Optimal Debt Contracts when Credit Managers are (Perhaps) Corruptible

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Boston College Working Papers in Economics, 2006

Originally posted on: http://ideas.repec.org/p/boc/bocoec/648.html
Optimal Debt Contracts when Credit Managers are (Perhaps) Corruptible

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First version: January 1995
This version: August 1999

Abstract

The paper derives the optimal organizational response of a bank (the principal) which faces a risk of collusion between the credit manager (the agent) and the credit-seeking firms. The bank can deter collusion either through internal incentives or by distorting the credit contracts. The model thus explicitly takes into account the interaction between internal (collusion) risks and external (default) risks in the optimal design of the internal organization as well as of the credit contracts. We investigate this question in two settings. In the first one, we adopt the standard assumption that the agent is always willing to collude (is corruptible) if that increases his monetary payoff. In the second one, he is corruptible with some probability only, and honest otherwise. A novel feature of our approach is to allow for screening among corruptible and honest agents. We find that if the probability that the agent is honest is sufficiently large, collusion occurs in equilibrium.

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1 Introduction

There are many examples of banks having run into deep trouble because of corrupt actions in their credit activities: the French Crédit Lyonnais suffered huge losses in 1992-93, largely due to insolvent credits given to a number of the bank’s “friend firms”;\(^1\) in 1994, the main shareholder of the Mexican financial group Union-Cremi was held responsible for having “stolen” an amount near to USD 1 billion from the bank through credits to friends and to phantom enterprises\(^2\). Moreover, according to Scheepens (1995), Hill (1975) found that 38 out of 67 US bank failures between 1960 and 1974 were due to “internal causes” such as loans given to employees’ relatives, business associates or friends, weak audit and control systems. Thus, an important reason for bank failure seems to be that the employees in charge of handling the credits collude with those who seek credits and thus grant some loans on a non-economic basis.

Obviously, bank failures entail heavy social costs, be it in form of uncertainty in the financial market or the taxpayers’ money being used to save the banks.\(^3\) That is why it is important to increase the understanding of collusion in the banking sector. The examples above can all be put in one of two categories: the corrupt action is initiated either by the bank owner, or by agents against the will of the owners. The first is a case for regulators who seek to protect the depositors’ interests. The second case is of concern to the bank owners (or managers, supposing that the owners have solved the problem of monitoring the managers) and is the subject of this paper. We study the case in which a firm bribes the credit manager to get a more favorable credit contract. The question is how the bank owners/superiors of the credit manager optimally cope with this problem. Intuitively, they can design internal incentives which deter collusion and/or alter the credit contracts so as to decrease the incentives for the firm to bribe the credit manager.\(^4\) Indeed, it is a main concern for bank owners to find the organiza-

\(^{1}\)See *The Economist* April 9th-15th 1994, “The bank that couldn’t say no”.


\(^{3}\)The French government plans to inject around $ 6 billion into Crédit Lyonnais, the Mexican government intervened to save Banco Union, etc.

\(^{4}\)An example of this is a Swedish bank which ran into deep trouble because of insolvent credits (due to collusion and other causes), and which chose to alter its internal organization in response to this incident: instead of letting each credit file be handled by one person, the organization was changed so that two persons of different hierarchical levels now have to agree before any credit is given. The reason given for this reorganization is that four eyes judge better than two, but obviously, there could
tion which optimally serves the objective to handle internal risks, such as collusion or fraudulent actions, together with external risks, such as default risks.\textsuperscript{5} The objective of this paper is to analyze a possible case for collusion between the credit manager of a bank and a credit-seeking firm. The paper belongs to the strand of literature which aims at understanding the effects of corruption on the design of organizations, by characterizing the optimal contracts given the threat of collusion.\textsuperscript{6}

We use the basic model of Bolton and Scharfstein (1990) to represent the credit relation between the bank and the firm. Given that it is difficult for the bank to observe the true realized profit of the firm, the bank faces the risk that the firm defaults. To avoid this, the bank has to incite the firm to reveal high profits. In a two-period setting, the bank induces the firm to reveal its realized profit by refusing to refinance the firm for the second period if a low profit is announced after the first period. We extend this model by supposing that when giving the credit, the bank does not know the expected profitability of the credit-seeking firm, i.e, an adverse selection parameter is added to the model of Bolton and Scharfstein (1990). The resulting credit contracts are such that the firm with a high profitability obtains a rent to reveal its type, and another rent to reveal its high profits, whereas the firm with a low profitability obtains the second rent only.

Then, introducing a credit manager (the agent) who sometimes observes the type of the firm, the stake of collusion appears. Indeed, if the credit manager observes that the firm has a high profitability, the firm is willing to bribe the credit manager in order to make him hide this information, and so obtain the credit contract with the additional rent referred to above. If the credit manager is corruptible, he accepts the bribe. This hurts the bank since it lowers its profits, so the bank has an incentive to fight collusion.\underline{be another reason as well, namely, the wish to render collusion between the credit manager and the firm more difficult.}

\textsuperscript{5}See Dupuch 1990 for a description of the internal organization of a bank in France.

\textsuperscript{6}The first model in this direction is due to Tirole (1986), analyzing job incentives in a three-tier hierarchy. See also Laffont (1988) for an early analysis of this setting. Other papers are Khalil and Lawarré (1995) and Kofman and Lawarré (1993) in the audit context; Laffont and Tirole (1993), Laffont and Martimort (1999) concentrate on regulation issues, the latter paper differing from the others in that it compares different organizational modes rather than sticking to an analysis of the distortions in the original contracts. See further Tirole (1992) for an extensive survey of this literature, as well as the survey of Andvig (1991).
We obtain the following results when characterizing the principal’s (the bank’s) optimal response to the collusion problem. If the collusion entails high transaction costs in the sense that the final bribe received by the credit manager is low, collusion is optimally deterred by internal incentives, i.e., compensations to the credit manager for revealing a high expected profit firm. If the transaction costs are low, this compensation scheme becomes too costly for the bank. Then the principal chooses instead to deter collusion by reducing the firm’s incentive to bribe the credit manager. This can be done in two ways, either by making the contract of the potentially bribing firm more advantageous, or by making the contract to which the bribing firm aspires less attractive.

To the best of our knowledge, there are only two other papers dealing with internal incentive problems and collusion in the banking sector, namely, Mitusch (1995) and Scheepens (1995). Scheepens studies the problem in which a credit manager might give a credit to an unprofitable firm against a bribe. The model deals exclusively with internal incentives (monetary compensations, audits, and the use of an incorruptible credit committee in the decision making above some threshold level of the credit amount), taking the credit contracts as exogenously given. As opposed to this, the present paper explicitly shows the interaction between internal and external risks that each bank has to deal with, through the distortions in the credit contracts that the collusion risk induces. Mitusch (1995) analyzes how banks manage to extract insider information from their client firms. The explanation he gives is that the bank optimally allows the credit manager to “collude” with the credit-seeking firm, the objective being to extract the firm’s private information. Mitusch (1995) thus analyzes one possible positive aspect of collusion.

Fighting collusion is of course costly for the bank. An immediate question is the following: is it possible that the bank finds it too costly to completely eradicate collusion? Indeed, observations of corruption cases in reality might suggest that the principal in fact finds that the expected cost of fighting collusion exceeds its expected gain, and thus finds it optimal to allow for some degree of collusion in equilibrium.

The aim of the last section is to study a case of collusion in equilibrium which was
first studied by Kofman and Lawarrée (1996), namely the coexistence of corruptible and uncorruptible agents: they find that if the proportion of honest agents is high enough, the expected cost of fighting collusion exceeds its expected gain and the principal prefers to run the risk of letting collusion occur. However, Kofman and Lawarrée (1996) seem to obtain their result by assuming that the principal cannot make the honest agent reveal his type, implying that the principal has to leave unnecessary informational rents to this type of agent. As opposed to this, we characterize an incentive scheme to make the honest type reveal himself. The important point is that this requires a precise definition of what an honest agent is. We propose two such definitions, the “partially honest” and the “totally honest” agent: the latter releases any information for free, including about his honesty, whereas the former releases the information about the firm for free, but claims that he is dishonest if that implies a higher salary. We show that collusion in equilibrium is obtained only if the honest agent is partially honest. We thus establish that the result of Kofman and Lawarrée (1996) does not rely on the restriction of the number of instruments available to the principal. The collusion in equilibrium obtained therefore seems to be unavoidable, in the sense that the principal can by no means replicate the bribe in the contract. We analyze this result closely in section 4.

The paper is organized as follows. Section 2 characterizes the optimal credit contract, first when the type of the firm is known, then when it is private information of the firm. Then we suppose that the credit manager is corruptible and determine the optimal collusion-proof contract (section 3). Last, we seek to define under which conditions there is collusion in equilibrium if there are corruptible and uncorruptible credit managers (section 4), before concluding in section 5.

2 The benchmark case - an incorruptible agent

2.1 Making the firm reveal its realized profit

In this subsection, we focus exclusively on the asymmetry of information on the realized profit. A firm of type $\theta$ needs to borrow an amount $F$ per period to finance its activities.
At the end of each period, the firm earns profits $\pi_1 > 0$ with probability $\theta$ and $\pi_2 > \pi_1$ with probability $(1 - \theta)$, the profits being uncorrelated between distinct periods.\(^7\) We suppose that the project is profitable in expected terms, but risky, i.e., the firm’s expected profits $\bar{\pi}(\theta)$ exceeds $F$, but the realized profit can be lower than the invested amount, $\pi_1 < F < \pi_2$. Without loss of generality, we consider a two-period model.\(^8\) Consider an investor (a bank) who is risk-neutral and who detains all bargaining power towards the firm.\(^9\) If the firm’s profits were observable, the bank would finance the project and require the maximal reimbursement compatible with the firm’s reservation net profit, as long as this would yield a positive expected gain to the bank. It is however quite reasonable to believe either that the profits are private information of the firm and thus not observable by the bank, or that they are observable by the bank but on a “soft” basis, i.e., they cannot be verified by a third party. In that case, the firm always claims to have realized $\pi_1$ and the bank makes a loss.

