Globalization and Income Distribution: A Specific Factors Continuum Approach*

James E. Anderson
Boston College and NBER

January 5, 2009

Abstract

Does globalization widen inequality or increase income risk? Globalization amplifies the effect of idiosyncratic relative productivity shocks. But wider markets reduce the effect of economy-wide supply shocks on world prices. Both forces are at work in the specific factors continuum model of this paper. Ex post equilibrium exhibits positive (negative) premia for export (import-competing) sector specific factors. Globalization widens inequality in North and South. Globalization increases personal income risk from idiosyncratic productivity shocks, but reduces aggregate shock risk acting on the factoral terms of trade. Both forces have their greatest impact on the poorest and least impact for the richest trading sectors, while the distribution in nontraded sectors is unaffected.

JEL Classification: F10.

Contact information: James E. Anderson, Department of Economics, Boston College, Chestnut Hill, MA 02467, USA.

*I would like to thank, without implicating, Karim Chalak, Arnaud Costinot, Fabio Ghironi, Philippe Martin and participants in seminars at Boston College and Paris School of Economics.
Globalization is often thought to have increased inequality. The model of this paper delivers the stark result that globalization raises inequality in both North and South. Globalization is also commonly thought to have increased personal income risk. New markets open to trade while in old markets like wine, French vintners are more exposed to supply innovations from Argentine and Australian vintners and vice versa. A contrasting economic intuition suggests that wider markets reduce the effect of national supply shocks on the variance of world prices and thus factor incomes. The two opposing forces are sorted out in this paper using a formal model that isolates the key elements while abstracting from inessential details.

The model embeds the specific factors model in the infinitely many goods continuum setup pioneered by Dornbusch, Fischer and Samuelson, complementing their analyses (1977, 1980) of the Ricardian and Heckscher-Ohlin models. For thinking about income inequality and insecurity, the specific factors model has several advantages over the Ricardian and Heckscher-Ohlin models. First, specific factors rationalize the tremendous heterogeneity of rents to otherwise observationally identical factors, especially the premium for export sector employment.\(^1\) Second, specific factors are essential to the theoretical and empirical success of the now-standard political economy of trade model (Grossman and Helpman, 1994; Goldberg and Maggi, 1999). Third, resource reallocation between firms within sectors in this paper is driven by a Darwinian gale blowing through the sectoral factor market. When paired with wage bargaining, the model can explain why larger and more productive firms pay higher wages for skilled labor. Finally, the specific factors model reverses the excessive specialization imposed by trade equilibrium in the Ricardian and Heckscher-Ohlin continuum models. Diversified production of traded goods occurs with measure zero in those models, while here diversification occurs with measure one. The latter is arguably a more useful metaphor.

The model is stripped down to focus on the distributional consequences of combining specificity, random productivity and globalization. After the endowment of potentially skilled labor is allocated across sectors, specific skills are acquired and the skilled labor combines with intersectorally mobile unskilled labor to produce output as efficiently as possible given the

\(^{1}\)This regularity was given prominence by Katz and Summers (1989). The phenomenon is well documented in the US and other developed countries. Sparser available evidence finds the same pattern in poorer countries as well — see Milner and Tandrayen (2006) on sub-Saharan Africa and Tsou, Liu and Huang (2006) on Taiwan.
realizations of productivity shocks. All sectors have identical ex ante potential production functions, an assumption that shuts off Heckscher-Ohlin and more importantly Stolper-Samuelson distributional properties for simplicity. The pure Ricardian continuum model is contained within the model as a special case, when the production function assigns zero marginal product to the specific factor or when the specific factor allocation is perfectly efficient. As with the original Dornbusch-Fischer-Samuelson Ricardian model, sharp implications are obtained that appear likely to obtain in more general cases. The tractability of the model suggests that it it is a good platform on which to build extensions.

The paper first characterizes the ex post equilibrium production and trade patterns. Familiar comparative static results with respect to growth, transfers and trade costs are reviewed, echoing Dornbusch, Fischer and Samuelson. A more novel result shows that in the presence of aggregate productivity shocks, globalization reduces the variance of the factorial terms of trade and hence the dispersion of personal incomes.

The allocation of the specific factors is given ex post, but a choice variable ex ante. In the face of productivity shocks, the best that can be done through efficient capital markets is to equalize ex ante expected returns. The ex post returns differ from their expectation due to realized productivity shocks and the efficient reallocation of the mobile factor to accommodate them.

The equilibrium internal income distribution given the efficient allocation of skills exhibits higher earnings for export sectors (those receiving high productivity realizations) than for import competing sectors (those receiving low productivity realizations). Globalization widens income inequality in each country, raising the top, lowering the bottom and narrowing the middle. Viewed ex ante, the model formalizes the popular sense that personal incomes are more risky in globalizing world with purely idiosyncratic productivity shocks.

When productivity shocks include an economy-wide component that shifts relative productivity between countries, there is a countervailing effect on income risk. Globalization reduces the factorial terms of trade variance, reducing the ex ante expected dispersion of personal incomes. The model implies that both effects of globalization are biggest for the poorest specific factors.

The effect of globalization on income distribution has previously been studied, but in models for which theory does not fit well with empirics. For example, the factor proportions model applications surveyed in Feenstra (2004) have income distributions of low dimension, in contrast to empirical
distributions with high dimensionality characterized by factor earnings in export industries greater than earnings of similar factors in import competing industries. In the Heckscher-Ohlin continuum model application of Feenstra and Hanson (1999), globalization raises the skill premium by increasing the average skill intensity of the production mix in North and South through reallocation on the extensive margin of production. There are no import-competing sectors. The empirical model is forced to treat sectoral wage premia as random shocks. The model of this paper is in contrast focused on the determination of sectoral wage premia via reallocation on the intensive margin. The model in its most general form developed in the Appendix shows that globalization can raise or lower the average skill premium in North and South depending on whether the average skill intensity of production rises or falls, itself ordinarily determined by whether the elasticity of substitution is less or greater than one. The text model sets the elasticity of substitution equal to one, neutralizing globalization’s effect on the average skill premium and substantially simplifying the analysis of the model.

New papers by Blanchard and Willman (2008) and Costinot and Vogel (2008) are similar to this paper in featuring continuum income distributions with heterogeneous workers who sort into industries of varying skill intensity. In contrast to the present paper these models do not explain locational rents to otherwise observationally identical factors. Moreover, they imply that globalization widens inequality in one economy while reducing inequality in the other economy, which is apparently counterfactual. Nevertheless, these two approaches should be viewed as complements in a fuller understanding of trade and income distribution.

The model is also related to a literature featuring productivity shocks. Eaton and Kortum (2002) derive the equilibrium implications of the Ricardian continuum model with sectoral productivity shocks. They solve the many country Ricardian continuum model by imposing a Frechet distribution on the productivity shocks. The present paper derives for the first time the specific factors model’s implications for the general equilibrium pattern of production, trade and income distribution in a two country world with productivity shocks. Judiciously imposing further restrictions on technology yields a closed form characterization of equilibrium. I speculate that restrictions on the productivity distribution may permit a closed form solution in

\[ The \text{ US rise in inequality is widely documented. For evidence on rising Mexican and Brazilian inequality see Calmon et al. (2002).} \]
the many country case.

Anderson (2008) develops the many country version of the specific factors model when gravity determines the bilateral trade patterns. That model nests the Eaton and Kortum model (and its extension by Costinot and Koumujer, 2008) when Dixit-Stiglitz monopolistic competition determines the number of varieties in each sector, the Frechet distribution determines the productivity draws, and, most important, skilled labor is allocated after the productivity draws instead of before.

Section 1 presents the basic production model. Section 2 derives the global equilibrium of the two trading countries. Section 3 deals with the equilibrium comparative statics of the model. Section 4 derives the distributional implications. Section 5 derives efficient ex ante allocation of the specific factor. Section 6 analyzes heterogeneous firms within sectors. Sections 7 and 8 conclude with speculation on extensions to dynamics and empirical work.

1 The Basic Production Model

There is a continuum of goods, each with an identical potential production function that is increasing, homogeneous of degree one and concave in skilled and unskilled labor. Maximal potential output in sector \( z \) is reduced by the realization of a productivity shock \( 1/a(z) \), the total factor productivity parameter, \( a(z) \geq 1 \). \( a(z)y(z) \) gives maximal potential production where \( y(z) \) is the output of sector \( z \), \( z \in [0, 1] \). A Cobb-Douglas potential production function is assumed in the text to generate sharp results. The general neo-classical production function case is analyzed in the Appendix, and the main qualitative results on income distribution continue to hold.

