Agency in Project Screening and Termination Decisions: Why is Good Money Thrown after Bad?

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Why is Good Money Thrown after Bad?
Chong-en Bai*  Yijiang Wang**

Abstract

We construct an agency model in which the planner (agent) makes project starting and termination decisions on behalf of the state (principal) to reflect the practice of socialist economies. The model shows that asymmetric information between the state and the planner regarding the quality of projects started leads to the persistence of unprofitable projects in most cases. Since in the model it is assumed that the state's objective is to maximize economic profit and the state has full power to dictate and enforce the optimal contract, the finding of the model has the implication that hardening budget constraints in socialist economies is difficult even under an "ideal" setting when these economies are free of social considerations and political frictions.

JEL classification: P51, D82
Key Words: Soft Budget, Agency, Project Screening [and Termination], Information.

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1. Introduction

Since Kornai's (1980) seminal work, it has been widely recognized that socialist economies are flawed in allocating investment funds and suffer from the "soft-budget" problem. Particularly puzzling is the question of why money-losing projects are not terminated. The purpose of this paper is to argue that the phenomenon of not terminating unprofitable projects can be explained by agency problems in the sequential decision process of project screening and termination. In this introduction, we first briefly describe the main idea of our argument and then compare it with explanations offered by others.

We model an economy in which there are many projects, each requiring an initial and a subsequent investment before the return can be realized. The projects have continuously distributed returns, some of them ex ante (before the initial investment) profitable, others ex ante unprofitable but ex post (after the initial investment) profitable, and still others unprofitable both ex ante and ex post. The principal in the model is the state, and the agent is the planner (the bureaucrat). The agent first examines the profitability of the projects, the only action that requires an effort, and then decides which projects to undertake. The number of unprofitable projects successfully identified increases with the agent's screening effort. Once the projects to be

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1 The problem caused enormous inefficiency in former socialist economies and contributed greatly to the eventual political collapse of many of them. As a significant economic phenomenon, the problem has not disappeared with the reforms undertaken in socialist economies. In China, for example, after 17 years of market-oriented reforms, poor financial performance continues to plague the state sector. In recent years, fully one-third of state-owned enterprises persistently lost money. Another one-third are believed to have made money solely due to their receipt of various forms of state subsidies.

2 Kornai (1980, p.197) described the problem in the following words: "From the claimant's point of view investment is a long campaign with many battles. But the whole campaign has only one life-and-death battle and that is at the beginning, since approval must be obtained for starting the investment. Once started, it will end in some way and at some time. That is exactly why it is possible to underestimate, without much hesitation, expected costs, and to forget about complementary investments. If costs are higher, or if investments above the plan are necessary, money will surely be raised in one way or another. Perhaps the claimant will be blamed for erroneous calculations, perhaps work will slow down for a while to wait for financial cover, but an investment project that has been started will not be stopped for good."

3 The terms "ex ante" and "ex post" here have the same meaning as in Dewatripont and Maskin (1995). A project is ex ante profitable if its return is greater than the total of the initial and the second investment. It is ex post profitable if its return is greater than the second investment.
undertaken have been chosen, and the initial investments made, the agent obtains perfect information about the quality of all started projects; the principal does not. The agent then decides which of these projects to terminate. Projects that are not terminated will receive the second investment and, after that, realize returns. The main question of interest is: What should the state's contract with the planner say about project termination decisions?

If the principal does not impose restrictions on the agent's project-termination decisions, and ties the agent's income only to aggregate net output, then the agent will terminate all *ex post* unprofitable projects (projects that have a return smaller than the second investment) to maximize net output. However, as we will show, by requiring the agent to continue some *ex post* unprofitable projects in the *ex ante* incentive contract, the principal can expect a higher profit. The reason for this is that, if an *ex post* unprofitable project is not screened out, it costs the agent more if it can not later be terminated than if it can. Therefore, requiring the agent to continue some *ex post* inefficient projects has the effect of increasing the marginal benefit of effort to the agent. This effect relaxes the incentive constraint for the agent's effort and, given the sharing rule, induces a higher effort. The higher effort leads to the benefit of more unprofitable projects being identified in the initial screening so that investments are not wasted on them. Restricting the agent's flexibility in project termination is desirable as long as the cost of continued financing of *ex post* unprofitable projects is smaller than the benefit of the resulting higher screening effort. This is usually true in the case of infinite projects with continuously distributed returns because of the envelope theorem.