Several papers have investigated the means by which the bank can force the firm to reveal its profits. In the one-period models of Diamond (1984) and Gale and Hellwig (1985), the incentive used is a threat of auditing beneath a threshold level of profits. Bolton and Scharfstein (1990) let the relationship between the bank and the firm last two periods. By doing so, they offer the bank the possibility to reconsider the credit after the first period and thus to make the likelihood of reinvesting contingent on the profits announced at that time. Consequently, it is the threat of not refinancing which induces the firm to reveal its profits. This choice of model can be justified by the fact that some banks do use this threat of cutting off investments: “Banks are more and more severe, and do not hesitate to end the relationship at the first reimbursement incident”\(^10\).

\(^7\)In a model with a continuum of profit levels, the results are not altered qualitatively - see Faure-Grimaud (1997).

\(^8\)This is without loss of generality compared to any finite $n$ number of periods, as shown by Faure-Grimaud (1997). The model however breaks down with an infinite number of periods - see Gromb (1994).

\(^9\)As is shown by Faure-Grimaud (1994), the assumption that the firm detains all bargaining power leads to the same qualitative results, the only difference being that the firm keeps a larger part of its profits. Hence, this assumption is not very restrictive in the benchmark case. It is however not innocuous in the case of a corruptible agent, since there might be scope for signaling if the firm detained the bargaining power. To avoid complications, we therefore stick to the original assumption.

\(^10\)Dupuch (1990), p 28; our translation.
The credit contract consists of three elements: the reimbursements $R_i$ and $R^i$ after the first and second period respectively, and a probability $\beta_i$ of refinancing, where $i = 1, 2$ corresponds to the profit announced after the first period, $\pi_1$ or $\pi_2$. The timing is the following:

- **Date 0** - the bank proposes a contract $\{R_i, R^i, \beta_i\}$, $i = 1, 2$; the bank invests $F$ if it is accepted;
- **Date 1** - the first period profit is realized, the firm announces a profit $\tilde{\pi}_i$, $i = 1, 2$, reimburses $R_i$ and is refinanced for the second period with probability $\beta_i$;
- **Date 2** - if the firm has been refinanced, the second profit is realized and the firm reimburses $R^i$.

In order to find the optimal incentive scheme, the bank maximizes its expected profits under the constraints that the firm accepts the contract (the individual rationality constraint [IR]) and that the firm reveals $\pi_2$ (the incentive compatibility constraint [IC]). We assume that the firm is protected by limited liability and that the firm does not detain any other capital or collateral. Therefore, the bank also has to respect one limited liability constraint for each period: the first-period reimbursement cannot exceed the first-period profits [LL1], and the second-period reimbursement cannot exceed the sum of the surplus from the first period, $(\pi_i - R_i)$, and the profit that is always announced at the end of the second period, i.e., $\pi_1$ [LL2].

Supposing that the firm is risk neutral, that its reservation profit is zero, and that the discount rate is zero, the bank solves the following problem:

$$\max_{\{R_i, R^i, \beta_i\}} - F + \theta[R_1 + \beta_1(R^1 - F)] + (1 - \theta)[R_2 + \beta_2(R^2 - F)]$$

subject to

$$[IC] \quad \pi_2 - R_2 + \beta_2[\bar{\pi}(\theta) - R^2] \geq \pi_2 - R_1 + \beta_1[\bar{\pi}(\theta) - R^1]$$

\[11\] Notice that the second-period reimbursement is independent of the second-period profit (the index $i$ means that it is contingent on the profit announced after the first period); since the bank has no way of inducing the firm to reveal its profit in the second period, the firm would always announce $\pi_1$ at the end of the second period.

\[12\] The firm wants to conceal its high profits and thus we consider only the incentive constraint for $\pi_2$. We will check afterwards that a firm having realized $\pi_1$ doesn’t want to announce $\pi_2$. 

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\begin{align*}
[\text{LL1}] & \quad \pi_i \geq R_i, \quad i = 1, 2 \\
[\text{LL2}] & \quad \pi_i - R_i + \pi_1 \geq R^i, \quad i = 1, 2 \\
[\text{IR}] & \quad \theta[\pi_1 - R_1 + \beta_1[\bar{\pi}(\theta) - R^1]] + (1 - \theta)[\pi_2 - R_2 + \beta_2[\bar{\pi}(\theta) - R^2]] \geq 0 \\
[\beta] & \quad \beta_i \in [0,1], \quad i = 1, 2
\end{align*}

Bolton and Scharfstein (1990) show that \([\text{IC}]\) is binding at the optimum, and that there exists an optimal contract in which \(R^1 = R^2 = \pi_1\). By then replacing \(R_2\) by the expression given by \([\text{IC}]\) and \(R^i, i = 1, 2\) by \(\pi_1\), they immediately obtain the following solution:

\textit{Bolton and Scharfstein (1990)}

\textit{When the firm’s realized profit is private information, the optimal credit contract with conditional refinancing is such that:}

\begin{itemize}
  \item the firm is refinanced only if it announces a high profit after the first period;
  \item the firm gets an informational rent by reimbursing less than its profit if it announces a high profit.
\end{itemize}

\textit{More precisely:} \(\{R_1, R_2, \beta_1, \beta_2, R^1, R^2\} = \{\pi_1, \bar{\pi}(\theta), 0, 1, \pi_1, \pi_1\}\)

\textit{The bank invests at date 0 if and only if} \(F \leq \bar{\pi}(\theta) - \frac{\bar{\pi}(\theta) - \pi_1}{2 - \theta}\).

The firm is refinanced if the project yields \(\pi_2\) in the first period. The firm then reimburses \(R_2 = \bar{\pi}(\theta)\),\(^{13}\) which means that the ex ante informational rent that the bank must leave to the firm to induce it to reveal \(\pi_2\) is \((1 - \theta)(\pi_2 - \pi_1)\). It is because of this rent that the bank does not finance all firms whose expected profits exceed \(F\).\(^{14}\) Note that the threat of not refinancing is credible, since the bank always makes a loss in a one-period setting. We must however assume that the bank can commit to refinance in case of a high realized profit, since this refinancing is unprofitable for the bank, once the first period profit has been revealed.

\(^{13}\)Because of limited liability and since \(\pi_1 < \bar{\pi}(\theta)\), the omitted incentive constraint is automatically verified.

\(^{14}\)Moreover, it is necessary to assume that \(\pi_2 - \pi_1 < F\). Otherwise, the firm can announce \(\pi_1\) although it realized \(\pi_2\) and refinance itself in the second period.
In order to simplify the computations and the presentation in the rest of the paper, we make one additional assumption and one remark before proceeding to the next step of the analysis:

1) Assumption Bolton and Scharfstein (1990) show that there exists an optimal contract in which \( R_i = \pi_1, i = 1, 2 \) provided that \([IC]\) is binding. To simplify the presentation, we henceforth assume that \( R_i = \pi_1, i = 1, 2 \), although in one case, the incentive constraint imposed for \( \pi_2 \) to be revealed is not binding. We therefore omit \( R_1 \) and \( R_2 \) in the sequel, and a credit contract is defined by the quadruple \( \{ R_1, \beta_1, R_2, \beta_2 \} \). This assumption is however not restrictive. Moreover, this implies that \([LL2]\) is equivalent to \([LL1]\) and henceforth we label the limited liability constraint \([LL]\).

2) Remark We have assumed that the firm’s reservation profit is zero. This combined with limited liability renders the participation constraint \([IR]\) unnecessary.

2.2 Making the firm reveal its expected and its realized profits

Let us now suppose that \( \theta \) is private information of the firm. \( \theta \) can take one of two values, \( \theta \) or \( \bar{\theta} \), with \( \theta < \bar{\theta} \). Both types of firms are profitable in expected terms, i.e., \( \bar{\pi}(\theta) > \bar{\pi}(\bar{\theta}) > F \). The type of any firm is drawn once and for all, so it does not change between period one and period two. The bank only knows that the firm yields a high expected profit, i.e., is of type \( \theta \), with probability \( (1 - \alpha) \), and a low expected profit with probability \( \alpha \).

The bank thus suffers from two informational problems: ex ante, the type of the firm is unknown, and ex post, the realized profit is unobservable.\(^{15}\) The revelation principle applies, so the bank maximizes its expected profit under the relevant incentive constraints. To make the firm reveal the two parameters, the bank manager designs a menu of two contracts between which the firm chooses. The timing is the following:

- **Date 0** - the firm learns its type \( \theta \), announces \( \tilde{\theta} \in \{ \theta, \bar{\theta} \} \), and obtains the contract \( \{ R_1(\tilde{\theta}), \beta_1(\tilde{\theta}), R_2(\tilde{\theta}), \beta_2(\tilde{\theta}) \} \); the bank invests \( F \).

\(^{15}\)An alternative timing by which the firm learns its type and its first-period profit simultaneously leads to the same results. In that situation, the firm can lie both about its profit and about its type at the same time, implying that the principal has to respect incentive constraints that prevent the firm to make “double deviations” in order to induce truth-telling. Such “double deviations” are not possible with the timing adopted in the main text, since \( \theta \) is known and revealed before the first-period profit is known.
• Date 1 - the firm learns its first period profit, announces $\tilde{\pi}_i \in \{\pi_1, \pi_2\}$, reimburses $R_i(\tilde{\theta})$ and is refinanced with probability $\beta_i(\tilde{\theta})$.
• Date 2 - the firm reimburses $\pi_1$ if it has been refinanced; otherwise, nothing happens.

