c_i = s_i \frac{C}{\beta_i}, \quad (3.25) \]

which implies that for Home varieties consumed in Home are:

\[ c(h) = \frac{\gamma \ln \mu(h)}{\rho(h)} C, \quad (3.26) \]

and Foreign varieties consumed in Home are:

\[ c(f) = \frac{\gamma \ln \mu(f)}{\rho(f)} C. \quad (3.27) \]

Combining the markup equations (3.10) and (3.11) and their Foreign counterparts with pricing equations (3.15)-(3.20) gives the following expressions for the four markups as functions of numbers of Home and Foreign varieties, wages and exchange rate (details of these derivations are in Appendix C): the markup charged by Home
firms in Home:

$$\ln \mu(h) = \frac{1}{\gamma N} + \frac{N^*}{2N} \ln(TOL(1 + \tau)),$$

(3.28)

the markup charged by Home firms in Foreign:

$$\ln \mu(h)^* = \frac{1}{\gamma N} - \frac{N^*}{2N} \ln \frac{1 + \tau}{TOL},$$

(3.29)

the markup charged by Foreign firms in Foreign:

$$\ln \mu(f)^* = \frac{1}{\gamma N} + \frac{N}{2N} \ln \frac{1 + \tau}{TOL},$$

(3.30)

and the markup charged by Foreign firms in Home:

$$\ln \mu(f) = \frac{1}{\gamma N} - \frac{N}{2N} \ln(TOL(1 + \tau)),$$

(3.31)

where $TOL$ denotes 'terms of labor' and is defined as:

$$TOL = \frac{w^* \epsilon / z^*}{w / z}.$$

(3.32)

In equilibrium with free entry, optimal investment into new varieties implies that entry costs must equal operating profits: $fw = d$. Combining equation (3.22) and its Foreign counterpart with expressions for consumption of Home and Foreign varieties (equations (3.26), (3.27) and their Foreign counterparts), and using (3.23) the zero profit condition for Home firms is:

$$\frac{(\mu(h) - 1) \ln \mu(h)}{\mu(h)} w^{\psi-1} + \frac{(\mu^*(h) - 1) \ln \mu^*(h)}{\mu^*(h)} w^\psi = \frac{f}{L\gamma},$$

(3.33)
and for Foreign, the zero profit condition is:

\[
\frac{(\mu^*(f) - 1) \ln \mu^*(f)}{\mu^*(f)} w^* \psi^{-1} + \frac{(\mu(f) - 1) \ln \mu(f)}{\mu(f)} \frac{w^*}{\psi - 1} + \frac{\epsilon}{w^*} = f^* L\gamma. \tag{3.34}
\]

Since trade is balanced, Home imports have to equal Home exports:

\[
N^* Lc(f) \rho(f) = \epsilon N L^* c^*(h) \rho^*(h), \tag{3.35}
\]

which can be rewritten using the expressions for consumption as:

\[
N^* L w^\psi \ln \mu^*(f) = \epsilon N L^* w^* \psi \ln \mu^*(h). \tag{3.36}
\]

Finally, labor market clearing conditions requiring that labor supplied in each country be demanded either for production or for innovation, close the model. For Home, the labor market condition is:

\[
f^i N + \frac{y}{z} N = Lw^\psi w^{-1}, \tag{3.37}
\]

which, using the expression for output for Home firms (3.21) and rearranging, can be rewritten as:

\[
f^i N + \frac{L \gamma \ln \mu(h)}{\mu(h)} w^\psi w^{-1} N + \frac{L^* \gamma \ln \mu^*(h)}{\mu^*(h)} \frac{w^* \psi}{w} N = Lw^\psi w^{-1}. \tag{3.38}
\]

\(f^i\) (for fixed cost of innovation) is the labor effort required to introduce new varieties in the economy. Each individual firm treats it as given \((f)\) however in
equilibrium it is given by the following expression:

\[ f^i N = \frac{\alpha Lzw^\psi - 1}{w^\beta}, \]  

(3.39)

where \( \beta > 0 \) and \( 0 > \alpha \geq 1 \). The intuition behind this expression is that the share of total effective labor effort in the economy \( (Lzw^\psi - 1) \) that is used for innovation declines as wage increases.

Equilibrium then is described by the markup equations ((3.28)-(3.31)), zero-profit conditions ((3.33)-(3.34)), labor market clearing conditions ((3.38) and its Foreign counterpart), and the balanced trade condition ((3.36)). The endogenous variables of the model are: \( \mu(h), \mu^*(h), \mu^*(f), \mu(h), w, w^*, N, N^*, \epsilon \). The exogenous shocks are \( z \) and \( f \).

### 3.2.4 Steady State and Calibration

In this subsection I solve for the steady state, where \( z_t = z_t^* = 1 \) \( f_t = f_t^* = f_{ss} \) and all endogenous variables are constant. Since in steady state Home and Foreign are symmetric, it follows that \( w = w^* = w_{ss}, N = N^* = N_{ss}, \epsilon = 1 \). A symmetric steady state implies that markups changed by Home firms in Home in steady state would be equal to markups changed by Foreign firms in Foreign. We can refer to this as 'domestic markup':

\[ \ln \mu(h)_{ss} = \ln \mu^*(f)_{ss} = \frac{1}{2\gamma N_{ss}} + \frac{\ln(1 + \tau)}{4}, \]  

(3.40)

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and markups changed by Home firms in Foreign in steady state would be equal to markups changed by Foreign firms in Home (‘export markup’):

\[
\ln \mu^*(h)_{ss} = \ln \mu(f)_{ss} = \frac{1}{2\gamma N_{ss}} - \frac{\ln(1 + \tau)}{4}.
\] (3.41)

As these equations show, domestic markups are higher than export markups due to existence of trade costs.

