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Abstract

We use the Michigan Panel Study on Income Dynamics to decompose the well-known rise in the cross-sectional variance of individual male earnings in the U.S. into permanent and transitory components. We find that about half of the increase has arisen from an increase in the variance of the permanent component of earnings and half from an increase in the variance of the transitory component over the period 1969-1991. In contrast to previous work by the authors, we employ a formal model for earnings dynamics. We also show that our results are robust to alternative models for earnings dynamics used in past studies in the literature. Finally, we draw out the implications of our analysis for the study of earnings mobility and show that the findings obtained in recent studies of trends in mobility are contained in our earnings dynamics model.
It has now been firmly established that the cross-sectional dispersion of male earnings in the U.S. has experienced a secular upward trend beginning in the late 1960s (see Levy and Murnane, 1992 for a review). Inequality in earnings grew not only from an increase in returns to education and experience but also from an increase in inequality within groups of workers of similar age and education. Furthermore, the increase in inequality appears to have occurred throughout the earnings distribution, for the proportion of high-earnings workers as well as of low-earnings workers increased during the 1970s and 1980s. An increase in the dispersion of wage rates has accompanied that of earnings.

This study decomposes the increase in the cross-sectional variance of male earnings and wage rates into permanent and transitory components. Traditionally it has been assumed that the rise in inequality has been a result of an increase in the dispersion of permanent earnings ("the rich getting richer," "the poor getting poorer," etc.) but mathematically a cross-sectional variance can also increase if the variance of year-to-year, transitory fluctuations rises. The latter could occur if earnings variability—or labor market instability, more generally—increases. To estimate permanent and transitory components requires using panel data on workers rather than the cross-sectional data sets (like the Current Population Survey) that have been used for most of the analysis of trends in inequality. We use the Michigan Panel Study of Income Dynamics (PSID) over the period 1969-1991 and we examine both white and black males.
In a prior paper (Gottschalk and Moffitt, 1994), we provided an initial analysis of this question by dividing the PSID data into two eleven-year periods (1969-1979, and 1980-1990) and then, within each, computing each individual’s mean earnings and deviation from that mean to obtain estimates of permanent and transitory components, respectively. That procedure was crude and involved arbitrary choice of endpoints, and ran the risk of obscuring time trends within each of the intervals (see Dickens, 1994 and Katz, 1994 for critiques of the methods and other issues). In this study, we instead use a formal model of earnings dynamics and consequently conduct a more reliable decomposition, and we use the data by individual year to avoid arbitrary choice of endpoints. We also conduct a more comprehensive analysis of alternative measures of earnings and wages, and of earnings mobility (see below) in this study, as compared to our prior one.

Finally, we discuss the relationship between models of earnings dynamics such as ours and models of earnings mobility—most commonly models of quantile mobility matrices. Recent studies of trends in earnings mobility in the U.S. form a somewhat separate literature from that on earnings dynamics and the relationship between the two has yet to be established. We show that, under certain distributional assumptions (e.g., multivariate normality), an earnings dynamics model completely determines the structure of mobility matrices. We therefore conduct an examination of quantile mobility to determine whether such an analysis yields the same findings as that from our earnings dynamics models. We also compare our results to those from recent mobility studies.

We find that the increase in the cross-sectional variance of individual earnings and wage rates in the U.S. since 1969 has been
roughly equally composed of increases in the variances of the permanent and transitory components of earnings. Because most of the theoretical explanations for the increase in inequality have been aimed at explaining increases in the variance of the permanent components (increases in the price of skill, for example), our finding that a substantial fraction of the inequality increase has arisen from an increase in the variance of the transitory component is surprising and unexpected.

I. Data and Variable Construction

We use the Panel Study on Income Dynamics (PSID), a longitudinal survey that has followed a sample of households from the civilian non-institutional population of the U.S. since 1968. Approximately 5,000 households were interviewed in the initial year of the survey, including a supplementary low-income sample (the SEO) which we also include in our analysis in order to increase sample size (therefore sample weights must be used). Members of the original 1968 households and their offspring have been interviewed annually; the most recent year of data available at the time this analysis was conducted is 1992. The primary advantage of the PSID is its long period of coverage and its conformity with cross-sectional measures of inequality. A

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1 Aside from small sample size, a problem with using only the SRC sample (i.e., that excluding the SEO) is that the weights in the PSID which have been constructed to adjust for sample attrition over time cannot be used with the SRC sample alone, only with the combined SRC-SEO sample. As a result, the SRC sample alone shows generally more attrition bias than the weighted SRC-SEO sample. See Fitzgerald et al. (1998).

2 In prior work, we have treated each wave of the PSID as an independent cross section and we have compared trends in earnings differentials to those in the CPS (Gottschalk and Moffitt, 1992). We
disadvantage of the PSID is that relatively little information is available on the education and earnings of individuals who are not heads of households.

Following the practice of most previous studies of inequality, we analyze only males; the study of females is left to future work (studying the latter group requires addressing the problem of nonworkers). We restrict our sample to heads of household 20-59 who had positive hours of work and earnings in the year prior to interview and who were not in school at the date of interview. Our sample--prime-age male heads of household--is traditionally regarded as the most stable group in the labor force and hence is likely to have a smaller transitory earnings variance than other groups in the population, implying that our findings could be considered as providing a lower bound estimate of transitory earnings variability in the labor force as a whole.

We include every annual observation for each individual for which these restrictions are met; thus individuals sometimes drop out and reappear in the sample over time. The advantage of this approach is that it enormously increases the sample size; restricting the sample to individuals who worked and were present in the sample for all years of the PSID would reduce the sample by a large amount. A possible disadvantage is that it subjects the analysis to possible bias from attrition and labor force turnover. Unfortunately, restricting the sample to a continuous group of workers does not eliminate these sources of bias and can even increase the magnitude of bias. We

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found overall conformity of the direction of the trends in the two data sets in both within-group and between-group earnings differentials.
therefore follow the procedure that yields the greater sample size.\textsuperscript{3}

The earnings and wage measures we examine are the log of real annual earnings (wage and salary only) in the year prior to interview, and the log of real weekly earnings in that year; most of our work concentrates on the latter. We exclude the first two years of the survey, 1968 and 1969, because wage and salary earnings data asked in those years were bracketed. Thus our analysis includes the interview years 1970-1992 and our earnings and wage measures cover the period 1969-1991. The real figures are obtained by deflating the nominal values by the GNP personal consumption expenditure deflator (base 1982). We also trim the outliers in the data to reduce noise and to eliminate top-coded observations.\textsuperscript{4} Our final sample includes 2,781 individuals with a total of 25,194 person-year observations.

We use these data to construct cross-sectional variances of wages over time, but also to construct covariances of wages between pairs of years for the same individuals. As we show below, these autocovariances are the building blocks for the estimation of the variances of permanent and transitory components and for the

\textsuperscript{3} Attrition bias in the level of earnings is fortunately quite small in the PSID despite 50 percent attrition. Bias in higher order moments is more problematic, however. Restricting the sample to a continuous group of workers will eliminate attrition bias only if attrition is based on the permanent individual effect and not the transitory effect, but the evidence on attrition shows this is not the case. Thus such a restriction is likely to bias the sample toward those with small transitory variances. See Fitzgerald et al. (1998) for evidence.

\textsuperscript{4} We delete the top and bottom one-percent of the earnings and wage observations within each age-year covariance cell. This trimming eliminates the top-coded earnings observations in the PSID but is mainly aimed at reducing noise in the data. Necessarily, our findings should be interpreted as pertaining only to the central 98 percent of the U.S. male earnings distribution. Results on untrimmed data show the same patterns as those we present below but with larger standard errors.
estimation of earnings dynamic models more generally. In most of our analysis we examine only within-group variances and covariances, using groups defined by education and age. We take this approach for the traditional reason that the variance of overall wages can change if the variance of education itself (or, less likely, of age) changes, and we do not wish to include this source of change in our estimates. However, because between-group differences in wages can—and have—changed over time, and because these will affect trends in permanent and transitory variances, we also estimate our final models with overall wages. The results show our findings on relative trends of permanent and transitory components not to be sensitive to whether overall wages or wage residuals are used. The reason is that, while an increasing education differential in wages has tended to push up the variance of the permanent component, that differential—measured, in regression terms, by the regression coefficient on education—has simultaneously become more unstable around its upwardly trending mean. Thus the between-group differential has experienced an increase in its transitory component as well. This has contributed to the overall increase in cross-sectional wage inequality.

