A dynamic model of cultural assimilation

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Boston College Working Papers in Economics, 2002

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November, 2002

Abstract

The paper analyzes the population dynamics of a country that has two ethnic groups, a minority and a majority. Minority members can choose whether or not to assimilate into the majority. If the minority is small, the long-run outcome is full assimilation. When the minority is large, the unique long-run equilibrium is the initial situation. For intermediate minority sizes multiple equilibria are possible, including the full- and no-assimilation ones. The paper also solves the social planner's problem, which indicates that the country can end up in an inefficient steady state. Even if the steady state is the optimal one, the equilibrium path will be suboptimal. Two extensions to the basic model are considered. The first one allows for a comparison between a multicultural and a "melting pot" society. The second one introduces population growth and studies the interplay between exogenous and endogenous changes in the minority's size.

1 Introduction

Minority ethnic groups can coexist with the majority in a country for a long time, and then suddenly disappear. Minorities that seem on the verge of extinction suddenly bounce back. How can these phenomena be explained,

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and what are the determinants of which outcome is realized? This paper is an attempt to answer these questions.

The first goal of the paper is to analyze the positive aspects of population dynamics. It builds a dynamic model of assimilation, which captures some basic features of cultural exchange. In particular, belonging to the majority group is desirable because of scale effects, but assimilation is costly - both in monetary and mental terms. Thus when deciding about assimilation, minority members weigh the benefits and the costs. It is also important, however, to adopt a dynamic setting, because forward-looking agents take into account future gains when deciding. In particular, parents might - and seemingly do - decide to assimilate even if it imposes very large costs for them, because they believe their children will enjoy the benefits. A second reason why dynamics is important is that it can reveal the instability of a static equilibrium. For each generation, assimilation is likely to be partial, because some people have very high costs. But partial assimilation is unlikely to be stable in the long run, since incentives for assimilation will not be the same for successive generations. In other words, a static model does not take into account that the state of the world changes over time, and thus cannot describe the long-run patterns of the population.

The second goal of the paper is to examine the normative properties of the long-run equilibrium. There is a concern that it might not be efficient, given that the minority's assimilation decision also effects the majority (due to the scale effects in ethnicity size). Indeed, in a static model assimilation is in general suboptimal. In a dynamic framework, however, additional considerations can arise. As the model shows, the long-run equilibrium can be efficient, although the equilibrium path that leads to it is not. Also, there is an additional problem that might cause the equilibrium steady state to be inefficient, and this is a problem of coordination. This leads to the possibility of multiple equilibria, which (unlike in a static model) is inevitable for some population distributions. In this case, even if the full assimilation steady state is feasible, the equilibrium selected might be the no-assimilation one, because atomistic agents cannot coordinate to pick the "right" one. This can happen even if the former equilibrium Pareto-dominates the latter.

The final goal of the paper is to extend the basic model in order to discuss related questions. One important question concerns the desirability of a multicultural society as opposed to a "melting pot". The former is defined as one in which minority members can learn the culture of the majority without giving up their own. It is not a priori obvious which choice is better, either individually or for the society as a whole. Incorporating the possibility of becoming bilingual into the model sheds some light on this question. The second issue involves changes in population sizes that result from factors other than assimilation. A shrinking minority might stop further assimilation when it receives an infusion of immigrants, or if it experiences an increase in its natural growth rate. An interesting historical application to the latter is the so-called "Revanche du berceau", the conscious policy of French Canadians in the 19th century to "outgrow" the English speaking majority, or at least to preserve their own heritage. Incorporating exogenous population change can show under what conditions a policy if this kind might achieve its goal.

There is international evidence that cultural and linguistic affiliation matters for economic outcomes. For example, Hall and Jones (1996) find that belonging to a major language group improves the economic performance of a country, even after controlling for a wide variety of factors. For individual countries, Sowell (1996) documents the experience of various immigrant groups throughout history. A recurring theme is that immigrants and natives with different cultures and languages experience frictions in intergroup encounters. From a theoretical point of view, the question of cultural assimilation was first studied by Lazear (1995) in a static framework. This paper retains Lazear's assumption of random matching between ethnic groups, but introduces dynamics explicitly. This opens up new possibilities, and leads to a richer set of outcomes.

The rest of the paper is organized as follows. Sections 2.1 and 2.2 describe the basic model and derive the properties of the equilibrium. Section 2.3 solves the social planner's problem, and compares the outcome with the equilibrium solution. Section 3 introduces the possibility of learning, and Section 4 looks at the effects of exogenous population changes. Finally, Section 5 concludes.

2 A model of assimilation

2.1 Basic setup

Imagine a country with a population normalized to unity. The country has two distinct cultural groups at some starting date, a minority with population measure $L_0 < 1/2$ and a majority with population measure $1 - L_0$. For simplicity the question of how the ethnic structure was formed before time zero is not examined, for a model of immigration and culture see Kónya (1999). For the time being the model also abstracts away from changes in the population structure other than through assimilation. In particular the natural growth of populations is zero. I will relax this assumption later.

The minority is assumed to be small in the sense that there are no incentives for the majority to learn the culture of the minority. This assumption is made in order to avoid strategic considerations in the assimilation and learning decisions. Thus the majority does not have any active role in the model, and we can concentrate on minority decisions. For a static model that incorporates strategic learning see Kónya (2000).

People are assumed to live for one period, and they have exactly one offspring. Agents maximize dynastic utility, which is identical to having infinitely lived households. There are two possible choices a minority member can make. First, she may decide to assimilate into the majority culture completely. Second, she can remain in the minority and choose not neither to assimilate. In the latter case, her offspring still belongs to the minority and faces the same decisions next period. In the first case, the offspring (and each successive generation) becomes a member of the majority and all links to the minority are severed. As discussed above, there is no reverse assimilation.

