The Forward Rate Unbiasedness Hypothesis Revisited: Evidence from a New Test

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THE FORWARD RATE UNBIASEDNESS HYPOTHESIS 
REEXAMINED: EVIDENCE FROM A NEW TEST

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ABSTRACT

Under conditions of risk neutrality and rational expectations in the foreign exchange market, there should be a one-to-one relationship between the forward rate and the corresponding future spot rate. However, cointegration-based tests of the unbiasedness hypothesis of the forward rate have produced mixed findings. In order to exploit significant cross-sectional dependencies, we test the unbiasedness hypothesis using a new multivariate (panel) unit-root test, the Johansen likelihood ratio (JLR) test, which offers important methodological advantages over alternative standard panel unit-root tests. When applied to a data set of eight major currencies in the post-Bretton Woods era, the JLR test provides strong and robust evidence in support of a unitary cointegrating vector between forward and corresponding future spot rates. However, the orthogonality condition is satisfied only for three major currencies.

Keywords: Forward rate unbiasedness, panel unit-root tests, cointegration
JEL Classification: F30, F31

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I. Introduction

The forward rate unbiasedness hypothesis (FRUH) states that, under conditions of risk neutrality and rational expectations on the part of market agents, the forward rate is an unbiased predictor of the corresponding future spot rate. Assuming the absence of a risk premium in the foreign exchange market, it must hold true that

\[ E_t(S_{t+k}) = f_t \]  

(1)

where \( f_t \) is the log forward rate at time \( t \) for delivery \( k \) periods later, \( S_{t+k} \) is the corresponding log spot rate at time \( t+k \), and \( E_t(\cdot) \) is the mathematical expectations operator conditioned on the information set available at time \( t \).\(^1\) Assuming the formation of rational expectations (Muth (1960)),

\[ S_{t+k} = E_t(S_{t+k}) + u_{t+k} \]  

(2)

\(^1\) The justification for using logarithms as opposed to levels in (1) is connected to Siegel's paradox. Assuming for a moment that the relationship in (1) is expressed in levels (without taking logs), Siegel (1972) notes that such a relationship must hold true on both sides of the market, that is, it must also hold true that \( E_t(\frac{1}{S_{t+k}}) = \frac{1}{f_t} \). However, \( E_t(S_{t+k}) = f_t \) and \( E_t(\frac{1}{S_{t+k}}) = \frac{1}{f_t} \) cannot simultaneously hold when the variables are expressed in levels due to Jensen's inequality \( E(\frac{1}{X}) > \frac{1}{E(X)} \). This problem is avoided if spot and forward rates are defined in logarithmic form.
where \( u_{t+k} \), the rational expectations realized forecast error, must have a conditional expected value of zero and be uncorrelated with any information available at time \( t \) (the orthogonality condition). Substituting (1) into (2) yields

\[
S_{t+k} = f_t + u_{t+k}.
\]

The hypothesis in (3) is generally tested by running the "levels" regression

\[
S_{t+k} = \alpha_0 + \alpha_1 f_t + u_{t+k}. \tag{4}
\]

Given that \( S_{t+k} \) and \( f_t \) are first-order integrated, or \( I(1) \), processes, the FRUH requires that \( S_{t+k} \) and \( f_t \) be cointegrated with the cointegrating vector \((1, -1)\), that is, \( \alpha_0 = 0 \) and \( \alpha_1 = 1 \) in (4). \(^3\) Under these restrictions, the forward rate does not systematically under- or over-predict the future spot rate, that is, the forward rate is a conditionally unbiased predictor of the corresponding future spot rate. In order for the FRUH to be empirically supported, \( S_{t+k} \) and \( f_t \) should share one common stochastic trend and the realized forecast error \( S_{t+k} - f_t \) should be a stationary (that is, an \( I(0) \)) process. \(^4\)

The empirical evidence on the existence of cointegration between \( S_{t+k} \) and \( f_t \) is decidedly mixed. Baillie and Bollerslev (1989) and Hai et al. (1997) find that \( S_{t+k} \) and \( f_t \) form a cointegrated system with a unitary cointegrating vector. Evans and Lewis (1993) and Alexakis and Apergis (1996) fail to even find a long-run relationship between forward and corresponding future spot rates. Ngama (1992)

\(^2\) For alternative forms of testing the FRUH and a survey of the evidence and issues involved see Baillie and McMahon (1989) and Engel (1996).