We conduct our within-group analysis by the usual method of working with regression residuals. The log wage regressions are estimated separately for each calendar year and for 10-year age groups, for sample size reasons (20-29, 30-39, 40-49, 50-59), leading to a total of 78 regressions (19 years, 4 age groups). Each regression contains education splines for ranges 0-8, 9-11, 12, 13-15, and 16+ years. The regression coefficients are available upon

\[5\] We choose this particular level of disaggregation to guarantee a minimum of 250 observations per regression.
request.

We follow the same individuals over time and compute the covariances of their residuals between pairs of years in the PSID—between, say, years t and t', where t and t' range between 1969 and 1991. However, preliminary examination revealed that the covariances between wage pairs differed according to the age interval in question—that is, we found that there are life cycle patterns to the covariances. Therefore we broke out the computation of covariances between the pairs of wage residuals by age, although, for sample size reasons, only 10-year age intervals are used (e.g., the covariance between residuals of those 20-29 one year and 21-30 the next year). When all age-specific and year-specific cells are defined, this approach leads to a set of 76 variances—each for a particular age interval in a particular year—and 477 covariances, each of which corresponds to a pair consisting of a particular initial age and year, and a later age and year (for the same group of individuals). We present the full set of 553 data elements, along with a few more details on their construction, in Appendix A.

II. Estimation of Trends in Permanent and Transitory Variances
A Simple Heuristic Approach

Rather than begin by specifying a formal error components model of model of earnings dynamics, we begin instead with a simpler approach that allows the estimation of permanent and transitory variances, and their trends, with simple and intuitive OLS methods. This approach also allows us to illustrate trends in permanent and transitory variances graphically, and to immediately reach tentative conclusions on their relative contributions to the increase in overall cross sectional variances. This can be achieved without resorting to
a particular parametric error components model that will necessarily rest on extra assumptions.\(^6\)

The basis for the approach is the realization that the variance of permanent and transitory variances in the canonical permanent-transitory model can be calculated and easily directly from the data matrix of variances and covariances of wage residuals; and this can be formalized in a simple OLS framework. Let \( y_{ia} \) be the wage residual for individual \( i \) at age \( a \). Then

\[
y_{ia} = \mu_i + \nu_{ia} \tag{1}
\]

where \( \mu_i \) is a time-invariant individual component with variance \( \sigma^2_{\mu} \) and \( \nu_{ia} \) is a transitory component with variance \( \sigma^2_{\nu} \) which, in the canonical model, is serially uncorrelated. Therefore \( \text{Var}(y_{ia}) = \sigma^2_{\mu} + \sigma^2_{\nu} \) and \( \text{Cov}(y_{ia}, y_{ja}) = \sigma^2_{\mu} \) between two ages \( a \) and \( a' \). If the data provide us with a variance-covariance matrix whose diagonals give estimates of \( \text{Var}(y_{ia}) \) (which should be the same at all ages according to this simple model) and whose off-diagonal elements, each of which corresponds to a covariance between a different \( (a, a') \) pair, give estimates of \( \text{Cov}(y_{ia}, y_{ja}) \), then we can estimate \( \sigma^2_{\mu} + \sigma^2_{\nu} \) by the mean of the variances in the matrix and \( \sigma^2_{\mu} \) by the mean of the covariances in the matrix; and we can estimate \( \sigma^2_{\nu} \) by the difference in the means.

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\(^6\) Of course, we will reject the canonical model as a full description of earnings dynamics, but the reasons for rejection will be illustrated by this exercise. In addition, as it will turn out in our case, the conclusions on the relative contributions of permanent and transitory variances to inequality trends that will emerge from a formal error components model also hold in this simpler framework.
of the variances and the covariances, since \( \sigma_y = \text{Var}(y_{ia}) - \text{Cov}(y_{ia}, y_{ia}') \).\(^7\)

Figures 1(a)-(d) show trends in mean variances and covariances computed from the data matrix, both taken over the variances and covariances for all age groups (Figure 1a) or over age subgroups (Figures 1b-1d). All figures show an increase in cross-sectional variances over time, consistent with past work on the rise in inequality. The figures also show an increase in covariances as well, reflecting an increase in the persistence of earnings differences from year to year and hence a rise in the variance of permanent earnings (the covariances are shown separately for different lag lengths, i.e., different time gaps between the two year constituting the covariance).\(^8\)

However, it is clear visually from the graphs that the gap between the variance and the covariances has widened over time, particularly for the long-lag covariances. As we will show below, when serial correlation of the transitory component is present, it is the

\(^7\) In our prior work (Moffitt and Gottschalk, 1994), we used the more obvious simple method of simply averaging individual wages over time to estimate a permanent component, taking deviations in each year from that mean to obtain a transitory component, and then computing the variances of those two quantities in the data. Unfortunately, that requires the arbitrary choice of endpoints of intervals over which to compute the permanent component and, more to the point, requires the assumption that the permanent component is constant within the interval, which in turn can result in falsely attributing what are really gradual changes in the permanent component to changes in the transitory component around the endpoints (see Katz, 1994, for a discussion). The approach we are taking here does not suffer from this problem and allows a year-by-year charting of the components, but still in a simple way.

\(^8\) The convention taken in the figures is that each covariance point in the figure represents the "second" time point in the covariance. So, for example, a covariance for lag 4 in 1980 represents the covariance between earnings in 1976 and 1980, not 1980 and 1984. The choice does not affect the conclusions (note that the same convention is taken for the separation of age groups between Figures 1b, 1c, and 1d—the age in question is that at the second time point). The covariances at lags "1-4" are averages over the covariances at lags 1, 2, 3, and 4, and similarly for the other lags.
covariances at the longer lags that more accurately measure the variance of the permanent component, and the gaps between these and the overall variances have unmistakably risen according to the figures.

This approach also leads naturally to a simple OLS method of summarizing the Figures and of estimating the variances of permanent and transitory components and their trends. If we pool all the 553 variances and covariances in the data matrix into one data set and denote them by "y," and we if regress y on a diagonal dummy "D" equal to 1 if y is a variance (i.e., on the diagonal of the matrix) and 0 if not, then the intercept will represent an estimate of $\sigma^2_{\mu}$ and the coefficient on D will represent an estimate of $\sigma^2_{v}$.

Column (1) of Table 1 shows the results of such a regression, yielding a estimated covariance of approximately .13 and a transitory variance of .19, implying a total variance of approximately .31 and a correlation coefficient (=percentage of total variance composed of the permanent component) of .41, an estimate close to other estimates of random effects earnings models.\footnote{The standard errors in Table 1 are estimated in the same way as those in the formal error components models discussed below; see that discussion. We should note that the general framework for fitting a model to the variances and covariances of a regression with either OLS or nonlinear methods which we are using is drawn from Chamberlain (1982).} Column (2) allows the variances and covariances to be affected by a separately-entered time trend (t) and to interact with D, thereby permitting a test of whether the error variances are trending over time (the regression also controls for various functions of age). The coefficients on t and Dt are statistically indistinguishable, leading to the implication that permanent and transitory variances have been trending upward at the
same rate, on average, over the period, and hence each constitute 50 percent of the increase in overall inequality. Column (3) tests whether entering the unemployment rate into the regression, either at the first or second time point of the covariance in question, and demonstrates that at least one simple test of cyclical effects does not alter the basic similarity of the trends in permanent and transitory variances. The fourth column interacts $t$ with the length of the lag ($A_2 - A_1$) between the two time points of each covariance, and thus permits the permanent variance to trend at different rates accordingly. The negative coefficient on the interaction variable confirms, as the graphical evidence indicated, that longer lags are taken as better estimates of $\sigma^2\mu$, it appears that the transitory variance has increased even faster because the permanent variance has increased even less. The regression evidence shows the interaction to be statistically significant.

B. Formal Models of Earnings Dynamics

The attractive simplicity of the canonical permanent-transitory model comes at the price of restrictive assumptions that have been shown in past data sets to be false. There is a significant literature on the estimation of formal error components models of the earnings process, summarized by Atkinson and Bourguignon (1992); some of the more influential studies over the years are those of Lillard and Willis (1978), Lillard and Weiss (1979), Hause (1977,1980), MaCurdy (1982), and Abowd and Card (1982). This body of literature suggests that transitory components are indeed serially correlated and that permanent components are not as fixed as the canonical model implies, for there are either random walk or random-growth components to the "permanent" component. In all these cases the exercise
reported above can be misleading. For example, if there is serial
correlation in the transitory component, the one-year-apart
covariance, Cov(y_{ia},y_{i,a+1}) should not be taken as measuring $\sigma^2_\mu$ alone.
In fact, in the presence of serial correlation of the transitory
component, it would presumably be best to measure $\sigma^2_\mu$ by covariances
between earnings taken at very distant ages, after all transitory
shocks and their resulting covariances have died out. But then one
has to be sure that the permanent effect is constant over long
periods, and that is where random-walk and random-growth factors must
be accounted for.