It is interesting to relate these results to the literature on trade liberalization and competition. Equation (3.40) predicts that falling trade costs would reduce domestic markups. This is in line with the empirical evidence in Levinson (1993), Harrison (1994), Feenstra and Weinstein (2010) and others, according to which trade liberalization has pro-competitive effects among domestic producers as foreign competition forces them to reduce markups. Equation (3.41) predicts on the other hand that everything else unchanged trade liberalization would increase the market power of exporters. This is consistent with the findings of Atkeson and Burstein (2007,2008), Edmond, Midrigan and Xu (2012) and Arkolakis et al. (2012) which demonstrate that trade costs reduce exporter markups.

Steady state wages are given by:

\[
w_{ss}^{\psi - 1} = w_{ss}^{*\psi - 1} = \frac{f_{ss}}{\gamma L} \left( \frac{(\mu(h)_{ss} - 1) \ln \mu(h)_{ss}}{\mu(h)_{ss}^2} + \frac{(\mu(f)_{ss} - 1) \ln \mu(f)_{ss}}{\mu(f)_{ss}^2} \right)^{-1},
\] (3.42)

and the steady state numbers of varieties are expressed as:

\[
N_{ss} = N_{ss}^* = \frac{L}{f_{ss}} w_{ss}^{\psi - 1 - \beta}.
\] (3.43)

For analyzing the model’s responses to productivity shocks I calibrate the param-
eters to facilitate comparison with results of a benchmark CES model (described in Appendix E). In particular, the average of domestic and export markups in steady state is calibrated to match the constant markup of the CES model: value of $\gamma = 0.02$ corresponds to a standard value of 6 for the elasticity of substitution of the CES model $\sigma$. The rest of the parameters in both models are as follows: labor supply elasticity $\psi = 1.3$, iceberg trade costs parameter $\tau = 0.2$, and the parameter that governs the sensitivity labor allocation between production and innovation to wage $\beta = 0.5$, $\alpha = 1$.

### 3.3 Responses to Productivity Shocks

In this section I explore qualitative model responses to a positive Home productivity shock. The model is log-linearized around the steady state assuming that Home is hit by a positive productivity shock that increases the efficiency of production ($z$-shock: $\dot{z} > 0$); reduces the cost of entry: ($f$-shock: $\dot{f} < 0$) or both. There are no shocks to Foreign. The full log-linearized system is described in Appendix D, however, the dynamics of markups, which is central for the paper, is given below.

\[
\dot{\mu}(h) = \frac{1}{4} TOL - \frac{1}{2} \ln \mu(h)_{ss} \bar{N} - \frac{1}{2} \ln \mu(f)_{ss} \bar{N}^* \quad (3.44)
\]

\[
\dot{\mu}(h)^* = \frac{1}{4} TOL - \frac{1}{2} \ln \mu(f)_{ss} \bar{N} - \frac{1}{2} \ln \mu(h)_{ss} \bar{N}^* \quad (3.45)
\]

\[
\dot{\mu}(f)^* = -\frac{1}{4} TOL - \frac{1}{2} \ln \mu(h)_{ss} \bar{N} - \frac{1}{2} \ln \mu(f)_{ss} \bar{N}^* \quad (3.46)
\]

\[
\dot{\mu}(f)^* = -\frac{1}{4} TOL - \frac{1}{2} \ln \mu(h)_{ss} \bar{N} - \frac{1}{2} \ln \mu(f)_{ss} \bar{N}^* \quad (3.47)
\]

As these equations show, all markup responses depend negatively on both Home
and Foreign number of varieties, with appropriate weights attached to each. In particular, if competition in one market (domestic or export) is relatively weak then a change in the number of varieties competing in that market becomes more important for the markup. Further, markups charged by Home firms depend positively on terms of labor, i.e. when Home effective labor becomes relatively cheap, Home firms can enjoy higher markups in both domestic and export markets. Markups charged by Foreign firms, on the other hand depend negatively on terms of labor. The intuition is that when Home experiences a shock leading to a fall in its relative marginal cost of production (such as improvement in productivity) Home firms will capture some of the benefits through higher markups.\footnote{It is worth emphasizing that these expressions for markups are quite general in the sense that they only depend on the assumed preference specification and are independent of other components of the model.}

Since the rest of the model is relatively standard, it is interesting to see how using translog preferences instead of CES affects the qualitative responses of key variables to productivity shocks. To illustrate the role of translog preferences and endogenous markups, the results are compared to those from a benchmark CES model (described in Appendix E) as illustrated in Figure 3.1 (z-shock), Figure 3.2 (f-shock), Figure 3.3 (combined shock). Since the models are identical except for the preference specification, endogenous markups are the only source of differences in responses to shocks.

Both increased efficiency of production (z-shock) and lower cost of innovation (f-shock) result in higher wages as they increase the demand for labor both for production and for innovation. Since $\psi > 1$ labor supply also increases but not enough to offset the demand effect on wages. The number of varieties increases due to both lower entry cost (in case of Home only) and higher demand (in both Home and
Foreign), as consumption increases more than proportionately with wage provided that \( \psi > 1 \). A factor working in the opposite direction is that higher wages trigger labor to move more towards production and away from innovation. In case of Foreign, where entry cost does not decline, this leads to a fall in the number of varieties. The responses of wages and numbers of varieties are similar in CES and translog models, except that in the translog model the impact on Foreign variables is more pronounced due to the presence of an additional transmission mechanism, namely the markup dynamics.

As shown in Figures 3.1-3.3, Home productivity improvement causes all markups to decline in the translog model. In case of f-shock markups charged by Home firms decline due to both lower terms of labor and higher number of varieties, while in case z-shock they decline because the effect of higher number of varieties dominates. Similarly, markups charged by Foreign firms decline with z-shock due to both lower terms of labor and higher number of varieties, while in case f-shock they decline because the effect of higher number of varieties outweighs the gains that Foreign firms enjoy due to more favorable terms of labor.