\(^3\) A series is integrated of order \( d \), denoted by \( I(d) \), if it is rendered stationary after differencing it \( d \) times.

\(^4\) Strictly speaking, the FRUH requires that \( S_{t+k} - f_t \) be a white-noise process, a stronger condition than covariance stationarity. In that sense, the cointegration of \( S_{t+k} \) and \( f_t \) with a unitary cointegrating vector is a necessary condition for the FRUH.
finds that the evidence of cointegratedness between $S_{t+k}$ and $f_t$ and the specific cointegrating relationship varies across currencies and forward contract horizon. Evans and Lewis (1995) find evidence of cointegration between $S_{t+k}$ and $f_t$ but they reject the null of a unitary cointegrating vector. Through comprehensive testing among alternative VAR specifications with respect to treatment of the constant term and lag-length structures, Luintel and Paudyal (1998) find robust evidence of cointegration between $S_{t+k}$ and $f_t$ but they reject the unitary cointegrating vector in the $(S_{t+k}, f_t)$ cointegrating relation. Copeland (1991) and Lai and Lai (1991) reject the joint null of $\alpha_0 = 0$ and $\alpha_1 = 1$ in (4) in a cointegration analysis.

In this paper, we reexamine the FRUH using a new multivariate (panel) unit-root test recently proposed by Taylor and Sarno (1998), referred to as the Johansen likelihood ratio (JLR) test. We motivate the relevance of employing a panel unit-root test on the basis of significant cross-sectional dependencies suggested by cross correlation and principal components analyses. In that context, a panel unit-root test provides efficiency gains and greater test power, as it increases the span of the data by jointly testing for a unit root across a number of series. We employ the JLR test which, contrary to standard unit-root tests, has as its null that at least one of the series under consideration is nonstationary. Its null will be rejected if and only if all of the series in the panel are stationary processes. The JLR test therefore avoids the pitfall of incorrectly inferring joint stationarity of all the series in standard panel unit-root tests, when only a subset of the panel series are indeed realizations of stationary processes. Inference drawn from the JLR test should be much more reliable.

We apply the JLR test to a data set comprised of quarterly observations for the realized forecast-error series of eight major currencies in the post-Bretton Woods

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5 Deviation of $\alpha_1$ from unity has an important effect on the stochastic properties of the risk premium $r_{t,k} = f_t - E_t(S_{t+k}) = (1 - \alpha_1)f_t - \alpha_0 - E_t(u_{t+k})$. Given that $f_t$ is a nonstationary process, the risk premium $r_{t,k}$ exhibits nonstationary behavior if $\alpha_1 \neq 1$. 
era. Cross correlation and principal component analyses suggest that the series under consideration are cross sectionally dependent except for the Canadian dollar. For the full sample of eight currencies, as well as the subset excluding the Canadian dollar, we find strong and robust support to a unitary cointegrating relationship between forward rates and corresponding future spot rates. This evidence stands in sharp contrast to the mixed findings in the empirical literature. However, the orthogonality condition for the realized forecast-error series with respect to information sets consisting of past forecast errors or forward premium values, in both single- and multi-currency contexts, holds only for the Deutsche mark, Swiss franc, and Italian lira.

The remainder of the paper is constructed as follows. Section 2 outlines the multivariate unit-root test employed. In section 3 we report the test findings. Section 4 summarizes and concludes.