Previous explanations of the soft budget problem have focused largely on social and political grounds. Kornai (1980) attributes it to the "paternalistic" role of the socialist state. Bardhan (1993) attributes it to the lower tolerance of socialist than capitalist societies for mobility and unemployment. Li (1996) suggests that employment stability has a social value taken into

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4 Another way of understanding the result is as follows. When the agent's effort is higher, more bad projects are found by the agent before the initial investment and quality of projects that are started is higher. As a result, the cost of continuing an *ex post* unprofitable projects is lower. Therefore, imposing restrictions on project termination costs the agent less when effort is higher.
account by socialist but not by private capitalist employers. Also, the reports that workers of a shipyard in an East European country struck to force the government to back down from a decision to reduce subsidies suggest that soft budget constraints can be a result of political bargaining. The problem exists because socialist governments are responsible for overseeing project performance, but politically too weak to commit to terminating money-losing projects.5

Dewatripont and Maskin (1995) provide an economic rationale for the soft budget constraint by focusing on an informational feature of a centralized system. In their model, managers have the *ex ante* information of the profitability of projects and propose projects to be undertaken. Projects approved by the creditor, which is a government agency in socialist economies, are undertaken. Faced with limited opportunities, managers have an incentive to propose projects that are *ex ante* unprofitable, but *ex post* profitable. The creditor can try to discourage managers from proposing *ex ante* unprofitable projects by threatening not to refinance them. The threat, however, is not credible because, when the creditor learns about the quality of the project, the initial investment in the project is already sunk. It is then in the creditor's self interest to finish projects that are *ex post* profitable. Schaffer (1989) and Segal (1993) also offer explanations based on time-inconsistency of the central planner.

We see these different explanations as complements rather than substitutes to each other, each of them offering some insight, but, by itself, explaining only part of the complex phenomenon of the soft budget constraint. For example, while the "preference" of the socialist state (paternalism or employment preference) can sometimes lead the state to "bailout" money-losing projects, one could also argue that improved project performance under hard budget constraints would only enhance the state's ability to play its paternalistic role. The degree to which the government or people in a socialist country might want to trade the greater long-term instability and higher unemployment associated with a less efficient economy for increased short-term

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5We thank a seminar participant for emphasizing this view.
stability and employment is also not clear. Similarly, while the bargaining story is plausible in some cases, it does not seem to explain why soft budget constraints also prevailed in socialist economies during the times of Stalin, Mao or Deng when the KGB or troops would be sent to crush any resistance to the leader's will. Dewatripont and Maskin's work and those of Schaffer (1989) and Segal (1993) address the important question of why in socialist economies \textit{ex ante} inefficient projects are started and refinanced. However, since these explanations are based on the idea that the government can not commit to not pursuing \textit{ex post} efficiency, they do not address the question as why \textit{ex post} inefficient projects are not terminated.  

Some people may feel uncomfortable with our result that it is the state who in the optimal contract requires the planner to refinance unprofitable projects. Since the enormous inefficiency resulting from soft budget constraints is so obvious, it may seem to be more intuitive to believe that the state "desires" or "wants" to terminate money-losing projects. It is not doing so only because, for one reason or another (e.g., workers' resistance), the state fails to commit to an economically efficient solution. 