A notable difference between translog and CES results relates to the response of output per firm (firm size) or the so called intensive margin of adjustment. While in the CES model, firm size in log-linear terms is simply given by \( \hat{y} = \hat{z} + \hat{f} \) (and analogously for Foreign), in the translog model due to endogenous markups re-orientation between domestic export and markets as a result of productivity shocks becomes important. In the translog model Home firm size in log-linear terms is given by:

\[
\hat{y} = \hat{y}^d \frac{y_{ss}^d}{y_{ss}^d + y_{ss}^x} + \hat{y}^x \frac{y_{ss}^x}{y_{ss}^d + y_{ss}^x},
\]  

(3.48)
where the demand by the domestic market is:

\[
y^d = \hat{z} + (\psi - 1)\hat{w} + \frac{1}{\mu(h)_{ss} - 1}\mu(h),
\]

(3.49)

and demand by the export market is:

\[
y^e = \hat{z} + \hat{\epsilon} + \psi\hat{w}^* - \hat{w} + \frac{1}{\mu(f)_{ss} - 1}\mu^*(h).
\]

(3.50)

As shown in Figures 3.1-3.2, Home firm size increases with z-shock and declines with f-shock in both translog and CES models. In case of combined shock illustrated in Figure 3.3 (where productivity shocks affecting efficiency of production and innovation are assumed to be perfectly symmetric: \( \hat{z} = 1 \) and \( \hat{f} = -1 \)) the firm size is unchanged in the CES model. In the translog model instead, firm size increases as endogenous markups lead to an intensive margin of adjustment. Moreover, while in the CES model there is no adjustment on the intensive margin in Foreign, in the translog model Foreign firm size reacts to Home productivity improvement in the same direction (but smaller magnitude) as Home firm size.

Another interesting result is that the translog model generates a positive GDP co-movement, unlike the CES model. GDP is defined as:

\[
GDP = N\rho(h)c(h) + N\rho^*(h)c^*(h).
\]

(3.51)

It is useful to define:

\[
\tilde{y} = \rho(h)c(h) + \epsilon\rho^*(h)c^*(h),
\]

(3.52)

as GDP by firm, which is output by firm measured in units of consumption.\(^6\) Then

\(^6\)Note that all firms are identical, even though each firm charges a different price in Home market
(3.51) can be written as \( GDP = N\dot{y} \). In log-deviations from steady state GDP is:

\[
\widehat{GDP} = \widehat{N} + \gamma N_{ss}[\mu(h) + \mu^*(h) + \psi \ln \mu(h)_{ss} \bar{w} + \psi \ln \mu(f)_{ss}(\bar{w}^* + \varepsilon)]. \tag{3.53}
\]

The positive comovement between Home and Foreign GDPs is driven by the endogenous comovement of markups charged by Home and Foreign firms and the presence of intensive margin of adjustment in case of the translog model. Despite the fact that Foreign produces a smaller number of varieties as a result of Home productivity improvement, Foreign GDP still increases because each Foreign firm produces more.

### 3.4 Total Factor Productivity

This section explores the model’s implications for the measurement of technology shocks, the relation of exogenous productivity shocks to observed total factor productivity (TFP) dynamics and its correlation across countries.

Measured TFP or the Solow residual is GDP divided by total factor input:

\[
TFP = \frac{N\dot{y}}{Lw^{\psi-1}}, \tag{3.54}
\]

From the labor market condition (3.38), we have:

\[
N = \frac{Lw^{\psi-1}z}{fz + y}, \tag{3.55}
\]

where \( y = y^d + y^x \), \( y^d = Lc(h) \) is the demand for a Home-produced variety by Home consumers and \( y^x = (1 + \tau)L^*c^*(h) \) is the demand for a Home-produced variety by versus Foreign market.
Foreign consumers (i.e. Home exports) as in equation (3.21).

From the zero-profit condition (3.33) we have:

\[ f_z = y^d (\ln \mu(h)) + y^x (\ln \mu^*(h)). \]  
(3.56)

Using the last two expressions (3.54) can be rewritten as:

\[ TFP = \frac{z \bar{y}}{(\ln \mu(h) + 1) y^d + (\ln \mu^*(h) + 1) y^x}. \]  
(3.57)

or using the approximation: \( \ln \mu(h) = \mu(h) - 1 \) and \( \ln \mu(f) = \mu(f) - 1 \):

\[ TFP = \frac{z \bar{y}}{\mu(h) y^d + \mu^*(h) y^x}. \]  
(3.58)

This expression can be viewed as an open-economy model analogy to the result in Jaimovich Floetotto, 2008 for closed economy: \( TFP = z/\mu(N) \).

In log-linear deviation from its steady state in response to exogenous productivity shock \( z \), TPF is given by:

\[
\hat{TFP} = \hat{z} - \hat{\mu(h)} \frac{\mu(h)_{ss} y_{ss}^d}{\mu(h)_{ss} y_{ss}^d + \mu(f)_{ss} y_{ss}^x} - \hat{\mu^*(h)} \frac{\mu(f)_{ss} y_{ss}^d}{\mu(h)_{ss} y_{ss}^d + \mu(f)_{ss} y_{ss}^x} + \\
+ [\hat{y} - \hat{y}^d] \frac{\mu(h)_{ss} y_{ss}^d}{\mu(h)_{ss} y_{ss}^d + \mu(f)_{ss} y_{ss}^x} - [\hat{y}^d - \hat{\mu(f)_{ss} y_{ss}^x}] \frac{\mu(f)_{ss} y_{ss}^d}{\mu(h)_{ss} y_{ss}^d + \mu(f)_{ss} y_{ss}^x}.
\]
(3.59)

This result illustrates that the observed \( TFP \) change differs from the exogenous productivity shock \( z \) because there are also endogenous changes in observed productivity. These changes result from lower markups charged by Home firms in domestic and export markets (the second and third terms on the right-hand-side) and from
changes reallocation of resources (the last term on the right-hand-side).

Lower markups or increased degree of competition lead to higher measured $TFP$ because a positive technology shock lowers marginal cost of production and fixed cost of entry, leading to increased profit opportunities and increased number of producers as a result. Lower markups mean that the monopolistically competitive producers increase output per unit of input, i.e. efficiency gains materialize. Also note that domestic and export markups enter the equation for $TFP$ with appropriate weights: the coefficients represent the weight of the respective market (domestic or export) in the total sales of of a firm. The second source of endogenous productivity variation relates to changes in allocation of production between domestic market and exports induced by the productivity shock $z$.