II. The Johansen Likelihood Ratio (JLR) Test

Johansen (1992) suggests a maximum likelihood method to determine the number of common trends in a system of unit-root variables. Without any loss of generality, a $p$-dimensional vector autoregressive (VAR) process of $k$-th order can be written as follows:

$$\Delta X_t = \mu + \Theta_1 \Delta X_{t-1} + \cdots + \Theta_{k-1} \Delta X_{t-k+1} + \Pi X_{t-k} + \epsilon_t$$  \hspace{1cm} (5)$$

where $\Delta$ is the first-difference lag operator, $\mu$ is a $(p \times 1)$ matrix of constants, $X_t$ is a $(p \times 1)$ random vector of time-series variables with order of integration of at most one denoted by $I(1)$, $\epsilon_t$ is a sequence of zero-mean $p$-dimensional white noise vectors, $\Theta_i$ are $(p \times p)$ matrices of parameters, and $\Pi$ is a $(p \times p)$ matrix of
parameters, the rank of which contains information about long-run relationships among the variables in the VAR. Expression (5) is referred to as the vector error correction model (VECM).

If \( \Pi \) has full rank, that is, \( \text{rank}(\Pi) = p \), then all variables in the system are stationary. If the rank of \( \Pi \) is zero, then no cointegrating vectors exist. In the case of \( 0 < r < p \), \( r \) cointegrating vectors exist. In this case, there exist \((p \times r)\) matrices \( \alpha \) and \( \beta \) such that \( \Pi = \alpha \beta' \). \( \beta \) is the matrix of cointegrating vectors and has the property that \( \beta' X_t \) is stationary even though \( X_t \) may be individually \( I(1) \) processes. The null hypothesis that one or more of the system processes are nonstationary can take the form of

\[
H_0: \text{rank}(\Pi) < p
\]

and be tested against the alternative that all system processes are stationary, that is,

\[
H_1: \text{rank}(\Pi) = p.
\]

To test the hypothesis in (6), it suffices to test that the smallest of the characteristic roots of \( \Pi \) is zero, as a rejection necessarily implies that all characteristic roots of \( \Pi \) are nonzero and therefore \( \Pi \) possesses full rank. Such a test can be constructed on the basis of the following test statistic, referred to as the Johansen likelihood ratio (JLR) test statistic:

\[
JLR = -T \ln \left(1 - \lambda_p\right),
\]

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6 The rank of a matrix is equal to the number of its nonzero characteristic roots.
where \( \lambda_p \) is the smallest eigenvalue of the generalized eigenvalue problem

\[
\lambda \sum_{kk} - \sum_{k0} \sum_{00}^{-1} \sum_{0k} = 0. \quad (9)
\]

The \( S_{ij} \) matrices are residual moment matrices from the VECM in (5). Taylor and Sarno (1998) show that the JLR test statistic in (8) is asymptotically distributed as \( \chi^2(1) \) under the null hypothesis.

**III. Data and Test Results**

We analyze U.S. dollar spot and 3-month (90-day) forward rates for eight major currencies: Canadian dollar (CD), Deutsche mark (DM), British pound (BP), French franc (FF), Swiss franc (SF), Netherlands guilder (NG), Italian lira (IL), and the Japanese yen (JY). The sample period spans 1974:3 to 1996:4 at the quarterly frequency. As the sampling frequency and the maturity date of the forward contract are the same, complications arising from the use of overlapping data are avoided.\(^7\) The data represent noon-time bid prices from the New York foreign exchange market and were obtained from the Federal Reserve Board of Governors. The 3-month forward rates are matched with the corresponding future spot rates and the empirical analysis is implemented on the realized forecast-error series \( s_{t+k} - f_t \), where \( k \) is a 3-month period. That is, we impose the vector \( (1, -1) \) on the \( (s_{t+k}, f_t) \) system, implying the restrictions \( a_0 = 0 \) and \( a_1 = 1 \) in (4). If the forecast-error series