Output in sector \( z \) is given by

\[
y(z) = \left[\frac{1}{a(z)}\right]L(z)^\alpha K(z)^{1-\alpha}.
\]

(1)

\( \alpha \) is labor’s share parameter.\(^3\) \( K(z) \) denotes the quantity of sector specific capital and \( L(z) \) denotes the quantity of labor allocated to sector \( z \). The sector specific capital includes human capital, and until Section 6 on heterogeneous firms it is best to think of it as human capital only. Prior to

---

\(^3\)Imposed for analytic clarity, the identical Cobb-Douglas assumption generates the well know empirical regularity of constant labor shares of GDP, despite shifting production shares.
its allocation, the human capital is a unit of potentially skilled labor that subsequently adapts to sectoral requirements.

After the human capital is allocated (a decision modeled in Section 5), productivity shocks are realized and the mobile factor is allocated across sectors to maximize the GDP of the economy. The aggregate supply of labor is given by $L$, so the resource constraint is $\int_0^1 L(z)dz \leq L$. Efficient allocation of labor across sectors with price taking behavior by firms requires value of marginal product conditions for each sector, embedded in the (maximum value) gross domestic product function.

**Lemma 1** The gross domestic product (GDP) function for this economy is given by

$$g = L^\alpha K^{1-\alpha} G$$

where the GDP deflator $G$ is given by

$$G = \left[ \int_0^1 \lambda(z) \left( \frac{p(z)}{a(z)} \right)^{1/(1-\alpha)} \right]^{1-\alpha} dz,$$

$p(z)$ denotes the price of good $z$, $\lambda(z) = K(z)/K$ and $K = \int_0^1 K(z)dz$.

See Anderson (2008) for the derivation. The notation for specific capital anticipates a possible reallocation between sectors.\(^4\) ‘Real GDP’ is given by $R \equiv L^\alpha K^{1-\alpha}$.

The GDP function has a very convenient constant elasticity of transformation (CET) form.\(^5\) The GDP function is convex in prices, concave in $K, L, \{\lambda\}$ and homogeneity of degree one in $p$ and in $K, L$. Let a subscript with a variable name denote partial differentiation with respect to that variable. Then by Hotelling’s Lemma, $y(z) = g_p(z)$, while $g_L = w$, where $w$ denotes the wage rate. The GDP production shares\(^6\) are given by

$$s(z) = \lambda(z) \left\{ \left( \frac{p(z)}{a(z)} \right) \right\}^{1/(1-\alpha)}.$$

A country produces all goods for which it has a positive specific endowment because, due to the Cobb-Douglas assumption, the mobile factor has a very large marginal product in any sector where its level of employment is very small.

\(^4\)As allocation of the specific capital grows more efficient, the model converges onto a Ricardian model (since labor share parameters $\alpha$ are constant over $z$). In the limit, $g = L^\alpha K^{1-\alpha} \max_z \{p(z)/a(z)\}$.

\(^5\)The elasticity of transformation is equal to $\alpha/(1-\alpha)$.

\(^6\)The word ‘share’ is used here and in the remainder of the paper for intuitive clarity at the cost of some violation of mathematical precision. Share is a discrete concept. $s(z)$ is strictly a share density, as is $\lambda(z)$, with the share of GDP due to production in the interval $z_0, z_1$ being given by $\int_{z_0}^{z_1} s(z)dz$. 

5
2 Global Equilibrium

There is a foreign economy with Cobb-Douglas production functions characterized by the same parametric labor share $\alpha$ but differing productivity parameters $1/a^*(z)$ and differing specific factor endowments $K^*(z)$ and labor endowment $L^*$. This yields the foreign GDP function as $g^* = (L^*)^\alpha(K^*)^{1-\alpha}G^*$ where

$$G^* = \left[ \int_0^1 \lambda(z)(p^*(z)/a^*(z))^{1/(1-\alpha)} \right]^{1-\alpha}.$$

Tastes are identical across countries and characterized by a Cobb-Douglas utility function with parametric expenditure share for good $z$ given by $\gamma(z)$. The cumulative share of expenditure on goods indexed in the interval $[0, \bar{z}]$ is given by $\Gamma(\bar{z})$.

Trade is costly, with parametric markup factor $t > 1$. For goods exported by the home country, $p^*(z) = p(z)t$. For goods exported by the foreign country, $p(z) = p^*(z)t$. International trade will occur in equilibrium for a range of goods where the productivity differences between countries are large enough to pay the trade cost. There is a range of nontraded goods in between the ranges of imported goods and exported goods due to differences too small to overcome trade costs.

Markets must clear for each good. As shown in the next section, this effectively determines the price in each sector as a function of the multi-factoral terms of trade, the relative real GDP deflator, $G/G^*$. The multi-factoral terms of trade is determined by the trade balance condition. This structure nests the Dornbusch-Fischer-Samuelson Ricardian continuum model in which equilibrium boils down to determining the relative wage, converging to the Ricardian case when specific factor allocation is efficient.

2.1 Goods Market Equilibrium

First, the indexes $z$ are assigned to industries. As in the Dornbusch-Fischer-Samuelson model, relative labor productivities can be ranked by industry to create intuitive ranges of products. In the specific factors model it is

\footnote{An older literature used an intuitive empirical concept called the ‘double factorial terms of trade’. It was based on the 2 good Ricardian model equilibrium in which the price of home relative to foreign labor is equal to the relative price of home exports to imports times the relative productivity of home exports to foreign exports. In the special case of efficient allocation of specific factors, $G/G^*$ is equal to the double factorial terms of trade.}
convenient to order the index of products according to the relative labor productivity shift parameters:

\[ \Lambda(z) \equiv \frac{\lambda(z)/a(z)^{1/(1-\alpha)}}{\lambda^*(z)/a^*(z)^{1/(1-\alpha)}}, \quad (4) \]

where the indexes are assigned so that \( \Lambda \) is decreasing in \( z \), \( \Lambda_z < 0 \). Thus the lowest indexed goods give the home country the greatest relative labor productivity and conversely for the foreign economy. Let \( \bar{z} \) denote the upper end of the range of goods exported by the home country, those goods with index \( z \in [0, \bar{z}] \). Let \( \bar{z}^* \) denote the lower limit of the range of goods exported by the foreign country, those goods with index \( z \in \left[ \bar{z}^*, 1 \right] \). The nontraded goods range is given by \( z \in [\bar{z}, \bar{z}^*] \). \( \bar{z}, \bar{z}^* \) are determined in equilibrium.

Equilibrium prices for any good \( z \) that is internationally traded are determined by market clearance:

\[ s(z)g + s^*(z)g^* = \gamma(z)(g + g^*). \]

For nontraded goods, \( s(z) = \gamma(z) = s^*(z) \). It is convenient to normalize by the foreign GDP deflator \( (G^* \equiv 1) \), so in the solutions that follow, based on the preceding equations, \( G \) is the relative GDP deflator, or factorial terms of trade.

The equilibrium prices are determined in four ranges, one for home exports, one for foreign exports and one each for the nontraded goods of home and foreign. For home exports, the transform of the efficiency unit price is given by, \( \forall z \in [0, \bar{z}] \),

\[ [p(z)/a(z)]^{1/(1-\alpha)} = \frac{\gamma(z)}{\lambda(z)} \frac{GR/R^*}{G^{-\alpha/(1-\alpha)}R^*/R + t^{1/(1-\alpha)}/\Lambda(z)}. \quad (5) \]

Intuitively, the price is decreasing in the specific allocation \( \lambda(z) \), increasing in relative labor productivity \( \Lambda(z) \), increasing in the factorial terms of trade.

---

Footnote 8: The text expression for market clearance is built up from material balance using iceberg melting trade costs. For example, in the range \( z \in [0, \bar{z}] \), market clearance is given by

\[ y(z) - x(z) = t[x^*(z) - y^*(z)] \]

where \( x(z), x^*(z) \) denote consumption of good \( z \) in the home and foreign countries. The equation implies that for each unit imported by the foreign economy, \( t > 1 \) units must be shipped from the home economy, \( t - 1 \) units melting away en route. Multiply both sides by \( p(z) \), use \( p^*(z) = p(z)t \) and utilize the GDP and expenditure share definitions to obtain the text expression.
and decreasing in trade costs $t$. Less obviously, but also intuitively, the price is decreasing in relative country size $R/R^*$.\footnote{Differentiating (5) with respect to $R/R^*$ and simplifying yields an expression that can be shown to be negative for $z < \bar{z}$ using the export cutoff expression (13) below.}

For foreign exports, the transform of the efficiency unit price is given by,

$$z \in [\bar{z}, 1],\quad [p^*(z)/a^*(z)]^{1/(1-\alpha)} = \frac{\gamma(z)}{\lambda(z)} \frac{GR/R^* + 1}{G^{1/(1-\alpha)}t^{1/(1-\alpha)}R/R^* + 1}. \quad (6)$$