An important implication of the above decomposition of observed $TFP$ into exogenous and endogenous components in an open economy model is that there is comovement in observed $TFPs$ in Home and Foreign even when there is no correlation in exogenous technology shocks. This results primarily from comovement of markups that Home and Foreign firms charge in both domestic and export markets, that in turn depend on both Home and Foreign number of varieties. In other words, when Home experiences a positive productivity shock $z$, observed $TFP$ will also improve in Foreign due to decrease in monopoly power and reallocation of production towards the more competitive market in Foreign while there is no change in Foreign exogenous productivity $z^*$. 
3.5 International Relative Prices

This section focuses on terms of trade and real exchange rate derived from the translog expenditure functions in a two-country model. The results in this section are general in the sense that they follow only from the preference structure (from which the markup equations are derived) and standard pricing equations and are independent of other aspects of the model.

3.5.1 The Terms of Trade

Terms of trade for Home is the ratio of Home export price $P(H)^*$ to Home import price $P(F)$. With translog preferences and all goods traded, the respective indices are given by the following expressions.

\[
\ln P(H)^*_t = \sum_{i=1}^{N^*} \alpha_i \ln p^*_it(h) + \frac{1}{2} \sum_{i=1}^{N^*} \sum_{j=1}^{N^*} \gamma_{ij} \ln p^*_it(h) \ln p^*_jt(h) \tag{3.60}
\]

\[
\ln P(F)_t = \sum_{i=1}^{N} \alpha_i \ln p_it(f) + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{ij} \ln p_it(f) \ln p_jt(f) \tag{3.61}
\]

If symmetry and homotheticity conditions analogous to (3.3)-(3.5) are satisfied, it is straightforward to show that $\ln P(H)^*_t = \ln p(h)^*_t$ and $\ln P(F)_t = \ln p(f)_t$.

\[
\ln TOT = \ln p(h)^* + \ln \varepsilon - \ln p(f), \tag{3.62}
\]

and using the pricing equations (3.18) and (3.20) can be rewritten as:

\[
\ln TOT = \ln \mu^*(h) - \ln \mu(f) - \ln TOL, \tag{3.63}
\]
which clearly shows the role of endogenous markups in determining the terms of trade.

Using the log-linearized versions of markup equations gives the following equation
for terms of trade deviation from steady state as a result of an exogenous productivity shock:

$$\hat{TOT} = -\frac{1}{2}\hat{TOL} + \ln(1 + \tau)\left(\hat{N} - \hat{N}^*_s\right),$$

(3.64)

where: $\hat{TOL} = \hat{w} + \hat{w}^* + \hat{\epsilon} + \hat{\epsilon}$ as defined before. Using the expressions for domestic and export markups in steady state (3.40) and (3.41) (3.64) can be also rewritten as:

$$\hat{TOT} = -\frac{1}{2}\hat{TOL} + \frac{1}{2}\ln\left(\mu(h)_{ss} - \mu(f)_{ss}\right)\left(\hat{N} - \hat{N}^*_s\right).$$

(3.65)

This is a very simple and intuitive result, which can be viewed as an open-economy extension of the famous result in Feenstra, 2002 according to which optimal prices place one-half of their weight on marginal costs and the other half on competitors’ prices. Equation 3.65 states that in an open economy setting terms of trade move only half-way with the relative marginal cost and half-way with the relative competition in the two countries.

Clearly, (3.64) provides for a richer dynamics for terms of trade than the standard CES model (according to which $\hat{TOT} = -\hat{TOL}$) and thus has a greater potential to match moments observed in the data. In particular, while in a standard model with CES preferences a productivity improvement leading to improvement of Home’s relative cost of effective labor would automatically lead to deterioration of $TOT$, it may not be the case in this model. The sensitivity of $TOT$ to the relative marginal cost is less (by half) and the extensive margin of adjustment is explicitly in the equation (with a coefficient that is increasing in trade costs). In particular, this implies that $TOT$ may improve following a positive Home productivity shock that
leads to a fall in relative marginal cost (TOL) if Home to Foreign differential in the number of produced varieties increases enough.

Atkeson and Burstein (2008) document that manufacturing terms of trade are significantly less volatile than manufacturing PPI-based real exchange rate in the U.S. and other major developed countries. Equation (3.64) would fit this empirical finding, as it shows that: (i) regardless of the nature of the shock, terms of trade move only half-way with terms of labor, which is a theoretical counterpart of PPI-based real exchange rate measure that Atkeson and Burstein (2008) use; and (ii) as long as the Home to Foreign differential in the number of varieties comoves positively with terms of labor (as is likely to be the case with productivity shocks) the volatility of terms of trade would be smaller.

### 3.5.2 The Real Exchange Rate

The real exchange rate is the relative price of Foreign consumption in terms of Home consumption:

\[
\ln Q = \ln P^* + \ln \varepsilon - \ln P. \tag{3.66}
\]

As shown in Appendix F, it can be rewritten as:

\[
\ln Q = \ln TOT + \frac{1}{2} \ln TOL(1 + \frac{\gamma \ln(1 + \tau) NN^*}{N + N^*}) + \ln(1 + \tau) \frac{N - N^*}{2(N + N^*)}. \tag{3.67}
\]

It is worth noting that since there are no nontraded goods and consumers in Home and Foreign have access to the same set of varieties, welfare-based and data consistent measures of price indices are equivalent.\(^7\)

\(^7\)This is similar to the result in Cacciatore, Ghironi, Stebunovs (2015) where some varieties are exogenously non-traded and welfare-consistent and data-consistent real exchange rates are shown to
or in log-linear terms:

\[
\hat{Q} = TOL \frac{\ln(1 + \tau) \gamma N}{4} + (\hat{N} - \hat{N}^*) \frac{\ln(1 + \tau)}{2}.
\] (3.68)

Note that PPP holds and \( \hat{Q} = 0 \) when \( \tau = 0 \).

