A Welfare-Based Measure of Productivity Growth with Environmental Externalities

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OF
PRODUCTIVITY GROWTH
WITH
ENVIRONMENTAL EXTERNALITIES

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The negative impact of environmental regulations on productivity growth — as conventionally measured — is well documented in the literature (e.g., Gollop and Roberts (1983)). Since the conventional measure of productivity growth accounts for the higher production costs caused by the environmental regulations but not the benefits resulting from reduced pollution, this result is not surprising. It does, however, raise the issue of whether, in this context, the conventional measure of productivity growth is in fact measuring what we want it to measure. If we want to measure the rate of technical progress, then we would like somehow to net out the effects of environmental regulation. And if we want to measure how rapidly society is getting better at combining a given set of inputs to produce “welfare”, then we would want to include the benefits from pollution reduction.

The problem is illustrated diagrammatically in Figure 1 which plots the production-possibilities frontier before (PPF₀) and after (PPF₁) a productivity improvement, with respect to a generic numéraire consumption good (which includes investment) and air quality, holding inputs fixed. In the absence of environmental regulation, the pre-improvement equilibrium is at A since the producer price of air quality is zero, and the post-improvement equilibrium is at B. Environmental regulation is modeled simply as a floor on air quality, a. With environmental regulation fixed at a, the pre-improvement equilibrium is at C and the post-improvement equilibrium is at D. Finally, the efficient production point — where the indifference curve is tangent to the production-possibility frontier — is at E prior to the improvement and at F afterwards.

Suppose, for the sake of argument, that the environmental regulation is applied between the times at which PPF₀ and PPF₁ apply. Then the equilibrium over this time period shifts from A to D. The issue is how the productivity improvement should be measured. The conventional procedure is to ignore air quality, which is tantamount to according it a zero price in productivity.

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measurement. Accordingly, the productivity improvement over the period is measured as $A'D'$, and the proportional productivity improvement as $\frac{A'D'}{O'A'}$. If the environmental regulation had not been imposed, the corresponding magnitudes would be $A'B'$ and $\frac{A'B'}{O'A'}$. It is evident that the environmental regulation causes the measured productivity improvement to decrease.

Now suppose that in the pre-improvement situation a minimum air quality regulation of $\bar{a}$ is in place and that in the post-improvement situation the minimum regulated air quality $\bar{a}$ is higher. This scenario is shown in Figure 2. The equilibrium then shifts from C to G. According to the conventional measure which gives a zero price to air quality, the productivity improvement is $cg$ and the proportional productivity improvement $\frac{cg}{Oc}$. But at C, there is a marginal cost or producer price to producing air quality. If one were to use this producer price in evaluating national income, measured as the height of the intersection point of the price line tangent to C with the vertical axis, the pre-improvement national income would be $O'C'$ and the post-improvement national income $O'G'$, with a productivity gain of $G'C'$ and a proportional productivity gain of $\frac{G'C'}{O'C'}$. A problem with this procedure is that it measures national income with “minimum” air quality as a reference point. But there is no natural minimum air quality. A superior procedure is to measure national income relative to maximum air quality. This is superior for two reasons. First, there is a natural maximum air quality — clean air. Second, standard national accounting measures do not include clean air. Under this alternative procedure, pollution is treated as a bad and subtracted from GNP. Real income in the pre-improvement situation is then measured as the height of the intersection point of the price line tangent to C with the vertical line through O, $OC''$. Post-improvement national income is $OG''$, the productivity gain $C''G''$ and the proportional productivity gain $\frac{C''G''}{OC''}$. This measure of productivity gain fails to correct for the loss in productivity due to the stricter environmental regulations but does value the reduction in
pollution — at the pre-improvement producer price.

**INSERT FIGURE 2 HERE**

In contrast, when air quality is given a zero price, the proportional productivity gain is the same whether measured with reference to $O'$ or $O$. Accordingly, we advocate using $O$ as the appropriate origin. If this is done, it is natural to define “smoke” to be the quality of clean air minus the actual quality of the air. Figure 3 portrays this convention. There are then three measures of productivity improvement in the movement of the economy from $C$ to $G$, which differ according to how smoke is valued. The first is the conventional measure, which we denote by $I_0$ — the index of productivity improvement where smoke is priced at zero\(^1\) : $I_0 = \frac{gc}{Oc}$. The second is the measure using the (negative) producer price of smoke at the pre-improvement equilibrium, which we denote by $I_p$: $I_p = \frac{G'C'}{O'C'}$. And the third is the measure using the (negative) consumer price of smoke at the pre-improvement equilibrium, which we denote by $I_q$: $I_q = \frac{G''C''}{OC''}$. If the consumer price exceeds the producer price in absolute value, which seems reasonable empirically, then $G''C'' > G'C' > gc$ and $Oc > OC'' > OC'$, which together imply that $I_q > I_p > I_0$.