As before, we assume that the bank detains all the bargaining power. It thus maximizes its expected profits. Letting $\Pi(\theta, \tilde{\theta})$ denote the expected profit of the firm of type $\theta$ which announces $\tilde{\theta}$, and $W(\theta)$ the expected social surplus if the firm is of type $\theta$, the bank’s expected profit can be written as in the expected social surplus minus the expected profit of the firm, as in (1):

$$-F + \alpha [W(\tilde{\theta}, \beta_1(\tilde{\theta}), \beta_2(\tilde{\theta})) - \Pi(\tilde{\theta}, \tilde{\theta})] + (1 - \alpha)[W(\theta, \beta_1(\theta), \beta_2(\theta)) - \Pi(\theta, \theta)]$$

where $\Pi(\theta, \tilde{\theta})$ and $W(\theta)$ are defined as

$$\Pi(\theta, \tilde{\theta}) \equiv \theta[\bar{\pi}_1 - R_1(\tilde{\theta}) + \beta_1(\tilde{\theta})(\tilde{\pi}(\theta) - \pi_1)] + (1 - \theta)[\bar{\pi}_2 - R_2(\tilde{\theta}) + \beta_2(\tilde{\theta})(\tilde{\pi}(\theta) - \pi_1)]$$

$$W(\theta, \beta_1(\theta), \beta_2(\theta)) \equiv \tilde{\pi}(\theta) + [\theta\beta_1(\theta) + (1 - \theta)\beta_2(\theta)][\tilde{\pi}(\theta) - F]$$

Notice that the social surplus is increasing in the probability of refinancing $\beta_i$. A firm reveals its type truthfully only if the following incentive constraints are satisfied:

$$\Pi(\theta, \tilde{\theta}) \geq \Pi(\tilde{\theta}, \tilde{\theta})$$

$$\Pi(\tilde{\theta}, \tilde{\theta}) \geq \Pi(\theta, \theta)$$

These constraints guarantee that the expected profit (at date 0) induced by a truthful revelation of $\theta$ is greater than the expected profit of lying about $\theta$. Then, at date 1, when each firm has revealed its type by choosing the appropriate contract at date 0, the bank faces two incentive constraints intended to induce the truthful revelation of the first-period profit of each firm. Each of these constraints is equivalent to the constraint $[IC]$ in the previous problem:

$$R_2(\theta) \leq R_1(\theta) + (\beta_2(\theta) - \beta_1(\theta))[\tilde{\pi}(\theta) - \pi_1] \quad \theta \in \{\tilde{\theta}, \bar{\theta}\}$$

As before, the firms are protected by limited liability:
If the bank proposes the symmetric information contracts, the “good” firm $\theta$ chooses the “bad” firm’s contract, since it then gains $(1 - \theta)[R_2(\bar{\theta}) - R_2(\bar{\theta})] = (1 - \theta)[\bar{\pi}(\bar{\theta}) - \bar{\pi}(\bar{\theta})].$\textsuperscript{16} The good firm needs incentives to reveal its type. This is the usual situation in adverse selection problems: the incentive constraint of the good type is binding, whereas the bad one’s is not. However, in the present setting, the incentive constraint of the bad firm is also binding. In fact, the two types of firms obtain the same credit contract. Maximizing (1) subject to (2)-(5) yields (the proof and the definition of the cutoff value $\alpha^*$ referred to in the proposition are in appendix 1):

**Proposition 1 Credit contracts with two adverse selection parameters**

The high-profit firm obtains an informational rent for the revelation of its type only if there are sufficiently many low-profit firms:

- If the proportion of low-profit firms is sufficiently high ($\alpha > \alpha^*$), it is optimal for the bank to leave a rent to the high-profit firm for it to reveal its type, by proposing the full-information contract of the low-profit firm to both types of firms. Thus:

  \[ \{R_1(\bar{\theta}), \beta_1(\bar{\theta}), R_2(\bar{\theta}), \beta_2(\bar{\theta})\} = \{R_1(\bar{\theta}), \beta_1(\bar{\theta}), R_2(\bar{\theta}), \beta_2(\bar{\theta})\} = \{\pi_1, 0, \bar{\pi}(\bar{\theta}), 1\} \]

- If the proportion of low-profit firms is not high enough ($\alpha < \alpha^*$), the bank distorts the contract of the low-profit firms and offers the full-information contract to the high-profit firms who consequently do not obtain any rent to reveal their type. Thus:

  \[ \{R_1(\bar{\theta}), \beta_1(\bar{\theta}), R_2(\bar{\theta}), \beta_2(\bar{\theta})\} = \{\pi_1, 0, \bar{\pi}(\bar{\theta}), 1\} \]

  \[ \{R_1(\bar{\theta}), \beta_1(\bar{\theta}), R_2(\bar{\theta}), \beta_2(\bar{\theta})\} = \{\pi_1, 0, \pi_1, 0\} \]

For a sufficiently high proportion of good firms ($\alpha$ small), the bank prefers not to leave a rent to the those firms. The distortion of the bad firm’s contract in order to

\[ \pi_1 \geq R_1(\theta) \quad \theta \in \{\theta, \bar{\theta}\} \quad (5) \]

\textsuperscript{16}In this context, it may seem odd that the good firm wants to lie about its type. Intuition suggests that, when asking for a credit, bad firms want to appear better than they are in order to obtain the credit. Here, the problem is however not that of getting the credit or not, since both firms are profitable in expected terms. And since both firms are refinanced with the same probabilities after the first period, the only concern is the reimbursement, which differs between the contracts for the high profit announcement. The reimbursement being lower for the bad firm, the good firm would pretend it was bad if not prevented from doing so.
prevent the good firm from choosing it implies that the bank suffers a loss on the bad firms. The bank must however finance them; otherwise, the bad firms would pretend to have a high expected profit in order to be financed.

When the number of bad firms is high, the bank prefers to offer a profit-making contract to $\bar{\theta}$ and leave a rent to the high-profit firm. It can make $\bar{\theta}$’s contract more attractive by reducing $R_2(\theta)$, or by increasing $\beta_1(\bar{\theta})$ (and thus reducing $R_2(\bar{\theta})$, because of the constraint inducing the revelation of $\pi_2$). However, if $\beta_1(\bar{\theta})$ is increased above zero, the bad firm’s incentive constraint is violated. Indeed, in that case the bad firm gains $\beta_1(\bar{\theta})[\bar{\pi}(\bar{\theta}) - \pi_1]$ by choosing the good firm’s contract and it then never reveals $\pi_2$. For any $\beta_1(\bar{\theta}) > 0$, the incentive constraint of $\bar{\theta}$ is thus not satisfied, and given the assumptions, the only way to induce $\bar{\theta}$ to reveal its type is to reduce $R_2(\bar{\theta})$. It must be reduced to $\bar{\pi}(\bar{\theta})$, yielding the same contract for the two types of firms.\footnote{This is due to the assumption that the second-period reimbursements are equal to $\pi_1$. Relaxing this assumption yields a non-pooling pair of contracts in which a good firm is induced to reveal its type by obtaining an additional rent through a reimbursement lower than $\pi_1$ in the second period. The full solution is then to give the complete information contract to the firm of type $\theta$ and a contract with a downwards distortion of the second-period reimbursement to the firm of type $\bar{\theta}$. The incentive constraints of the good and of the bad firm are both binding. In terms of payoffs, it is equivalent to the solution described in proposition 1. The assumption that the second-period reimbursements are equal to $\pi_1$ is thus not restrictive, in the sense that it does not modify the remaining analysis qualitatively. Moreover, it simplifies the presentation.}

### 2.3 The threat of collusion

Assume now that a credit manager is handling the contracts with the firms, and that this manager sometimes is informed and sometimes uninformed about the firm’s type. The credit manager observes a signal $\sigma$ at zero cost (the firm also observes it). The signal reveals the true type with probability $\nu$ but reveals nothing (represented by $\emptyset$) with probability $(1 - \nu)$. The manager (the agent) then makes a report $r$ to his superior (the principal). This report states the information that he has about the credit-seeking firm. The choice of credit contract corresponds to the information in the report. This can interpreted by saying that the credit manager knows some of the firms from before, and can by previous experience tell the different types of firms apart. There are however new firms arriving, and the manager does not know their expected profits.
Hence, if the credit manager acts as a perfect agent of the principal, he makes the report \( r = \sigma \) and proposes the full information credit contract as described in proposition 2.1 if the signal revealed the type, and the pair of contracts as described in proposition 1 if the signal revealed nothing. The figure below summarizes these propositions.

However, if the credit manager is not a perfect agent of the principal, i.e., if he is corruptible, then there is a case for collusion. For simplicity, we assume that hard information can only be destroyed and not created: if \( \sigma = \emptyset \), the agent has no choice but to report that he has observed no evidence on \( \theta \), but if \( \sigma = \theta \), \( \theta \in \{\theta, \bar{\theta}\} \), he might
hide this information and make the report \( r = \emptyset \). Thus, if \( \sigma = \emptyset \) and if \( \alpha > \alpha^* \) so that the firm \( \theta \) obtains an informational rent if \( r = \emptyset \), the firm is willing to pay up to \((1 - \emptyset)[\bar{\pi}(\emptyset) - \bar{\pi}(\emptyset)]\). This amount corresponds to the stake of collusion, which is the expected gain from obtaining the contract proposed upon the (false) report \( r = \emptyset \) instead of the contract given when the report \( r = \emptyset \) is made. This collusion implies foregone profits for the bank, which may provide incentives in order to deter it. In the sequel, we restrict our attention to the case when \( \alpha > \alpha^* \), since when \( \alpha < \alpha^* \), collusion is not an issue, the stake of collusion being equal to zero.