Coefficients of both \( TOL \) and \((\hat{N} - \hat{N}^*)\) are strictly positive as long as trade costs are present. When home effective labor becomes cheaper, the relative price of Home consumption decreases because due to presence of trade costs Home consumers enjoy more of the benefits of cheaper home goods than Foreign consumers do. When the number of Home produced varieties increases relative to Foreign again Home consumption becomes relatively cheaper since the number of varieties on which Home consumers do not pay trade costs increases with a positive welfare effect.\(^8\)

Equation (3.68) can also be rewritten to illustrate the dynamics of the real exchange rate versus that of the term of trade:

\[
\hat{Q} = 2T\hat{O}T + T\hat{O}L(1 + \frac{\ln(1 + \tau) \gamma N}{4}),
\] (3.69)

which shows that for example when terms of trade deteriorate (Home exports become relatively cheap) the real exchange can still improve (the relative price Home consumption can decrease) if the relative cost of Home labor falls enough. Higher trade costs would make this more likely to happen.

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\(^8\)This result is parallel to the one in Cacciatore, Ghironi and Stebunovs, 2015 where in a model with Dixit-Stiglitz preferences the changes in real exchange rate is decomposed into changes in terms of labor and the varieties differential.

be the same and is different from the result in Ghironi and Melitz (2005) where some varieties are endogenously non-traded and welfare-consistent price indices have to be adjusted by removing the pure variety effect in order to obtain price data-consistent indices.
Finally, it is useful to rewrite equation (3.68) using the expressions for markups in steady state (3.40) and (3.41):

\[ \hat{Q} = \frac{1}{2} T \hat{Q}L \ln \frac{\mu(h)_{ss} - \ln \mu(f)_{ss}}{\ln \mu(h)_{ss} + \ln \mu(f)_{ss}} + (\hat{N} - \hat{N}^*) (\ln \mu(h)_{ss} - \ln \mu(f)_{ss}). \] (3.70)

Since in a symmetric steady state markups charged in domestic and export markets are different only because of trade costs, equation (3.70) shows that (i) the real exchange rate will move more with the varieties differential if the relative monopoly power in domestic market versus export market is higher; and that (ii) the real exchange rate will move more with the relative cost of home effective labor if the relative monopoly power in domestic market versus export market is higher while the average monopoly power in both markets is lower.

3.6 Conclusion

The role of pricing to market in explaining the behavior of international relative prices as well as national and international business cycle dynamics has gained recent attention. This paper seeks to contribute to this literature by departing from the common assumption of CES preferences and using instead translog preferences in an open economy setting. I show that in an open economy markups depend not only on the number of available varieties but also on the relative marginal cost in the two countries and trade costs. Endogenous markups imply a richer dynamics for international relative prices and in particular allow for the possibility of terms of trade to improve even when relative marginal costs fall.

To explore the relevance of endogenous markups for the international business
cycle I compare qualitatively the log-linear model responses to a positive Home productivity shock to those from a benchmark model that features CES preferences and find that endogenous markup dynamics resulting from translog preferences acts as a potentially important transmission mechanism. In particular, it amplifies the responses of Foreign variables to a Home productivity improvement and causes a positive GDP comovement across borders. Further, endogenous markups give rise to endogenous procyclical movements in measured TFP due to lower monopoly power and re-allocation of resources between domestic and export markets. This means that there will be a cross-country correlation of measured TPFs even when there is no cross-country correlation of exogenous productivity shocks.
Figure 3.1: **Responses to Home Productivity $z$-Shock in Translog versus CES Models**

- **Home Wage**: Comparison of TL and CES models.
- **Foreign Wage**: Comparison of TL and CES models.
- **Home Markup in Home Market**: Comparison of TL and CES models.
- **Home Markup in Foreign Market**: Comparison of TL and CES models.
- **Output per firm: Home**: Comparison of TL and CES models.
- **Output per firm: Foreign**: Comparison of TL and CES models.
- **Terms of Labor**: Comparison of TL and CES models.
- **Terms of Trade**: Comparison of TL and CES models.
- **Number of Varieties: Home**: Comparison of TL and CES models.
- **Number of Varieties: Foreign**: Comparison of TL and CES models.
- **Foreign Markup in Foreign Market**: Comparison of TL and CES models.
- **Foreign Markup in Home Market**: Comparison of TL and CES models.
- **GDP: Home**: Comparison of TL and CES models.
- **GDP: Foreign**: Comparison of TL and CES models.
- **Exchange Rate**: Comparison of TL and CES models.
Figure 3.2: Responses to Home Productivity f-Shock in Translog versus CES Models
Figure 3.3: Responses to Combined Home Productivity Shock in Translog versus CES Models
Appendix C

Translog Preferences, Expenditure Shares, and Markups

The translog expression for unit expenditure function for Home country can be written as:

\[
\ln P = \sum_{i=1}^{N} \alpha_i \ln p_i(h) + \sum_{i=1}^{N^*} \alpha_i \ln p_i(f) + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \gamma_{ij} \ln p_i(h) \ln p_j(f) \\
+ \frac{1}{2} \sum_{i=1}^{N^*} \sum_{j=1}^{N^*} \gamma_{ij} \ln p_i(h) \ln p_j(f) + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N^*} \gamma_{ij} \ln p_i(h) \ln p_j(f) \\
+ \frac{1}{2} \sum_{i=1}^{N^*} \sum_{j=1}^{N^*} \gamma_{ij} \ln p_i(h) \ln p_j(f),
\]

where \( \gamma_{ij} = \gamma_{ji} \). \( p_i(h) \) is the price of a Home produced variety and \( p_i(f) \) is the price of a Foreign produced variety.\(^1\) The unit expenditure function for Foreign can be defined in a similar fashion. Homogeneity of degree one requires that \( \sum_{i=1}^{\tilde{N}} \alpha_i = 1 \) and \( \sum_{i=1}^{\tilde{N}} \gamma_{ii} = 0 \), while requiring that all varieties enter ”symmetrically” into the expenditure function implies the following restrictions: \( \alpha_i = 1/\tilde{N} \), \( \gamma_{ii} = -\gamma(\tilde{N} - 1)/\tilde{N} \), \( \gamma_{ij} = \gamma/\tilde{N} \).