We certainly see the inefficiency resulting from refinancing money-losing projects. We also agree that, in socialist economies, political considerations are very important in microeconomic decision making and can contribute to the soft budget problem. However, while explanations like political bargaining have the merit that they are more specific of conflicts, frictions and political institutions in socialist economies, our approach of abstracting from these institutional details also

\footnote{Indeed, since communist governments do not face elections every few years, one could quite reasonably argue that they are better able to take measures that reflect the long-term interests of the people even though these measures may cause short-term political damage due to a higher level of short-term unemployment.}

\footnote{In these models, the government can pursue \textit{ex post} efficiency because \textit{ex post} there is no asymmetric information between the government and managers. In contrast, in our explanation, the state's behavior is time consistent. Asymmetric information regarding the profitability of projects persists after the initial investment. As we will discuss in detail later, because of this problem of \textit{ex post} asymmetric information, renegotiation does not qualitatively alter the result of our model.}

\footnote{Note that the commitment problem here is due to the government's lack of political power, which is different from that in Dewatripont and Maskin (1994), Schaffer (1989), and Segal (1993) where the failure is due to time inconsistency in the government's own behavior.}
has a benefit. It enables us to focus on a most fundamental feature of socialism: socialist economies are hierarchically managed with the means of production publicly owned (practically state owned). This ownership arrangement precludes the trading of productive equities and, thereby, eliminates the price mechanism for information about project quality. With an alternative mechanism to secure information yet to be found, asymmetric information exists between the state and the planner regarding the quality of ongoing projects. A valuable insight of our model is that this asymmetric information problem in the hierarchically managed socialist system is sufficient for the soft budget problem to arise. By assuming that the state's objective is to maximize profit and it can dictate optimal contracts, our model studies the soft budget problem in a socialist economy that is "perfect" except for asymmetric information between the state and the planner. The result obtained under these assumptions shows that the soft budget constraint is unavoidable in socialism even under an ideal setting when it is free of social considerations and political frictions. It is very suggestive of the limited extent to which the socialist system can improve the efficiency of investment fund allocation without doing away with some of its most fundamental features.

Our focus on the informational problem in socialist economies and the result derived from the model echo a view emphasized by Hayek (1945), who argued that a main problem of "designing an efficient economic system" is how to utilize "knowledge not given to anyone in its totality." He then suggested that it is in utilizing information "initially dispersed among all the people" through a price system that a decentralized economy enjoys a merit not shared by centralized planning. Following Hayek, Stiglitz (1994) also sees informational problems in

9Public ownership of the means of production is one of the most fundamental teachings of Marxism.

10Price signals from equity markets may not provide perfect information on project quality. Also, it is not clear if alternative institutions exist that can provide equally good or better information of ongoing projects. Practically, however, it seems that a more effective institution than the equity market is yet to be found.
socialism as "perhaps the most important reason" for failure (p.198). How to better utilize information in an economy is also a focal point in the discussion of market socialism, both historically and most recently.

The plan for the remainder of the paper is as follows. The model is introduced in Section 2. Section 3 discusses how the planner's effort and the restrictions on the planner's project starting- and termination-decision affect the expected gross value-added of the projects. The main result of the model is derived in Section 4. Section 5 discusses some of the assumptions made in the model. Section 6 concludes the paper with some additional remarks.

2. The Model

The principal in the model is the socialist state and the agent the planner (bureaucracy). The timing of events is given in Figure 1.

<table>
<thead>
<tr>
<th>Event</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>before time 0</td>
<td>The principal receives project proposals, hires the agent, and signs an incentive contract with him.</td>
</tr>
<tr>
<td>time 0</td>
<td>The agent examines the projects and decides which projects to fund.</td>
</tr>
<tr>
<td>first stage</td>
<td>The first-stage investment $c$ is made.</td>
</tr>
<tr>
<td>time 1</td>
<td>The profitability of funded projects is revealed to, and the termination decision is made by, the agent.</td>
</tr>
<tr>
<td>second stage</td>
<td>The second-stage investment $i$ is made.</td>
</tr>
<tr>
<td>time 2</td>
<td>Returns to the projects are realized and the agent is rewarded.</td>
</tr>
</tbody>
</table>

Figure 1

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11 On the role of price in designing an efficient economic system, Stiglitz (1994, p.202) points out that "...while the price system may be imperfect, it performs a number of vital roles." One of such roles is that "...prices...provide the basis of an incentive structure and a selection mechanism" in an economy.