\(^7\) When the observation and forecast periods do not coincide, that is, when the time to maturity for the forward contract exceeds the time interval between observations, the error term in the cointegrating regression in (4) is a noninvertible moving average process (Hansen and Hodrick (1980)). In that case, Moore (1992) shows that the Johansen cointegration methodology is inapplicable, as the Granger representation theorem breaks down in the presence of noninvertible moving average errors. Therefore, estimates and test results reported in previous empirical studies employing the Johansen technique are invalid when the observation period is shorter than the duration of the forward contact maturity. As suggested by a referee, violations of the FRUH will probably occur at greater than quarterly frequencies (such as weekly or daily). However, for the econometric reasons given above, we restrict our attention to quarterly data in order to ensure that observation and forecast periods coincide.
are found to be stationary processes, then the necessary condition for the FRUH is satisfied.\textsuperscript{8}

Summary statistics and other preliminary data analysis are reported in Table 1.\textsuperscript{9} There is no evidence of skewness, excess kurtosis, or conditional heteroscedasticity in any of the series. We next implement correlation and principal component analyses to identify cross-sectional dependencies among the system variables, thus suggesting that implementation of a panel unit-root test would yield substantial efficiency gains. As reported in Table 1, there are substantial correlations among the panel forecast-error series, with the notable exception of the Canadian dollar. The Lagrange multiplier test statistic for the null that the estimated residual covariance matrix of the system's variables is diagonal (for the absence of correlations among the forecast-error series) is 1409.90. Compared to the 5 per cent critical value of 41.34, this statistic massively rejects the null hypothesis of cross-sectional independence. In addition, a principal component analysis of the covariance matrix of the series under consideration is performed to extract evidence of common factors. Final communality estimates ($h^2$), which represent the proportion of each series' variance explained by common factors, are presented in Table 1. The results suggest that one common factor, or latent variable, can explain a significant proportion of the variation in each of the series except the Canadian dollar. Both cross correlation analysis and principal component analysis suggest that the forecast-error series under consideration exhibit significant commonalities in their time series behavior. The only exception is the Canadian dollar, which does not appear to be closely linked to the behavior of the remaining currencies.

\textsuperscript{8} This methodology (imposition of the cointegrating vector implied by theory, and evaluation of the resulting residual series) is termed the "restricted cointegration test" in Liu and Maddala (1992).

\textsuperscript{9} The $I(1)$ behavior of $S_{t+k}$ and $f_t$ has been established in numerous papers (see, e.g., Meese and Singleton (1982), Baillie and Bollerslev (1989)). We establish similar evidence for the spot and forward rate series in our sample using Augmented Dickey-Fuller and Phillips-Perron unit-root tests. Given that this evidence is a widely accepted stylized fact, it is not reported here but it is available upon request.
Given the above results, the application of a panel unit-root test to the system of forecast-error series is bound to lead to substantial efficiency gains in estimation by exploiting the cross-equation dependencies. Standard panel-unit root tests suggested by Levin and Lin (1992, 1993), Im et al. (1995), O’Connell (1998), and others, have as their null hypothesis that all variables in the panel are realizations of unit-root processes. This null will be rejected if even one of the series in the panel is stationary. Under these conditions, rejection of the null leads to the misleading inference that all series in the panel are realizations of stationary processes. The rejection frequencies for such tests are sizable even when there is a single stationary process (with a root near unity) in the panel (Taylor and Sarno (1998)). The JLR test differs fundamentally from the standard panel-unit root tests in how it establishes its null and alternative hypotheses. In the JLR test the null is that at least one of the series in the panel is a unit-root process with the alternative being that all series in the panel are stationary processes. Rejection of the null hypothesis implies that all (not some) of the series are realizations of stationary processes. Therefore, the JLR test provides a useful alternative to standard panel unit-root tests as it avoids the pitfalls of incorrect inference inherent in the standard tests. Through Monte Carlo simulations, Taylor and Sarno (1998) show that the JLR test has good size and power properties.