For home nontraded goods the transform efficiency price is given by,

$$z \in [\bar{z}, \bar{z}^*],\quad [p(z)/a(z)]^{1/(1-\alpha)} = \frac{\gamma(z)}{\lambda(z)} G^{1/(1-\alpha)}. \quad (7)$$

For the foreign nontraded goods the transform efficiency price is given by,

$$z \in [\bar{z}, \bar{z}^*],\quad [p^*(z)/a^*(z)]^{1/(1-\alpha)} = \frac{\gamma(z)}{\lambda(z)}. \quad (8)$$

The equilibrium production shares, based on the equilibrium prices in (5)-(8), are as follows. For the range of goods exported by the home country, $z \in [0, \bar{z}]$,

$$s(z) = \frac{\gamma(z)}{\lambda(z)} \frac{GR/R^* + 1}{GR/R^* + G^{1/(1-\alpha)t^{1/(1-\alpha)}}/\Lambda(z)}. \quad (9)$$

$$s^*(z) = \frac{\gamma(z)}{\lambda(z)} \frac{GR/R^* + 1}{G^{1/(1-\alpha)}t^{1/(1-\alpha)}\Lambda(z)R/R^* + 1}. \quad (10)$$

For the range of goods exported by the foreign country, $z \in [\bar{z}^*, 1]$,

$$s(z) = \frac{\gamma(z)}{\lambda(z)} \frac{GR/R^* + 1}{GR/R^* + G^{1/(1-\alpha)t^{1/(1-\alpha)}}/\Lambda(z)}. \quad (11)$$

$$s^*(z) = \frac{\gamma(z)}{\lambda(z)} \frac{GR/R^* + 1}{G^{1/(1-\alpha)}t^{1/(1-\alpha)}\Lambda(z)R/R^* + 1}. \quad (12)$$

For nontraded goods, $s(z) = \gamma(z) = s^*(z), z \in [\bar{z}, \bar{z}^*].$

The margins of non-tradeability are determined by $s(\bar{z}) = \gamma(\bar{z})$ and $s^*(\bar{z}^*) = \gamma(\bar{z}^*)$. These solve for

$$G = \Lambda(\bar{z})^{1-\alpha}/t \quad (13)$$
and

\[ G = \Lambda(\bar{z}^*)^{1-\alpha}t. \]  \hspace{1cm} (14)

Thus the factorial terms of trade determines the dividing lines between imports and exports. \( \bar{z}^* \) is implicitly a function \( Z^*(\bar{z}, t) \) that is increasing in \( \bar{z} \) and \( t \) in equilibrium, by (13)-(14).

### 2.2 Factoral Terms of Trade

It remains to determine the factorial terms of trade using the balanced trade condition, a special case of the international budget constraint. Home GDP is equal to the value of shipments to all destinations, valued at destination prices. This setup implies that home factor payments include the cost of shipment, iceberg trade costs imply a technology of distribution that utilizes factors proportionately to their production cost. Let \( \Gamma(\bar{z}) \equiv \int_0^{\bar{z}} \gamma(z)dz \) and let \( \Gamma^*(\bar{z}^*) \equiv \int_{\bar{z}^*}^1 \gamma(z)dz. \) The expenditure share on nontraded goods is given by \( 1 - \Gamma - \Gamma^* \). The budget constraint is

\[ \Gamma g + (1 - \Gamma - \Gamma^*)g + \Gamma g^* = g, \]

stating that expenditure on home produced goods is equal to home GDP. Solve for \( G \) to yield

\[ G = \frac{\Gamma(\bar{z})}{\Gamma^*[Z^*(\bar{z}, t)]} \frac{R^*}{R}. \]  \hspace{1cm} (15)

(13), (14) and (15) are displayed in Figure 1. The intersection of (13) and (15) determines equilibrium \( \bar{z} \).

**Lemma 2** Provided trade costs are not too high, a unique trading equilibrium exists on \( z \in [0, 1] \).

If equilibrium exists, it is unique because (15) is increasing in \( z \) while \( \Lambda(z) \) is decreasing in \( z \). Nonexistence arises when the absolute penalty of trade cost is too large relative to the absolute advantage schedule \( \Lambda(z) \). There are two intuitive aspects. If \( t \) is too large for a given \( \Lambda(z) \) schedule, the two downward sloping schedules in Figure 1 are too far apart and there is no value of \( \ln G \) for which both \( \bar{z} \) and \( \bar{z}^* \) are in the unit interval. If \( \Lambda(z) \) is too large relative to a given \( t \), both the downward sloping schedules in Figure 1 are shifted upward and there is no trade because the foreign disadvantage is too large to overcome the trade cost.

It is useful to emphasize the points of difference with the familiar Ricardian continuum model. With specific factors there is generally positive
production in every sector in each country in contrast to the pure Ricardian model where there is specialization: home exports are not produced by the foreign economy and vice versa. The pure Ricardian case is contained in the present model as the special case in which $\alpha \to 1$, implying that $\ln \Lambda(z) \to \ln a^*(z)/a(z)$.

---

Figure 1. Equilibrium Factoral Terms of Trade

\[
\log G = (1-\alpha)\log \Lambda + \log t + \log \left\{ \frac{\Gamma(z)}{\Gamma^*(z,t)} \right\} + \log \left( \frac{R^*}{R} \right)
\]
3 Comparative Statics

Comprehension of the properties of the model is aided by reviewing its comparative static responses to changes in factor endowments, trade costs and international transfers. These are familiar from Dornbush, Fischer and Samuelson (1977, 1980). A more novel comparative static result deals with effect of globalization on the variance of the factorial terms of trade.

A key distributional property of the model is that the average skill premium $g_K/w - 1$ is independent of international equilibrium forces. The average skill premium rises with the skill bias of technology ($\alpha$) and falls with relative skill abundance $(K/L)$. The wage rate is given by

$$w = g_L = \alpha(K/L)^{1-\alpha}G.$$ 

The return to the group of specific factors (the value of marginal product of an equiproportionate increase in all specific factors) is given by

$$g_K = (1-\alpha)(L/K)^{\alpha}G.$$ 

$g_K$ is shown below to be the average specific factor return. Then

$$g_K/g_L = \frac{1-\alpha}{\alpha} \frac{L}{K}.$$ 

The invariance of the average skill premium to external prices in this paper conveniently isolates the distributional effects of globalization from Stolper-Samuelson effects. In the 2x2 factor proportions model, globalization causes the skill premium to rise in the skill abundant country and fall in the skill scarce country. In the Heckscher-Ohlin continuum model, globalization causes the average skill intensity of production to rise in both North and South, driving up the skill premium in both. More general neoclassical production functions in the specific factors setting imply that the average skill premium may rise or fall in both North and South due to globalization,\footnote{Linkage between openness and capital accumulation or technology will also violate the invariance property.} depending on whether the average skill intensity of production rises or falls. The Appendix develops the details and argues that with CES production functions the skill premium ordinarily rises (falls) as the elasticity of substitution is less (greater) than one.
3.1 Growth

Relative returns to the relatively slow-growing factor rise.

Neutral growth causes factorial terms of trade deterioration. A rise in \( R/R^* \) shifts the left hand side of (15) downward. Figure 1 reveals that the downward shift must lower the equilibrium value of the home country’s factorial terms of trade \( G \) and raise \( \bar{z} \), increasing the range of goods exported by the home country and reducing the range of goods exported by the foreign country. These effects are very similar to those of the Ricardian continuum model.

Neutral growth raises average real income in the home country. The utility (real income) of a representative agent who receives a per capita share of national income is represented by \( \ln u = \ln GR - \ln P \), where \( P \) is the price index. As to the first term on the left hand side, home nominal income in terms of foreign factor prices, \( GR \), must rise, as can be illustrated by the effect of a rise in \( R \) on the budget constraint function in Figure 1. A fall in \( G \) that fully offset the rise in \( R \) would imply a constant \( \bar{z} \), hence the rise in \( \bar{z} \) induced in the new equilibrium must imply a higher \( GR \).