To highlight the role of intercultural exchange, assume that production takes the form of random matching. To produce, people are arranged in pairs according to a random device. Depending on the characteristics of a pair, a surplus is generated and shared equally. A match between members of the same group generates a surplus of one for both parties. Matches between majority members and minority members involve a "cultural" transaction cost, and generate a surplus of $\theta < 1$. Agents' period utility is given by the probability weighted sum of possible match outcomes, and the probability weights are simply the population measures. Given this description of the production process, the period surpluses that correspond to each minority choice are easily obtained. Using π_n and π_a to indicate the surplus of a minority and majority member, these are:

$$\pi_n = L + \theta(1 - L)$$
$$\pi_a = 1 - L + \theta L,$$

Random matching is a very crude but efficient way to capture the plausible assumption that belonging to a culture is subject to increasing returns to scale. In particular, a larger culture offers many more opportunities in occupational, cultural and recreational choices. It is possible to derive functional forms similar to the ones above from a fully specified trade model (Kónya 2000), but one has to make very specific functional assumptions. In this context I choose to sacrifice some of the micro-foundations for the easy handling of additional parameters and for simple functional forms.

In addition to the static considerations, agents' choices are also influenced by dynamic factors. First, assimilation changes the relative size of the two populations, and thus influences the future value of the different choices. Second, assimilation is a costly activity, and the cost might differ across generations. Let $G(c;t)$ be the c.d.f. of costs associated with assimilation at time t. At each time period the asset value of not assimilating depends on three arguments: the period gain π_n, the "capital" gain $\dot{V}_n$ and on the expected (or option) value of the choices of the next generation given the evolution of costs and benefits. At each period, agents pick the choice with the higher value.

The general model described above is not very tractable. Thus for the sake of analytical clarity, it is useful to make special assumptions on the intergenerational cost linkages. These assumptions do not influence the main qualitative conclusions, but they lead to simple functional forms. They are the following:

Assumption 1. *Assimilation costs are not inherited. That is, for each generation the cost of assimilation is drawn from the time-invariant distribution* $G(c)$*,* $c \in [K, \infty]$.

Given this assumption, the asset equations for the two choices are relatively simple. These determine the asset value of belonging to the minority or the majority. Let V_n and V_a indicate the value functions for the two choices. The optimal choice for agents will be to assimilate if and only if $c < V_a - V_n$. Let $q_a = V_a - V_n$, then the asset equations are:

$$rV_a = 1 - L + \theta L + \dot{V}_a \tag{2.1}$$

$$\begin{aligned} rV_n &= L + \theta(1-L) + \dot{V}_n + G(q_a)\left[q_a - \int_K^{q_a} c\,\frac{dG(c)}{G(q_a)}\right] \\ &= L + \theta(1-L) + \dot{V}_n + \max\left\{0, \int_K^{q_a} G(c)\,dc\right\} \end{aligned} \tag{2.2}$$

2.2 Equilibrium

The equilibrium of the model can be described by the Euler equations. Define $\gamma(x)$ as the last part in (2.2), and subtract (2.2) from (2.1). This yields, after rearranging:

$$\dot{q}_a = rq_a + \max\{0, \gamma(q_a)\} - (1-\theta)(1-2L). \tag{2.3}$$

The evolution of the state variable (L) depends on the aggregate assimilation outcome:

$$\dot{L} = -G(q_a)L. \tag{2.4}$$

The law of motion of the system is given by the two differential equations (2.3) and (2.4). Notice that $q_a \leq K$ implies $\dot{L} = 0$. This means that there are two steady states, defined by

$$\begin{aligned} r\bar{q}_a + \gamma(\bar{q}_a) &= 1-\theta \\ \bar{L} &= 0 \end{aligned} \tag{2.5}$$

and

$$\begin{aligned} r\tilde{q}_a &= (1-\theta)(1-2L_0) \\ \tilde{L} &= L_0 \end{aligned} \tag{2.6}$$

The second steady state is just the initial situation as far as assimilation is concerned. It is important to know the initial conditions under which this steady state is feasible and under which it is unique. The answer to the first question boils down to the comparison between $\tilde{q}$ and K: the initial situation is a feasible steady state if

$$\tilde{q}_a < K \quad \Rightarrow \quad (1-\theta)(1-2L_0) < rK.$$

This defines a cutoff level in L_0, given by

$$L_l \equiv \frac{1}{2} - \frac{1}{2}\frac{rK}{1-\theta}. \tag{2.7}$$

Thus the initial situation is a feasible steady state if and only if $L_0 > L_l$.

The second question concerns the uniqueness of the initial steady state. Even if it is feasible, it is possible that the other steady state with full assimilation can also arise. If the initial steady state is unique, it must be the