3 The collusion-proof contract

Since the credit manager is sensitive to monetary compensations (he is willing to manipulate information in exchange of a bribe), the principal may fight collusion by offering monetary compensations \( S^r \), where \( r \) stands for the report \( r \in \{\emptyset, \theta, \bar{\theta}\} \). Apart from compensating the agent for a truthful report, the principal may also reconsider the terms of the contracts offered to the firm to avoid collusion, by reducing the firm’s incentive to bribe the credit manager, i.e., to reduce the stake of collusion. Therefore, the credit contracts also depend on the report \( r \) made by the credit manager. The timing is the following:

- Date 0 - the firm learns its type \( \theta \), and the credit manager and the firm observe the signal \( \sigma \in \{\emptyset, \theta, \bar{\theta}\} \); collusion takes place or not\(^{18} \); the agent makes the report \( r \in \{\sigma, \emptyset\} \), obtains \( S^r \) and signs the credit contract \( \{R_1^r(\theta), \beta_1^r(\theta), R_2^r(\theta), \beta_2^r(\theta)\} \) with the firm (for \( r = \emptyset \), it is a menu of two contracts which the firm chooses between, in which case the contract which is signed is also a function of the type that the firm announces); the bank invests \( F \).
- Date 1 - the firm announces \( \bar{\pi}_i \) and the contract is implemented.
- Date 2 - the firm reimburses \( \pi_1 \) if it has been refinanced.

\(^{18}\)Since we have assumed that the firm detains no cash, we must here assume that the agent accepts to collude in exchange of an expected bribe from the firm. Indeed, it is only if \( \pi_2 \) is realized that the firm gains anything from collusion and is able to give a bribe to the agent. If \( \pi_1 \) is realized after collusion having taken place, the firm did not gain by colluding and the agent thus receives no bribe. We also assume the firm can commit to giving the bribe to the agent if \( \pi_2 \) is realized.
Let $\Pi_r(\theta, \tilde{\theta})$ denote the expected profit of a firm of type $\theta$ who announces $\tilde{\theta}$ within the set of contracts corresponding to the report $r$ of the credit manager (if $r$ is equal to $\bar{\theta}$ or $\tilde{\theta}$, the firm does not announce its type, in which case $\tilde{\theta} \equiv r$). The expected profit of the principal is consequently:

$$
-F + \nu\{\alpha[W(\bar{\theta}, \beta_{1}^\theta, \beta_{2}^\theta) - \Pi_\emptyset(\bar{\theta}, \bar{\theta}) - S^\emptyset] + (1 - \alpha)[W(\theta, \beta_{1}^\theta, \beta_{2}^\theta) - \Pi_\emptyset(\theta, \theta) - S^\emptyset]}
+ (1 - \nu)\{\alpha[W(\bar{\theta}, \beta_{1}^\theta(\bar{\theta}), \beta_{2}^\theta(\bar{\theta})) - \Pi_\emptyset(\bar{\theta}, \bar{\theta})]
+ (1 - \alpha)[W(\bar{\theta}, \beta_{1}^\theta(\bar{\theta}), \beta_{2}^\theta(\bar{\theta})) - \Pi_\emptyset(\bar{\theta}, \bar{\theta}) - S^\emptyset]\}

(6)
$$

The bank maximizes this with respect to the terms of the four contracts and the monetary compensations. There are three incentive problems which prevent the principal from reaching the first best solution. First, for each contract, there is the profit revelation constraint, as well as the limited liability constraint for each contract:

$$
R_r^r + (\beta_2^r - \beta_1^r)[\pi(r) - \pi_1] \geq R_r^\emptyset \quad r \in \{\bar{\theta}, \bar{\theta}\}

(7)
$$

$$
R^\emptyset_1(\theta) + (\beta_2^\theta(\theta) - \beta_1^\theta(\theta))[\pi(\theta) - \pi_1] \geq R^\emptyset_2(\theta) \quad \theta \in \{\theta, \tilde{\theta}\}

(8)
$$

$$
\pi_1 \geq R^r_1 \quad r \in \{\bar{\theta}, \bar{\theta}\}

(9)
$$

$$
\pi_1 \geq R^\emptyset_1(\theta) \quad \theta \in \{\theta, \tilde{\theta}\}

(10)
$$

Second, the menu of contracts for $r = \emptyset$ must be incentive compatible, in that each type of firm reveals itself when $r = \emptyset$. The two incentive constraints are:

$$
\Pi^\emptyset(\theta, \bar{\theta}) \geq \Pi^\emptyset(\theta, \tilde{\theta})

(11)
$$

$$
\Pi^\emptyset(\bar{\theta}, \bar{\theta}) \geq \Pi^\emptyset(\bar{\theta}, \tilde{\theta})

(12)
$$

Thirdly, to prevent collusion, the bank must respect collusion incentive constraints, which guarantee that the agent does not accept to manipulate information against a bribe. Here it is indeed optimal for the principal to deter collusion, as will be implied by proposition 3. The loss in salary that the agent incurs by manipulating a report...
must exceed the amount that the firm is willing to pay to obtain the manipulation, or the stake of collusion. For the sake of generality, we allow for transaction costs in the side transfer, reducing the value of the bribe for the agent to \( k \) times the amount paid by the firm, with \((0 < k \leq 1)\). There are two collusion incentive constraints, since there are two possible information manipulations: if the manager observes either \( \underline{\theta} \) or \( \bar{\theta} \), he can manipulate the report and claim \( r = \emptyset \).

\[
S^{\underline{\theta}} - S^{\emptyset} \geq k[\Pi^{\emptyset}(\underline{\theta}, \emptyset) - \Pi^{\underline{\theta}}(\emptyset, \emptyset)] \tag{13}
\]

\[
S^{\bar{\theta}} - S^{\emptyset} \geq k[\Pi^{\emptyset}(\bar{\theta}, \emptyset) - \Pi^{\bar{\theta}}(\emptyset, \emptyset)] \tag{14}
\]

Nevertheless, only the first one is relevant. Indeed, it is easily verified ex post that the firm is not willing to bribe the manager to make the report \( r = \emptyset \) when the signal is \( \sigma = \bar{\theta} \), since the stake is null. Therefore, we omit the second collusion incentive constraint, and when mentioning the stake of collusion, we refer to the stake on the right hand side of constraint (13).\(^{19}\) Maximizing (6) subject to the constraints (7)-(14), and assuming that the agent’s reservation wage is zero, we obtain the results stated in the following proposition. In the proposition, the term benchmark refers to the situation where the credit manager is incorruptible, in which the credit contracts would be those defined on page 7 when the signal has revealed the type, and in proposition 1 when the signal has revealed nothing. The proof of the proposition as well as the definitions of the cutoff values are in appendix 2.

**Proposition 2** Optimal collusion deterrence

1. For \( k \) sufficiently small \((k < \hat{k})\), collusion is optimally deterred by giving a monetary incentive to the agent to prevent him from accepting the bribe: \( S^{\underline{\theta}} = k(1 - \theta)(\bar{\theta} - \underline{\theta})(\pi_2 - \pi_1) \) and \( S^{\emptyset} = S^{\emptyset} = 0 \). The credit contracts are undistorted compared to the benchmark case.

\(^{19}\)In fact, the right hand side of equation (14) is equal to zero in all cases of proposition 2. Had we obtained a strictly negative right hand side, there would have been a case for extortion: the agent could have threatened the firm to report \( r = \emptyset \) if \( \sigma = \bar{\theta} \). The firm for which \( \sigma = \bar{\theta} \) would then be willing to pay a strictly positive amount to avoid this. As Lambert Mogiliansky (1997) notes, however, this would not affect the principal’s profit, since extortion does not induce an actual manipulation of information - there is only a threat of manipulation. Hence, only the agent’s utility and the firm’s profit would have been affected and the principal would not care about that in the present model.
2. For \( k \) sufficiently large (\( k > k^* \)) and \( \nu \) sufficiently small (\( \nu < \hat{\nu} \)), collusion is optimally deterred by reducing the stake of collusion to zero, by distorting the credit contract connected to the report \( r = \bar{\theta} \):

\[
\{ R_1^\theta, \beta_1^\theta, R_2^\theta, \beta_2^\theta \} = \{ \pi_1, \bar{\theta} - \bar{\theta}, \pi(\bar{\theta}) - (\bar{\theta} - \bar{\theta})(1 - \bar{\theta})(\pi_2 - \pi_1), 1 \}
\]

The other credit contracts are undistorted compared to the benchmark case, and the credit manager receives no compensation: \( S^\theta = S^\emptyset = S^{\bar{\theta}} = 0 \).

3. When \( k \in [\hat{k}, k^*] \) or when \( k > k^* \) and \( \nu > \hat{\nu} \), the bank finances only the high-profit firm if \( r = \emptyset \), and the contract then is \( \{ \pi_1, 0, \pi(\bar{\theta}), 1 \} \), and offers the benchmark contracts when \( \sigma = \theta \). The credit manager receives no compensation: \( S^\emptyset = S^\theta = S^{\bar{\theta}} = 0 \).

For small \( k \)'s (case 1 of the proposition), i.e., for large transaction costs, collusion is optimally deterred by compensating the agent for a truthful report. The stake of collusion is unchanged, since the credit contracts are undistorted, but the firm of type \( \theta \) whose type has been revealed by the signal to the agent cannot induce the agent to manipulate the report. Indeed, its willingness to pay for it does not exceed the salary that the agent would lose from making the manipulation. When \( k \) is large enough, this solution is too costly since the compensations then are very large. It then is optimal to reduce the stake of collusion to save on monetary compensations. Because of the discrete character of the model, the stake and \( S^\theta \) are reduced to zero. The stake is reduced to induce the firm of type \( \theta \) whose type has been revealed by the signal to choose the contract connected to the truthful report \( r = \emptyset \).

Two cases arise: for \( \nu \) small enough, i.e., when the case \( \sigma = \emptyset \) occurs with a sufficiently high probability, the principal chooses to leave an additional rent to the firm for \( r = \emptyset \), by refinancing with positive probability if \( \pi_1 \) is realized in the first period: \( \beta_1^\emptyset = \bar{\theta} - \bar{\theta} > 0 \) (part 2 of the proposition).\(^{20}\) Of course, one may wonder how the bank

\(^{20}\) Notice the parallel with the adverse selection problem: the bank here wants to prevent a firm of type \( \bar{\theta} \) to choose the contract \( \{ \pi_1, 0, \pi(\bar{\theta}), 1 \} \) instead of \( \{ \pi_1, 0, \pi(\emptyset), 1 \} \). There is however one crucial difference: there is no incentive problem in the other direction here, i.e., from \( \emptyset \) to \( \bar{\theta} \), since information may not be created from nothing (in other words, the present problem is technically the same as in the previous section less the incentive constraint (3)). We thus obtain a solution following the reasoning on page 11, i.e., we can here leave a rent to the firm by increasing \( \beta_1 \) without threatening any incentive constraint in the other direction.
can implement a refinancing probability which is between 0 and 1. If we consider that the project is divisible, one may interpret it by saying that the bank finances a fraction \( \bar{\theta} - \theta \) of the project if the first-period profit is low. When \( \nu \) is large enough, it is too costly to leave the rent for \( r = \bar{\theta} \), and the bank prefers to distort the contract for \( \bar{\theta} \) when \( r = \emptyset \) compared to what is optimal given that \( \alpha > \alpha^* \).