Table 2 reports the JLR test results. As the JLR test statistic is asymptotically distributed as $\chi^2(1)$, its 5 per cent critical value is 3.84. To account for finite-sample bias, we also adjust the asymptotic critical value by the Reinsel-Ahn scale factor $\frac{T}{T-pk}$, where $T$ is the number of observations, $p$ is the number of series in the panel (dimension of the system), and $k$ is the lag order in the VECM in (5). Using response surface analysis, Taylor and Sarno (1998) show that such an adjustment produces a reasonable approximation to the finite-sample critical values, thus avoiding significant size distortions. The adjusted critical value at the 5 per cent
level is 4.6702. For the full panel of forecast-error series for the eight currencies, the value of the obtained JLR test statistic is 19.5084. Therefore, the null hypothesis that the matrix of long-run multipliers is less than full rank (implying that one or more of the system variables is a nonstationary process) is strongly rejected in favor of a full rank impact matrix, thus implying that all system variables under consideration are realizations of stationary processes.\(^{10}\)

We also apply the JLR test to the panel of series excluding the Canadian dollar, given the lack of significant cross-sectional interdependencies between the Canadian dollar and the remaining forecast-error series. The obtained JLR statistic is 21.7371 and, given an adjusted critical value of 4.5473 at the 5 per cent level, the null hypothesis is easily rejected in favor of the alternative that all series in the panel are stationary processes. Therefore, our collective findings unequivocally support the hypothesis that forward and corresponding future spot rates are one-to-one cointegrated for the eight major currencies.

Given the stationarity of the realized forecast-error series, we now conduct statistical tests to examine whether the temporal structure of the series in question is consistent with white-noise behavior. The FRUH in expression (3) states that \(u_{t+k}=S_{t+k} - f_t\), which is an approximate measure to the rate of return to speculation, should be unforecastable, that is, orthogonal to information available at time \(t\).\(^{11}\) We consider two time \(t\) information sets available to economic agents: i) lagged values of the forecast-error series in both single- and multi-currency contexts, and ii) forward premium values in both single- and multi-currency contexts.

Table 3 reports Box-Pierce serial correlation test statistics for each of the forecast-error series. At the 5 per cent level, there is evidence of serial dependence only for the JY while weaker evidence of serial correlation (at the 10 per cent level)

\(^{10}\) This inference is robust to the specification of lag order in the VECM.

\(^{11}\) A one-to-one cointegrating relationship between \(s_{t+k}\) and \(f_t\) is only a necessary condition for the FRUH.
is obtained for the BP and the NG. To further assess the statistical dependence and predictive power of any temporal structure in the stochastic behavior of the realized forecast-error series, we fit an autoregressive model of order two (AR(2)) to each of the series. The regression test results reported in Table 2 suggest that only for the BP and JY are the AR coefficients statistically significant. However, in these currencies’ AR models, the adjusted $R^2$ measures are very low (6.27% and 2.7%, respectively) thus suggesting limited explanatory power for this forecasting model.

We also analyze the conditional dependence of the realized forecast-error series in a multi-market context. In addition to each currency’s lagged values, we also consider one or two lagged values of other currencies’ forecast-error series in the time $t$ information set (analogous to a VAR model). There is no evidence of multi-currency correlations for any currency’s forecast-error series. The overall evidence appears to sustain the orthogonality principle when conditioning on lagged forecast-error values. In the limited cases where deviations from orthogonality are observed, the explanatory power of the underlying structure is very small.

We next examine the forecastibility of the realized forecast-error series conditional on time $t$ forward premium values in both single- and multi-currency contexts. A number of studies (e.g. Hodrick and Srivastava (1986), Bekaert and

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12 Coefficient estimates, F-statistics, and adjusted $R^2$ for the multi-market framework are not reported here but are available upon request.