As to the price index for the home economy, \( \ln P \) can be shown to rise less than proportionally to \( GR \). The Cobb-Douglas price index is given by \( \int_0^1 \gamma(z)lnp(z)dz \). Using the general equilibrium solution for prices (5)-(7), the price index becomes

\[
\frac{1}{1-\alpha} \ln P = k + \nu \ln GR + (1-\nu) \ln (GR/R^* + 1) - \Gamma \Pi - (1-\Gamma-\nu)(\Pi^* - \ln t),
\]

(16)

where

\[
\Pi \equiv \int_0^{\bar{z}} \frac{\gamma(z)}{\Gamma(\bar{z})} \ln [G^{\alpha/(1-\alpha)}R/R^* + t^{1/(1-\alpha)}/\Lambda(z)]dz,
\]

and

\[
\Pi^* \equiv \int_{\bar{z}^*}^{1} \frac{\gamma(z)}{\Gamma(\bar{z}^*)} \ln [G^{\alpha/(1-\alpha)}t^{1/(1-\alpha)}\Lambda(z)R/R^* + 1]dz,
\]

and \( \nu \equiv \int_{\bar{z}^*}^{\bar{z}} \gamma(z)dz \). Here, \( k \) is a constant term not dependent on \((G,t,R,R^*,\Lambda)\). Examining (16), it is clear that \( \ln P \) rises with \( GR \), but less than proportionately. For given \( \bar{z}, \bar{z}^* \), real income in the average sense must increase. The effect of changes on the extensive margins on the price index is ordinarily small compared to the effect of change in \( GR \), being proportional to the share densities \( \gamma(\bar{z}), \gamma(\bar{z}^*) \). Thus real income on average ordinarily rises with a rise in real GDP.
3.2 Fall in Trade Costs

A one percent fall in symmetric trade costs evidently shifts \( \ln \Lambda - \ln t \) up by one unit in Figure 1. The fall in \( t \) lowers \( \bar{z}^* \) by \( Z_t^* > 0 \) and thus lowers the function \( \Gamma R^*/\Gamma^* R \). The net effect on the factorial terms of trade is ambiguous, depending on the relative slopes of the two schedules on Figure 1 as well as the strength of the shift in \( \Gamma R^*/\Gamma^* R \). Normally the equilibrium will imply a rise in \( \bar{z} \), a fall in \( \bar{z}^* \) and a change in \( G \) that is contained by (13) and (14) under these conditions.

Real income on average in the home country must rise with a fall in trade costs. This is because the rise in the price index induced by a possible rise in \( G \) does not offset fully the rise in nominal GDP, while the direct effect of lower trade costs on the price of imports isolated in (16) increases the gain. As with the analysis of real GDP shifts, these inferences suppress the small effect of changes in \( \bar{z}, \bar{z}^* \) on price indexes for simplicity.

For the foreign country the real income effects essentially complement those of the home country, the foreign factorial terms of trade being the inverse of the home factorial terms of trade.

3.3 Transfers

Transfers affect the equilibrium factorial terms of trade in a standard way. Let \( B \) denote the transfer from the home country to the foreign country (in domestic price terms). The effect on equilibrium in any individual product market arises only through the factorial terms of trade, \( G \). This property reflects the well known special case of the 2 good model with equal marginal propensities. The factorial terms of trade \( G \) is solved from (15) shifted to reflect the effect of the transfer on aggregate spending:

\[
G = \frac{\Gamma}{\Gamma^*} \frac{R^*}{R} \left( 1 - \frac{1 - \Gamma - \Gamma^* B}{\Gamma^*} \frac{B}{R} \right).
\]  

The effect of a transfer is to lower the factorial terms of trade of the transferor and to reduce the range of goods exported. This secondary burden of the transfer arises on the extensive margin only. In the absence of trade costs that create a range of nontraded goods, \( 1 - \Gamma = \Gamma^* \) and there is no secondary burden due to the identical Cobb-Douglas tastes assumption that implies identical marginal propensities to spend on the traded goods.
3.4 Income Risk and Globalization

Does a more open world economy experience greater income risk? There are two aspects of this question, the variation of national incomes across economies in the world system and the dispersion of personal incomes within an economy. This subsection deals with external variation while the next section deals with internal variation.

The variation of incomes across economies is driven by variation in relative country size due to differential growth rates, relative productivity shifts or transfers. Between countries, relative incomes are determined by the factoral terms of trade $G$.

**Proposition 1** Globalization reduces the variance in $G$ induced by small shocks in relative productivity or relative country size.

The rationale is simple — wider markets damp the effect of aggregate supply shocks on relative prices. Aggregate relative productivity risk enters the model as a multiplicative scalar random variable $\mu$ with unit mean, applied to the schedule of relative labor productivities. $\Lambda(z)$ is replaced in this section by $\mu\Lambda(z)$. The equilibrium comparative statics with respect to $\ln \mu$ are used to derive the variance of $G$ in the neighborhood of $\mu = 1$. Then it is shown that the variance is decreased by reductions in $t$.

Equilibrium $\bar{z}$ is solved from combining the marginal export condition $G = \mu\Lambda(\bar{z})^{1-\alpha}t$ with the trade balance condition $G = \Gamma(\bar{z})R^*/\Gamma(\bar{z}^*)R$:

$$
\mu\Lambda(\bar{z})^{1-\alpha}t = \frac{\Gamma(\bar{z})}{\Gamma(Z^*(\bar{z}, t))} \frac{R^*}{R}.
$$

Taking logs and differentiating (18) with respect to $\ln \mu$,

$$
\frac{d\bar{z}}{d\ln \mu} = \frac{1}{\gamma(\bar{z}) + \frac{\gamma(\bar{z}^*)}{\Gamma(\bar{z}^*)}Z_\bar{z}^* - (1 - \alpha) \frac{\Lambda(\bar{z})}{\Lambda(\bar{z})}} > 0.
$$

Relative productivity risk $\mu$ affects $G$ via its effect on $\bar{z}$. Using the expression for $G$ on the right hand side of (18),

$$
\frac{d\ln G}{d\ln \mu} = \left\{ \frac{\gamma(\bar{z})}{\Gamma(\bar{z})} + \frac{\gamma(\bar{z}^*)}{\Gamma(\bar{z}^*)}Z_\bar{z}^* \right\} \frac{d\bar{z}}{d\ln \mu} > 0.
$$

The variance of $\ln G$ in the neighborhood of the mean is given by

$$
V(\ln G) = \left\{ \frac{d\ln G}{d\ln \mu} \right\}^2 V(\ln \mu).
$$
The effect of trade costs on the variance of $G$ is given by

$$2V(\ln \mu) \frac{d \ln G}{d \ln \mu} \frac{\partial d \ln G}{\partial t},$$

where

$$\frac{\partial d \ln G}{\partial t} = \frac{\gamma(\bar{z}^*) \{ d \ln G \}}{\Gamma(\bar{z}^*) \{ d \ln \mu \}} \left[ \frac{-(1 - \alpha) \Lambda_z(\bar{z})/\Lambda(\bar{z})}{[\gamma(\bar{z})/\Gamma(\bar{z} + Z^*_z)\gamma(z^*_z)/\Gamma(z^*_z)]^2} \right]^2 Z^*_{zt} > 0.$$

Thus the variance of $G$ is increasing in trade costs $t$.

A numerical example developed in the Appendix demonstrates the potential quantitative importance of the variance damping property of globalization. There is scope to reduce variance by 1/3 to 1/2 in the example.

The aggregate-risk-damping property of globalization due to market widening obtains much more generally than in the present model where market widening is on the extensive margin of trade. The Appendix provides an example where market widening acts exclusively on the intensive margin.\(^1\)

### 4 Income Distribution

The main goal of this paper is to derive equilibrium income distribution properties, in particular the comparative statics of income distribution with respect to trade costs, growth in endowments and transfers. A necessary component is the assignment of the potentially specific factors to sectors. In this section, an arbitrary assignment is eventually imposed that matches the allocation of skilled workers to demand patterns. This assignment generates sharp comparative static results. The pattern of persistent positive (negative) premia for export (import competing) sectors obtains. In the next section the arbitrary assignment is rationalized as an efficient ex ante allocation of skills investments in rational expectational equilibrium. More realistically, errors will arise but the efficient benchmark serves to indicate central tendency.

The analysis of distribution focuses exclusively on the home country because the specific factor income distribution in the foreign country is the mirror image of the home distribution.

\(^1\)The numerical example also implies that the variance of income is less for relatively smaller economies. This benefit of smallness implication is likely to hold in a wider class of models, but will be less robust than the market widening implication.
The specific factor return in sector \( z \) is given by

\[
r(z) = \frac{s(z)}{\lambda(z)} g_K,
\]

where the equilibrium value of \( s(z) \) is given by (9) and (11) for traded goods and \( s(z) = \gamma(z) \) for nontraded goods. The average sector specific return is equal to \( g_K: \int_0^1 r(z) \lambda(z) dz = g_K \).

(19) in combination with (9) and (11) implies that for given factoral terms of trade and trade costs, relatively high returns are associated with good relative productivity draws, high demand and a relatively low amount of competing sector specific investment.

\[
r(z) / g_K = \frac{\gamma(z)}{\lambda(z)} \frac{(GR/R^* + 1)}{GR/R^* + G^{1/(1-\alpha)} + 1/(1-\alpha)/\Lambda(z)}, z \in [0, \bar{z});
\]

\[
r(z) / g_k = \frac{\gamma(z)}{\lambda(z)}, z \in [\bar{z}, \bar{z}^*];
\]

\[
r(z) / g_K = \frac{\gamma(z)}{\lambda(z)} \frac{(GR/R^* + 1)}{GR/R^* + G^{1/(1-\alpha)} + 1/(1-\alpha)/\Lambda(z)}, z \in (\bar{z}^*, 1].
\]

Because the distribution of specific factor returns \( r(z) \) is governed by \( s(z)/\lambda(z) \), the distribution depends on the ex post inefficiency of allocation. In the limit of perfect efficiency, the factoral income distribution collapses onto \( g_K \). With the benchmark allocation \( \gamma(z)/\lambda(z) = 1, \forall z \).