Compared to previous literature on collusion, the present results are not new: collusion is deterred either by giving the equivalent of the bribe to the agent, or by reducing the stake of collusion. The main interest of the results are to suggest that the credit contracts offered by a bank may be distorted because of the potential collusion between the credit manager and the credit-seeking firm.

4 Uncertainty about the agent’s type - collusion in equilibrium?

Fighting collusion is costly for the bank. May it ever be too costly? That is, may it be optimal for the bank to let collusion occur in equilibrium? In his survey on collusion, Tirole (1992) lists several possible causes of collusion in equilibrium. One of them was first modeled by Kofman and Lawarrée in an article of 1990, published in 1996.\(^{21}\) They suppose that there is a fraction \( \gamma \) of corruptible or opportunistic agents and a fraction \( (1 - \gamma) \) of incorruptible or honest agents. If the principal could tell those types apart, he would leave a rent to avoid collusion only to the corruptible agents. However, it is assumed that the principal cannot make a scheme to make the agents reveal their type, so collusion in equilibrium is obtained if there is a sufficiently large proportion of honest agents. The result is due to the fact that to deter collusion, the principal leaves a rent to the agent even when he would not need to do so, i.e., with probability \( (1 - \gamma) \), which becomes too costly under some threshold level \( \gamma^* \).

\(^{21}\)The other causes listed by Tirole (1992) are non-separabilities between collusion incentive constraints if the variable subject to information manipulation can take more than two values, and the fact that collusion is socially desirable if the agent is better at collecting funds than the government in the regulation context. In Laffont and Tirole (1993, ch 11), collusion arises in equilibrium because it helps revealing information. In Acemoglu (1994), collusion in equilibrium is due to the fact that the size of the punishment inflicted to the agent in case of collusion is determined exogenously, thereby restricting the number of instruments.
Two questions emerge. First, is the result of Kofman and Lawarrée (1996) due to the restriction of the number of instruments available to the principal, i.e., to the assumed inability of the principal to screen among agents? We model such a screening device and obtain collusion in equilibrium, which suggests that the answer to this question is “no”. Second, how should the result of collusion in equilibrium be interpreted? The existence of non-revealing equilibria could possibly threat the existence of a general collusion-proofness principle, the equivalent of the revelation principle when agents are allowed to collude. In the second part of this section, we discuss this point more closely, claiming that we do not believe that it constitutes such a threat.

4.1 Collusion in equilibrium due to the coexistence of corruptible and incorruptible agents - confirmed

As opposed to Kofman and Lawarrée (1996) we propose an explicit modeling of the screening device designed to make the credit manager reveal whether he is corruptible or not. The manager’s type is represented by the parameter $k$. The manager is honest, $k = 0$, with probability $1 - \gamma$, and dishonest, $k = \bar{k} > 0$, with probability $\gamma$. $k = 0$ means that the bribe has no value to the manager, that he has infinitely high moral costs for accepting a bribe, or that he is very risk averse so that he does not want to take even a small risk to be caught colluding with the firm. This last interpretation would be suitable with a dynamic model or with a model in which the principal makes controls and is able to find out that there has been collusion. The value $\bar{k}$ is known by the bank, as is the distribution of honest and dishonest agents, but the principal does not know a priori of which type the agent is.

The fact that $k$ is revealed to the principal means that the credit contracts offered to the firms can be made contingent not only on the report $r \in \{\sigma, \emptyset\}$, but also on the announcement that the agent makes about his type. For each type $k \in \{0, \bar{k}\}$, the principal thus proposes a set $C_k \equiv \{R_1^r(\theta), \beta_1^r(\theta), R_2^r(\theta), \beta_2^r(\theta)\}_{k}$, $r \in \{\emptyset, \bar{\theta}, \emptyset\}$, $\theta \in \{\emptyset, \bar{\theta}\}$, of four contracts. In each set, there is one contract for each report $r = \emptyset$, $\theta \in \{\emptyset, \bar{\theta}\}$, and a menu of two contracts for $r = \emptyset$. The timing is the same as in the previous section, except for an additional event at date 0: the manager announces its type by choosing
a set of credit contracts $C_k$, $k \in \{0, \bar{k}\}$, together with some compensation scheme to be defined later, before observing the signal on the firm’s type.

There are four incentive problems, three of which have been treated earlier in the paper. Let us recapitulate the different constraints on the contracts in order to get a clear picture of the situation:

• Each individual credit contract must be profit-revealing (this is the information problem treated in section 2.1);
• When $r = \emptyset$, the menu of contracts which is proposed to the firm must be type-revealing (treated in section 2.2);
• Possibly, the set of credit contracts together with the compensation scheme are such that collusion is deterred (the case when it is deterred is treated in section 3);
• The two sets of credit contracts (the stakes of collusion arising from them) together with the compensation schemes must be such that the manager reveals its type.

For the first three information problems, we refer to the preceding sections for the detailed constraints. Let us now turn to the constraints corresponding to the last incentive problem. Before stating the constraints exactly, though, there is some preliminary work on the variables which appear in these constraints.

As in the previous section, the only relevant case for collusion is the possible manipulation of information from $r = \theta$ to $r = \emptyset$. There is thus one relevant stake of collusion for each set of contracts:

$$\Phi_{k=0} \equiv \Pi_{k=0}^{\emptyset}(\theta, \theta) - \Pi_{k=0}^{\theta}(\theta, \theta)$$

$$\Phi_{k=\bar{k}} \equiv \Pi_{k=\bar{k}}^{\emptyset}(\theta, \theta) - \Pi_{k=\bar{k}}^{\theta}(\theta, \theta)$$

The first one is the stake of collusion which emerges in the set of contracts intended for the incorruptible type of manager. It is important to define it, although an incorruptible manager never colludes with the credit-seeking firm, since it will appear in the incentive constraints for the revelation of the manager’s type.
Now, let us turn to the compensation scheme. Let $S^r(\bar{k}) \geq 0$ be the date-1 monetary compensation to the manager who at date 0 announces that he is of type $\bar{k} \in \{0, \bar{k}\}$ and who makes the report $r \in \{\emptyset, \theta, \bar{\theta}\}$ at date 1. For collusion to be deterred, the compensation to a manager of type $\bar{k}$ who makes the report $r = \bar{\theta}$ must therefore be greater than $\bar{k}$ times the stake of collusion defined by the set of credit contracts attributed to this type of manager:

\[ S^{\theta}(\bar{k}) \geq \bar{k}\Phi_{k=\bar{k}} \quad (15) \]

Of course, there is no such constraint for the type $k = 0$, since, by definition, the incorruptible type of manager does not accept to collude with the credit-seeking firm.

We are now ready to state a first incentive constraint for the revelation of the credit manager type. To allow for the possibility of collusion in equilibrium, we propose a stochastic incentive mechanism for the revelation of $k = \bar{k}$. A manager of type $\bar{k}$ who has made the report $r = \theta$ obtains the compensation $S^{\theta}(\bar{k})$ with probability $p(\bar{k})$; with probability $[1 - p(\bar{k})]$, he receives no compensation from the bank and accepts a bribe $\bar{k}\Phi_{k=\bar{k}}$. We can conclude that there is collusion in equilibrium if the solution contains $p(\bar{k}) < 1$ together with a stake strictly greater than zero, $\Phi_{k=\bar{k}} > 0$.

Given this scheme, the following constraint ensures that type $\bar{k}$’s expected profit if he chooses the set of contracts and compensation scheme intended for him, exceeds the bribe he can get by choosing the set of contracts of the manager of type $k = 0$.

\[ p(\bar{k})S^{\theta}(\bar{k}) + [1 - p(\bar{k})]\bar{k}\Phi_{k=\bar{k}} \geq \bar{k}\Phi_{k=0} \quad (16) \]

What about the honest type? Are there incentive constraints to make him reveal his type as well? In fact, as a novelty compared to the analysis in Kofman and Lawarré (1996), we propose two different interpretations of the term honesty. Depending on the interpretation, there is a different set of incentive constraints and different results concerning the optimality of deterring collusion.
4.1.1 “Partial honesty”

Assume that the agent is honest in the sense that he never accepts to manipulate information against a bribe, but he picks the dishonest’s set of contracts $C_{k=\bar{k}}$ if it is connected to a better compensating scheme than $C_{k=0}$.$^{22}$ The incentive constraint for the honest type is then:

$$S(0) \geq p(\bar{k})S(\bar{k})$$  (17)

Moreover, this case entails an additional incentive constraint for the dishonest type:

$$p(\bar{k})S(\bar{k}) + [1 - p(\bar{k})]\bar{k}\Phi_{k=\bar{k}} \geq S(0)$$  (18)

With the definition chosen here of an honest manager, we obtain a confirmation of the result of Kofman and Lawarrée (1996), namely, that there is collusion in equilibrium (for some parameter values) if there are sufficiently few corruptible managers.$^{23}$

**Proposition 3** When the honest agent is partially honest in the sense that he never accepts to be bribed, but that he would lie about his type in order to obtain a higher wage, there is collusion in equilibrium if $\gamma < \bar{k}$ when collusion is deterred by monetary compensations, which is optimal under some parametric configurations.