We specify an error components model to be consistent with the
covariance patterns in our own data but also which incorporates
elements used in past models. In our own data, the notable features
of the autocovariance and autocorrelation structures are that (i)
covariances and correlations fall with distance from the diagonal
(i.e., length of the lag); (ii) they drop off rapidly at one-year lags
and then more gradually; (iii) they do not reach zero at our longest
lag but instead appear to asymptote at a positive value; and (iv) both
variances and covariances and correlations rise, on average, with
age.\textsuperscript{10} Each of these features can be captured. Feature (i) can be
captured with any of a variety of ARMA specifications, but the AR(1)
model with positive correlation coefficient is the standard. Feature
(ii) can be captured by the addition of a MA(1) or MA(2) component.
Feature (iii) can be captured by the presence of $\mu_i$ itself. Feature
(iv) can be captured either with a random walk in $\mu_i$ or a random
growth individual effect or both, since both such a pattern.

\textsuperscript{10} We show these patterns in detail in Moffitt and Gottschalk
A model which captures all these components is:

\[
Y_{ia} = \mu_{ia} + \nu_{ia} \tag{2}
\]
\[
\mu_{ia} = \mu_{i,a-1} + \omega_{ia} \tag{3}
\]
\[
\nu_{ia} = \rho\nu_{i,a-1} + \xi_{ia} + \theta\xi_{i,a-1} \tag{4}
\]

Equation (2) shows the log earnings (or earnings residual) of person \(i\) at age \(a\) to be composed of an individual effect \((\mu_{ia})\) and a transitory effect \((\nu_{ia})\). The individual effect follows a random walk as shown in (3) and the transitory effect follows the ARMA\((1,1)\) process shown in (4). As conventional in these models, we assume the forcing variables \(\omega_{ia}, \xi_{ia}\), and the initial value of the individual effect \((\mu_{i1})\) to be independently distributed over age and time and w.r.t. each other. In other results, we tested ARMA\((1,2)\) and ARMA\((2,1)\) specifications for the transitory effect while maintaining the random-walk specification for the individual effect. In neither case was the fit significantly improved and in neither case was the additional parameter significant. An ARMA\((1,1)\) with a random-walk individual effect hence fits our data adequately.\(^{11}\)

Our main interest is in allowing the parameters of the process to change with calendar time. To introduce time-varying parameters into the model, we therefore modify the specification to allow the

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\(^{11}\) We make no attempt to explicitly identify measurement error components although such error is unquestionably present and affects our parameter estimates. Classical measurement error would primarily affect our estimates of the MA error variance, but recent work on error in earnings reports suggests that measurement error is serially correlated (Bound et al., 1990; Bound and Krueger, 1991). Hence our parameters \(\theta_t\) and \(\rho_t\) presumably pick up some measurement error as well. Unfortunately, the Bound-Krueger and Bound et al. studies only had two periods of validated earnings data, and hence AR and MA components of measurement error could not be separately identified.
parameters to vary with calendar time \((t)\):

\[
Y_{iat} = \alpha_t \mu_{iat} + \nu_{iat} \tag{5}
\]

\[
\mu_{iat} = \mu_{i,a-1,t-1} + \omega_{iat} \tag{6}
\]

\[
\nu_{iat} = \rho_t \nu_{i,a-1,t-1} + \xi_{iat} + \theta_t \xi_{i,a-1,t-1} \tag{7}
\]

The individual effect \((\mu_{iat})\) now has a time-varying factor loading \((\alpha_t)\), consistent with the interpretation of the individual effect as representing latent unobservable human capital whose price \((\alpha_t)\) shifts with calendar time. Aside from the variance of the initial individual effect, there are five parameters in the model--\(\alpha_t\), \(\rho_t\), \(\theta_t\), and the variances of \(\omega_{iat}\) and \(\xi_{iat}\)--which, together, determine the pattern of variances and covariances. We permit all five to vary linearly with calendar time (and, subsequently, with year dummies):

\[
\alpha_t = 1 + b_1 t \tag{8}
\]

\[
\rho_t = c_0 + c_1 t \tag{9}
\]

\[
\theta_t = d_0 + d_1 t \tag{10}
\]

\[
\text{Var}(\omega_{iat}) = e_0 + e_1 t \tag{11}
\]

\[
\text{Var}(\xi_{iat}) = f_0 + f_1 t \tag{12}
\]

The factor loading \(\alpha_t\) is normalized to 1 at \(t=0\) (1969 in our data), and we let \(\text{Var}(\mu_{iit})=\sigma_\mu^2\) to establish the baseline variance of the individual effect.

A "permanent" effect in this model is not permanent in the literal sense since the individual effect is permitted to shift over the life cycle and with calendar time. The distinction between the two components in (5) is, instead, based upon a decomposition of
shocks into those that are mean-reverting and those that are not. Our
decomposition defines permanent shocks to be those that are non-mean-
reverting and transitory shocks to be those that are mean-reverting.

We estimate the model by minimum distance using the form
suggested by Chamberlain (1984) for the estimation of covariance
structures. The mapping of the model into the variances and
covariances necessary for the estimation is given in Appendix B.
Robust standard errors are computed from the empirical covariance
matrix of the residuals in the moment equations.

Table 2 shows estimates of the model. Initial testing revealed
that the time trend coefficients were significant only for $\alpha_t$ and the
variance of $\xi_{iat}$, so column (1) shows a specification with only these
two time effects allowed. The year coefficient for $\alpha_t$ is .019,
implying that its factor loading (or the "price of permanent
unobserved human capital") increased by approximately 42 percent over
the 18-year period 1969-1991 ($1.42 = 1 + .019 \times 22$). Thus the model
strongly confirms the existence of an increase in the variance of the
permanent component. At the same time, the variance of $\xi_{iat}$—which is
a two-period transitory component—more than doubled over the period,
increasing from .112 in 1969 to .244 in 1991 (.244 = .112 + .006 \times 22).
Thus the model also confirms that there was a strong increase in the
transitory component. However, because of the presence of the
autoregressive process, the increase in the transitory variance
persists over time in its effect on the variance of the total
transitory component, $\nu_{iat}$. But this effect dies out at the rate
$\rho^2$, implying, at our estimates of that parameter, that the impact is
negligible after three years.

The second column in Table 2 shows that the time trends in the
other three parameters of the covariance matrix are insignificant. The magnitude of the trend coefficient for $\rho$ is not trivial, implying an increase from .657 to .767 over the period and hence a strengthening of the low-order covariances and a longer persistence of transitory shocks. However, the large standard error on the coefficient makes this result fairly uncertain.\textsuperscript{12}

These estimates provide an interpretation of figure 1 and an explanation for the differing rates of growth of low-order and high-order covariances. The estimates imply that covariances of earnings within three years of one another reflect not only the permanent component but also the serially-correlated transitory component. Thus they imply that it is incorrect to associate off-diagonal elements with the permanent variance per se, as the simple canonical model assumes. The more rapid increase in the low-order covariances in figure 1 (and table 1) than in the high-order covariances simply reflects the fact that the former captures the increasing transitory variance as well as the increasing permanent variance whereas the latter reflects only the increasing permanent variance. This also implies that, within the simple permanent-transitory model discussed earlier in this section, it is the gap between the variance and the high-order covariances—not the low-order covariances—that measures the total transitory variance, and this has clearly risen in Figure 1.