The share of specific factor payments in sector \( z \) is given by,

\[
\rho(z) = s(z)(1 - \alpha).
\]

The return and the income share are related by (19)-(23). For the uniform allocation of specific capital, the distribution of \( r(z) \) mimics that of \( \rho(z) \). The richest specific factor owners are in the most advantaged sectors in the benchmark case.

The connection from the factoral distribution of income to the personal distribution requires knowledge of ownership patterns. The most convenient interpretation of the specific factor is human capital, in which case the personal and factoral distributions are tightly linked.\(^\text{12}\)

\(^\text{12}\)Let \( F(\tilde{z}) \) denote the proportion of capital owners who own the residual returns to industries richer than \( \tilde{z} \). A common measure of income inequality is the share of total
The comparative static implications of the model for income distribution can now be drawn. Consider first the effect of improvements in the factoral terms of trade $G$. For example, two underlying drivers of such improvements are foreign relative growth and a transfer into the home country. Both $\rho$ and $r$ vary directly with $s$. Examining (9) and (11), $s$ is decreasing in $G$ for both exports and imports (a formal development follows below), while for nontraded goods $s$ is independent of $G$. Increases in the factoral terms of trade $G$ thus redistribute specific factor income from traded goods to nontraded goods. There is no effect on the relative shares of mobile vs. the average return to specific factors.

As to the distribution of specific factor returns $r(z)$ within the traded goods sectors, it is convenient to focus first on returns relative to the mean, factor income received by some specific target for $F$ such as the richest 10 per cent. This measure is implemented by solving the ownership distribution for $\tilde{z}$:

\[ F(\tilde{z}) = 0. \]

10. Let

\[ S(\tilde{z}) \equiv \int_0^{\tilde{z}} \rho(z)dz = (1 - \alpha) \int_0^{\tilde{z}} s(z)dz, \tag{24} \]

define the specific factor income share of the sectors with returns higher than $r(\tilde{z})$, then solve for $S(\tilde{z})$, an index of inequality focused on the upper tail. Suppose for example that half the population owns no human capital. Then 20 per cent of the total capital will be owned by the richest 10 per cent in equilibrium. With the uniform allocation of capital, this means $\tilde{z} = 0.20$. Then for the case where the export industries alone contain the richest owners $S$ is evaluated as

\[ S(\tilde{z}, G) = (1 - \alpha)(GR/R^* + 1) \int_0^{\tilde{z}} \gamma(z)[GR/R^* + (Gt)^{1/(1-\alpha)}A(z)]^{-1}dz \]

where $A(z) \equiv [a(z)/a^*(z)]^{1/(1-\alpha)}$.

The model is completed by specifying distributions for home and foreign productivities and for tastes. Suppose these are uniform with productivities being independent draws on $[a_{\min}, a_{\max}]$ and $[a^*_{\min}, a^*_{\max}]$ respectively. Then $A(z)$ is uniform on $[A_{\min}, A_{\max}] = [(a_{\min}/a^*_{\max})^{1/(1-\alpha)}, (a_{\max}/a^*_{\min})^{1/(1-\alpha)}]$. (24) has a closed form solution given $G$ in this case, given by $S = (1 - \alpha)(GR/R^* + 1)M$ where

\[ M = \ln [(GR/R^* + (Gt)^{1/(1-\alpha)}((1 - \tilde{z})A_{\min} + \tilde{z}A_{\max})] - \ln [(GR/R^* + (Gt)^{1/(1-\alpha)}A_{\min})] \]

While the uniform distribution for tastes and productivities is convenient in yielding a closed form solution, the qualitative properties of the model are invariant to more general distributions of $\gamma(z)$ and $A(z)$.
\[ r(z)/g_K = s(z)/\lambda(z). \] Then

\[ \frac{\partial \ln r(z)/g_K}{\partial \ln G} = \frac{\partial \ln s(z)}{\partial \ln G}. \]

For nontraded goods, \( s(z) = \gamma(z) \), which is independent of \( G \). For traded goods the term on the right is given by:

\[ \frac{\partial \ln s(z)}{\partial \ln G} = \frac{1}{1 + GR/R^*} - \frac{\alpha}{1 - \alpha} \frac{H(z)}{GR/R^* + H(z)} < 0 \]

where \( H(z) \equiv \Lambda(\bar{z})/\Lambda(z) \in [0, 1], z \leq \bar{z}; H(z) \equiv \Lambda(\bar{z}^*)/\Lambda(z) \geq 1, z \geq \bar{z}^*; \) and \( H' > 0 \). The export cutoff equations are used above to simplify the derivatives of (9) and (11). The relative returns of trade-exposed specific factors fall with \( G \) everywhere, and most for the least productive sectors.

The specific factor return levels respond to \( G \) according to

\[ \frac{\partial \ln r(z)}{\partial \ln G} = \frac{GR/R^*}{1 + GR/R^*} - \frac{\alpha}{1 - \alpha} \frac{H(z)}{GR/R^* + H(z)}. \tag{25} \]

The right hand side of (25) is negative (positive) for

\[ H(z) \geq (\leq) \frac{(GR/R^*)^2}{GR/R^* + \alpha/(1 - \alpha)}. \]

The intuition for these results is that specific factor returns are run by the response of equilibrium goods prices to the demand and supply shifts arising from the change in \( G \). In the nontraded goods sectors, a rise in \( G \) raises both the willingness-to-pay and the short run unit cost function in proportion to \( G \). Therefore the price rises in proportion to \( G \) and all sector specific factor returns rise in proportion to \( G \). In tradable sectors, the rise in \( G \) results in price movements governed by (5) and \( t \) times (6). The uniform increase in willingness-to-pay is less than proportional to \( G \), while the short run cost functions still rise in proportion to \( G \). The equilibrium price scaled by \( G \) (and hence \( r(z)/g_K \)) falls relatively less in the most productive sectors because there the uniform demand rise interacts with the smallest general equilibrium supply elasticities.\(^{13} \) Thus:

**Proposition 2** the redistributive effects of factoral terms of trade changes on trade exposed sectors are larger the less relatively productive the sector,

\(^{13} \)The general equilibrium supply elasticity is given by \( G_{pp}/G = (1 - s(z))\alpha/(1 - \alpha). \)
while nontraded sectors are completely insulated from the factorial terms of trade.

The intuition suggests that Proposition 2 is a property of specific factors models that holds more widely than in the special Cobb-Douglas structure that permits such sharp results. The more that the mobile factors crowd the sector specific factor, the closer that something like a capacity constraint approaches and the lower the sectoral supply elasticity will tend to be, driving up the rise in the specific factor return from given additions of mobile labor. Non-traded sectors are no longer completely insulated because the skill premium changes. See the Appendix for more details.

Figure 2 illustrates the effect of a rise in $G$ and a fall in $t$ (analyzed below) on the distribution of $r$ for the benchmark case of the allocation $\lambda(z) = \gamma(z) = \lambda^*(z)$. A 1 percent rise in $G$ lowers the $\ln r(z)/g_K$ schedules for traded goods by $-1/[(1 - \alpha)(GR/R^* + 1)]$. A 1 percent fall in $t$ raises export relative incomes by the (absolute value of the) expression on the right hand side of (26) and lowers import sector relative incomes by the expression on the right hand side of (27). The figure is drawn assuming that $G < t$ so that a one percent fall in $t$ has a bigger impact than a one percent rise in $G$ for import competing sectors, but this ranking is arbitrary and without significance for the analysis. The complication of non-uniform $\gamma(z)/\lambda(z)$ does not affect the elasticities of returns with respect to $G$, but it alters the one-to-one relationship between $r(z)$ and $s(z)$ imposed in Figure 2. The distribution schedule in Figure 2 is best thought of as indicating central tendency, with a confidence interval enclosing it.
When aggregate productivity risk $\mu$ is present, the preceding result implies that personal income risk due to terms of trade risk is zero for the middle nontraded goods sectors and among traded goods it is greatest for the most disadvantaged sectors and least for the most advantaged sectors. $\mu$ has no direct effect on shares because $G^{1/(1-\alpha)} \rho(z)/(1-\alpha) / \Lambda(z) = \Lambda(\bar{z}) / \Lambda(z)$ which is invariant to $\mu$ save through its effect on equilibrium $\bar{z}$. With the
benchmark allocation of specific capital, \( r(z) \) is declining in \( z \) so aggregate risk hits the poorest sectors the hardest.