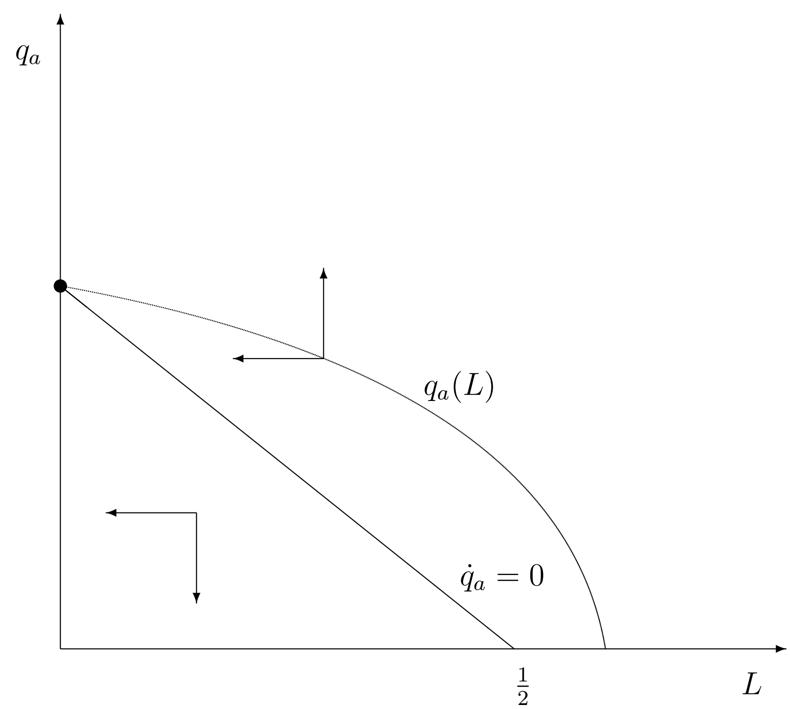

Figure 1: Phase diagram for the assimilation path

case that assimilation is not profitable even if the system would proceed on the assimilation path. In other words, the initial value of assimilation - given that the system converges to the full assimilation steady state - must be less than K. In order to check that condition, (2.3) and (2.4) has to be solved. The two equations are non-linear and thus do not yield an analytical solution, but a qualitative characterization of the solution is readily available. The Jacobian of the system evaluated at the steady state is

$$J_a = \begin{bmatrix} -G(\bar{q}_a) & 0 \\ 2(1-\theta) & r + G(\bar{q}_a) \end{bmatrix},$$

and the eigenvalues associated with it are $-G(\bar{q}_a)$ and $r + G(\bar{q}_a)$. Thus the assimilation steady state is saddle path stable, and there is a unique policy function $q_a(L)$ such that $q_a(0) = \bar{q}_a$ and $q_a'(L) < 0$.

It can also be shown that when $q_a(L) = 0$ either $L = 1/2$ or $L < 1/2$ and $q_a'(L) = -\infty$. Using the time elimination method, the slope of the policy function is given as

$$q_a'(L) = -\frac{rq_a(L) + \gamma[q_a(L)] - (1 - \rho\theta)(1 - 2L)}{G[q_a(L)]L}.$$

Suppose that $q_a(L) = 0$. This means that the denominator of the expression is zero, which implies that the slope is minus infinity, unless the numerator is also zero. For this latter case to hold, it is necessary that $L = 1/2$.

The phase diagram that graphically illustrates these results can be seen on Figure 1. The initial steady state is unique if

$$q_a(L_0) < K \quad \Rightarrow \quad L_0 > L_h \geq 0. \tag{2.8}$$

The condition under which L_h is positive is that $\bar{q}_a > K$ or $rK < 1 - \theta$. This will be the case when the discount rate is not very large, learning costs are moderate (as captured by their lower limit K) and cultural differences are sizable.

These results show that the outcome of the assimilation model depends on the initial size of the minority, L_0. Proposition 1 summarizes the possibilities:

Proposition 1. *If the minority's initial share of the population is large ($L_0 > L_h$), this share will be stable over time. If the minority's initial share is small ($L_0 < L_l$), the only equilibrium outcome is full assimilation. Finally, when $L_l \leq L_0 \leq L_h$, multiple equilibrium paths exist, including the no assimilation and full assimilation ones.*

Proof. The only thing left to show is that $L_l \leq L_h$, which proves the existence of an equilibrium and the possibility of multiple equilibria. To prove this, use (2.3) to write

$$\frac{dq_a(L_0)}{dt} = r[q_a(L_0) - \tilde{q}_a] + \gamma[q_a(L_0)] - \gamma(\tilde{q}_a).$$

Suppose $q_a(L_0) < \tilde{q}_a$, then from above it follows that $dq_a(L_0)/dt < 0$. But this is impossible, since $q_a(L_0) \leq \bar{q}_a$ and $q_a(L)$ converges towards the steady state $\bar{q}_a$. This implies that $q_a(L_0) \geq \tilde{q}_a$, which in turn yields that $L_l \leq L_h$. Figure 1 illustrates the result, since $q_a(L)$ must be above the $\dot{q}_a = 0$ line that defines $\tilde{q}_a$. □

The propositition shows that when the minority's size is in the intermediate range, multiple equilibria exists. The obvious outcomes are the full assimilation path and the initial steady state, but there are other possible outcomes. For example, it is possible that assimilation starts, but at a future time it stops. Thus even without an outside shock, the model is capable to generate a path where assimilation is a tempoprary phenomenon. Another possibility is a "zig- zag" path, where assimilation switches on and off. It must be noted, however, that such a cyclical trajectory can only be temporary. Once the minority's size gets below L_l, assimilation is inevitable. Thus limit cycles cannot arise in this framework.

2.3 The social planner's problem

This section looks at the solution for the social planner's problem, who maximizes the country's welfare. For simplicity welfare is just the sum of individual surpluses, which implicitly assumes the possibility of compensation. The first step to the solution is to notice that the planner either chooses a full assimilation path or a non-assimilation one, but not a combination of the two. As it will be shown later, the value of both paths is unique at $t = 0$, the value of assimilation increases along the full assimilation path, and the value of assimilating is constant along the no assimilation trajectory. Thus the optimal outcome is given by the one of the two that gives a higher value at the initial position.