**INSERT FIGURE 3**

Several comments are in order. First, all three indexes measure the proportional productivity improvement as the change in real income divided by real income, where real income is measured as the value of consumption and investment minus the cost of smoke; they differ in how they cost smoke. Second, what is the “true” proportional productivity improvement? This reduces to the question of what is “true” real income. Weitzman (1976) provides one answer for the situation where there is no smoke: True real income is the current value of the Hamiltonian where the objective is the maximization of the present value of consumption, and therefore equals consumption plus investment. How might this measure be adapted to include smoke? One possibility is to replace consumption in the definition by consumption minus the utility cost of

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\(^1\) The subscripts on the productivity indexes are chosen using the optimal tax theory convention that $p$ denotes a producer price and $q$ a consumer price.
smoke, or equivalently that level of consumption which, in the absence of smoke, would provide
the same level of utility as the current situation. According to this definition, real national income
in the pre-improvement situation would be \( OC \) and in the post-improvement situation would be
\( OG \), and the index of productivity improvement would be \( \frac{G \cdot C}{O \cdot C} \). This measure is of conceptual
interest but, because it depends on the properties of the utility function outside the range of
observation, is not empirically implementable.

The above discussion lays out the basic theory on the simplifying assumption that inputs
are fixed. The next section provides empirically-implmentable measures of \( I_0 \), \( I_p \), and \( I_q \), which
account for changes in the economy’s inputs.

II. Empirical Measures

We employ the following notation

- \( p_j \) market price of good \( j, j = 1, ..., m \)
- \( y_j \) output of good \( j \)
- \( w_i \) market price of input \( i, i = 1, ..., n \)
- \( x_i \) quantity of input \( i \)
- \( s \) smoke
- \( \varepsilon \) marginal cost of reducing smoke by one unit
- \( \sigma \) marginal benefit of reducing smoke by one unit
- \( I \) index of total factor productivity growth

The conventional measure of growth in total factor productivity is

\[
I^* = \sum_j \left( \frac{p_j y_j}{M} \right) \frac{\dot{y}_j}{y_j} - \sum_i \left( \frac{w_i x_i}{M} \right) \frac{\dot{x}_i}{x_i},
\]

(1)

where \( M = \sum p_j y_j = \sum w_i x_i \) is real national income. The first term on the right-hand side is the
value-share weighted average of the growth rate of outputs, and the second term is the value-share
weighted average of the growth rate of inputs. The measure is derived in Jorgenson and Griliches
(1967) and Jorgenson, Gollop, and Fraumeni (1987), inter alia, on the assumptions that
production exhibits constant returns to scale, all markets are competitive, and all outputs are marketed.

The issue at hand is how this measure needs to be modified when there is an unmarketed output. The traditional procedure is simply to ignore the unmarketed output, in which case (1) continues to apply, so that

$$I_o = I^r$$  \hspace{1cm} (2)

Pittman (1983) proposed a measure that treats smoke as an undesirable output. National income is defined as

$$M_p \equiv \sum_j p_j y_j - \varepsilon s,$$

where $\varepsilon > 0$ is the marginal cost of reducing smoke by one unit. National income is defined using clean air as the reference point, and values smoke as the negative of its marginal cost or producer cost. We refer to this as the producer-cost measure of growth in total factor productivity:

$$I_p = \left( \frac{\sum_j p_j y_j}{M_p} \right) y_j - \left( \frac{\varepsilon s}{M_p} \right) s - \sum_i \left( \frac{w_i x_i}{\sum_j w_i x_i} \right) x_i$$  \hspace{1cm} (3)

Note that, because of general equilibrium effects, the measured growth in total factor productivity will differ depending on whether the reduction in smoke is achieved through taxation (with the revenue being returned to consumers as a lump sum) or regulation. First, the market-clearing prices will differ. Second, since (under the assumptions of constant returns to scale and perfect competition) the prices of all goods equal the corresponding unit costs: with regulation alone,

$$\sum_j p_j y_j = \sum_i w_i x_i;$$

with taxation alone

$$\sum_j p_j y_j = \sum_i w_i x_i + \tau s,$$

where $\tau = \varepsilon$ is the tax rate on smoke,

and with both taxation and regulation

$$\sum_j p_j y_j = \sum_i w_i x_i + \tau s$$ where $\tau \neq \varepsilon$ in general.

The measure we propose is analogous to Pittman’s except that smoke is valued at its (negative) shadow consumer price, $\sigma$ — the marginal rate of substitution between smoke and a numéraire composite commodity. Thus,

$$I_q = \left( \frac{\sum_j p_j y_j}{M_q} \right) y_j - \left( \frac{\sigma s}{M_q} \right) s - \sum_i \left( \frac{w_i x_i}{\sum_j w_i x_i} \right) x_i$$  \hspace{1cm} (4)

where $M_q = \sum_j p_j y_j - \sigma s$. When the level of smoke is controlled only by regulation,
\[ \sum_{j} p_j y_j = \sum_{i} w_i x_i, \text{ etc. — as above.} \]