12 See Bardhan and Roemer (1993) for a review of the history and a collection of recent works on market socialism.
Before time 0, the principal receives a continuum of project proposals. Without loss of generality, we assume that the projects are uniformly distributed on the unit square \([0,1] \times [0,1]\), represented by area OCKH in Figure 2. The expected revenue (type) of a project, denoted by \(\alpha\),

Ex post unprofitable projects

Ex ante profitable projects

Area OAIH is the set of ex post unprofitable projects
Area ABJI is the set of ex ante unprofitable but ex post profitable projects
Area BCKJ is the set of ex ante profitable projects
Area OCDG is the set of projects that are reviewed at time 0
Area OBEG is the set of ex ante unprofitable projects that are identified at time 0

Figure 2

13 We will discuss the case in which there are finite projects in section 4.

In this paper, we assume that the managers under the administration of the planner are each endowed with a project of given profitability. As long as a project is started, the manager will receive a positive private benefit that cannot be taken away. We also assume that the manager has no personal wealth and thus cannot be penalized for proposing a poor project. Under these assumptions, the manager always proposes the project to the planner, regardless of its quality.

These assumptions about the managers are made so that we can focus on the planner's agency problem. In a companion paper, we add to our consideration the manager's agency problem in project searching effort and the possibility of penalizing the manager.
is given by the horizontal coordinate of the point representing it. Therefore, \( \alpha \) is uniformly
distribution on \([0,1]\) and there are infinitely many projects of each type. The projects have the
technological feature that, to realize the return of a project, two stages of investment are needed:
investment of amount \( \$c \) in the first stage, and \( \$i \) in the second. For simplicity, we assume that
the interest rate is zero. We call the difference between the revenue and the investment costs the
value-added of a project. If the project is completed, its expected value-added is \( \alpha - c - i \). If it is
terminated after the first investment, then its value-added is \( -c \). A project is \textit{ex ante} (at time 0)
profitable if its expected value-added is non-negative, that is, \( \alpha \geq c + i \). In Figure 2, area BCKJ is
the set of \textit{ex ante} profitable projects. Projects with \( \alpha \in [i, c+i) \) are \textit{ex ante} unprofitable because
\( \alpha < c + i \), but they are \textit{ex post} (at time 1) profitable because \( \alpha \geq i \) --- by then the first investment
\( \$c \) is sunk. In Figure 2, area ABJI is the set of \textit{ex ante} unprofitable but \textit{ex post} profitable projects.
Finally, projects with \( \alpha \in [0, i) \) (in area OAIH in Figure 2) are unprofitable both \textit{ex ante} and \textit{ex post}.
We assume that
\[
E(\alpha) \geq c + i ,
\]
so that it is desirable to carry out a project of unknown profitability.\(^{14}\)

Also before time 0, the principal hires an agent to screen the projects and make investment
decisions on the principal’s behalf. The principal signs an incentive contract with the agent to
maximize net revenue, which is the gross revenue net of investment costs and payment to the
agent.

At time 0, the planner examines the project proposals. We assume that each proposal is
reviewed with probability \( e \), which is normalized to be the planner’s effort level. The range of \( e \) is
of course \([0,1]\). When a project is reviewed, its true profitability is identified. The set of such

\(^{14}\)We will discuss this assumption in section 4.
projects is area OCDG in Figure 2. No new information is acquired about projects that are not reviewed (in area GDKH in Figure 2).\(^{15}\)

After the initial screening, the planner decides which projects to start, and invests $c$ in each of them. Let \(s\) be the number of projects started.\(^{16}\)

At time 1, the profitability of all the projects that are funded in the first stage is revealed to the planner, but not to the principal. Some of the projects are \textit{ex post} unprofitable, as expected. The planner then chooses to terminate some of them. Let \(t\) be the number of projects terminated. \(s\) and \(t\) are public information.\(^{17}\)

In the second stage, an additional investment of $i$ is required for each project that is not terminated at time 1 in order to realize its return.