The social planner solves the following problem:

$$\max \int_0^\infty e^{-rt} \left\{ L[L + \theta(1 - L)] + (1 - L)(1 - L + \theta L) - L \int_K^{\bar{c}} c \, dG(c) \right\} dt$$

$$\text{s.t.} \quad \dot{L} = -G(\bar{c})L \qquad \text{and} \qquad \bar{c} \geq K,$$

where $\bar{c}$ is the cutoff for assimilation. One way to find the optimal outcome is to first solve the problem assuming assimilation, then impose the inequality condition to find the cutoff between the assimilation and no- assimilation solutions. After simplifying the instantaneous surplus function, the current value Hamiltonian, with λ_a as the *negative* of the usual dynamic multiplier, is written as

$$\mathcal{H} = L^2 + (1 - L)^2 + 2\rho\theta L(1 - L) - L \int_K^{\bar{c}} c \, dG(c) + \lambda_a G(\bar{c})L.$$

The first order condition for the control variable $\bar{c}$ is

$$\bar{c} = \lambda_a.$$

Using this in the other conditions yields

$$\begin{aligned}\dot{L} &= -G(\lambda_a)L \\ \dot{\lambda}_a &= r\lambda_a + \gamma(\lambda_a) - 2(1-\theta)(1-2L).\end{aligned}$$

Just as in the equilibrium case, the solution can be characterized qualitatively. The steady state is given by

$$\begin{aligned}\bar{L} &= 0 \\ \bar{\lambda}_a + \gamma(\bar{\lambda}_a) &= 2(1-\theta),\end{aligned} \tag{2.9}$$

and the Jacobian that corresponds to it is

$$J_p = \begin{bmatrix} -G(\bar{\lambda}_a) & 0 \\ 4(1-\theta) & r + G(\bar{\lambda}_a) \end{bmatrix}.$$

The eigenvalues are $-G(\bar{\lambda}_a)$ and $r+G(\bar{\lambda}_a)$, so the steady state is saddle path stable with a monotonically decreasing policy function $\lambda_a(L)$. Assimilation will be optimal if and only if

$$\lambda_a(L_0) > K \quad \Rightarrow \quad L_0 < L_p.$$

The equilibrium and optimal outcomes now can be compared. Proposition 2 describes the welfare properties that correspond to the possible scenarios:

Proposition 2. *If the initial size of the minority is large ($L_0 \geq L_p$) the equilibrium - no assimilation - is efficient. If the initial size of the minority is small ($L_0 \leq L_l$), the equilibrium steady state - full assimilation - is also efficient. If $L_h < L_0 < L_p$, the equilibrium steady state is inefficient, whereas if $L_l < L_0 < L_h$ the steady state may or may not be efficient. Even if the equilibrium steady state is efficient, the rate of assimilation on the full assimilation equilibrium path is too slow.*

Proof. To prove all the claims it is enough to verify that $\lambda_a(L) > q_a(L)$. To see this, first note that $\bar{\lambda}_a > \bar{q}_a$. This follows from the fact that $rx + \gamma(x)$ is an increasing function of x. Second, for any L,

$$\dot{\lambda}_a - \dot{q}_a = r(\lambda_a - q_a) + \gamma(\lambda_a) - \gamma(q_a) - (1-\theta)(1-2L).$$

Suppose $\lambda_a(L) < q_a(L)$, then $\dot{\lambda}_a - \dot{q}_a < 0$. But this means that $\bar{\lambda}_a < \bar{q}_a$, which leads to a contradiction. Thus it is necessary that $\lambda_a(L) > q_a(L)$, which implies that $L_h < L_p$.

□

The inefficiency of the full assimilation equilibrium trajectory - even when the steady state is efficient - follows from the positive external effect assimilation has on the majority. Due to the random matching assumption and the equal sharing of the surplus, majority members benefit from meeting an assimilated minority member, but the latter do not take this into account. The possible inefficiency of the no-assimilation steady state arises partly from this externality (when $L_h < L_0 < L_p$), and from the coordination problem that leads to multiple equilibria when $L_l < L_0 < L_h$. Since both the no-assimilation and assimilation paths are equilibrium ones, there is nothing to guarantee that individual decisions lead to the socially optimal choice.

2.4 Comparative dynamics

Comparative dynamics looks at the effects of parameters on the path of the endogenous variables. In the current case, the parameters are θ and r. Figure 2 shows how an increase in θ changes the policy function $q_a(L)$. If the two cultures are more similar (θ large), assimilation is less attractive. Formally, it is easy to show that in the steady state

$$\frac{\partial \bar{q}_a}{\partial \theta} = -\frac{1}{r + G(\bar{q}_a)} < 0.$$

Now compare the policy function for $\theta_1 < \theta_2$. If $q_a(L_1, \theta_1) < q_a(L_1, \theta_2)$ for some $L_1 > 0$, then by continuity there must exist $\hat{L} < L_1$ such that $q_a(\hat{L}, \theta_1) = q_a(\hat{L}, \theta_2)$ and $q_a(\hat{L}, \theta_1)$ is steeper than $q_a(\hat{L}, \theta_2)$ (see Figure 2). Using $q_a(\hat{L})$ for the common value at the intersection point, the difference in the two slopes is given by

$$\dot{q}_a(\hat{L}, \theta_1) - \dot{q}_a(\hat{L}, \theta_2) = \frac{(\theta_2 - \theta_1)(1 - 2\hat{L})}{G(\hat{L}) q_a(\hat{L})} > 0.$$