The proof is in appendix 3, but we here provide an outline of it. This outline also gives the intuitive explanation of the result. It is immediately seen that (17) is binding, implying that (18) can be omitted. We also show that (15) and (16) are binding. Notice that (17) binding implies that the compensation to the honest manager is strictly positive if the expected monetary compensation to the corruptible one is strictly positive. If $\bar{k}$ is sufficiently small, we know that the bank prefers to deter collusion through a monetary compensation than by eliminating the stake of collusion. The monetary compensation both to the corrupt and the honest agent is

$^{22}$An interpretation of this is that the agent has infinitely high moral costs for accepting bribes, but he always requires the same wage as his colleagues.

$^{23}$Note that if $\gamma = 1$, which is the case in section 3, there is no collusion in equilibrium, so it is optimal to impose the collusion incentive constraints when $\gamma = 1$. 
thus $k \Phi_k > 0$. This is the expected cost of fighting collusion. The benefit of fighting collusion is that the corruptible managers do not manipulate the information, i.e., the stake of collusion is not transferred from the bank to the firm. Thus, the expected gain is the stake multiplied by the probability that the manager is corrupt: $\gamma \Phi_k > 0$. Comparing the expected cost and the expected gain yields that the cost exceeds the gain when $\gamma < k$, in which case it is better not to fight collusion, $p(k) = 0$.

4.1.2 “Pure honesty”

Assume now that the agent is purely honest, i.e., he neither accepts to be bribed, nor to lie about his type, or expressed differently, he always releases information freely. In that case, the only incentive constraint for the revelation of the agent’s type is given in (16), and we obtain (proof in appendix 3):

**Proposition 4** When the honest type is purely honest in the sense that he never accepts a bribe and reveals his type independently of the dishonest type’s compensating scheme, there is no collusion in equilibrium.

Here, the honest agent needs no incentives at all to reveal his private information, so no rents are left unnecessarily to this credit manager or to the firms he handles. So there is no collusion in equilibrium.

4.2 Interpreting the results

These results show that collusion in equilibrium due to uncertainty about the agent’s type rests on the assumption that the honest type demands the same expected wage as the dishonest type. It is therefore not due to the restriction of the number of instruments available to the principal. Hence this constitutes a case in which collusion, and therefore non-revelation of the true information when $\sigma = \theta$, is unavoidable for some parameter values. In particular, this means that if $\gamma$ is small, the problem of the principal cannot be solved by imposing a simple collusion incentive constraint which imposes collusion deterrence. The question is whether this threatens the existence of a general collusion-proofness principle, i.e., the equivalent of the revelation principle.
In adverse selection models with one principal and agents who cannot collude by assumption, the revelation principle "states that the principal [who designs the mechanism in step 1] can content herself with ‘direct’ mechanisms, in which the message spaces are the type spaces, all agents accept the mechanism in step 2 regardless of their types, and the agents simultaneously and truthfully announce their types in step 3". The practical importance of this principle is that it makes it very easy to solve adverse selection problems: it is optimal for the principal to solve the problem by imposing incentive constraints which induce truth-revelation, and so the objective function can be written as a function of the true types.

A general collusion-proofness principle would characterize under which conditions the principal finds it optimal to impose a collusion incentive constraint, implying the same practical value as the revelation principle. It is therefore important to analyze cases of collusion in equilibrium in order to understand if they really constitute cases in which such a general principle would not apply. The results of the present section suggest that the coexistence of corruptible and incorruptible agents define a particular case in which such a principle would not hold. Nevertheless, we do not believe that it constitutes a threat for a future general collusion-proofness principle. Indeed, we believe that it is possible to construct a model without collusion which is mathematically equivalent to the situation depicted in this section and which yields a non-revealing equilibrium for some parameter values. That would mean that the existence of non-revealing equilibria is not a result of the potential collusion per se, but of something else. Let us briefly sketch how such a model would be constructed.

It is a game between a principal and one agent. At date 0, the principal proposes a contract to the agent, who will discover his own type at date 1 only. The contract describes the variables (for instance, the quantity to be produced, the salary) as functions of the type that the agent will announce at date 1. Then, suppose that the agent can have two attitudes towards the announcement at date 1: either he lies at date 1 if that procures a greater utility, he is opportunistic; or he does not lie, he is honest. Let us give an example. The principal is an employer who hires the agent at date 0. He proposes several contracts. The type to be announced at date 1 could correspond

\[ \text{24 Fudenberg and Tirole (1991), p. 255.} \]
to some information concerning the productivity of the agent (he learns how suitable he really is for the job). An opportunistic agent announces the level of productivity which gives him the highest utility given the contract he chose at date 0. This contract will thus typically include informational rents. An honest agent who always reveals his productivity automatically needs no informational rents, so his contract does not contain any such rents. However, the honest agent might not be honest at date 0. Indeed, let us suppose that there are two different kinds of honest agents. The first kind is partially honest in the sense that he always reveals his productivity at date 1 (he enjoys working and cannot hide a high productivity), but at date 0 he chooses the contract which yields the highest expected utility knowing this. Thus, he chooses the contract with the informational rent. We conjecture that if the probability that the agent is opportunistic is sufficiently low, the principal prefers to leave no rent at all, thereby taking the risk that some agents (the opportunistic ones) do not reveal their productivity at date 1. The second kind of honest agent is purely honest in the sense that he does not hide at date 0 that he enjoys working, and he does not want to be associated to lazy and opportunistic people. He picks the contract without rent. In that case, the principal should always find it optimal to make the opportunistic agents reveal their productivity at date 1.

To conclude, we conjecture that when the agent is partially honest, and if the proportion of opportunistic agents is sufficiently low, there is some non-revelation of information at date 1. It would mean that this setting constitutes a case in which the revelation principle does not apply for the revelation of the information at date 1. Such a result would clearly indicate that the collusion in equilibrium obtained in the present section is not a consequence of collusion per se. Such a result would therefore mean that the results of this section do not imply pessimism about a future existence of an equivalent to the revelation principle when agents can collude. No general result is available for the moment, though, and further research is presently being undertaken in this direction.
5 Conclusions

Because agents in banks handle large sums of money, the internal organization of a bank should be such that all sorts of fraudulent actions, including collusion between agents and firms, are avoided to a large extent. An account of the internal organization of banks in France (Dupuch (1990)) is full of descriptions of diverse mechanisms used for that end, e.g., internal and external audits, delegation rules, paperwork rules, etc. This paper studies one particular decision taken in connection with a bank’s credit activities, namely the choice of credit contract. We study the possible collusion between the credit seeking firm and the credit manager of the bank aimed at providing a better contract to the firm. Compared to earlier literature, the particularity of the present paper is to take explicitly into account the terms of the credit contracts as the real source of the collusion problem, and as one possible instrument to fight collusion. However, some questions remain. For instance, it would be interesting to allow for other types of internal incentives, e.g., a separation of the information handling between several agents, at the same or at different hierarchical levels (see Laffont and Martimort (1999) and Laffont and Martimort (1998) for this type of anti-collusion measures in the regulation context).

The analysis of this paper has also led to a result on a more theoretical level. We show that the result on collusion in equilibrium obtained by Kofman and Lawarrée (1996) is not due to the restriction that they impose on the number of instruments available to the principal. Indeed, allowing the principal to screen between credit managers of different types, we show that it is optimal for the principal to allow for collusion in equilibrium if the honest managers demand the same wage as the dishonest ones. Therefore, this seems to constitute a genuine case in which there is some non-revelation of information in equilibrium. The principal finds it too costly to replicate the rents implied by the opportunistic behavior of the agents in the contract. Nevertheless, and this is a topic for further research, intuition suggests that this result could be obtained in a model without collusion as well. This would imply that the non-revelation of information obtained by Kofman and Lawarrée (1996) and in this paper is not induced by collusion per se, but that remains to be proved formally.
Appendix 1

Using the definition of $\Pi(\theta, \bar{\theta})$, rewrite (1):

\[
-F + \alpha[\bar{\theta}R_1(\bar{\theta}) + (1 - \bar{\theta})R_2(\bar{\theta}) - \bar{\theta}[\beta_1(\bar{\theta}) + (1 - \bar{\theta})\beta_2(\bar{\theta})(F - \pi_1)]
+ (1 - \alpha)[\theta R_1(\theta) + (1 - \theta)R_2(\theta) - \theta[\beta_1(\theta) + (1 - \theta)\beta_2(\theta)(F - \pi_1)]
\]

First, notice that the constraints (5) are binding at the optimum, since the expected profit and the right hand side of the constraint (4) is increasing in $R_1(\bar{\theta})$ and/or $R_1(\theta)$. Then, suppose that the profit revelation constraints (4) are binding for the two types of firms. The expected profit of the bank thus becomes:

\[
-F + \alpha[\bar{\pi}_1 + (1 - \bar{\theta})\beta_2(\bar{\theta})(\bar{\pi}(\bar{\theta}) - F) - \bar{\theta}((1 - \bar{\theta})^2(\pi_2 - \pi_1) + \bar{\theta}(F - \pi_1)]
+ (1 - \alpha)[\pi_1 + (1 - \theta)\beta_2(\theta)(\bar{\pi}(\theta) - F) - \theta((1 - \theta)^2(\pi_2 - \pi_1) + \theta(F - \pi_1)]
\]

Using (4) and (5) binding, the “downwards” incentive constraint (2) can be written:

\[
\beta_1(\theta)[\bar{\pi}(\theta) - \pi_1] \geq \beta_1(\bar{\theta})[\theta \bar{\pi}(\theta) + (1 - \theta)\bar{\pi}(\theta) - \pi_1] + (1 - \theta)\beta_2(\bar{\theta})[\bar{\pi}(\theta) - \bar{\pi}(\bar{\theta})]
\]

Since the profit is increasing in $\beta_2(\bar{\theta})$ and decreasing in $\beta_1(\bar{\theta})$ it is optimal to bind this constraint. Thus, replacing $\beta_1(\bar{\theta})$ by the expression obtained by binding this constraint, one obtains that the expected profit is an increasing function of $\beta_2(\bar{\theta})$, and a decreasing function of $\beta_1(\bar{\theta})$, so it is optimal to choose $\beta_2(\bar{\theta}) = 1$ and $\beta_1(\bar{\theta}) = 0$. For $\beta_2(\bar{\theta})$, there are parameter values ($\alpha$ low) for which the expected profit is decreasing; then choose $\beta_2(\bar{\theta}) = 0$. In that case, the above constraint binds for $\beta_1(\bar{\theta}) = 0$. Hence, (4) implies $R_2(\bar{\theta}) = R_1(\bar{\theta}) = \pi_1$. Then, choose the maximal $R_2(\bar{\theta})$ which is compatible with the constraints: $R_2(\bar{\theta}) = \bar{\pi}(\bar{\theta})$. This yields the second part of proposition 1.