One way of assessing the relative importance of the increase in the variance of the permanent component ($\alpha_t\mu_{iat}$) and the transitory component ($\nu_{iat}$) is to calculate what the increase in the total

\textsuperscript{12} The chi-squared statistics for both specifications are far above the one-percent critical values of 600-624, implying considerable unexplained variation in the model. However, replacing the time trends with year dummies, whose results will be discussed momentarily, reduce the chi-squareds to 739.
variance would have been from 1969 to 1991 had each parameter increased separately. Table 3 shows the results of such an exercise, obtained by calculating the variance of $y_{i,t}$ assuming no change in the parameters from 1969-1991, and by then calculating what the 1991 variance would have been had each of the parameters increased by the magnitudes implied by the coefficients in the second column of Table 2.\textsuperscript{13} The results show that the increase in the permanent variance accounted for approximately 40 percent of the increase in total variance and the increase in the transitory variance accounted for approximately 50 percent, with the remainder accounted for by changes in other parameters.\textsuperscript{14} Thus, although the change in the transitory variance accounts for slightly more of the change than that of the permanent variance, the two are roughly equal in importance for practical purposes.\textsuperscript{15}

The estimates thus far have restricted year effects to a linear trend; yet, at minimum, there may be different trends in the 1970s and

\textsuperscript{13} The order of parameter change in Table 3 does not materially affect these conclusions because the permanent and transitory components are additive in the total variance and hence do not interact. For example, introducing the change in the variance of $\xi$ first increases the four variances from their 1969 values to .277, .305, .317, and .328 for the four respective ages shown in the Table.

\textsuperscript{14} The change induced by the trend coefficient for the variance of the random walk is negligible in magnitude and hence is not broken out separately; the changes in the last row are entirely due to the change in $\rho$.

\textsuperscript{15} An alternative computation is to compute the "steady-state" variances implied by the values of the parameters in 1991, and to compare these to the 1969 steady-state variances shown in the first row of Table 3. The 1991 variances in Table 3 reflect the historical experience of the shocks from 1969 to 1991 and, because of the autoregressive structure of the model, do not represent the steady-state values. However, because the autoregressive lag in the variances is so short--of negligible importance after three years--the 1991 steady-state variances differ from those in the last row of Table 6 only at the second decimal place.
1980s. We therefore reestimate the model in column (1) of Table 2, allowing \( a_t \) and \( \text{Var}(\xi_{it}) \) to take on different values in each year 1969-1991. As shown in Figure 2, the increase in the two parameters occurred in quite different periods. While the permanent variance grew, on average, through about 1982 or 1983, it leveled off or fell subsequently. The transitory variance, on the other hand, showed essentially no trend until 1980 or 1981, when it began to rise. Although it showed a slight decline after 1984, it was still unambiguously higher in the late 1980s than in the early 1980s, opposite to the pattern for the permanent variance. Thus we find additional evidence indicating relatively higher growth rates of the permanent variance in the 1970s and of the transitory variance in the 1980s.\(^{16}\)

III. Mobility

Mobility, defined as a change in individual ranks within a distribution, is closely related to the covariance structure. For example, an increase in an earnings covariance between any two points

\(^{16}\) The increase in transitory variance could in principle be the result of increasing measurement error in the PSID, but there is no evidence that it did so or any reason to think it would have increased more in the 1980s. Nor is there any evidence that there has been a change in the accuracy of earnings data in the PSID. The fraction of earnings observations that are imputed, combining what the PSID calls "major" and "minor" imputations, is only 1.6 percent in our sample averaged over all years. This low percent probably reflects better reporting among prime-age white males than other population groups. More important, the fraction has not changed over the period, varying only between .6 percent and 2.1 percent, with a slight downward trend over time. As a consequence, estimates of the model shown in Table 6 change only at the third or fourth decimal place when imputed earnings observations are deleted. In addition, there has been no change in the coding procedures used to detect "erroneous" earnings. Those procedures are documented for coders, and the same documents have been used for the entire PSID.
in time will necessarily lower mobility because earnings in the two periods are more closely related. However, a stronger statement than this can be made. In Appendix C we show that if earnings follow a joint normal distribution, the probability of a change in individual ranks between any two points in time is a function only of the correlation coefficient between earnings at those two points, and not a function of the absolute levels of either of the variances at the two points in time or the covariance.

The intuition for this result is particularly strong in the canonical permanent-transitory model, where the correlation coefficient between earnings at any two points is equal to the fraction of the variance accounted for by the permanent component, or \( \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\nu^2} \). The degree of mobility in this model thus hinges only on the relative sizes of the permanent and transitory variances. A rise in the permanent variance, which increases the average distance between the earnings of different individuals, lowers the chance of a change in rank; a rise in the transitory variance, on the other hand, makes the chance of a change in rank more likely. But a proportionate increase in the permanent and transitory variances has no effect on mobility; the two effects exactly cancel. Therefore, to the extent that the permanent and transitory variances have risen at about the same rate, as suggested by our previous results, this model would show little change in mobility.

We should note that the value of the correlation coefficient in a more realistic model, such as one with serially-correlated transitory components, varies depending on the distance between the two points under consideration. With serially-correlated but mean-reverting
transitory components, correlation coefficients fall with that distance and hence mobility is likely to be greater over longer periods. In addition, if mobility is defined instead on the basis of average earnings over multiple years rather earnings in a single year, and if it is a change in the rank of mean earnings that is considered, mobility is likely to be lower since the transitory component is a smaller portion of the total variance when earnings are averaged over multiple years.\textsuperscript{17}

Since our estimated error components model reported in the last section provides a full accounting of the changes in correlation coefficients (i.e., over different distances and intervals) that have occurred during the 1970s and 1980s, a mobility analysis may at first blush appear redundant; that is, our estimated error components model should by itself determine trends in mobility. However, an examination of transition rates between quantiles of the earnings distribution can provide more detail on whether any changes in mobility have occurred at different parts of the distribution (e.g., at top and bottom). We therefore provide a simple quantile analysis of mobility in this section.

Our mobility analysis uses the same data set and covariance structure as used in the previous analyses except that variance elements are eliminated since they are not relevant to mobility. This leaves us with 477 observations, each of which corresponds to a pair of ages in two particular years. Instead of computing covariances for each such cell, we compute quantile mobility rates using five

\textsuperscript{17} Over the entire lifetime, for example, an increase in the permanent variance must both increase the variance of lifetime earnings and lower mobility between average earnings in the first part of the life cycle and in the second part, at least as long as transitory components die out within those parts.
quantiles (i.e., quintiles).\textsuperscript{18}

Table 4 shows the year-to-year rates of mobility in the sample between quintiles, pooled over all years and ages. Mobility at the upper and lower quintiles is less than in the middle quintiles.\textsuperscript{19} At the upper and lower ends there is an approximate one-third chance of changing rank from one year to the next, as opposed to an approximately fifty-fifty chance for the middle quintiles. The mobility table is also remarkable for its symmetry.

Our interest is, once again, in how these mobility rates have changed over time conditional on age. As we discussed previously, the overall shape of mobility trends should follow those of the covariance analysis closely, but should depend primarily upon trends in the correlation coefficients rather than in the covariances. Figures 3(a)-(b) show the trends in both the correlation coefficient and the mobility rate between the illustrative ages 35 and 36 ("short") and between 35 and 40 ("long").\textsuperscript{20} The measure of mobility we use is the sum of the off-diagonal elements in each row of Table 4 (i.e., one minus the probability of staying in the same quintile). This measure is the inverse of what is known as the "immobility ratio" (Atkinson et al., 1992). As expected, the correlation coefficients and mobility rates in both diagrams show an extremely close inverse relationship.

\textsuperscript{18} The limited number of observations in the sample prevent us from disaggregating the quantiles further. Some prior analyses in the literature have been able to use a finer set of quantiles by pooling the data across years and across ages. However, not only have we found pooling across years to be incorrect, both our descriptive and error components analysis showed the necessity of conditioning on age.

\textsuperscript{19} This is to be expected since persons in the upper (lower) quintiles can only move down (up), whereas persons in the middle quintiles can move in either direction.

\textsuperscript{20} Given our age grouping, "35" stands for 30-39, "36" stands for 31-40, and "40" stands for 35-44.
The one-year-apart correlation coefficient between ages 35 and 36 shows a slight upward trend in the 1970s but a steeper trend in the 1980s, reflecting the pattern of the transitory variance. Correspondingly, there was very little trend in one-year-apart mobility until the late 1970s, when short-term mobility dropped sharply. The five-year-apart correlation coefficient rose steadily over the late 1970s, albeit with considerable fluctuation, but leveled off in the 1980s; correspondingly, five-year mobility dropped steadily in the 1970s but leveled off in the 1980s. These patterns closely reflect the relative patterns of the transitory and permanent variances discussed previously, and, though showing some changes in mobility, they are quite small in magnitude.