Globalization is modeled as decreases in symmetric trade costs. On the extensive margin, globalization widens inequality as it narrows the range of nontraded goods \([\bar{z}, \bar{z}^*]\) that is sheltered from external competition. Most popular discussion focuses on the extensive margin effect. A key contribution of this paper is to show that globalization redistributes specific factor income to exports from both nontraded goods and imported goods for any given factorial terms of trade \( G \). The effect of a change in \( t \) on the distribution of specific factor income relative to the mean is given by

\[
\frac{\partial \ln r(z)}{\partial \ln t} = \frac{\partial \ln s(z)}{\partial \ln t} = -\frac{1}{G^{-\alpha/(1-\alpha)}t^{-1/(1-\alpha)}\Lambda(z)R/R^* + 1} \cdot \left(\frac{1}{1 - \alpha}\right), z \leq \bar{z} ;
\]

and

\[
\frac{\partial \ln r(z)}{\partial \ln t} = \frac{1}{G^{-\alpha/(1-\alpha)}t^{1/(1-\alpha)}\Lambda(z)R/R^* + 1} \cdot \left(\frac{1}{1 - \alpha}\right), z \geq \bar{z}^* .
\]

For nontraded goods, sector specific factor incomes are invariant to \( t \). For exported goods, the relative income is increased by fall in \( t \) by more the more productive the sector while for imported goods the relative income is reduced by more the less productive the sector. The results are illustrated in Figure 2. Thus

**Proposition 3** Globalization at given factorial terms of trade reduces the specific factor income of import-competing sectors by more the less relatively productive the sector, increases the specific factor income of exporting sectors by more the more productive the sector, while nontraded sectors are completely insulated from globalization.

Notice that inequality increases in both countries, and that this property does not require restricting the distributions of productivity draws. It is a feature of factor specificity and the assumed benchmark allocation of factors. The effect of globalization on the factorial terms of trade is ambiguous, but any improvement due to the fall in trade costs will redistribute income to nontraded sector specific factors from traded sector specific factors.

In the presence of aggregate productivity risk, Proposition 1 showed that globalization reduces the induced variance of the factorial terms of trade, thus tending to offset the increased variance of ex post specific factor incomes in traded goods sectors. The size of the reduction in variance of relative income
varies by sector in proportion to the square of
\[
\frac{\partial \ln r(z)/gK}{\partial \ln G}.
\]

With the benchmark allocation, Proposition 2, illustrated by Figure 2, shows
that this offset in aggregate risk is most important for the poorest factors,
least important for the richest factors and irrelevant for the middle nontraded
sector factors. Proposition 3, also illustrated by Figure 2, shows that glob-
alization increases idiosyncratic risk and is likewise most important for the
poorest factors ex post, least important for the richest factors and irrelevant
for the middle income nontraded sector specific factors.

The model implies that increases in the spread of the distribution of
relative productivities serve to raise the dispersion of relative specific factor
incomes because they raise the absolute value of the slope of \( \Lambda(z) \) in the
relevant range. Stretching the interpretation of the model, increases in the
spread of the relative productivity distribution can be seen as an aspect of
globalization reflecting the integration of much poorer countries into a world
previously dominated by trade between rich countries.

5 Equilibrium Investment Allocation

Assume that specific factor investments are made through a Diamond stock
market. Agents are risk neutral, so the equilibrium equalizes expected re-
turns in all sectors for each country.\(^\text{14}\) A stock of wealth \( K \) is allocated
among the sectors. The exact nature of the specific capital is not important
formally, but the most useful interpretation is as human capital created by
the investment of \( K \) as individual workers choose to acquire sector specific
skills. The distribution of the returns to capital is then part of the earnings
distribution.

Investments are made prior to receiving productivity draws. Productivity
in each sector \( z \) is independently drawn from a common distribution \( D(a) \) in

\(^{14}\)Anderson and Riley (1976) point out that the Diamond stock market decentralizes
equilibrium in a trading economy under uncertain prices or technology shocks. Helpman
and Razin (1978) develop the implications of international trade in securities when there
is aggregate risk. Both papers develop the important resource allocation implications of
risk aversion.
Lemma 3 The equilibrium allocation of specific capital is $\lambda(z) = \gamma(z) = \lambda^*(z), \forall z$.

Proof: For this allocation the ex post relative factor returns are given by

$$r(z)/g_K = \frac{(GR/R^* + 1)}{GR/R^* + G^{1/(1-\alpha)} t^{1/(1-\alpha)}/\Lambda(z)}, z \in [0, \bar{z}); \quad (28)$$

$$r(z)/g_k = 1, z \in [\bar{z}, \bar{z}^*]; \quad (29)$$

$$r(z)/g_K = \frac{(GR/R^* + 1)}{GR/R^* + G^{1/(1-\alpha)} t^{1/(1-\alpha)}/\Lambda(z)}, z \in (\bar{z}^*, 1], \quad (30)$$

where $\Lambda(z) = \frac{[a^*(z)/a(z)]^{1/(1-\alpha)}}{}$. The right hand side of the equations is independent of the allocation $\{\lambda(z), \lambda^*(z)\}$, implying that there is no incentive to deviate. For any other allocation in which $\lambda(z) = \lambda^*(z)$, the terms on the right hand side are multiplied by $\gamma(z)/\lambda(z)$ (and similarly for the foreign relative returns distribution. Then it would always be possible to raise expected returns by reallocating more capital to sectors $z$ where $\gamma(z)/\lambda(z) > 1$ from sectors $z'$ where $\gamma(z')/\lambda(z') < 1$. Allocations where $\lambda(z) \neq \lambda^*(z)$ provide more complex opportunities to gain from reallocation.

The intuition is simple. The sectors are all ex ante identical from the production side while they differ systematically from the demand side whenever the expenditure shares are not uniform. It pays to allocate more investment to sectors with higher expenditure shares.

The comparative statics of income distribution are given in the preceding section. The elasticity of $r(z)/g_K$ with respect to $G$ is negative for traded goods. This implies that a fall in $G$ shifts income from the nontraded goods middle into the tails. Moreover, the shift is larger for the lower tail, the import-competing goods. By previous comparative static results on drivers of $G$, an economy growing less fast than its partner thus experiences increasing inequality while the faster growing partner experiences decreasing inequality. Also, a transferee will experience increasing inequality while its partner the transferor experiences decreasing inequality.

Globalization represented by a fall in $t$ increases inequality at constant $G$ by reducing income in the lower tail and raising it in the upper tail. The amplification occurs through a second channel as well because the range of

---

15In long run equilibrium with no productivity shocks or with complete mobility of capital, the model converges onto the Ricardian model due to the identical Cobb-Douglas production function restriction.
nontraded goods shrinks, the more investments are trade sensitive. Improvements in factoral terms of trade, if they occur when driven by the fall in trade costs, will further increase inequality. Increases in the dispersion of relative productivities serve to increase the share of income earned by the richest, as in the preceding section.

Introducing relative productivity risk as in Section 3.4 introduces aggregate risk in the factoral terms of trade $G$. All the basic setup of this section remains valid, understanding that agents’ expectations include expectations of $G$.

Pushing the model very hard to reflect the effect of individual skilled workers’ errors in location choice, Figure 2 can be interpreted as giving the expected value of the income distribution with a confidence interval around it. Errors in $\ln \frac{\gamma(z)}{\lambda(z)}$ have zero mean and constant variance. They displace the distribution schedule vertically. That is the end of the story if foreign and domestic errors are perfectly correlated. Otherwise, the $\ln \Lambda(z)$ schedule is shifted and more structure is required to pin down effects.

6 Heterogeneous Firms

Recent research emphasizes that firms are very heterogeneous within sectors, identified with idiosyncratic productivity draws. Trade has a systematic impact tending to raise the average productivity of a sector by weeding out low productivity firms. In the Melitz (2003) model, a fixed cost of exports for each firm is combined with a variable cost of trade. A fall in the variable trade cost results in upward pressure on wages as more labor is devoted to entering exporting. The wage pressure causes low productivity firms to exit, raising the average productivity. The model of this paper can readily be extended to incorporate the endogenous productivity effect. More novel, however, the current setup implies a mechanism of endogenous productivity response to trade that amplifies productivity differences between export and import-competing industries even in the absence of fixed export costs. Trade does not cause average productivity changes, in contrast to the Melitz model, but exports are correlated with higher exit of the least productive firms and expansion of the most productive firms, with nontraded goods sectors having less churning and import-competing sectors the least churning.