13 The forward premium is defined as $s_t - f_t$.

14 In order for the regression of the forecast-error series on the forward premium series to be "balanced", or non-spurious, the forward premium series must be of the same integration order as the forecast-error series, that is, it must be a stationary process (given the obtained evidence of stationarity for the forecast-error series). There is a debate in the literature regarding the integration order of the forward premium series. Crowder (1994) finds $s_t - f_t$ to be a unit-root process whereas Hai et al. (1997) reach the opposite conclusion. Baillie and Bollerslev (1994) find $s_t - f_t$ to be a fractionally integrated process. We apply the JLR test to our panel of forward premium series for the eight currencies and document strong evidence supporting that the forward premium series under consideration are realizations of stationary processes. These results are not reported here but are available upon request. Therefore, the regression of forecast-error series on the forward premium series is balanced, and non-spurious.
Hodrick (1992)) find that the forecast-error series is a function of forward premium values. The test results reported in Table 3 show that the forward premium enters significantly in the CD, BP, SF, NG, and JY equations in the single-currency framework. In the multi-currency framework, there is evidence, at the 5 per cent level, of cross-market dependencies for the CD and NG.

The combined evidence indicates that the necessary conditions for the FRUH, that is, the one-to-one cointegrating relationship between \( S_{t+k} \) and \( f_t \), is sustained for all currencies under consideration. However, the orthogonality condition, with respect to information sets consisting of either past forecast-error values or forward premia, implied in equation (3) is maintained only for the DM, FF, and IL.

### IV. Conclusions

We apply the JLR test to test the hypothesis that the forward rate is an unbiased predictor of the corresponding future spot rate. The usage of a panel unit-root test increases the test power as it exploits significant cross-sectional dependencies found among the panel series. The JLR test offers important

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15 It must be noted that the predictive power of the forward premium is not sizable, as suggested by the low values of the adjusted \( R^2 \) reported in Table 3.

16 Expanding the lag structure in both single- and multi-currency regression equations does not yield any additional statistically significant correlations.

17 This analysis does not preclude the presence of a time-varying stationary risk premium in the forward rate. Maintaining the rational expectations assumption but now assuming that agents are risk averse implies that \( f_t = E_t(S_{t+k}) + \eta r_{t,k} \) and \( S_{t+k} = f_t - \eta r_{t,k} + u_{t+k} \), where \( \eta r_{t,k} = f_t - E_t(S_{t+k}) \) is the rational expectations risk premium. Risk aversion entails that, in equilibrium, the logarithm of the forward exchange rate is the conditional expectation of the logarithm of the future spot rate plus a risk premium on forward contracts that expose economic agents to foreign exchange risk.

Consequently, the finding that \( S_{t+k} \) and \( f_t \) are cointegrated with a unitary cointegrating vector is consistent with the presence of a time-varying risk premium as seen in the equation \( S_{t+k} = f_t - \eta r_{t,k} + u_{t+k} \). However, the time-varying risk premium, if it exists, must be stationary in nature. In that case, the forward rate would be a conditionally biased forecast of the future spot rate if the risk premium is forecastable on the basis of time \( t \) information available to market participants. For example, time-varying currency risk premia based on equilibrium models reflect moments of relevant macroeconomic variables (see, for example, Hodrick and Srivastava (1984)). It must be noted that the JLR test is asymptotically immune to the omission of a stationary risk premium.
methodological advantages over alternative standard panel unit-root tests. Applied to a panel of forecast-error series for eight major currencies over the post-Bretton Woods era, the evidence overwhelmingly supports a slope coefficient of unity in the cointegrating regression between forward and corresponding future spot rates. This strong and robust finding contrasts with the mixed evidence in the current literature. However, the forward rate unbiasedness hypothesis is rejected for all currencies but the Deutsche mark, Swiss franc, and Italian lira due to failure of the orthogonality condition.
References


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Table 1. Preliminary Analysis of Forecast Error Series