The expenditure share of each variety can be computed by differentiating the expenditure function with respect to the price of that variety.

\[
s_i = \frac{\partial \ln P}{\partial \ln p_i} = \alpha_i + \sum_{j=1}^{\tilde{N}} \gamma_{ij} \ln p_j,
\]

\(^1\)The notation difference from (3.2) is due to grouping Home and Foreign prices.
Since we care only about real variables, it is useful to rewrite the expenditure function in real terms. Using the shorter notation below:

\[
\ln P = \sum_{i=1}^{\tilde{N}} \alpha_i \ln p_i + \frac{1}{2} \sum_{i=1}^{\tilde{N}} \sum_{j=1}^{\tilde{N}} \gamma_{ij} \ln p_i \ln p_j, \tag{C.3}
\]

and subtracting \( \ln P \) from both sides, the first term on right-hand-side becomes:

\[
\sum_{i=1}^{\tilde{N}} \alpha_i \ln p_i - \ln P = \sum_{i=1}^{\tilde{N}} \alpha_i \ln p_i - \sum_{i=1}^{\tilde{N}} \alpha_i \ln P = \sum_{i=1}^{\tilde{N}} \alpha_i (\ln p_i - \ln P) = \sum_{i=1}^{\tilde{N}} \alpha_i \ln \rho_i, \tag{C.4}
\]

where

\[
\rho_i = \frac{p_i}{P}. \tag{C.5}
\]

The second term on right-hand-side of (C.3) can be rewritten as:

\[
\frac{1}{2} \sum_{i=1}^{\tilde{N}} \sum_{j=1}^{\tilde{N}} \gamma_{ij} (\ln p_i - \ln P)(\ln p_j - \ln P) = \frac{1}{2} \sum_{i=1}^{\tilde{N}} \sum_{j=1}^{\tilde{N}} \gamma_{ij} (\ln p_i \ln p_j) - \\
\frac{1}{2} \sum_{i=1}^{\tilde{N}} \sum_{j=1}^{\tilde{N}} \gamma_{ij} (\ln P \ln p_j) - \frac{1}{2} \sum_{i=1}^{\tilde{N}} \sum_{j=1}^{\tilde{N}} \gamma_{ij} (\ln P \ln P). \tag{C.6}
\]

Since the last three terms of this expression are zero, the following is true:

\[
\frac{1}{2} \sum_{i=1}^{\tilde{N}} \sum_{j=1}^{\tilde{N}} \gamma_{ij} \ln p_i \ln p_j = \frac{1}{2} \sum_{i=1}^{\tilde{N}} \sum_{j=1}^{\tilde{N}} \gamma_{ij} \ln \rho_i \ln \rho_j, \tag{C.7}
\]

yielding the following for the expenditure function:

\[
0 = \sum_{i=1}^{\tilde{N}} \alpha_i \ln \rho_i + \frac{1}{2} \sum_{i=1}^{\tilde{N}} \sum_{j=1}^{\tilde{N}} \gamma_{ij} \ln \rho_i \ln \rho_j. \tag{C.8}
\]
Further, the equation for expenditure share of each variety (C.2) can be rewritten as:

\[ s_i = \alpha_i + \sum_{j=1}^{\tilde{N}} \gamma_{ij} \ln \rho_j, \quad (C.9) \]

where the following was used:

\[ \sum_{j=1}^{\tilde{N}} \gamma_{ij} \ln \rho_j = \sum_{j=1}^{\tilde{N}} \gamma_{ij} (\ln p_j - \ln P) = \sum_{j=1}^{\tilde{N}} \gamma_{ij} \ln p_j - \ln P \sum_{j=1}^{\tilde{N}} \gamma_{ij} = \sum_{j=1}^{\tilde{N}} \gamma_{ij} \ln p_j, \quad (C.10) \]

and which is intuitive because the share of expenditure on each variety is a real variable.

In symmetric equilibrium, the share of each Home produced variety respectively becomes:

\[ s(h) = \alpha_i + \ln \rho(h)((N - 1)\gamma_{ij} + \gamma_{ii}) + \ln \rho(f)N^*\gamma_{ij}, \quad (C.11) \]

and for each foreign produced variety:

\[ s(f) = \alpha_i + \ln \rho(h)N\gamma_{ij} + \ln \rho(f)N^*\gamma_{ij}. \quad (C.12) \]

Using the above parameter restrictions, these can be rewritten respectively as:

\[ s(h) = \frac{1}{N} + \frac{\gamma^N}{N}(\ln \rho(f) - \ln \rho(h)), \quad (C.13) \]

\[ s(f) = \frac{1}{N} + \frac{\gamma^N}{N}(\ln \rho(h) - \ln \rho(f)). \quad (C.14) \]

The expressions can be also be rewritten using the difference between the price of
a specific variety and average price as in Feenstra 2002.

\[ s_i = \frac{1}{\tilde{N}} + \gamma (\ln \bar{p} - \ln p_i) \]  \hspace{1cm} (C.15)

where

\[ \ln \bar{p} = \frac{\sum_{i=1}^{\tilde{N}} \ln p_i}{\tilde{N}} = \frac{N}{\tilde{N}} \ln p(h) + \frac{N^*}{\tilde{N}} \ln p(f) \]  \hspace{1cm} (C.16)

For home produced varieties \( i = 1, \ldots, N \)

\[ s_i = \frac{1}{N} + \gamma \left( \frac{N}{N} \ln p(h) + \frac{N^*}{N} \ln p(f) - \ln p(h) \right) = \frac{1}{N} + \frac{\gamma N^*}{N}(\ln p(f) - \ln p(h)) \]  \hspace{1cm} (C.17)

and for Foreign produced varieties \( i = N+1, \ldots, \tilde{N} \)

\[ s_i = \frac{1}{N} + \gamma \left( \frac{N}{N} \ln p(h) + \frac{N^*}{N} \ln p(f) - \ln p(f) \right) = \frac{1}{N} + \frac{\gamma N^*}{N}(\ln p(h) - \ln p(f)) \]  \hspace{1cm} (C.18)