At time 2, the returns of the retained projects are realized and the planner is rewarded according to the incentive contract signed with the principal before time 0.

The expected gross value-added thus depends on screening effort \(e\), projects started, \(s\), and projects terminated, \(t\). We denote it by \(y(e, s, t)\). To compute \(y(e, s, t)\), let us look at Figure 3. In the figure, NS is the set of projects that are not started, T is the set of projects that are terminated at time 1, and C is the set of projects that are completed. NS, T, and C are mutually exclusive and collectively exhaustive. Let \(m\) denote the probability measure in the space of projects (square OCKH in Figure 2). The measure (area) of T is \(m(T) = t\) and the measure (area) of C is \(m(C) = s - t\). Then,

\[
y(e, s, t) = \int_{(a,e)\in C} \alpha d\alpha de - (c + i)m(C) - cm(T).
\]

\(^{15}\)Such a screening technology is very similar to that used by many to study the monitoring problem in hierarchies, e.g., Calvo and Wellisz (1978) and Qian (1994), called "imperfect supervision and monitoring but perfect observation".

\(^{16}\)Since the set of projects is not countable, the term "number" is abused. The more proper terms are "the probability measure" or "the proportion". However, we ask the reader to tolerate this abuse of the term because "the probability measure" is a mouthful expression and "the proportion" can be confusing due to changes in the reference population over time.

\(^{17}\)Starting or terminating a project is usually a high profile public event.
NS is the set of projects that are not started, T is the set of projects that are terminated at time 1, and C is the set of projects that are completed.

Note that the shapes of NS and T depend on s and t, and can be different from those depicted in Figure 3.

We assume that the gross value-added is given by

$$x = y(e, s, t) + \theta,$$

where $\theta$ is a random variable with mean 0 and probability density function $g(\theta)$. The value of $\theta$ is realized at time 2. The principal can observe the realization of $x$, but not that of $\theta$ or $y(e, s, t)$.

Besides $x$, $s$, and $t$, the principal observes the total investment budget, denoted by $b$.

However, since $b$ is uniquely determined by $s$ and $t$ --- $b = sc + (s - t)i$, the principal only needs to include $x$, $s$, and $t$ in the incentive contract; that is, the payment to the planner is $w(x, s, t)$.

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18 $\theta$ is a common shock to all projects under the planner’s jurisdiction. Without this common shock, the first-best effort of the planner can be implemented by “selling the economy to the planner”, i.e. demanding a certain payment from the planner and giving him all the residual value-added. When there is no uncertainty, this arrangement is efficient even when the planner is risk averse or/and has limited liability. Idiosyncratic shocks are not sufficient, because the number of projects is large and, by the law of large numbers, idiosyncratic shocks will be averaged out.

19 When there is more than one planner, the state can use relative performance evaluation, as discussed by Lazear and Rosen (1981) and Holmstrom (1982), among others. Our results are robust to this possibility. According to Holmstrom (1982), the optimal incentive scheme depends on a planner’s performance alone if the $\theta$’s are independent across planners. Relative performance evaluation will not change the qualitative features of our conclusion as long as $\theta$ has a component that is idiosyncratic to individual planners. Note that the idiosyncrasy here is one reason why the state cannot observe $\theta$, and is different from that in the last footnote, where it is with respect to individual projects, as opposed to individual planners.
Assume that the planner's utility function is 
\[ v(w) - d(e), \]
where \( w \) is the income of the planner and \( d(e) \) the disutility of effort \( e \). Assume that 
\[ v' > 0, \quad v'' < 0, \quad d' > 0, \quad d'' > 0, \quad d'(0) = 0, \quad \text{and} \quad d'(1) = \infty. \]
Let \( w(x,s,t) \) be the principal's wage offer to the agent. Then the expected utility of the agent and the expected net revenue of the principal are, respectively,
\[ u(e,s,t) = E_\theta \{v[w(x(e,s,t,θ),s,t)] - d(e)] \]
and
\[ \pi = E_\theta \{x(e,s,t,θ) - w(x(e,s,t,θ),s,t)\}. \]