Table 5 shows the results of a regression analysis of the mobility rates for all quintiles, all ages, and lag orders. The first row shows that while there was only a small net decline in overall mobility (over all lag orders), a significant decline in mobility occurred in the top and the bottom two quintiles. The subsequent rows of the table show overall mobility rates consistent with Figure 3, falling significantly only for short-term mobility in the 1980s and only for long-term mobility in the 1970s. However, as in the first row, the trends seem to be concentrated in the upper and lower tails of the distribution. Indeed, for the lowest fifth of earners even short-term mobility declined in the 1970s, which is an indirect indication that the variance of serially-correlated transitory shocks has been increasing for that group over the entire period, not just over the 1980s.

IV. Additional Issues
Weekly Wages and Weeks of Work. An important secondary question is the extent to which the increase in instability in earnings profiles signified by the increase in transitory variances has been a result of increasing instability in wage rates or in employment. The literature on the overall increase in cross-sectional dispersion of earnings indicates that a majority of that increase has arisen from increases in the cross-sectional dispersion of wage rates rather than of weeks of work, hours of work, and employment in general (Levy and Murnane, 1992; Burtless, 1990, Table 7). However, there is no necessary reason for the lesser importance of dispersion in cross-sectional employment measures to follow through for the relative importance of permanent and transitory variances. In fact, the literature on life cycle labor supply analysis and on business cycle fluctuations indicates that employment fluctuates with a greater variance than wages, suggesting that transitory components in employment might be considerably stronger than permanent components.

Figures comparable to Figure 1 but with trends in the variances and covariances of log real weekly wages and the log of annual weeks worked (available upon request) indicate that both variances and covariances of log weekly wages rose and that they did so in the same pattern as for log real earnings (i.e., with the same relative patterns for high-order and low-order covariances). Clear evidence of increases in the variances and covariances of weeks of work also appear, although the increases in covariances are much weaker than for earnings or wages. This pattern is consistent with a greater relative importance of transitory factors for weeks worked.

Table 6 shows estimates of several models for log real weekly wages and log annual weeks worked. The descriptive regressions show that the increasing variance of log real weekly wages is equally
shared between diagonal and off-diagonal elements, as was the case for annual earnings. However, the coefficients are only approximately two-thirds the magnitude of the earlier results, thus confirming a role for increasing dispersion in weeks of work. This is further confirmed by the results for log and absolute weeks worked in the table, which indicate increasing diagonal and off-diagonal elements but greater relative trends for the diagonal elements. This pattern is also consistent with a greater relative importance of increases in transitory variances for weeks worked. The estimates of the error components models shown in the lower half of the table confirm this and show that increases in transitory variances were particularly marked for weeks of work. However, it should be stressed that transitory variances have increased for real weekly wages as well.

These analyses exclude any consideration of changes in the proportion of the population with no weeks worked at all during the year. Those percentages are relatively small for our sample of prime-age white males but have increased over the period. For those 20-29, for example, the percent without work at all during the year increased from 0.4 percent in 1969 to 2.7 percent in 1987. The corresponding percents for those 30-39, 40-49, and 50-59 are, respectively, 0 to 1.7, 1.8 to 2.2, and 8.0 to 11.3.21 Our results thus far already indicate increases in the variance of weeks worked in the worker subsample, and our data indicate even larger increases when nonworkers are included.

21 These trends by themselves have no necessary implication for our prior results. Mean weeks of work have fallen in our sample whether these zero-weeks observations are included or not (although they have fallen more when the zeros are included than when they are not). More important, a change in mean weeks worked, negative or positive, has no necessary implication for changes in variances.
Estimates of the descriptive and error components models for absolute weeks of work inclusive of zeros, comparable to those shown in the last column of Table 6, show stronger trends in the permanent variance and weaker trends in the transitory variance. We speculate that an entire year without work may be an indication of a serious wage or employment problem that reflects a permanent condition.

**Between-Group Trends.** The analysis thus far has been conducted entirely on the residuals from earnings and wage regressions, regressions containing education dummies and estimated separately by year and age interval. An important question is whether our results on the relative importance of trends in the variances of the permanent and transitory components of these within-cell earnings components apply as well to log earnings itself. The answer depends upon the relative importance of trends in the permanent and transitory variances of the between-cell components, which in our case are the components accounted for by education and age differences in earnings.

There is a much larger literature on trends in education and age differences in earnings than on trends in the within component, the literature showing markedly different trends in both over the 1970s and 1980s for both within and between components (see Levy and Murnane for a review). Our education and age coefficients follow the same general pattern over time as those in the past literature, which have been mainly estimated on the CPS, and therefore we do not present

---

22 For the descriptive regressions, the off-diagonal and diagonal coefficients are 1.296 and 1.109, respectively, both significant at the 10 percent level. The magnitudes are considerably larger than those for conditional weeks worked because variances and covariances showed larger absolute increases over the period. Estimates of the error components model reduce the magnitude and significance level of the trend in the transitory component, and increase them for the permanent component.
them. Instead, we take a simpler approach to this question by reestimating the models we reported in Section IV on log earnings itself rather than on the regression residuals; the difference in results will be an indirect indication of the importance of trends in the between-group variances. Thus, we work with 553 cells of a covariance matrix of log annual earnings over all years, age groups, and lag orders, constructed as described previously for the regression residuals.

The estimates of the descriptive regressions (not shown) indicate that the permanent variance is considerably more important when the "between" is included. Estimates of average permanent and transitory components for the specification in column (1) of Table 1 are .172 and .185, respectively, implying a correlation coefficient of approximately .48 as opposed to our prior estimate of .41. This is to be expected since education levels in our sample are essentially constant for each individual and, therefore, will mainly contribute to the permanent component of earnings. Estimates of trend coefficients comparable to those in columns (2) and (3) of Table 1 show, moreover, approximately the same coefficients on Dt (.0057 and .0056 in the two columns) but somewhat higher coefficients on D (the average covariance) of .0066 and .0068. This higher value reflects a net increase in educational differentials over the period. Estimates of column (4) for the new covariance matrix reveals, however, the same pattern of greater increases of high-order covariances than low-order covariances as found previously.

In order to contrast the within-group and total results we

23 A detailed comparison of the PSID and CPS in this dimension can be found in our prior benchmarking exercise (Gottschalk and Moffitt, 1992).
estimate the error components model on both and simulate the implied permanent and transitory variances. The steady-state values in 1969 and 1987 are shown in Table 7.\textsuperscript{24} As suggested by the descriptive analysis, the results show a higher level of the permanent variance for total log earnings. In addition, there was a slightly greater rate of increase in the permanent variance when the between is included. However, the magnitudes of the changes induced by including trends in the between are not large, and hence none of our substantive findings (e.g., that upward trends in transitory variance are important) are affected.\textsuperscript{25}

V. Summary and Conclusions

In this paper we have examined the source of the increasing cross-sectional variance of male earnings in the U.S. over the 1970s and 1980s by determining its origins in the autocovariance structure of earnings. Using data from the Michigan Panel Study of Income

\textsuperscript{24} The sum of the permanent and transitory components of the within values are almost identical to the values given in Table 6. As noted previously, the steady-state predicted values from the model are essentially identical to the values predicted historically because the latter only include the influence of "history"—that is, the fact that variances have been growing over time and hence have not been at their steady-state value for the whole period—but history is unimportant after approximately three years.

\textsuperscript{25} To some extent the small magnitude of the change induced by including the between simply reflects the relatively small R-squareds in all log earnings regressions when only education and age are the explanators; hence trends in the covariance structure of total log earnings are dominated by trends in the covariance structure of the residuals. Note as well that the correlation coefficients in Table 11 imply even less reduction in mobility than was found for the within analysis. The trends in the correlation coefficients arise from the size of the proportionate increase in the permanent variance, not its absolute size, and the proportionate increase in that variance is smaller for the between than for the within.
Dynamics from 1969-1991 for white males, we find that about half of the increase in variance within education and age groups has arisen from an increase in the variance of the permanent component of earnings and half from an increase in the variance of the transitory component, where the transitory component reflects shocks that die out within three years. We thus find that increases in transitory shocks are of equal importance to increases in the dispersion of permanent earnings in explaining recent increases in earnings inequality. Indeed, the increase in transitory shocks was especially great in the 1980s. Other results show that the increase in transitory shocks appears in weekly wages as well as annual earnings, although even greater in annual weeks of work. We also find that transitory shocks are still very important when trends in the variance across education and age groups are included.

Our investigation of earnings mobility indicates that mobility changed very little over the period, but with a slight fall in long-term mobility in the 1970s and a slight fall in short-term mobility in the 1980s, the latter reflecting the increase in short-term covariances arising from a higher variance of serially-correlated transitory shocks. These mobility declines are concentrated in the top and bottom quintile of the earnings distribution.