The elasticity of demand is defined as:

\[ \eta_i = 1 - \frac{d \ln s_i}{d \ln \rho_i} \]  \hspace{1cm} (C.19)

which, following Feenstra, 2002 it can be approximated as:

\[ \eta_i = 1 + \frac{\gamma}{s_i} \]  \hspace{1cm} (C.20)

Subsequently, the markup is is given by the following expression:

\[ \mu_i = 1 + \frac{1}{\eta_i - 1} = 1 + \frac{s_i}{\gamma} \]  \hspace{1cm} (C.21)

This implies that the markups for Home and Foreign produced varieties can be

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expressed respectively as:

\[
\ln \mu(h) = \frac{1}{\gamma N} + \frac{N^*}{N} (\ln \rho(f) - \ln \rho(h)), \tag{C.22}
\]

\[
\ln \mu(f) = \frac{1}{\gamma N} + \frac{N}{N} (\ln \rho(h) - \ln \rho(f)), \tag{C.23}
\]

where the approximation \( \ln \mu \approx \mu - 1 \) was used.

Note that combining (C.21) with (C.22) and (C.23) the following expressions for expenditure shares for Home and Foreign produced varieties respectively can be obtained:

\[
s(h) = \gamma (\ln \mu(h)), \tag{C.24}
\]

\[
s(f) = \gamma (\ln \mu(f)). \tag{C.25}
\]

Subtracting (C.23) from (C.22) gives:

\[
\ln \mu(h) - \ln \mu(f) = \ln \rho(f) - \ln \rho(h). \tag{C.26}
\]

On the other hand, from pricing equations (3.16)-(3.20) the ratio of Foreign to Home varieties for Home consumer is:

\[
\frac{\rho(f)}{\rho(h)} = \frac{\mu(f)(w^*\epsilon^*/z^*)(1 + \tau)}{\mu(h)(w/z)}, \tag{C.27}
\]

or, using the notation \( TOL = \frac{w^*\epsilon^*/z^*}{w/z} \):

\[
\frac{\rho(f)}{\rho(h)} = \frac{\mu(f)}{\mu(h)} (1 + \tau)TOL. \tag{C.28}
\]
Substituting (C.26) back into (C.22) and (C.23) gives the following key result:

\[
\left( \frac{\rho(f)}{\rho(h)} \right)^2 = (1 + \tau)TOL = \left( \frac{\mu(h)}{\mu(f)} \right)^2,
\]  
(C.29)

using which the markup equations can be rewritten as:

\[
\ln \mu(h) = \frac{1}{\gamma N} + \frac{N^*}{2N} \ln(TOL(1 + \tau)), \quad (C.30)
\]

\[
\ln \mu(f) = \frac{1}{\gamma N} - \frac{N}{2N} \ln(TOL(1 + \tau)), \quad (C.31)
\]

and similarly for markups in Foreign:

\[
\ln \mu(f)^* = \frac{1}{\gamma N} + \frac{N}{2N} \ln \frac{1 + \tau}{TOL}, \quad (C.32)
\]

\[
\ln \mu(h)^* = \frac{1}{\gamma N} - \frac{N^*}{2N} \ln \frac{1 + \tau}{TOL}. \quad (C.33)
\]
Appendix D

The Stylized Model in Log-linear Form

\( \hat{\mu}(h) = \frac{1}{4} TOL - \frac{1}{2} \ln \mu(h)_{ss} \hat{N} - \frac{1}{2} \ln \mu(f)_{ss} \hat{N}^* \) \hspace{1cm} (D.1)

\( \hat{\mu}(h)^* = \frac{1}{4} TOL - \frac{1}{2} \ln \mu(f)_{ss} \hat{N} - \frac{1}{2} \ln \mu(h)_{ss} \hat{N}^* \) \hspace{1cm} (D.2)

\( \hat{\mu}(f)^* = -\frac{1}{4} TOL - \frac{1}{2} \ln \mu(f)_{ss} \hat{N} - \frac{1}{2} \ln \mu(h)_{ss} \hat{N}^* \) \hspace{1cm} (D.3)

\( \hat{\mu}(f)^* = -\frac{1}{4} TOL - \frac{1}{2} \ln \mu(h)_{ss} \hat{N} - \frac{1}{2} \ln \mu(f)_{ss} \hat{N}^* \) \hspace{1cm} (D.4)

\( \hat{N} = (\psi - 1 - \beta) \hat{w} - \hat{f} \) \hspace{1cm} (D.5)

\( \hat{N}^* = (\psi - 1 - \beta) \hat{w}^* \) \hspace{1cm} (D.6)

\( \psi(\hat{w} - \hat{w}^*) - (\hat{N} - \hat{N}^*) - \hat{\epsilon} - \frac{\mu(f)_{ss}}{\mu(f)_{ss} - 1} \hat{\mu}^*(h) + \frac{\mu(f)_{ss}}{\mu(f)_{ss} - 1} \hat{\mu}(f) = 0 \) \hspace{1cm} (D.7)

\( \hat{w}((\psi - 1) \frac{(\mu(h)_{ss} - 1)^2}{\mu(h)_{ss}} - \frac{(\mu(f)_{ss} - 1)^2}{\mu(f)_{ss}}) + \hat{w}^*(\psi) \frac{(\mu(f)_{ss} - 1)^2}{\mu(f)_{ss}} + 
+ \frac{(\mu(f)_{ss} - 1)^2}{\mu(f)_{ss}} + \hat{\mu}(h)(\mu(h)_{ss} - 1) + \mu(h)^*(\mu(f)_{ss} - 1) + \frac{1}{\mu(f)_{ss}} = f \frac{L\gamma}{w_{ss} \psi - 1} \hat{f} \) \hspace{1cm} (D.8)