Our assumption that the profitability of the projects is not revealed to the principal after the initial investment is critical. Otherwise, the principal would be able to deduce the effort of the agent from the observed profitability and base a contract on the effort level. Then, by mandating the first-best screening effort and paying a fixed wage in the incentive contract, the principal can achieve first-best efficiency. The contract leads to efficient project-starting and -termination decisions. In summary, we have,

**Proposition 1:** If the principal knows the profitability of the projects at time 1, then she will know the agent’s effort in screening the projects and can implement first-best screening effort and efficient project-starting and -termination decisions.

We now proceed to study and characterize the optimal contract when there is asymmetric information about the profitability of the projects at time 1. We first show that the optimal contract can take a special form.

The general form of the contract is for the principal to offer a wage function \( w(x,s,t) \). Given this wage function, the agent chooses his effort, \( e \), the number of projects to start at time 0, \( s \), and the number of projects to terminate at time 1, \( t \), to maximize his expected utility. Since \( s \) and \( t \) are verifiable, the principal can mandate \( s = \tilde{s} \) and \( t = \tilde{t} \), for some \( \tilde{s} \) and \( \tilde{t} \). Together with a wage function \( w(x) \), they constitute a wage contract of the following special form:
Given such a contract, the agent chooses the effort level \( e \) to maximize his utility. We have the following standard result:

**Proposition 2:** Restricting the wage contracts to the special form given in (SF) does not reduce the principal's optimal payoff; that is, the optimal contract can be chosen to be of the special form.\(^{20}\)

**Proof:** Suppose the optimal wage contract is \( w^*(x,s,t) \), and under this contract, \( e^* \), \( s^* \), and \( t^* \) are induced. If the principal mandates \( s^* \) and \( t^* \) and offers a wage contract

\[
\hat{w}(x) = w^*(x,s^*,t^*),
\]

then the agent chooses \( e \) to maximize

\[
\max_{e} u(e,s^*,t^*) = \max_{e} E_{\theta} [v[w^*(x(e,s^*,t^*,\theta),s^*,t^*)] - d(e)].
\]

By the definition of \( e^* \), \( s^* \), and \( t^* \), the agent's optimal effort is \( e^* \). That is, mandating \( s^* \) and \( t^* \) and offering \( \hat{w}(x) \) induces the same effort as offering \( w^*(x,s,t) \). The principal gets

\[
\pi = E_{\theta}[x(e^*,s^*,t^*,\theta) - w(x(e^*,s^*,t^*,\theta),s^*,t^*)],
\]

which is the net revenue that the principal gets under the optimal contract \( w^*(x,s,t) \). That is, the maximum net revenue that the principal can attain by choosing the optimal contract from all possible contracts is attained by choosing one,

\[
w(x,s,t) = \begin{cases} 
\hat{w}(x) & \text{if } s = s^* \text{ and } t = t^*; \\
-\infty & \text{otherwise,}
\end{cases}
\]

from the special class of contracts. \( \text{Q.E.D.} \)

The result of Proposition 2 makes it much easier to set up the principal's problem of choosing the optimal incentive contract. Since \( x = y(e,s,t) + \theta \) and the probability density function of the random variable \( \theta \) is \( g(\theta) \), the probability density function of \( x \) is

\[
f(x;e,s,t) \equiv g(x - y(e,s,t)).
\]

\(^{20}\) \( w \) will depend on \( t \) smoothly if the state only observes an imperfect signal of \( t \), and/or if the value of \( \theta \) is at least partially realized before the project-termination decision is made. A similar remark applies to \( s \).
Given \( s, t, \) and \( w(x) \), the agent’s utility is

\[
    u(e,s,t) = \int v(w(x)) f(x;e,s,t) dx - d(e). 
\]