Next, when $\alpha$ is large, the expected profit is increasing in $\beta_2(\bar{\theta})$. If one chooses $\beta_2(\bar{\theta}) > 0$, then one must also set $\beta_1(\bar{\theta}) > 0$ to satisfy the downward incentive constraint (2). This is costly for the bank. It is costly not only because it increases the
loss at date 2, but also because $R_2(\bar{\theta})$ then has to be reduced in order to respect the profit-revelation constraint. Moreover, (3) is violated (with $\beta_1(\bar{\theta}) = 0$, $\beta_2(\bar{\theta}) = 1$ and $\beta_2(\bar{\theta}) = 1$, (2) binding implies $\beta_1(\bar{\theta}) = \bar{\theta} - \bar{\theta}$, so (3) becomes $0 \geq \bar{\theta} - \bar{\theta}$, which is false).

(3) is violated in that manner for any $\beta_2(\bar{\theta}) > 0$, given that we have assumed that (4) binding for $\theta$. By looking at the constraint (4) for the type $\theta$, which must hence be relaxed, one sees that there is an alternative way of increasing the rent to this type of firm, namely, by reducing $R_2(\bar{\theta})$ without increasing $R_1(\theta)$. Thus, set $\beta_2(\bar{\theta}) = 1$, $\beta_1(\bar{\theta}) = 0$, $R_1(\theta) = \pi_1$, and choose the contract for firm $\bar{\theta}$ which maximizes the expected profit under the constraint (4) for that firm, i.e., \{ $R_1(\bar{\theta}), \beta_1(\bar{\theta}), R_2(\bar{\theta}), \beta_2(\bar{\theta})$ \} = \{ $\pi_1, 0, \bar{\pi}(\bar{\theta}), 1$ \}. In order to induce the good firm to reveal itself through a reduction in $R_2(\bar{\theta})$, it must be reduced to $\bar{\pi}(\bar{\theta})$, thus yielding the first part of the proposition.

$\alpha^*$ is easily obtained by comparing the expected profit of the bank for the two solution candidates, yielding:

$\alpha^* = \frac{(1 - \theta)(\bar{\theta} - \bar{\theta})(\pi_2 - \pi_1)}{(1 - \theta)(\bar{\theta} - \bar{\theta})(\pi_2 - \pi_1) + (1 - \theta)[\bar{\pi}(\bar{\theta}) - F]}$

**Appendix 2**

We immediately see that constraint (14) is never binding. Indeed, originally, there is no risk for collusion from $r = \bar{\theta}$ to $r = \emptyset$, since $[\Pi(\bar{\theta}, \emptyset) - \Pi(\bar{\theta}, \bar{\theta})]$ is non-positive. Moreover, intuition suggests that the bank can either leave an additional rent to the firm for which $\sigma = \theta$, or deteriorate the contract for the firms for which $\sigma = \emptyset$ to prevent information manipulation from $r = \theta$ to $r = \emptyset$. Consequently, $[\Pi(\bar{\theta}, \emptyset) - \Pi(\bar{\theta}, \bar{\theta})]$ remains non-positive. Optimally, we then have \{ $R_1^0, \beta_1^0, R_2^0, \beta_2^0$ \} = \{ $\pi_1, 0, \bar{\pi}(\bar{\theta}), 1$ \}. Furthermore, the bank needs not encourage the report $r = \emptyset$. Thus, $S^0 = 0$, and consequently $S^\emptyset = 0$. Furthermore, (13) is binding at the optimum, since the profit is decreasing in $S^\emptyset$. Thus, $S^\emptyset = k[\Pi^0(\theta, \emptyset) - \Pi^2(\theta, \emptyset)]$.

Now, notice the similarity with the problem solved in appendix 1: the basic incentive problem is the same (making a firm of type $\theta$ reveal itself), except for two important things. First, there is no “upward” incentive constraint (equivalent to con-
straint (3)), since a firm whose type is not revealed by the signal (i.e., \(\sigma = \emptyset\)) cannot create information. Second, there is the compensation \(S^g\). Proposition 1 together with some intuition rapidly gives an idea of the structure of the solution.

1. When \(k\) is small, it is cheap to deter collusion by compensating the manager. Then, do not distort the contracts, i.e, propose the contract described in proposition 2.1 when the signal has revealed the type, and the contract described in part 2 of proposition 1 when the signal has revealed nothing. Thus:

\[
\{R^\theta_1, \beta^\theta_1, R^\theta_2, \beta^\theta_2\} = \{\pi_1, 0, \pi(\theta), 1\}, \theta \in \{\theta, \bar{\theta}\}
\]

\[
\{R^\theta_1(\theta), \beta^\theta_1(\theta), R^\theta_2(\theta), \beta^\theta_2(\theta)\} = \{\pi_1, 0, \pi(\theta), 1\}, \theta \in \{\theta, \bar{\theta}\}
\]

\[
S^g = k(1 - \theta)(\bar{\theta} - \theta)(\pi_2 - \pi_1)
\]

When \(k\) is greater, this becomes too costly. One can show with straightforward calculus that it is then optimal to set \(S^g = 0\). So, the stake of collusion has to be reduced to zero in order to satisfy the constraint (13). Two cases (2 and 3) emerge.

2. When \(\nu\) is small enough (i.e., the probability that \(\sigma = \emptyset\) is big enough), it is better to increase \(\Pi^g(\theta, \bar{\theta})\) than to reduce \(\Pi^g(\theta, \bar{\theta})\) (since this last solution implies that losses are made on the contract for \(\bar{\theta}\) when \(r = \emptyset\) - see case 3). Here, it is possible to increase \(\Pi^g(\theta, \bar{\theta})\) by increasing \(\beta_1\) and keep the profit revelation constraint binding, which is strictly better than not having it bind (in appendix 1, this was not a possible distortion, since it caused a violation of the upward incentive constraint, which here does not exist). Thus, (13) binding together with \(\beta^g_1 = 0, \beta^g_2 = 1\) and \(\beta^g_2 = 1\) implies \(\beta^g_1 = \bar{\theta} - \theta\). The binding profit revelation constraint (7) for \(\bar{\theta}\) in turn implies \(R^\theta_2 = \bar{\pi}(\theta) - (\bar{\theta} - \theta)(1 - \theta)(\pi_2 - \pi_1)\). Hence:

\[
\{R^\theta_1, \beta^\theta_1, R^\theta_2, \beta^\theta_2\} = \{\pi_1, \bar{\theta} - \theta, \bar{\pi}(\theta) - (\bar{\theta} - \theta)(1 - \theta)(\pi_2 - \pi_1), 1\}
\]

\[
\{R^\theta_1, \beta^\theta_1, R^\theta_2, \beta^\theta_2\} = \{\pi_1, 0, \bar{\pi}(\theta), 1\}
\]

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\[
\{ R^0_1(\theta), \beta^0_1(\theta), R^0_2(\theta), \beta^0_2(\theta) \} = \{ \pi_1, 0, \bar{\pi}(\theta), 1 \}, \theta \in \{ \bar{\theta}, \bar{\theta} \}
\]

\[ S^0 = 0 \]

3. Last, check whether it is preferable not to finance \( \bar{\theta} \) when \( r = \emptyset \) and propose the contracts

\[
\{ R^0_1(\theta), \beta^0_1(\theta), R^0_2(\theta), \beta^0_2(\theta) \} = \{ \pi_1, 0, \bar{\pi}(\theta), 1 \}, \theta \in \{ \bar{\theta}, \bar{\theta} \}
\]

\[
\{ R^0_1(\bar{\theta}), \beta^0_1(\bar{\theta}), R^0_2(\bar{\theta}), \beta^0_2(\bar{\theta}) \} = \{ \pi_1, 0, \bar{\pi}(\bar{\theta}), 1 \}
\]

\[
\{ R^0_1(\bar{\theta}), \beta^0_1(\bar{\theta}), R^0_2(\bar{\theta}), \beta^0_2(\bar{\theta}) \} = \{ \pi_1, 0, \pi_1, 0 \}
\]

\[ S^0 = 0 \]

The following threshold values are found by comparing the expected profit of the bank with the three different solution candidates (they are all strictly between 0 and 1).

\[
k^* = \frac{\theta(F - \pi_1)}{(1 - \theta)(\pi_2 - \pi_1)} + (1 - \theta)
\]

\[ \hat{k} \equiv \frac{(1 - \nu)[\alpha(1 - \bar{\theta})(\bar{\pi}(\bar{\theta}) - F) - (1 - \theta)(1 - \alpha)(\bar{\theta} - \theta)(\pi_2 - \pi_1)]}{\nu(1 - \alpha)(1 - \theta)(\theta - \bar{\theta})(\pi_2 - \pi_1)} \]

\[ \hat{\nu} \equiv \frac{1}{1 + \frac{(1 - \alpha)(\theta - \bar{\theta})[\theta(\pi_1 - F) + (1 - \theta)(1 - \theta)(\pi_2 - \pi_1)]}{\alpha(1 - \theta)(\bar{\pi}(\bar{\theta} - F) - (1 - \alpha)(1 - \theta)(\bar{\theta} - \theta)(\pi_2 - \pi_1))}} \]
Appendix 3

At this point, there is a large number of variables: for each credit contract, there are four variables (the reimbursements and the probabilities of refinancing), and there are four credit contracts plus a compensation scheme for each type of credit manager (the corruptible and the incorruptible). Some can immediately be eliminated from the analysis, namely the credit contracts and the compensations for \( r = \bar{\theta} \) since there is no risk of collusion in that case, and the compensations for the report \( \emptyset \) which can be set equal to zero, since setting such a compensation strictly greater than zero would only make it more costly to deter information manipulation from \( r = \emptyset \) to \( r = \emptyset \). Thus, we focus on the credit contracts for \( r = \emptyset \) and \( r = \emptyset \), as well as the compensations \( S_\emptyset(\bar{k}) \) (together with the probability \( p(\bar{k}) \)) and \( S_\emptyset(0) \). In total, there are thus 27 variables. It is not necessary to write the whole maximization problem. Instead, let us begin by making two remarks:

1. First, the expected profit of the bank is a decreasing function of the compensations to the corruptible manager \( S_\emptyset(\bar{k}) \) and to the incorruptible manager \( S_\emptyset(0) \).
2. Second, in the absence of potential collusion, the bank would choose credit contracts such that the stake of collusion is strictly positive. Let \( \Phi^* \) be this “optimal” stake: \( \Phi^* \equiv (1 - \theta)(\bar{\theta} - \emptyset)(\pi_2 - \pi_1) > 0 \).