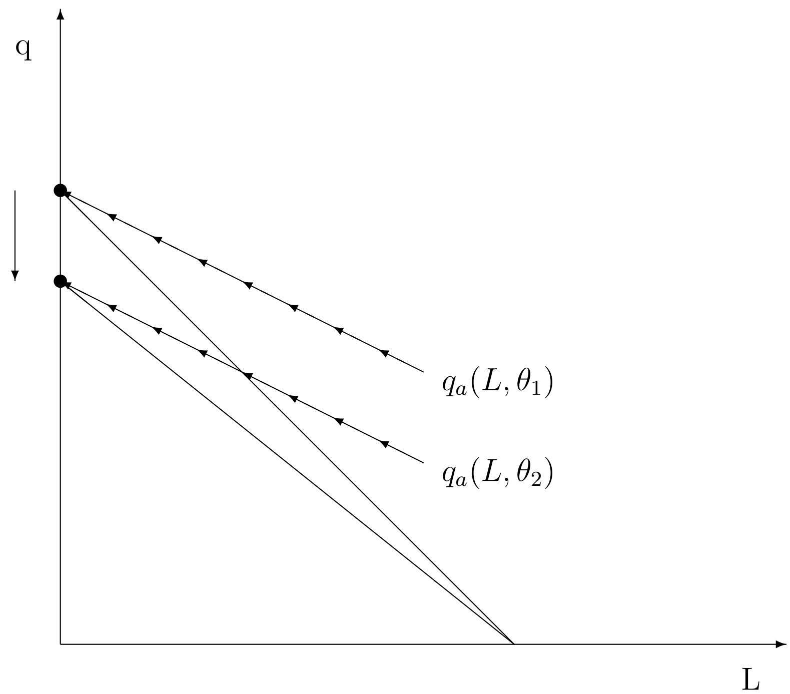

Figure 2: The effect of θ on $q_a(L)$

But the policy functions are downward sloping, so this result would imply that the policy function at θ_2 is steeper, which is a contradiction. Thus the two policy functions cannot intersect, hence $q(L)$ is decreasing in θ.

One consequence of this result is that as θ increases, assimilation slows down, since the speed of convergence is given by $G(q_a)$. Second, it is easy to see that $\tilde{q}_a$ is also decreasing in θ (see [2.6]). Together with the previous result, these imply that the cutoff levels L_l and L_h also decline with θ. Thus when the cultural difference between the minority and the majority is smaller (θ large), assimilation is less likely, since the range of the unique initial steady state expands, whereas the range of the unique assimilation steady state shrinks. Without explicitly solving for the policy function $q_a(L)$, the effect of an increase in θ on the range of multiple equilibria cannot be determined.

The intuition behind this result is that gains from assimilation diminish when the two groups are more similar. This has two interesting implications. First, worries over the effects of globalization on minority cultures might not be justified. If globalization increases θ, the direct effect is for the majority and minority cultures to become more similar. On the other hand, the indirect effect is to make assimilation less likely. Under some parameter values a large enough increase in θ can stop assimilation entirely. Thus the model can explain the experience of minority groups such as the Scottish and Welsh in Britain, the Catalans and Basques in Spain, or the Quebecois in Canada, whose identity have become stronger in the last decades.

The second implication of the results is that an increase in θ might have an ambiguous effect on the majority welfare. If the initial equilibrium is the assimilation path, more cultural similarity leads to more efficient matches across groups, but also to slower (or no) assimilation. This second, indirect, effect has a negative impact on majority welfare. Given that the equilibrium is not socially optimal (due to the external effects on natives), the Envelope Theorem *cannot* be invoked to ignore the behavioral response. Thus more cultural similarity can actually make natives worse off.

The second parameter of the model is r, the discount rate. Its effects can be analyzed exactly the same way as the effects of θ, with similar conclusions. An increase in the discount rate will lower both $q_a(L)$ and $\tilde{q}_a$, since future gains from assimilation are discounted more heavily. Thus the cutoff levels in L_0 decrease, and assimilation slows down on the full assimilation path. Because of less assimilation, majority welfare decreases. The interpretation of these results is less clear cut then for θ, so it is left to the imaginative reader.

The effect of the parameters on the social optimum can also be analyzed. Since the calculation of $\lambda_a(L)$ is entirely analogous to the determination of $q_a(L)$, the comparative dynamics results are also the same. Thus an increase in θ or r decreases $\lambda_a(L)$, which implies a bigger range for non-assimilation and a lower assimilation rate along the full assimilation path. In contrast to the equilibrium, an increase in θ must increase total welfare along the optimal path. This is just a consequence of the Envelope Theorem, which applies for the planner's problem.

3 Multiculturalism or a melting pot?

This section extends the analysis to include a third choice for minority members. This choice is to become bilingual, but to retain identity as a minority member - an option that will be referred to as learning. A further modification of the model involves the inefficiency parameter θ. It is reasonable to assume that becoming bilingual eliminates some, but not all aspects of the inefficiency, perhaps because the minority is physically separated from the majority. In particular, assume that there are two inefficiency parameters, ρ and θ, where the latter can be eliminated by learning. Thus a match involving a bilingual person and a majority member yields a surplus of ρ, and a match between other minority members and majority members yields $\rho\theta$.

The costs for learning and the evolution of these costs for future generations also needs to be specified. There could be many different formulations, but one yields relatively straightforward analytical results. Thus let the initial cost of learning - nobody else was bilingual in the family before - is the same as the cost of assimilation. For future generations, however, assume that the cost of remaining bilingual is zero. One possible justification for this second assumption is that bilingual parents can (and usually do) teach their children both languages at a very young age, when the costs of learning are very low. Next, assume that there is a switching cost for bilingual families that is sufficiently high to prevent them from assimilation. This statement will be clarified later. Finally, now it is sufficient to look at the case of $K = 0$.

Here attention will be restricted to the social planner's solution, as the question about bilinguality versus assimilation is an important policy problem. The equilibrium solution is very similar to that in the basic section, and the equilibrium can be characterized by ranges in L_0. If the initial size of the minority is small, the unique equilibrium path is assimilation. If the initial size of the minority is large, the unique equilibrium trajectory is becoming bilingual for the whole minority. Finally, in middle ranges of L_0 multiple equilibria are possible, including the two extreme cases.