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<tr>
<th>Test Statistics</th>
<th>CD</th>
<th>DM</th>
<th>BP</th>
<th>FF</th>
<th>SF</th>
<th>NG</th>
<th>IL</th>
<th>JY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0004</td>
<td>-0.0002</td>
<td>-0.0026</td>
<td>-0.0057</td>
<td>-0.0002</td>
<td>-0.0013</td>
<td>-0.0071</td>
<td>-0.0033</td>
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<td>0.0639</td>
<td>0.0586</td>
<td>0.0607</td>
<td>0.0741</td>
<td>0.0642</td>
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<td>0.1910</td>
<td>0.1985</td>
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<tr>
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<td>-0.4922</td>
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<td>1.1328</td>
<td>1.7036</td>
<td>1.4502</td>
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<td></td>
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<td>(0.9712)</td>
<td>(0.8890)</td>
<td>(0.7900)</td>
<td>(0.8354)</td>
<td>(0.9613)</td>
<td>(0.9091)</td>
<td>(0.4276)</td>
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Correlations

<table>
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<tr>
<th></th>
<th>CD</th>
<th>DM</th>
<th>BP</th>
<th>FF</th>
<th>SF</th>
<th>NG</th>
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<th>JY</th>
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<td>DM</td>
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<td>FF</td>
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<td>0.6416</td>
<td>0.5086</td>
<td>0.6342</td>
<td>0.6653</td>
<td>0.6275</td>
<td>0.4849</td>
<td>1.000</td>
</tr>
</tbody>
</table>

LM(diagonal) 1409.90

$h^2$ for Factor Analysis

<table>
<thead>
<tr>
<th></th>
<th>1 Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0510</td>
</tr>
</tbody>
</table>

Notes: The data are the forecast error series $s_{t+k} - f_t$, where $f_t$ is the log forward rate at time $t$ for delivery date $t+k$, where $k$ is a 3-month (90-day) period, and $s_{t+k}$ is the corresponding future spot rate at time $t+k$, for the Canadian dollar (CD), Deutsche mark (DM), British pound (BP), French franc (FF), Swiss franc (SF), Netherlands guilder (NG), Italian lira (IL), and the Japanese yen (JY). The data are quarterly observations covering the period 1974:3 to 1996:4. ARCH(4) is Engle’s (1982) LM test statistic for ARCH effects of order 4 (marginal significance levels are given in parentheses). The $h^2$ is the final communality estimate of a particular forecast-error series and represents the part of its variance that is related to the common factors. The LM(diagonal) test is the Lagrange multiplier statistic $T \sum \sum r_{ij}^2$, where $r_{ij}^2$ is the $ij$th residual correlation coefficient; it is $\chi^2$ distributed with $p(p-1)/2$ degrees of freedom, where $p$ is the number of series, and it is $\chi^2(28) = 41.34$ in our case.
### Table 2. Johansen Likelihood Ratio (JLR) Test Statistics

<table>
<thead>
<tr>
<th>Currencies</th>
<th>Test Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD, DM, UK, FF, SF, NG, IL, JY</td>
<td>19.5084</td>
</tr>
<tr>
<td>DM, UK, FF, SF, NG, IL, JY</td>
<td>21.7371</td>
</tr>
</tbody>
</table>

**Notes:** See notes in Table 1 for data details. The null hypothesis in the JLR test is that at least one of the system variables under consideration is a unit-root process with the alternative being that all system variables are stationary processes. The Vector Error Correction Model (VECM) estimated is of order two, which results in serially uncorrelated residual vectors in all system equations. Inference is not sensitive to the specification of lag structure used in the VECM. The asymptotic distribution of the JLR test is $\chi^2(1)$; consequently, the 5% asymptotic critical value is 3.84. The 5% critical value adjusted for finite-sample bias is given by $\chi^2(1) \frac{T}{T - pk}$, where $T$ is the number of observations, $p$ is the number of system variables (dimension of the system), and $k$ is lag order in the VECM. At the 5% level, the adjusted critical value is 4.6702 (4.5473) for the system of eight (seven) currencies.
Table 3. Dependence Test Results for the Forecast Error Series