The following constraints are the relevant ones for the proof of proposition 3:

\[
[CIC] \quad S_\emptyset(\bar{k}) \geq \bar{k}\Phi_{k=\bar{k}} \\
[IC_{k=\bar{k}}^1] \quad p(\bar{k})S_\emptyset(\bar{k}) + [1 - p(\bar{k})]\bar{k}\Phi_{k=\bar{k}} \geq \bar{k}\Phi_{k=0} \\
[IC_{k=\bar{k}}^2] \quad p(\bar{k})S_\emptyset(\bar{k}) + [1 - p(\bar{k})]\bar{k}\Phi_{k=\bar{k}} \geq S_\emptyset(0) \\
[IC_{k=0}] \quad S_\emptyset(0) \geq p(\bar{k})S_\emptyset(\bar{k})
\]

Of course, we also keep in mind that the compensations \( S_\emptyset(k) \), \( k \in \{0, \bar{k}\} \) cannot be negative.
• Prove that $\Phi_k \leq \Phi^*$, $k \in \{0, \bar{k}\}$: it cannot be optimal to choose $\Phi_k > \Phi^*$ for any $k \in \{0, \bar{k}\}$, since then it would be possible to choose $\Phi^*$ which is more profitable than any $\Phi_k > \Phi^*$, and which moreover enables the bank to set lower compensations, thus increasing its expected profit. So, $\Phi_k \leq \Phi^*$.

• Prove that $[CIC]$ binds: suppose that $[CIC]$ is satisfied with slack. If $\Phi_{k=\bar{k}} = \Phi^*$, then $\Phi_{k=0} = \Phi^*$ also since nothing hinders that. Thus, if $p(\bar{k}) > 0$, $[IC_{k=\bar{k}}]$ is satisfied with slack also, and it would then be possible to decrease $S^\theta(\bar{k})$ without jeopardizing any constraint, thereby increasing the bank’s expected profit, until it equals $\Phi_{k=\bar{k}}$, contradicting the slackness of $[CIC]$. If $\Phi_{k=\bar{k}} < \Phi_{k=0} \leq \Phi^*$, and even if $[IC_{k=\bar{k}}]$ is binding, it is possible to increase the expected profit by increasing $\Phi_{k=\bar{k}}$ and decreasing $S^\theta(\bar{k})$ until equalizing them, implying a contradiction again. If $\Phi_{k=\bar{k}} > \Phi_{k=0}$, and even if $[IC_{k=\bar{k}}]$ is binding, it is possible to increase the expected profit by increasing $\Phi_{k=\bar{k}}$ and decreasing $S^\theta(\bar{k})$ until the equalization of $\Phi_{k=\bar{k}}$ and $\Phi_{k=0}$. If $[IC_{k=\bar{k}}]$ is slack then, decrease $S^\theta(\bar{k})$ until it binds this constraint. But that implies that $[CIC]$ also binds. So $S^\theta(\bar{k}) = \bar{k} \Phi_{k=\bar{k}}$.

• Next, note that $[IC_{k=0}]$ obviously binds, implying $S^\theta(0) = p(\bar{k})S^\theta(\bar{k}) = p(\bar{k})\bar{k} \Phi_{k=\bar{k}}$. This also implies that $[IC_{k=0}]$ is satisfied as long as $\Phi_{k=0} \geq 0$. We can thus omit this constraint, since it cannot be optimal for the principal to reduce the stake beneath zero.

• $[IC_{k=\bar{k}}]$ reduces to $\Phi_{k=\bar{k}} \geq \Phi_{k=0}$. This constraint is binding since it is the only constraint on $\Phi_{k=0}$, and the expected profit of the bank is increasing in this term up to $\Phi^*$ and we know that $\Phi_{k=\bar{k}} \leq \Phi^*$.

Using these elements, the term multiplying $p(\bar{k})$ in the objective function is the following:

$$
\nu(1 - \alpha) \{(\gamma - \bar{k})\Phi_{k=\bar{k}} + \gamma[W(\theta, \beta^\theta_1(\bar{k})], \beta^\theta_2(\bar{k})) - W(\theta, \beta^\emptyset_1(\theta, \bar{k}), \beta^\emptyset_2(\theta, (\bar{k})))\} \tag{19}
$$

where $\beta^\emptyset_i(\theta, \bar{k})$ corresponds to the probability of refinancing the firm if the profit $i$
is announced, given that the report \( r \in \{ \theta, \bar{\theta}, \emptyset \} \) has been made by a credit manager of type \( \bar{k} \). Moreover, if \( r = \emptyset \), \( \beta \) also depends on the announced type of firm, which is truthful and therefore equal to \( \theta \).

We know from proposition 2 that the cheapest ways to deter collusion are either to keep the stake at \( \Phi^* \) and pay the compensation to the credit manager, or to reduce the stake to zero. It would not be rational for the bank to choose another way of deterring collusion, given that it is possible to choose those devices.

If the first option is chosen (case 1 of proposition 2), the second term of (19) is equal to zero, since \( \beta_{1}^0(\bar{k}) = \beta_{1}^0(\theta, \bar{k}) \) and \( \beta_{2}^0(\bar{k}) = \beta_{2}^0(\theta, \bar{k}) \). Thus, the term multiplying \( p(\bar{k}) \) is \((\gamma - \bar{k})\Phi_{k=\bar{k}}\). The stake \( \Phi_{k=\bar{k}} \) being strictly greater than zero, this term is strictly negative if \( \gamma < \bar{k} \), in which case it is optimal to set \( p(\bar{k}) = 0 \), yielding collusion in equilibrium.

If the second option is chosen, the second term of (19) is either strictly positive (if the bank reduces the stake by increasing \( \beta_{1}^0(\bar{k}) \) above zero, case 2 of proposition 2) or equal to zero (if the stake is reduced by increasing \( R_{2}^0(\theta, \bar{k}) \) up to \( R_{2}^0(\bar{k}) \), case 3 of proposition 2). The first term is equal to zero, so the bank chooses \( p(\bar{k}) = 1 \), and there is no collusion in equilibrium in that case.

Taking this into account, comparisons of the values of the objective function show that if it is not the case that \( \nu < \hat{\nu} \) and \( \gamma > k^* \) and \( \bar{k} > k^* \), or \( \nu > \hat{\nu} \) and \( \gamma > \bar{k} \) and \( \bar{k} > \hat{k} \) (see appendix 2 for the definitions of \( \hat{\nu}, k^* \) and \( \hat{k} \)), choosing the first option is strictly better than the second option. So, there exist parametric configurations for which it is optimal for the bank to allow for collusion in equilibrium if \( \gamma < \bar{k} \). \textbf{Q.E.D.}

For the proof of proposition 4, the following constraints are the relevant ones, since \( S_{\theta}(0) \) can immediately be set to zero:

\[
[CIC] \quad S_{\emptyset}(\bar{k}) \geq \bar{k}\Phi_{k=\bar{k}}
\]

\[
[IC^{1}_{k=\bar{k}}] \quad p(\bar{k})S_{\emptyset}(\bar{k}) + [1 - p(\bar{k})]k\Phi_{k=\bar{k}} \geq \bar{k}\Phi_{k=0}
\]
With similar arguments as above, \([CIC]\) and \([IC^1_{k=\bar{k}}]\) can be shown to be binding, so \(S^\emptyset(\bar{k}) = \bar{k}\Phi_{k=\bar{k}} = \bar{k}\Phi_{k=0}\). Using this, the term multiplying \(p(\bar{k})\) in the objective function is:

\[
\nu(1 - \alpha)\{\gamma(1 - \bar{k})\Phi_{k=\bar{k}} + \gamma[W(\theta, \beta^\emptyset_1(\bar{k}), \beta^\emptyset_2(\bar{k})) - W(\theta, \beta^\emptyset_1(\bar{k}, \bar{k}), \beta^\emptyset_2(\theta, \bar{k}))]\} \tag{20}
\]

1. If collusion is deterred by setting \(S^\emptyset(\bar{k}) = \Phi_{k=\bar{k}}\) with \(\Phi_{k=\bar{k}} = \Phi^*\), then the second term in (20) is equal to zero. The term multiplying \(p(\bar{k})\) is thus strictly positive, so the bank sets \(p(\bar{k}) = 1\).

2. If collusion is deterred by setting \(S^\emptyset(\bar{k}) = \Phi_{k=\bar{k}} = 0\), then the first term in (20) is equal to zero, and the second term is equal to or strictly greater than zero, so the bank sets \(p(\bar{k}) = 1\).

Therefore, there is no collusion in equilibrium when the honest credit manager is “purely honest”, whatever the size of \(\gamma\). Q.E.D.
References