A perfect capital market finances the startup of a mass of firms in each
sector \( z \). The firms subsequently draw their productivities. The productivity draw has a sector specific component, so that the ex post distribution of firms’ draws differs by sector. Some low productivity firms will exit in each sector \( z \). The surviving firms in sector \( z \) have an average productivity that appears in the preceding sections as \( 1/a(z) \).

In each sector, the firms compete for the sector specific capital. That implies a harsher winnowing process in the sectors that have the better average draws; low productivity firms are faced with hiring more expensive capital. Thus the endogenous productivity effect acts to increase the average productivity of the exporting sectors relative to import competing sectors. In the import competing sectors, their relatively cheap specific capital softens the winnowing process, reducing the impact of the endogenous productivity effect. As in Melitz (2003), the Darwinian force comes through the factor market and acts to raise the average productivity of surviving firms in every sector, but differentially so in the specific factors model.

To preserve some heterogeneity of firms within sectors in equilibrium, assume (realistically) that skilled worker movement is costly. The cost represents a firm specific component of skill that is lost when the worker moves. For simplicity, assume that one unit of original capital becomes \( \phi < 1 \) units of usable capital in the hiring firms. The (inverse) productivity draw of a firm is the sum of a sectoral component and an idiosyncratic component: \( a(z, h) = a(z) + b(h) \) for firm \( h \) in sector \( z \). Suppose that the firm level \( h \) dimension is ordered such that \( b_h > 0 \). In any sector \( z \), the ex post value of marginal product of the specific factor is thus decreasing in \( h \). When capital can be reallocated within the sector, the highest productivity firm hires specific capital away from low productivity firms in its sector. Provided that \( \phi \) is not too small, this process drives the lowest productivity firms out of business.

In equilibrium, the least productive surviving firm, located at \( h^{\text{max}} \), can

---

\( ^{16} \) The description of equilibrium allocation is slightly more complicated than in preceding sections because the expected return is more complex. A full development is suppressed here because the equilibrium capital allocation remains uniform due to the complete \textit{ex ante} symmetry of all firms in all sectors. ‘The firm’ can be thought of as owning the residual claim to operate the process it draws, employing skilled and unskilled labor for that purpose. The higher productivity processes earn rents. \textit{Ex ante}, the potential firms bid for the right to receive a draw from the productivity distribution, in essence buying an option to operate. The expected profits from buying the option to operate are equal to zero, incorporating expectations of the equilibrium returns that include the winnowing process.
pay enough to offset the value of marginal product of the specific capital transferred to the most productive firm.\textsuperscript{17} This implies

\[ \phi = \frac{[a(z) + b(h_{\text{max}})]}{[a(z) + b(0)]}. \]  \hspace{1cm} (31)

All draws of productivity \( b(h) \geq b(h_{\text{max}}) \) result in the capital being resold to the firm at the upper end of productivity. This results in an average productivity of surviving firms equal to

\[ \bar{a}(z) = a(z) + D(h_{\text{max}})b(0) + [1 - D(h_{\text{max}})]E[b|h \leq h_{\text{max}}]. \]

Here \( D \) is the probability of an idiosyncratic draw with worse productivity than the marginal firm.

To sort out the implications for endogenous productivity and trade, it helps to consider an additional ordering condition \( a_z > 0 \). \( a_z > 0 \) is met only in an average sense because the ordering of \( z \) in general equilibrium depends on domestic productivity relative to foreign productivity. Under \( a_z > 0 \), differentiating (31) yields

\[ h_{z_{\text{max}}}^\text{max} = -a_z \frac{[1 - \frac{1}{a(z) + b(0)}]}{b_h} < 0, \forall z > 0. \]

Here the sign follows from the natural normalization \( a(0) + b(0) = 1 \). The implication is that the endogenous productivity effect is most powerful in the most productive sectors. The endogenous productivity response is such that exports (imports) are correlated with high (low) exit of firms and high (low) expansion of the most productive firms. On average, the Darwinian force is most strong in the export sectors, weakest in the import-competing sectors and in the middle for the nontraded goods sectors.

Trade does not cause endogenous productivity changes in this model because the Darwinian force operates even in the absence of trade. It is intuitively appealing to think that globalization strengthens the Darwinian force, but this hypothesis would be true only with very strong assumptions because the average endogenous productivity change is a convex combination of bigger export and smaller import-competing changes.

\textsuperscript{17}More realistic but more complex reallocations from a set of low productivity to a set of high productivity firms follow when there are diminishing returns to the transfer due either to a fixed managerial input for the firm or convex adjustment costs. Alternatively, more firms expand if there are heterogeneous adjustment costs (\( \phi \)’s) not perfectly negatively correlated with productivity.
Turning to the distributional implications, the endogenous productivity effect amplifies the dispersion of productivity and therefore amplifies the dispersion of ex post factor incomes. The effect of globalization, a fall in trade costs, is to amplify the endogenous sectoral productivity response to intra-sectoral differences in productivity and thus to amplify the inter-sectoral dispersion of factor returns beyond what arises with exogenous average productivities in each sector.

A further twist on the model provides an explanation for the well documented within sector link between productivity, firm size and wages. The highest productivity firms in each sector earn quasi-rents relative to the lowest productivity firm that remains in business. Suppose that the firms are subject to wage bargaining such that the rents are shared with the skilled workers of each firm. Then the highest productivity (and biggest) firms will pay the highest skilled wages within each sector. The dispersion of within sector wages will be least in the highest productivity sectors because the stronger Darwinian force compresses the productivity distribution of the surviving firms. Formalizing these points, the zero profit condition for the least productive firm in sector $z$ implies that it can pay skilled workers

$$r_{\text{min}}(z) = \left( \frac{p(z)}{a(z) + b[h^{\text{max}}(z)]} \right)^{1/(1-\alpha)} w^{-\alpha/(1-\alpha)}.$$

The more productive firms share their profits with the skilled workers according to

$$r(z, h) = r_{\text{min}}(z) + \theta(z)[p(z) - (a(z) + b(h))w^\alpha r_{\text{min}}(z)^{1-\alpha}]; \theta(z) \in [0, 1].$$

The higher is $r_{\text{min}}(z)$, smaller is the within-sector dispersion of skilled wages.

7 Toward Dynamics

The purely static analysis of this paper is a platform for interesting dynamics. The specificity of factors is transitory. Adjustment to a longer run equilibrium will have interesting and important economic drivers. An earlier literature (for example, Neary, 1978) provides a thorough analysis of adjustment to a one time shock. In the present setup it is natural to think of productivity draws arriving each period. Serial correlation in the draws would induce persistence in comparative advantage with potentially interesting implications for investment patterns and income distribution. Labor
market evidence reveals that young workers are much more likely to relo-
cate in response to locational rents, suggesting that overlapping generations
models might usefully be deployed.

A significant extension of the model would focus on capital market im-
perfection. One approach would focus on the credit constraints that workers
face in acquiring new specific human capital.\textsuperscript{18}

\section{Conclusion}

The combination of specific factors and productivity shocks provides a struc-
tural rationale for the premia of earnings in export industries. In a global
economy, the \textit{comparatively} best productivity draws become export industries
and are associated with high returns to the specific factor while the worst
productivity draws become import competing industries and are associated
with low returns.

Globalization necessarily increases the ex post dispersion of factor in-
comes within economies. Viewed ex ante, specific factor incomes are more
risky due to idiosyncratic productivity shocks. In contrast, globalization
damps the effect of aggregate supply side shocks on personal incomes. Both
effects of globalization are largest for the poorest specific factors.

The results suggest that globalization reduces income risk for mobile fac-
tors while it may reduce or increase risk for specific factors. This insight
may hold up in a much wider class of models than those examined here. The
result that the poorest specific factors are affected most by globalization is
likely to be less robust.

The model is rich with suggestions for empirical work. Inefficient skilled
labor allocations blur the sharp predictions of Section 4, but should ordinarily
result in something like a confidence interval around the distribution schedule
of Figure 2. Anderson (2008) develops empirical implications further in a
model where the gravity equation characterizes trade costs. Distributional
predictions are less general and clean.

The complementary work of Blanchard and Willman (2008) and Costinot
and Vogel (2008) on income distribution based on worker heterogeneity sug-
gests that a combination of ex ante heterogeneity and ex post locational

\textsuperscript{18}Other interesting extensions would allow a role for risk aversion and the magnitude of
risk. One tack could take the line of Helpman and Razin (1978), who deploy the Diamond
stock market model to analyze related issues.
premia can go far toward fitting the extremely rich empirical regularities of actual income distributions. Matching frictions are a promising way to dig more deeply into the structure of random productivities. The analytic simplicity of their models and the specific factors continuum model suggests that analytic solutions may be feasible.
9 References


Blanchard, Emily and Gerald Willman (2008), “Trade, Education and the Shrinking Middle Class”, University of Virginia.


Costinot, Arnaud and Jonathan Vogel (2008), “Matching, Inequality and the World Economy”, MIT.