It might appear that the measure we propose is ad hoc. It can, however, be derived from basic principles, which strengthens its appeal. Start with a representative individual’s utility function, \( U = U(y_1, \ldots, y_m, s) \). Totally differentiate the utility function with respect to time.

\[ \dot{U} = \sum_{j} U_j \dot{y}_j + U_s \dot{s}. \quad (5) \]

It makes no difference for a measure of productivity growth which good is taken as the numéraire; let it be good one. Then using the first-order conditions from the individual’s optimization problem:

\[ \frac{\dot{U}}{U_1} = \sum_{j} p_j \frac{\dot{y}_j}{y_j} - \sigma \frac{\dot{s}}{s} \]

\[ = \sum_{j} \left( p_j y_j \right) \frac{\dot{y}_j}{y_j} - \left( \sigma s \right) \frac{\dot{s}}{s}. \quad (6) \]

If we define \( M_q \) to be real national income, then (6) is growth of national income evaluated at consumer prices (including the consumer shadow price of smoke). And

\[ \frac{\dot{U}}{U_1 M_q} = \sum_{j} \left( \frac{p_j y_j}{M_q} \right) \frac{\dot{y}_j}{y_j} - \left( \frac{\sigma s}{M_q} \right) \frac{\dot{s}}{s} \]

\[ \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad

This welfare-based measure of productivity growth has a nice feature. Holding inputs fixed, the rate of productivity growth has the same sign as the direction of change of utility or welfare. Neither \( I_o \) nor \( I_p \) has this property.

We now relate the various productivity measures. To do so, note that all the productivity measures can be written as

\[ I(\rho) = \frac{A - \rho \dot{s}}{B - \rho s} - C, \quad (8) \]

\[ A \equiv \sum_{j} p_j \dot{y}_j, \quad B \equiv \sum_{j} p_j y_j, \quad \text{and} \quad C \equiv \frac{\sum w_i \dot{x}_i}{\sum w_i x_i}, \quad \text{and} \quad \rho \text{ is the shadow price of smoke used in the} \]
productivity measure, with \( \rho = 0 \) for \( I_0 \), \( \rho = \varepsilon \) for \( I_p \), and \( \rho = \sigma \) for \( I_q \). Then

\[
\frac{dI(\rho)}{d\rho} = \frac{-\dot{s}}{B - \rho s} + \frac{(A - \rho \dot{s})s}{(B - \rho s)^2} = \frac{-B \dot{s} + As}{(B - \rho s)^2}
\]

\[
= \frac{sB}{(B - \rho s)^2} \left( -\frac{\dot{s}}{s} + \frac{A}{B} \right)
\]  

(9)

If, therefore, the rate of growth of market output, \( \frac{A}{B} \), exceeds (falls short of) the rate of growth of smoke, then measured productivity growth is increasing (decreasing) in \( \rho \), the shadow price of smoke used in the productivity measure. In the case of central interest, where environmental regulation is becoming increasingly stringent so that \( -\frac{\dot{s}}{s} + \frac{A}{B} > 0 \), \( I_q > I_p > I_0 \). Also, using (8)

\[
I(\rho_1) - I(\rho_0) = \frac{A - \rho_1 \dot{s}}{B - \rho_1 s} - \frac{A - \rho_0 \dot{s}}{B - \rho_0 s}
\]

\[
= \frac{sB(\rho_1 - \rho_0)(-\frac{\dot{s}}{s} + \frac{A}{B})}{(B - \rho_1 s)(B - \rho_0 s)}
\]  

(10)

for any pair of values of \( \rho_0 \) and \( \rho_1 \).

Which measure of productivity growth is superior depends on the intent of the measure. If the intent is to measure the rate at which welfare is increasing, netting out the growth rate of inputs, then \( I_q \) is the appropriate measure.

III. Concluding Comments

The paper developed a measure of total factor productivity growth designed to account for the beneficial effects of environmental regulation — which we termed a welfare-based measure of productivity growth.

To make the essential points as starkly as possible, we employed a very simple description of an economy. The theory of productivity measurement has been extended to include taxes and subsidies, intermediate inputs, public services, imports and exports, non-constant returns to scale, and non-competitive pricing. The welfare-based measure of productivity growth we have presented could be generalized to treat all these extensions.

To operationalize the welfare-based measure of productivity growth requires not only
making these extensions, but also measuring the marginal benefit from environmental improvement, which is notoriously difficult. Nevertheless, the usefulness of the measure has already been demonstrated in practical applications (Chaston (1997) and Swinand (1997)).
REFERENCES


Figure Legends

Figure 1: Pre- and post- improvement production-possibility frontiers.

Figure 2: Productivity improvement using producer prices with strengthening environmental regulation.

Figure 3: Four measures of productivity improvement with strengthening environmental regulation: conventional \( \frac{g_c}{O_c} \), using producer price for smoke \( \frac{G''C''}{O'C''} \), using consumer price for smoke \( \frac{G''C''}{O'C''} \) and using utility \( \frac{\hat{G'C}}{O'C} \).