Our study has been largely a statistical accounting exercise aimed at determining the relative contributions of different error component variances to the upward trend in overall cross-sectional variances, rather than a search for causes. We have conducted a rudimentary exploration of the latter type in Gottschalk and Moffitt (1994), where we found that while some of the increasing transitory variance is a result of decline in unionization (union jobs have lower transitory variances) and industry shifts, these do not provide a
sufficient explanation by themselves--transitory variances have increased within unionized and non-unionized jobs, within all industrial sectors, and even for workers who have stayed with the same firm for up to 10 years. Further work in exploring the sources of increased variability would therefore appear warranted.
Appendix A
The Autocovariance Matrix Constructed from the Data

The cells used to construct the autocovariance matrix of log wage residuals from the data are broken out separately by age and year. The need to group the data into age intervals is the only complication that requires discussion. We group the data from age 20 through 59 into 4 ten-year age groups (20-29, 30-39, 40-49, 50-59). In each year t of the data, we follow the individuals in each of these four groups through to year t+1, year t+2, etc. until either the end of our data is reached (1991) or until the age interval in question reaches beyond age 59 (e.g., the 40-49 cohort in 1969 can be followed through to 1979, when the individuals are 50-59, but no further). Covariances are then calculated between the initial year, t, and each subsequent year using, for each covariance calculation, only those individuals present in both years. A fresh set of cohorts is begun in each year, starting in 1969, and continuing through 1991; the four cohorts (i.e., age groups) started in each year are again followed over time.

This method of grouping ensures that every individual variance and covariance in the panel is included uniquely in one cell. There

\[\text{Note that there is no necessary relationship between the age intervals used in the log wage regressions, and the age intervals used for the grouping of the residuals; they are separate exercises. However, we choose the same age intervals because they both are convenient for yielding adequate sample sizes.}\]

\[\text{The individuals in age group 50-59 cannot be followed at all. However, their wage residuals are used to construct variances in each year.}\]
are many alternative methods of grouping the data and computing the autocovariance matrix, but there is no reason for any one to be preferred to another except for convenience. Our method is designed to make trends over time, holding age constant, particularly easy to discern. As the text demonstrates, we plot such age-constant covariance trends as a way to simply illustrate trends in permanent and transitory components.
Appendix B

The Minimum Distance Method
and the Covariance Model Mapping

Minimum Distance Method. Let \( s_{im} = y_{ij}y_{ik} \) where \( y_{ij} \) and \( y_{ik} \) are log earnings (or residuals) for individual \( i \) and age-year "locations" \( j \) and \( k \), and where \( m=1,...,M \) is the moment generated by the product of residuals at locations \( j \) and \( k \). In our case, \( M=553 \). Posit the model

\[
 s_{im} = f(j,k;\theta) + \varepsilon_{im} \quad \quad i=1,...,N, \quad m=1,...,M \tag{B1}
\]

where \( \theta \) is a \( L \times 1 \) vector of parameters. Then the set of \( M \) equations in (B1) constitutes an SUR system whose efficient estimation requires an initial consistent estimate of the covariance matrix of the \( \varepsilon_{im} \).

However, following the findings and recommendations of Altonji and Segal (1991) on bias in estimating covariance structures of this type, we employ the identity matrix for the estimation.\(^{28}\) Hence we choose \( \theta \) to minimize the sum of squared residuals:

\[
 \min_{\theta} \sum_{i=1}^{N} \sum_{m=1}^{M} [s_{im} - f(j,k;\theta)]^2 \tag{B2}
\]

or, equivalently, since \( f \) is not a function of \( i \),

\[
 \min_{\theta} \sum_{m=1}^{M} [\bar{s}_m - f(j,k;\theta)]^2 \tag{B3}
\]

\(^{28}\) Abowd and Card (1989) also used the identity matrix because of problems similar to those discussed by Altonji and Segal.
where $\bar{s}_m$ is the mean (over i) of $s_{im}$ (i.e., a covariance).

To obtain standard errors, we apply the extension of Eicker-White methods in the manner suggested by Chamberlain, using the residuals from (B1), each of which we denote $e_{im}$. Let $\Omega$ be the $M \times M$ covariance matrix of the $e_{im}$, each element of which is estimated by:\footnote{Each individual in our data set contributes to only a subset of the moments in $\Omega$ (see Appendix A); we do not adjust the notation in (B4) for this, but instead leave it implicit.}

$$\hat{\sigma}_{mm'} = (1/N) \sum_{i=1}^{N} e_{im} e_{im'} \quad (B4)$$

Let the $NM \times NM$ covariance matrix of individual residuals be:

$$
\Delta = \begin{bmatrix}
\Omega & 0 & 0 \\
0 & \Omega & 0 \\
0 & 0 & \Omega \\
\end{bmatrix}
$$

Then

$$\text{Cov}(\hat{\theta}) = (G'G)^{-1}G'\Delta G(G'G)^{-1} \quad (B5)$$

where $G$ is the $NM \times L$ matrix of gradients $\partial \ell(j,k;\theta)/\partial \theta$.

We experienced considerable difficulty in obtaining positive definite covariance matrices from (B5) in some of our larger models because of the amount of noise in the $\Omega$ matrix, which has over 150,000 unique elements. Consequently, we set many elements of that matrix to zero and we smoothed many others with polynomial functions of age and year to obtain our standard error estimates.
Mappings. For Table 1 and related tables in the paper, \( f \) is just a linear regression function of age, year, and other variables corresponding to the location of the element in the covariance matrix. For the models presented in Table 2, a mapping from the assumed error-components process to \( f \) is required. The models in Table 2 are a function of calendar time which means that the covariances (\( f \)) implied are not easily obtained. We derive \( f \) at each age and year by recursively deriving successive variances and covariances over the life cycle for each cohort beginning at the beginning of the life cycle (age 20) and proceeding forward simultaneously in age and year. No initial conditions problems per se are presented in our model because the life cycle has a finite start date and end date by assumption; no stationarity assumptions are made or required in either age or calendar time.

For brevity we will present the mapping only for the most complex model, that in equations (5)-(7). Letting \( \delta_t = \text{Var}(\xi_{iat}) \) and \( \psi_t = \text{Var}(\omega_{iat}) \), we have:

\[
\begin{align*}
\text{Var}(y_{iat}) &= \alpha_t^2 \text{Var}(\mu_{iat}) + \text{Var}(\nu_{iat}) \\
\text{Cov}(y_{iat}, y_{i,a-s,t-s}) &= \alpha_t \alpha_{t-s} \text{Cov}(\mu_{iat}, \mu_{i,a-s,t-s}) \\
&\quad + \text{Cov}(\nu_{iat}, \nu_{i,a-s,t-s}) \\
\text{Var}(\mu_{ilt}) &= \sigma_\mu^2 \\
\text{Var}(\mu_{iat}) &= \text{Var}(\mu_{i,a-1,t-1}) + \psi_t, \quad a > 1 \\
\text{Cov}(\mu_{i2t}, \mu_{i1,t-1}) &= \sigma_\mu^2 \\
\text{Cov}(\mu_{iat}, \mu_{i,a-s,t-s}) &= \text{Cov}(\mu_{i,a-1,t-1}, \mu_{i,a-s,t-s})
\end{align*}
\]
\[
\text{var}(v_{il}) = \delta_t
\]
\[
\text{var}(v_{iat}) = \rho_t^2 \text{var}(v_{i,a-1,t-1}) + \delta_t + \theta_t^2 \delta_t + 2\rho_t \theta_t \delta_{t-1}, \quad a > 1
\]  \hfill (B12)
\[
\text{cov}(v_{iat}, v_{i,a-1,t-1}) = \rho_t \text{var}(v_{i,a-1,t-1}) + \theta_t \delta_{t-1}
\]  \hfill (B13)
\[
\text{cov}(v_{iat}, v_{i,a-s,t-s}) = \rho_t \text{cov}(v_{i,a-1,t-1}, v_{i,a-s,t-s}) + \theta_t \delta_{t-1}, \quad s > 1
\]  \hfill (B14)
\[
\text{cov}(v_{iat}, v_{i,a-s,t-s}) = \rho_t \text{cov}(v_{i,a-1,t-1}, v_{i,a-s,t-s}) + \theta_t \delta_{t-1}, \quad s > 1
\]  \hfill (B15)