\( \hat{w}^*((\psi - 1) \frac{(\mu(h)_{ss} - 1)^2}{\mu(h)_{ss}} - \frac{(\mu(f)_{ss} - 1)^2}{\mu(f)_{ss}}) + \hat{w}(\psi) \frac{(\mu(f)_{ss} - 1)^2}{\mu(f)_{ss}} - 
- \frac{(\mu(f)_{ss} - 1)^2}{\mu(f)_{ss}} + \hat{\mu}(f)(\mu(h)_{ss} - 1) + \mu(h)^*(\mu(f)_{ss} - 1) = 0 \) \hspace{1cm} (D.9)
Appendix E

The Benchmark CES Model

With CES preferences, the consumption, the price index is given by:

\[
P = \left[ \int_0^N p(h)^{1-\sigma} dh + \int_0^{N*(1-\delta)} p(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}} \tag{E.1}
\]

where \( \sigma > 1 \) is the elasticity of substitution and all markups are constant and equal to \( \frac{\sigma}{\sigma - 1} \). Consumption of Home and Foreign varieties is given by the following relations:

\[
c(h) = \rho(h)^{-\sigma} C \tag{E.2}
\]

\[
c(f) = \rho(f)^{-\sigma} C \tag{E.3}
\]

Substituting these expressions into the equation for profits yields:

\[
d = (\mu - 1) \frac{w}{z} y \tag{E.4}
\]

which, combined with the free entry condition, gives the following equation for the size of the firm:

\[
y = \frac{f z}{\mu - 1} \tag{E.5}
\]

Incorporating also the labor market clearing condition and the trade balance equation, The system can be summarized by the following five equations in five endogenous
variables: $N, N^*, w, w^*, \epsilon$.

$$N = w^\psi - \frac{1 - \beta}{f} L_z \mu - 1 \frac{1}{\mu}$$  \hfill (E.6)

$$N^* = w^{*\psi} - \frac{1 - \beta}{f^*} L^*_{z^*} \mu - 1 \frac{1}{\mu}$$  \hfill (E.7)

$$\frac{fz}{\mu - 1} = L(\mu \frac{w}{z})^{-\sigma} w^\psi + L^*(1 - \phi)(1 + \tau)^{1 - \sigma}(\mu \frac{w}{\epsilon z})^{-\sigma} w^{*\psi}$$  \hfill (E.8)

$$\frac{f^{*z^*}}{\mu - 1} = L^*(\mu \frac{w^*}{z^*})^{-\sigma} w^{*\psi} + L(1 - \phi)(1 + \tau)^{1 - \sigma}(\mu \frac{w^* \epsilon}{z^*})^{-\sigma} w^\psi$$  \hfill (E.9)

$$N^*L(\mu \frac{w^* \epsilon}{z^*})^{-1 - \sigma} w^\psi = NL^*(\mu \frac{w}{\epsilon z})^{-1 - \sigma} w^{*\psi} \epsilon$$  \hfill (E.10)
Appendix F

The Real Exchange Rate

The real exchange rate is the relative price of Foreign consumption in terms of Home consumption.

\[ \ln Q = \ln P^* + \ln \varepsilon - \ln P \]  \hspace{1cm} (F.1)

Using the translog expenditure function as defined above and the symmetry assumption (in Home all Home produced varieties have the same price and all Foreign produced varieties have the same price) the Home price index can be rewritten as:

\[ \ln P = \frac{N}{N_N} \ln p(h) + \frac{N^*}{N_N} \ln p(f) - \frac{1}{2} \frac{\gamma N N^*}{N_N} (\ln p(h) - \ln p(f))^2 \]  \hspace{1cm} (F.2)

And Foreign price index can be rewritten as:

\[ \ln P^* = \frac{N^*}{N^*_N} \ln p^*(f) + \frac{N}{N^*_N} \ln p^*(h) - \frac{1}{2} \frac{\gamma N N^*}{N^*_N} (\ln p^*(f) - \ln p^*(h))^2 \]  \hspace{1cm} (F.3)

Note that:

\[ \frac{N}{N_N} \ln p(h) + \frac{N^*}{N_N} \ln p(f) = \frac{N}{N_N} (\ln p(h) - \ln p(f)) + \ln p(f) \]  \hspace{1cm} (F.4)

And:

\[ \frac{N^*}{N^*_N} \ln p^*(f) + \frac{N}{N^*_N} \ln p^*(h) = \frac{N^*}{N^*_N} (\ln p^*(f) - \ln p^*(h)) + \ln p^*(h). \]  \hspace{1cm} (F.5)
Plugging these expressions into (3.66) yields:

\[
\ln Q = \ln TOT + \frac{N^*}{N + N^*} \left(\ln p^*(f) - \ln p^*(h)\right) - \frac{N}{N + N^*} (\ln p(h) - \ln p(f)) - \frac{1}{2} \frac{\gamma NN^*}{N + N^*} ((\ln p^*(f) - \ln p^*(h))^2 - (\ln p(h) - \ln p(f))^2),
\]  

(F.6)

where \(\ln TOT = \ln p(h)^* + \ln \varepsilon - \ln p(f)\) was used. Further, using the following expressions for relative prices in Home and Foreign respectively:

\[
\ln p(h) - \ln p(f) = -\frac{1}{2} (\ln TOL + \ln(1 + \tau)), \quad (F.7)
\]

\[
\ln p^*(f) - \ln p^*(h) = \frac{1}{2} (\ln TOL - \ln(1 + \tau)), \quad (F.8)
\]

the real exchange rate can be written as:

\[
\ln Q = \ln TOT + \frac{1}{2} \ln TOL (1 + \frac{\gamma \ln(1 + \tau) NN^*}{N + N^*}) + \ln(1 + \tau) \frac{N - N^*}{2(N + N^*)}. \quad (F.9)
\]

In log-linear terms:

\[
\hat{Q} = TOL \frac{\ln(1 + \tau) \gamma N}{4} + (\hat{N} - \hat{N}^*) \frac{\ln(1 + \tau)}{2}. \quad (F.10)
\]
Bibliography