The social planner's problem is to choose whether the country should follow the learning or the assimilation path, and then to find the optimal trajectories for the chosen direction. Similarly to the basic model, the optimal paths that correspond to a particular direction can be obtained, and then compared to find the cutoff between the two possibilities. The planner maximizes the sum of utilities over time, where the period surpluses are given by

$$\begin{aligned}\pi_n &= L + \rho\theta(1-L)\\ \pi_a &= 1 - L + \rho b L + \rho\theta(1-b)L\\ \pi_b &= L + \rho(1-L),\end{aligned}$$

where b is the share of bilinguals in the minority population. The dynamic constraints are

$$\dot{L} = -G(\bar{c})L$$

if assimilation occurs and

$$\dot{b} = G(\bar{c})(1-b)$$

if learning takes place. Let $\bar{c}$ be the cutoff for either learning or assimilation on the appropriate optimal path.

The problem of solving for the optimal learning path can be summarized by the following current value Hamiltonian (ignoring constant terms):

$$\mathcal{H} = 2[\rho b(1-L_0) + \rho\theta(1-b)(1-L_0)] - (1-b)L_0 \int_0^{\bar{c}} c\, dG(c) + \lambda_b G(\bar{c})(1-b)L_0,$$

where the constraint is written this way for reasons of comparability. The first-order conditions can be simplified to yield

$$\begin{aligned}\dot{b} &= G(\lambda_b)(1-b)\\ \dot{\lambda}_b &= r\lambda_b + \gamma(\lambda_b) - 2\rho(1-\theta)(1-L_0).\end{aligned}$$

Since L_0 is constant and its law of motion is independent of b, λ_b jumps immediately to its steady state value, implicitly defined by

$$r\lambda_b + \gamma(\lambda_b) = 2\rho(1-\theta)(1-L_0). \tag{3.1}$$

To calculate the full assimilation path, the current value Hamiltonian can be defined as

$$\mathcal{H} = L^2 + (1-L)^2 + 2\rho\theta L(1-L) - L\int_0^{\bar{c}} c\, dG(c) + \lambda_a G(\bar{c})L,$$

where for future convenience the dynamic multiplier is the negative of the usual one. The first-order conditions are

$$\begin{aligned}\dot{L} &= -G(\lambda_a)L\\ \dot{\lambda}_a &= r\lambda_a + \gamma(\lambda_a) - 2(1-\rho\theta)(1-2L).\end{aligned}$$

The system is non-linear, but it is easy to characterize the solution qualitatively. In fact, the properties of the system are the same as in the basic model, therefore a unique policy function $\lambda_a(L)$ exists that is decreasing in L. The size of the minority approaches zero, and the steady state value of $\lambda_a(L)$ is defined by

$$r\bar{\lambda}_a + \gamma(\bar{\lambda}_a) = 2(1 - \rho\theta). \tag{3.2}$$

The phase diagram of the system looks exactly the same as the phase diagram on Figure 1.

The planner selects the full assimilation path if and only if $\lambda_a(L_0) > \lambda_b$. It is easy to check that the condition is satisfied when $L_0 = 0$, since $\rho < 1$. Thus for small minority sizes the optimal solution is assimilation. For large minority sizes, assimilation may or may not dominate multiculturalism. But it is possible to show that the $\lambda_a(L)$ and the λ_b schedules can intersect at most once, so that a clear separation of the optimal outcomes exists. Proposition 3 shows the result:

Proposition 3. *There exists a cutoff, $0 < L_m \leq 1/2$, such that when $L_0 < L_m$ the optimal choice is full assimilation, and when $L_0 > L_m$ the optimal path is the bilingual one. In other words, the planner chooses assimilation if and only if the minority is relatively small.*

Proof. It is sufficient to prove that the two schedules cannot intersect more than once. It has already been showed that $\lambda_a(0) > \lambda_b(0)$, so the statement is equivalent with the proposition that at a potential intersection point λ_a is steeper than λ_b. Assume that there are more than one intersections at points $L_1 < L_2 < ... < L_n$. It is evident that the slope statement must hold in L_1, given that $\lambda_a(0) > \lambda_b(0)$. In any intersection $\lambda_a = \lambda_b$, therefore

$$\begin{aligned}
|\lambda_a' - \lambda_b'| &= \frac{r\lambda_b + \gamma(\lambda_b) - 2(1-\rho\theta)(1-2L)}{G(\lambda_b)L} - \frac{2\rho(1-\theta)}{r + G(\lambda_b)} \\
&= \frac{2\rho(1-\theta)(1-L) - 2(1-\rho\theta)(1-2L)}{G(\lambda_b)L} - \frac{2\rho(1-\theta)}{r + G(\lambda_b)} \\
&= 2\frac{[2-\rho(1+\theta)]L - (1-\rho)}{G(\lambda_b)L} - \frac{2\rho(1-\theta)}{r + G(\lambda_b)} \\
&> \frac{2(1-\rho)}{G(\lambda_b)}\left(2 - \frac{1}{L}\right),
\end{aligned}$$

where the second equality utilizes (3.1), and the inequality follows from the fact that $r > 0$. The sign of the last expression only depends on L, and it was shown to be positive at L_1. Given that $L_1 < L_2 < ... < L_n$, it must be the case that in all other intersections λ_a is steeper than λ_b. The only way this can be satisfied is that $n = 1$, i.e. there is only one intersection. Thus $\lambda_a > \lambda_b$ for small L_0, and the opposite holds for large L_0. See Figure 3 for a graphical illustration. □