Equations (B6) and (B7) provide recursion relationships for the variances and covariances, respectively, of equation (2) in the text. Equations (B8)-(B11) are recursion relationships for the variances and covariances of the permanent component, while (B12)-(B15) are recursion relationships for the variances and covariances of the transitory component. Conditional on values of the initial permanent variance \(\sigma_\mu^2\) and the five parameters \(\alpha_t, \rho_t, \theta_t, \psi_t, \text{ and } \delta_t\), all variances and covariances can be calculated by starting at \(a = 1\) for each cohort (each cohort begins at \(a = 1\) at a different calendar time, \(t\)) and by moving recursively forward over the life cycle using the formulae.
Appendix C
Relation of Mobility to Covariance Structure

Assume we have a random sample of \( n \) individuals with earnings observed at two points in time. We denote the earnings of individual \( i \) at time \( t \) as \( y_{it} \) \((i=1, \ldots, n; t=1,2)\). Although earnings are independent across individuals we assume that they are correlated over time for the same individual. We also assume that the two earnings observations for each individual follow a bivariate normal distribution, with means zero and with \( \text{Var}(y_{it}) = \sigma^2_y \) and \( \text{Cov}(y_{it}, y_{it'}) = \rho \sigma^2_y (t \neq t') \). Let \( P \) denote the probability that there are no changes in rank in the distribution from \( t=1 \) to \( t=2 \). We shall demonstrate that \( \partial P / \partial \rho > 0 \) and that \( P \) is independent of \( \sigma^2_y \).

Ordering the individuals from \( i=1 \) to \( i=n \) by rank, we have:

\[
P = n! \, \text{Prob}(y_{11} < y_{21} < y_{31} < \ldots < y_{n1}, y_{12} < y_{22} < y_{32} < \ldots < y_{n2}) \quad (A1)
\]

since there are \( n! \) possible orderings of the \( n \) individuals, each ordering with the same probability.

It is sufficient to compare only the change in relative rank for any given pair of individuals \( i \) and \( j \), since the result will generalize to all pairs. Let

\[
P' = \text{Prob}(y_{i1} < y_{j1}, y_{i2} < y_{j2}) + \text{Prob}(y_{i1} > y_{j1}, y_{i2} > y_{j2})
= 2 \cdot \text{Prob}(y_{i1} < y_{j1}, y_{i2} < y_{j2}) \quad (A2)
\]

Defining
\[ w_1 = y_{11} - y_{12} \]  
\[ w_2 = y_{21} - y_{22} \]  

we have that \( w_1 \) and \( w_2 \) are distributed bivariate normal with means zero and with \( \text{Var}(w_t) = 2\sigma_y^2 \) and \( \text{Cov}(w_1, w_2) = 2\sigma_y^2 \rho \). Hence

\[
P' = \int_{-\infty}^{0} \int_{-\infty}^{0} (2\sigma_y^{-1}) \ b(w_1, w_2; \rho) \ dw_1 \ dw_2 \tag{A5}
\]

\[
= \int_{-\infty}^{0} \int_{-\infty}^{0} b(\hat{w}_1, \hat{w}_2; \rho) \ dw_1 \ dw_2 \tag{A6}
\]

where \( b \) is the unit bivariate normal density and where \( \hat{w}_j = w_j / \sqrt{2\sigma_y} \). Thus \( P' \) is only a function of \( \rho \) and not a function of \( \sigma_y^2 \).

The partial derivative of a bivariate normal cumulative distribution function w.r.t. \( \rho \) is equal to the bivariate density evaluated at the upper limits. Hence \( \partial P' / \partial \rho = b(0,0; \rho) > 0 \).
REFERENCES


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"The Growth of Earnings Instability in


Table 1
Descriptive Covariance Regressions for Log Annual Earnings

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<td>-.0027*</td>
<td>-.0027*</td>
<td>-.0027*</td>
</tr>
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<td></td>
<td></td>
<td>(.0006)</td>
<td>(.0006)</td>
<td>(.0006)</td>
</tr>
<tr>
<td>(A2-A1)</td>
<td>-</td>
<td>-.0145*</td>
<td>-.0144*</td>
<td>-.0102*</td>
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<tr>
<td></td>
<td></td>
<td>(.0014)</td>
<td>(.0015)</td>
<td>(.0015)</td>
</tr>
<tr>
<td>(A2-A1)^2/100</td>
<td>-</td>
<td>.0343*</td>
<td>.0343*</td>
<td>.0573*</td>
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<td></td>
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<td>(.0084)</td>
<td>(.0083)</td>
<td>(.0112)</td>
</tr>
<tr>
<td>U2</td>
<td>-</td>
<td>-</td>
<td>.0027*</td>
<td>.0012*</td>
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<td></td>
<td></td>
<td></td>
<td>(.0006)</td>
<td>(.0007)</td>
</tr>
<tr>
<td>DU2</td>
<td>-</td>
<td>-</td>
<td>.0046</td>
<td>.0045</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(.0036)</td>
<td>(.0036)</td>
</tr>
<tr>
<td>U1</td>
<td>-</td>
<td>-</td>
<td>.0022</td>
<td>.0038*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(.0013)</td>
<td>(.0014)</td>
</tr>
<tr>
<td>t(A2-A1)/10</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-.0046*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(.0011)</td>
</tr>
<tr>
<td>Chi-squared^a</td>
<td>2272</td>
<td>1029</td>
<td>1138</td>
<td>1101</td>
</tr>
<tr>
<td>(df)</td>
<td>(551)</td>
<td>(545)</td>
<td>(542)</td>
<td>(539)</td>
</tr>
</tbody>
</table>

Notes:
*: significant at the 10 percent level
n=553

Unemployment rate is for all U.S. male civilians 20 and over.
D=diagonal dummy; A2 = the older age minus 20; A1 = the younger age
minus 20; t = year at age A2 minus 1969; U2 = unemployment rate at
age A2; U1 = unemployment rate at age A1

^a Statistic=ne'S-1e, where e is the vector of estimated residuals and S
is their empirical covariance matrix.
### Table 2

Error Components Models for Log Annual Earnings
with Calendar Time Effects

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_t$:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year</td>
<td>.019*</td>
<td>.018*</td>
</tr>
<tr>
<td></td>
<td>(.003)</td>
<td>(.008)</td>
</tr>
<tr>
<td>$\text{Var}(\xi_{iats})$:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year</td>
<td>.006*</td>
<td>.006*</td>
</tr>
<tr>
<td></td>
<td>(.001)</td>
<td>(.001)</td>
</tr>
<tr>
<td>Constant</td>
<td>.112*</td>
<td>.109*</td>
</tr>
<tr>
<td></td>
<td>(.010)</td>
<td>(.010)</td>
</tr>
<tr>
<td>$\rho_t$:</td>
<td></td>
<td>.005</td>
</tr>
<tr>
<td>Year</td>
<td>-</td>
<td>(.010)</td>
</tr>
<tr>
<td>Constant</td>
<td>.720*</td>
<td>.657*</td>
</tr>
<tr>
<td></td>
<td>(.055)</td>
<td>(.153)</td>
</tr>
<tr>
<td>$\theta_t$:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year</td>
<td>-</td>
<td>-.007</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.010)</td>
</tr>
<tr>
<td>Constant</td>
<td>-.448*</td>
<td>-.357*</td>
</tr>
<tr>
<td></td>
<td>(.059)</td>
<td>(.157)</td>
</tr>
<tr>
<td>$\text{Var}(\omega_{iats})$:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year</td>
<td>-</td>
<td>.001</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(.010)</td>
</tr>
<tr>
<td>Constant</td>
<td>.117*</td>
<td>.119*</td>
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<tr>
<td></td>
<td>(.035)</td>
<td>(.037)</td>
</tr>
<tr>
<td>$\sigma^2_{\mu}$</td>
<td>.059*</td>
<td>.060*</td>
</tr>
<tr>
<td></td>
<td>(.005)</td>
<td>(.009)</td>
</tr>
<tr>
<td>Chi-Squared</td>
<td>1082</td>
<td>1076</td>
</tr>
<tr>
<td>(df)</td>
<td>(546)</td>
<td>(543)</td>
</tr>
</tbody>
</table>

Notes: Standard errors in parentheses
*: significant at 10 percent level
Year=0 in 1969, -1 in 1970, etc.
**Parameters multiplied by 100.**
Table 3