<table>
<thead>
<tr>
<th>Test Statistics</th>
<th>CD</th>
<th>DM</th>
<th>BP</th>
<th>FF</th>
<th>SF</th>
<th>NG</th>
<th>IL</th>
<th>JY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q(2)</td>
<td>0.5118</td>
<td>2.1106</td>
<td>5.8362*</td>
<td>2.4824</td>
<td>1.6571</td>
<td>3.0933</td>
<td>2.8828</td>
<td>3.9021</td>
</tr>
</tbody>
</table>

Model: \( S_{t+k} - f_t = \alpha_0 + \sum_{j=1}^{2} \alpha_j L^j (S_{t+k} - f_t) + u_{t+k} \)

\( \alpha_0 \)  
0.0001 | 0.0002 | -0.0025 | -0.0042 | 0.0014 | -0.0008 | -0.0063 | -0.0033 |
\( \alpha_1 \)  
0.0715 | 0.1145 | 0.2631** | 0.1574 | 0.0933 | 0.1559 | 0.1692 | 0.2236** |
\( \alpha_2 \)  
-0.0288 | -0.1350 | -0.2001* | -0.0979 | -0.1137 | -0.1513 | -0.1196 | -0.0874 |
Adjusted R\(^2\) 
-0.0177 | 0.0065 | 0.0627 | 0.0075 | -0.0025 | 0.0190 | 0.0142 | 0.0270 |

Model: \( S_{t+k} - f_t = \alpha_0 + \alpha_1 (S_t - f_t) + u_{t+k} \)

\( \alpha_0 \)  
0.0057* | -0.0068 | 0.0094 | -0.0017 | -0.0155 | -0.0080 | -0.0006 | -0.0230** |
\( \alpha_1 \)  
1.5879*** | 1.2288 | 1.8990** | 0.6115 | 1.7560** | 1.9507** | 0.3934 | 2.9624*** |
Adjusted R\(^2\) 
0.0729 | 0.0142 | 0.0480 | 0.0035 | 0.0414 | 0.0460 | -0.0043 | 0.1273 |

Model: \( S_{t+k}^{i} - f_t^{i} = \alpha_0 + \sum_{j=1}^{8} \alpha_j (S_t^{i} - f_t^{i}) + u_{t+k}^{i} \)

Adjusted R\(^2\)  
CD 0.2600 | 0.0006 | 3.6533*** | -1.6188* | -0.7545 | -0.0639 | -0.1261 | 0.7671 | -0.6627** | 0.6325 |
DM 0.0332 | -0.0208 | 0.1857 | -4.1567 | 0.6656 | 1.4523 | 1.2882 | 3.3361 | -1.6368* | 1.0892 |
BP 0.06346 | -0.0068 | -2.6715 | -2.7926 | 3.1808* | -0.2389 | 1.3020 | 1.6121 | -0.8531 | -0.3300 |
FF 0.0521 | -0.0278 | -2.2457 | -4.1329 | 1.3973 | 1.7170 | 2.3963 | 2.1577 | -1.4224* | 0.7165 |
SF 0.0292 | -0.0414 | 0.7143 | -5.1852 | -0.9925 | 1.0033 | 3.4249 | 1.7940 | -1.1866 | 2.3734 |
NG 0.0596 | -0.0178 | -0.2332 | -5.7866** | 0.8359 | 1.4608 | 1.8226 | 4.7600** | -1.4827* | 0.6912 |
IL -0.0436 | 0.0068 | -1.0717 | -1.9854 | 2.1252 | 0.4371 | 1.5254 | -0.1388 | 0.0283 | -0.4524 |
JY 0.0790 | -0.0332 | 1.3044 | -1.3994 | -1.6559 | 0.9972 | -0.5290 | 1.0570 | -0.5589 | 4.4205*** |

Notes: See notes in Table 1 for data details. \( Q(k) \) is the Box-Pierce test statistic for serial dependence of order \( k \). Adjusted R\(^2\) is the adjusted coefficient of determination. \( j = 1, \ldots, 8 \) represent the currencies CD, DM, BP, FF, SF, NG, IL, and JY, respectively. ***. **, and * denote statistical significance at the 1, 5, and 10 per cent levels, respectively.