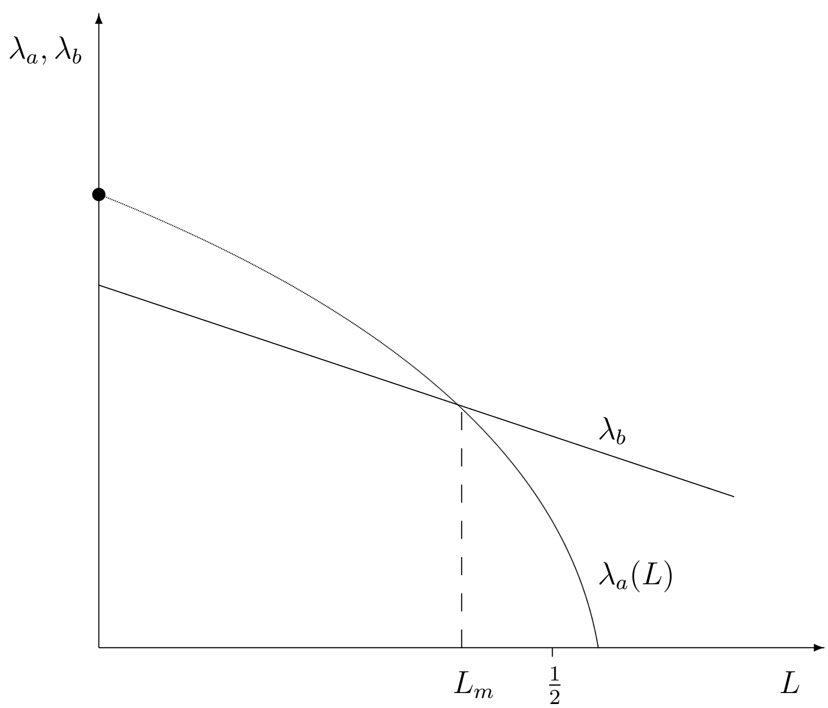

Figure 3: The value of assimilation and learning

Thus the first conclusion in this section is that small minorities should assimilate, whereas large ones should not. Even without a lower bound on assimilation costs, assimilation is *not* always optimal. This result urges caution in the debate on multiculturalism vs. melting pot: the choice between the two policies depends on the minority size. The conclusion does not depend on the cost assumption for learning (children of bilingual parents become bilingual costlessly), although the exact cutoff level does. The crucial assumption behind the result is that bilingual minority members have higher utility than assimilated ones, which is guaranteed for L large.

It is interesting to conduct comparative dynamics exercises, especially regarding the distance parameters ρ and θ. It is easy to check that λ_b increases with ρ (physical closeness), and decreases with θ (cultural similarity). Using the same method as in Section 2.4, one can show that $\lambda_a(L)$ declines with both distance measures. Thus an increase in θ has an ambiguous effect on the cutoff level L_m, although it will slow convergence in both regimes. The shifts of the two schedules in the full assimilation steady state, however, can be compared. Since $\lambda_a(0) > \lambda_b(0)$, the learning schedule will shift down *more* with an increase in θ. Assuming this holds for other values of L, an increase in cultural similarity will make assimilation more likely relative to learning. The result follows from the convexity of the $\gamma(\cdot)$ function: since the long-run prospects of assimilating are superior to learning (it is only discounting that makes learning more attractive for large L), its option value of waiting falls less with θ than the option value of waiting for learning.

An increase in ρ has a very different effect: it leads to an unambiguous decrease in L_m. A decrease in physical distance has therefore an asymmetric effect on assimilation and learning: it makes the latter more attractive relative to the former. If globalization mostly means an increase in ρ, this implies that globalization makes a multicultural society more, and a melting-pot society less attractive. This is an interesting result, and perhaps not immediately obvious. The intuition behind it is that less physical separation from the majority makes it less costly to maintain minority status, and hence assimilation is not as attractive. Gains from an intercultural interaction, however, become larger, so that learning is encouraged. More cultural similarity, on the other hand, decreases incentives to eliminate the cultural inefficiency. This has the same effect on both assimilation and learning, leaving the end result ambiguous.

These implications can be combined with the results from the basic model. With a positive lower bound on assimilation and learning costs ($K > 0$), more cultural similarity would increase the regions for non-assimilation and assimilation to the expense of learning. Thus more cultural similarity eliminates the “middle ground”: the outcome is likely to be either full or no assimilation. An increase in ρ, on the other hand, would expand the learning region to the expense of the other two. Thus the optimal response to globalization hinges crucially on whether it involves physical or cultural convergence.

An important application of the results above is the current debate about the integration of immigrants in North America and Europe. Immigration is, of course, endogenous to government policy, but at least in Europe the earlier large flows of guest workers and people from the ex-colonies have mostly dried up. Thus the model can be used to answer the question of how best these groups can be integrated (if at all) into the majority culture, ignoring future immigrant flows that would add to the minority size and composition. The interesting implication of the current model is that today multiculturalism is the likely optimal solution, whereas a hundred years ago it was most likely the "melting pot". The reasons for this change follow from the comparative dynamics, and from the observations that:

- globalization reduces the physical isolation of ethnic groups, and
- the recent arrivals are more culturally distinct from the majority group.

Both of these effects make learning more likely to be optimal relative to assimilation.

In the United States, the above argument justifies the efforts for bilingual education, at least in a form that does not discourage the children of immigrants from learning English. Another caveat concerns the response of future immigration to existing policies, see Kónya (1999) on this topic. Finally, all these results depend on the assumption that the social planner cares about both the majority and the minority. Given that both groups are part of the society (and both vote), this seems to be a reasonable postulate.

4 La revanche du berceau[1]

So far the assumptions did not allow for population changes apart from assimilation. This section examines an interesting extension that allows for faster natural growth among the minority. The inspiration is provided by the experience of French Canadians (Quebecois) in the 19th century, who thought to avoid assimilation into the ruling British culture by faster population growth. This policy, "La revanche du berceau", was actively fostered by the

[1] "The revenge of the cradle"

Catholic Church, which had a large influence on the French speakers. The goal of this section is to examine under what conditions can such a policy reverse the full assimilation outcome.