Effects of Parameter Changes on Log Annual Earnings Variances, 1969-1987

<table>
<thead>
<tr>
<th>Variances by Age</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>1969 Values of all</td>
<td>.179</td>
<td>.199</td>
<td>.210</td>
<td>.221</td>
</tr>
<tr>
<td>Parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1987 Value of $\alpha_\ell$ only</td>
<td>.240</td>
<td>.273</td>
<td>.296</td>
<td>.318</td>
</tr>
<tr>
<td>1987 Values of $\alpha_\ell$ and Variance of $\xi_{iat}$</td>
<td>.338</td>
<td>.379</td>
<td>.401</td>
<td>.423</td>
</tr>
<tr>
<td>1987 Values of $\alpha_\ell$, Variance of $\xi_{iat}$, and $\theta_\ell$</td>
<td>.338</td>
<td>.372</td>
<td>.394</td>
<td>.416</td>
</tr>
<tr>
<td>1987 Values of all</td>
<td>.338</td>
<td>.401</td>
<td>.423</td>
<td>.445</td>
</tr>
<tr>
<td>Parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4
One-Year Quintile Mobility Rates for Log Annual Earnings: All Years and Ages

<table>
<thead>
<tr>
<th>Quintile at t-1</th>
<th>Sun</th>
<th>Bottom Fifth</th>
<th>Next to Bottom Fifth</th>
<th>Middle Fifth</th>
<th>Next to Top Fifth</th>
<th>Top Fifth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom Fifth</td>
<td>100</td>
<td>67</td>
<td>21</td>
<td>8</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Next to Bottom Fifth</td>
<td>100</td>
<td>20</td>
<td>49</td>
<td>22</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>Middle Fifth</td>
<td>100</td>
<td>7</td>
<td>21</td>
<td>44</td>
<td>22</td>
<td>6</td>
</tr>
<tr>
<td>Next to Top Fifth</td>
<td>100</td>
<td>4</td>
<td>7</td>
<td>20</td>
<td>47</td>
<td>22</td>
</tr>
<tr>
<td>Top Fifth</td>
<td>100</td>
<td>2</td>
<td>3</td>
<td>7</td>
<td>20</td>
<td>69</td>
</tr>
</tbody>
</table>

Notes: 477 observations per row
Table 5
Year Coefficients in Quintile Mobility Regressions

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Initial Quintile Location</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Bottom Fifth</td>
<td>Next to Bottom Fifth</td>
<td>Middle Fifth</td>
<td>Next to Top Fifth</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All Lag Orders</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year</td>
<td>-.0020*</td>
<td>-.0040*</td>
<td>-.0028*</td>
<td>-.0010</td>
<td>-.0007</td>
<td>-.0020*</td>
</tr>
<tr>
<td></td>
<td>(.0004)</td>
<td>(.0008)</td>
<td>(.0008)</td>
<td>(.0007)</td>
<td>(.0007)</td>
<td>(.0007)</td>
</tr>
<tr>
<td>Lag Orders 1-4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1969-1980</td>
<td>-.0013</td>
<td>-.0051*</td>
<td>-.0008</td>
<td>.0001</td>
<td>.0006</td>
<td>-.0017</td>
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<tr>
<td></td>
<td>(.0011)</td>
<td>(.0019)</td>
<td>(.0019)</td>
<td>(.0019)</td>
<td>(.0017)</td>
<td>(.0017)</td>
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<tr>
<td>1981-1987</td>
<td>-.0025*</td>
<td>-.0022</td>
<td>-.0038</td>
<td>-.0026</td>
<td>-.0017</td>
<td>-.0026</td>
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<td></td>
<td>(.0014)</td>
<td>(.0023)</td>
<td>(.0024)</td>
<td>(.0024)</td>
<td>(.0021)</td>
<td>(.0022)</td>
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<td>Lag Orders 5+</td>
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<td>1969-1980</td>
<td>-.0067*</td>
<td>-.0083*</td>
<td>-.0035</td>
<td>-.0014</td>
<td>-.0087*</td>
<td>-.0108*</td>
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<tr>
<td></td>
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<td>(.0035)</td>
<td>(.0034)</td>
<td>(.0030)</td>
<td>(.0034)</td>
<td>(.0034)</td>
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<tr>
<td>1981-1987</td>
<td>-.0005</td>
<td>-.0027</td>
<td>-.0044*</td>
<td>-.0004</td>
<td>.0021</td>
<td>.0020</td>
</tr>
<tr>
<td></td>
<td>(.0009)</td>
<td>(.0018)</td>
<td>(.0018)</td>
<td>(.0015)</td>
<td>(.0018)</td>
<td>(.0018)</td>
</tr>
</tbody>
</table>

Notes: Standard errors in parentheses
*: significant at the 10 percent level
Sample sizes are 477 for all-lag-order sample, and 198 and 279 for the 1-4 lag-order and 5+ lag-order samples, respectively.
Dependent variable: fraction of population in the relevant age-year-quintile cell that changed quintiles over the lag orders shown.
Independent variables in addition to time trends: A2, A3-A1, and (A2-A1) squared (see notes to Table 2 for definitions).
Table 6

Covariance Analysis of Real Weekly Wages and Annual Weeks of Work

<table>
<thead>
<tr>
<th></th>
<th>Log Real Weekly Wage</th>
<th>Log Annual Weeks Worked</th>
<th>Annual Weeks Worked$^a$</th>
</tr>
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<tbody>
<tr>
<td></td>
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<tr>
<td><strong>Descriptive Regressions$^b$</strong></td>
<td></td>
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</tr>
<tr>
<td>$t$</td>
<td>.0044*</td>
<td>.0008*</td>
<td>.485*</td>
</tr>
<tr>
<td></td>
<td>(.0004)</td>
<td>(.0002)</td>
<td>(.110)</td>
</tr>
<tr>
<td>$Dt$</td>
<td>.0043*</td>
<td>.0021*</td>
<td>.846*</td>
</tr>
<tr>
<td></td>
<td>(.0008)</td>
<td>(.0004)</td>
<td>(.215)</td>
</tr>
<tr>
<td><strong>Error Components Model$^c$</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_t$</td>
<td>.024*</td>
<td>.075</td>
<td>.013</td>
</tr>
<tr>
<td></td>
<td>(.002)</td>
<td>(.065)</td>
<td>(.014)</td>
</tr>
<tr>
<td>$\text{Var}(\xi_{iat})$</td>
<td>.003*</td>
<td>.002*</td>
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<tr>
<td></td>
<td>(.001)</td>
<td>(.000)</td>
<td>(.283)</td>
</tr>
</tbody>
</table>

Notes:  
*: significant at the 10 percent level  
$^a$: For positive weeks of work  
$^b$: For specification in third column of Table 2  
$^c$: Trend coefficients. For specification in first column of Table 5
Table 7
Steady-State Variance Components
Implied by the Estimated Error Components Models

<table>
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<tr>
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<th>Total</th>
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<tr>
<td></td>
<td>Permanent</td>
<td>Transitory</td>
</tr>
<tr>
<td><strong>Age 20</strong></td>
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<td></td>
</tr>
<tr>
<td>1969</td>
<td>0.061</td>
<td>0.118</td>
</tr>
<tr>
<td>1987</td>
<td>0.122</td>
<td>0.216</td>
</tr>
<tr>
<td><strong>Age 30</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1969</td>
<td>0.072</td>
<td>0.127</td>
</tr>
<tr>
<td>1987</td>
<td>0.144</td>
<td>0.258</td>
</tr>
</tbody>
</table>

Notes: The permanent variance is the estimated value of Var(\(\nu_{i,t}\)) at 1969 and 1987 values. The transitory variance is the estimated value of Var(\(\nu_{i,t}\)) at 1969 and 1987 values of the parameters. The model estimates in column (1) of Table 2 and the analogous estimates for total log earnings are used.
Log Earnings Variances and Covariances by Year and by Lag Order

**Figure 1(a): All Ages**

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariance</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
</tr>
</tbody>
</table>

**Figure 1(b): Ages 25-34**

<table>
<thead>
<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariance</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
</tr>
</tbody>
</table>

**Figure 1(c): Ages 35-44**

<table>
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<th></th>
</tr>
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<tbody>
<tr>
<td>Covariance</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
</tr>
</tbody>
</table>

**Figure 1(d): Ages 45-54**

<table>
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</thead>
<tbody>
<tr>
<td>Covariance</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
</tr>
</tbody>
</table>
Figure 3: Mobility Rates and Correlation Coefficients by Year and Age

Figure 3(a): Ages 35-36

Figure 3(b): Ages 35-40