10 Appendix

10.1 A Numerical Example of Aggregate Risk and Globalization

Assume a uniform distribution of tastes, hence $\gamma = 1$ and $\Gamma(z) = z$ and $\Gamma^*(z) = 1 - z^*$. Let $\Lambda(z) = \bar{\Lambda}/z$. The two export cutoff equations imply $Z^*(\bar{z},t) = \bar{z}t^{2/(1-\alpha)}$.\(^{19}\) Then

$$\frac{d\ln G}{d\ln \mu} = \frac{1/\bar{z} + t^{2/(1-\alpha)}/(1 - \bar{z}^*)}{1/\bar{z} + t^{2/(1-\alpha)}/(1 - \bar{z}^*) + (1-\alpha)/\bar{z}}.$$

Suppose that equilibrium implies symmetry, such that $\bar{z} = 1 - \bar{z}^*$. Then

$$\frac{d\ln G}{d\ln \mu} = \frac{1 + t^{2/(1-\alpha)}}{1 + t^{2/(1-\alpha)} + (1-\alpha)}.$$

Suppose $1 - \alpha = 0.33$, reflecting the roughly constant labor share of 0.67 that has long been a stylized fact of aggregate income accounting. Then for a frictionless equilibrium ($t = 1$), $d\ln G/d\ln \mu = 0.858$ and $V(\ln G) = 0.737V(\ln \mu)$. For $t = 1.74$, $d\ln G/d\ln \mu = 0.989$ and $V(\ln G) = 0.977V(\ln \mu)$. $t = 1.74$ is the benchmark value reported in Anderson and van Wincoop (2004) for OECD countries, with much larger values being appropriate for some developing countries. Evidently, as trade costs increase without bound, $d\ln G/d\ln \mu \to 1$. The effect of unequal country size or relative productivity is reflected in $(1 - \bar{z}^*)/\bar{z}$. Suppose for example that the home country (the South) exports one tenth as many products as the foreign country (the North). In frictionless equilibrium, $(d\ln G/d\ln \mu)^2 = 0.589$, hence $V(\ln G) = 0.589V(\ln \mu)$. The example shows that there is scope in the model for globalization to decrease income variance by $1/3$ or more.

\(^{19}\)A trading equilibrium always exists in this case. The equilibrium $\bar{z}$ is solved from

$$\frac{\bar{z}R^*/R}{1 - \bar{z}t^{2/(1-\alpha)}} = \left(\frac{\bar{\Lambda}}{\bar{z}}\right)^{1-\alpha} \frac{1}{t}.$$

This equation always has a solution $\bar{z} \in [0, 1]$ for any positive $\bar{\Lambda}$ and $t > 1$. 

32
10.2 Income Variance and Globalization on the Intensive Margin

As an example of market widening exclusively on the intensive margin, consider a generic two good two country general equilibrium trade model with symmetric iceberg trade costs. The home relative price of good 2 in terms of good 1 is $p$, the foreign relative price is $p^*$ and arbitrage equilibrium implies $p^* = pt^2$ when the home country exports good 2. Suppose that $t^2 = \tau \epsilon$ where $\epsilon$ reflects a small random shock to the productivity of distribution with unit mean and $\tau$ is the mean value of $t^2$. The market clearance equilibrium condition is given by $X(p) = M^*(p^*)$ where $X$ is the upward sloping export supply schedule of the home country and $M^*$ is the downward sloping foreign import demand schedule. Then the variance of $p$ is given by

$$\left\{ \frac{M^*_p \tau}{X_p - M^*_p} \right\}^2 \text{Var}(\epsilon).$$

This expression is decreasing in $\tau$. The variance of $p$ drives the variance of home factor incomes, so incomes are less risky as mean trade costs fall. The same setup can be reinterpreted as variance in incomes induced by random relative productivity differences represented by $\epsilon$, using the concept of ‘efficiency prices’ set out in Section 1.

10.3 General Production Function Case

The main distributional implications of the paper continue to hold when the Cobb-Douglas production function is replaced with a general neoclassical degree one homogeneous and concave differentiable production function $f(K(z), L(z))$. The gross domestic product function is given by $g(p(z)/a(z), \lambda(z)K, L) = \max_{\{L(z)\}} \int_0^1 [p(z)/a(z)] f(\lambda(z)K, L(z)) dz : \int_0^1 L(z)dz = L.$

The first order conditions give the value of marginal product condition for unskilled labor in each sector. No closed form solution is generally available for $g$ and its derivatives.

The return to skilled labor is residually determined in each sector. In combination with the value of marginal product condition, this implies that

$$\frac{r(z)}{gK} = \frac{s(z) 1 - \alpha(z)}{\lambda(z) 1 - \bar{\alpha}}$$

(32)
where $\alpha(z) \equiv wL(z)/[p(z)/a(z)]y(z)$ and $\bar{\alpha} \equiv \int_0^1 s(z)\alpha(z)$, the sectoral and average labor shares.

The rational expectations equilibrium allocation of skilled labor for the home and foreign economies remains $\lambda(z) = \gamma(z) = \lambda^*(z)$. This follows because the production functions in all sectors are ex ante identical. With risk neutrality, certainty equivalence obtains. When the draws of the productivities are all at their expected values, $a^*(z)/a(z) = \bar{a}^*/\bar{a}, \forall z$, there is no trade, and efficient allocation requires $\lambda(z) = \gamma(z)$.

Substituting $\gamma$'s for $\lambda$'s in (32),

$$\frac{r(z)}{g_K} = \frac{s(z)}{\gamma(z)} \frac{1 - \alpha(z)}{1 - \bar{\alpha}}.$$

The material balance conditions generate the same market clearance conditions as in the text case: $s(z)g + s^*(z)g^* = \gamma(z)(g + g^*)$ for traded goods and $s(z) = \gamma(z) = s^*(z)$ for nontraded goods. Home exports constitute all goods for which $s(z) - \gamma(z) > 0$ while foreign exports constitute all goods for which $s(z) - \gamma(z) < 0$. Then (32) implies that, leaving aside the variation in labor shares, export sectors have high relative returns, non-traded sectors have intermediate returns and import-competing sectors have low relative returns.

The influence of endogenous labor shares on income distribution is clarified by considering the CES production function case. For this case

$$\alpha(z) = \beta \left( \frac{w}{p(z)/a(z)} \right)^{1-\sigma},$$

where $\beta \in (0, 1)$ is the labor distribution parameter and $\sigma$ is the elasticity of substitution parameter. For $\sigma < 1$, $\alpha(z) < (> \bar{\alpha}$ in exporting (import-competing) industries, and the more so for the higher returns industries. This force amplifies the inequality-inducing effects of good or bad productivity draws and the associated personal income risk. For $\sigma > 1$, the opposite is true, the inequality effects are damped down. For non-CES production functions, these statements apply more loosely to average returns in the export, nontraded and import competing ranges.

The average skill premium, $g_K/g_L$ generally depends on the vector of equilibrium prices, unlike the Cobb-Douglas case.

$$\frac{g_K}{w} = \frac{L}{K} \frac{1 - \bar{\alpha}}{\bar{\alpha}},$$
and similarly for the foreign skill premium $g_k^*/w^* = (1 - \bar{\alpha}^*)/\bar{\alpha}^*$.

Globalization in the form of trade cost reductions will narrow the range of nontraded goods, expose more of the economy to external idiosyncratic risk and widen inequality for given terms of trade. (As in the Cobb-Douglas case, globalization has ambiguous effects on the terms of trade, but unlike the Cobb-Douglas case, no ready definition of the factorial terms of trade is available.) Globalization ordinarily would have some effect on the average skill premium, but general analytic results appear to be precluded. For the CES production function with $\sigma > 1$, globalization at constant terms of trade ordinarily raises both $\bar{\alpha}$ and $\bar{\alpha}^*$ and thus the average skill premium ordinarily falls in both North and South. This property arises from consideration of

$$\bar{\alpha} = \int_0^{\bar{z}} s(z)\alpha(z)dz + \int_{\bar{z}}^{\bar{z}^*} \gamma(z)\alpha(z)dz + \int_{\bar{z}^*}^{1} s(z)\alpha(z)dz.$$ 

Export sectors experience rising $s$ and rising $\alpha$ while contracting sectors experience falling $s$ and falling $\alpha$. So the first and third terms on the right hand side of the above equation must rise. The middle term should ordinarily not change much because the mobile factor flows from import-competing to export sectors mainly. Finally, the fall in $t$ should intuitively raise $\bar{z}$ and lower $\bar{z}^*$, further raising $\bar{\alpha}$. For the case of $\sigma < 1$, the effects through $s$ and $\alpha$ in the first and third terms reverse in sign, and the effect of globalization should ordinarily raise the average skill premium in North and South.