The first question to look at is whether a permanent increase in the minority population growth rate leads to a stable population distribution. Since there are no aggregate scale effects in the model, it is simpler to work with the population shares, and assume that the natural growth in the share of the minority is n. Ignoring the possibility of learning - just as in the basic model -, the equations that characterize the laws of motion are modified to

$$\begin{aligned} \dot{L} &= [n - G(q_a)]L \\ \dot{q}_a &= rq_a + \gamma(q_a) - (1-\theta)(1-2L) \\ q_a &\geq K. \end{aligned} \tag{4.1}$$

There are now three steady states: full assimilation, no assimilation[2], and stable population shares. The last of the three is characterized by the two equations

$$\begin{aligned} G(\bar{q}_a) &= n \\ r\bar{q}_a + \gamma(\bar{q}_a) &= (1-\theta)(1-2\bar{L}). \end{aligned} \tag{4.2}$$

The Jacobian of the system evaluated in this steady state can be written as

$$J = \begin{bmatrix} 0 & -g(\bar{q}_a)\bar{L} \\ 2(1-\theta) & r + G(\bar{q}_a) \end{bmatrix},$$

and the eigenvalues are

$$\mu_{1,2} = \frac{r + G(\bar{q}_a) \pm \sqrt{[r + G(\bar{q}_a)]^2 - 8g(\bar{q}_a)\bar{L}(1-\theta)}}{2}.$$

It is easy to check that the real values of the eigenvalues are both positive, therefore the constant population shares steady state is unstable. This means that a minority with a permanently higher population growth rate either assimilates at a rate higher than n, or eventually becomes the majority.

[2]This steady state is characterized by the eventual disappearance of the initial majority. Once the minority becomes the majority, the natural assumption would be to reverse the direction of possible assimilation. Here the complications that arise from this issue are ignored.

This still leaves open the possibility that a *temporary* increase in the minority growth rate stabilizes population shares. To illustrate this, assume that the goal of population policy is to increase the minority share to a sustainable size, $\bar{L} \geq L_l$. Then a possible functional form for the natural growth in the minority's share is

$$n(\bar{L} - L),$$

which decreases monotonically to zero as the minority's share approaches $\bar{L}$. The modified system can be written as

$$\begin{aligned} \dot{L} &= n\bar{L} - [n + G(q_a)]L \\ \dot{q}_a &= rq_a + \gamma(q_a) - (1-\theta)(1-2L) \\ q_a &\geq K. \end{aligned}$$

The question is, what are the combinations of $\bar{L}$ and n that lead to a sustainable non-assimilation outcome.

Given that there is no assimilation, the evolution of the system is described by

$$\begin{aligned} \dot{L} &= n(\bar{L} - L) \\ \dot{q}_a &= rq_a - (1-\theta)(1-2L). \end{aligned} \tag{4.3}$$

This is a linear system, and it can easily be solved to yield

$$\begin{aligned} L(t) &= \bar{L} - (\bar{L} - L_0)\, e^{-nt} \\ q_a(t) &= \frac{(1-\theta)(1-2\bar{L})}{r} + \frac{2(1-\theta)(\bar{L} - L_0)e^{-nt}}{r+n}. \end{aligned} \tag{4.4}$$

To simplify the formula, assume that the goal is to reach parity with the majority, so that $\bar{L} = 1/2$. The no-assimilation steady state is stable at the initial minority size if $q_a(0) < K$, which leads to the condition

$$L_0 > \frac{1}{2} - \frac{1}{2}\frac{(r+n)K}{1-\theta} \qquad \Rightarrow \qquad n > \frac{(1-\theta)(1-2L_0)}{K} - r \equiv \bar{n} \tag{4.5}$$

Thus the no-assimilation equilibrium can be sustained by population growth even when the initial minority size is below L_l (see [2.7]). The lower the initial size of the minority, the higher its population growth must be in order to survive. On the other hand, the more similar the two cultures are, the less important it is to have a high population growth. This follows from the property of the model that gains from assimilation are smaller if the cultural difference is smaller. This raises the intriguing possibility that the French Canadian society could have survived without fast population growth by moving closer to the majority culture. The increase in nationalistic sentiment in the last decades, which coincided with the decline in the power of the Catholic Church and the emergence of a modern French speaking middle class, might confirm this prediction.

5 Conclusion

This paper has examined the population dynamics of a country with two ethnic groups, a majority and a minority. It showed that small minorities are likely to assimilate, whereas large ones are not. There is, however, a middle ground, where both outcomes (and various others in between) can occur, depending on the self-fulfilling expectations of minority members.

The long-run equilibrium is efficient in many cases, although in the case of multiple equilibria the "wrong" one might be selected. The transition path to full assimilation - if that is the efficient steady state - is, however, not optimal. In particular, assimilation is too slow, because minority members do not take into account the positive external effect of their decision on the majority.

The paper then proceeded to examine an important question, that of the choice between a multicultural or a melting-pot society. If cultural (but not physical) distance can be overcome by learning, for large minority groups it is optimal to be bilingual, but not to assimilate. The choice between the two outcomes also depends on the costs of interaction. It was shown that cultural convergence between the two groups can actually halten the assimilation of the minority. This can explain the recent strengthening of identity in many minority groups.

Finally, the paper analyzed an interesting question concerning faster natural population growth in the minority. It was shown that permanently higher birth rates lead to either of two extremes: the minority either fully assimilates or it becomes the majority. If the demographic boom is temporary, it can stabilize the minority population that was on the path of assimilation. This outcome is consistent with the experience of French Canadians in the 19th century.

Possible extensions include allowing for learning in the majority. When the minority and the majority are of similar size, strategic considerations are important to include. It would also be interesting to look at more than two ethnic groups, and look at the question of multiculturalism vs melting pot again. Finally, rational immigration and native response in immigration policy would lead to further insights. These, and possible other extensions are left for future research. Hopefully, the reader is convinced that this paper already contains interesting results, and it can form as a basic for future work in this area.

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