Then, the principal’s problem can be reformulated as

\[
    \max_{s,t,e} \Pi(s,t,e) 
\]

where,

\[
    \Pi(s,t,e) = \max_{w(x)} \int [x - w(x)] f(x;e,s,t) dx 
    \text{ s.t. } \int v(w(x)) f(x;e,s,t) dx - d(e) \geq 0 
    \text{ for all } e', \quad (IC) 
\]

\[
    \text{ and } \int v(w(x)) f(x;e,s,t) dx - d(e) \geq 0 \quad (IR) 
\]

\( \Pi(s,t,e) \) is the maximum expected net revenue when the principal chooses to induce the effort level \( e \) and mandates \( s \) and \( t \). (IC) is the incentive compatibility constraint. It requires that the wage contract chosen by the principal must make it optimal for the agent to choose the intended \( e \). (IR) is the agent’s individual rationality constraint; i.e., the wage contract must be acceptable to the agent.

3. Effort and Project Starting and Termination Decisions.

In this section, we study how the expected gross value added, \( y(e,s,t) \), changes with effort \( e \), the number of projects started at time 0, \( s \), and the number of projects terminated at time 1, \( t \). Lemmas 1 and 2 prepare for Lemma 3, which states that the marginal benefit of effort decreases with \( t \) and \( s \). This is the key to understanding the main result of the model that it is \textit{ex ante} optimal for the principal to restrict project starting and termination. Lemma 4 says that \( y(e,s,t) \) increases with effort, \( e \), in the relevant range of the variables, and will be used to prove that the optimal wage function is increasing. The proofs of the lemmas in this section are in the appendix.

Let \( \hat{s}(e) \) be the number of projects that are not identified as \textit{ex ante} unprofitable at time 0. \( \hat{s}(e) = 1 - m(\text{OBEG}) = 1 - (c + i)e \), where OBEG is an area in Figure 2. \( \hat{s}(e) \) decreases with \( e \); as screening effort increases, more \textit{ex ante} unprofitable projects are identified at time 0.
Let \( \hat{t}(s,t,e) \) be the number of ex post unprofitable projects at time 1. For example, when \( s = \hat{s}, \hat{t}(s,t,e) = m(GFIH) = i(1 - e) \), where GFIH is an area in Figure 2. For other values of \( s \), the set of ex post unprofitable projects at time 1, denoted by \( U_p^1 \), is given in Figures 4 and 5 in the appendix. When \( s > 1 - eE(\alpha) \), \( \hat{t} \) is independent of \( t \) because, given the outcome of screening, project-starting decisions do not depend on \( t \); the agent chooses to start \( s \) projects with the highest expected returns. This case is illustrated by Figures 4(a), 4(b), and 5(a). When \( s < 1 - eE(\alpha) \), however, \( \hat{t} \) depends on \( t \). Refer to Figure 5(b). In such a case, the planner needs to forego some projects with expected revenue greater than \( E(\alpha) \). If \( t > 0 \), the planner may choose to start some projects with their returns uncertain (projects above Line \( e + h \) in Figure 5(b)) and forego some projects known to have returns greater than \( E(\alpha) \) (projects in area ABCD in Figure 5(b)). The reason for this is that a project with an uncertain return has an option value. The planner can terminate the project at time 1 if it turns out to be one of low return, or retain it if its return is high. This option value depends on \( t \), and thus \( \hat{t} \) also depends on \( t \). In the remainder of this paper, we base the analysis on the assumption that \( s > 1 - eE(\alpha) \) so that all projects inspected at time 0 and found to have returns greater than \( E(\alpha) \) are started.\(^{21}\)

Now, under the assumption that \( s > 1 - eE(\alpha) \) and, therefore, \( \hat{t} \) is independent of \( t \), we proceed to discuss the relationship between the number of ex post unprofitable projects \( \hat{t} \) and effort \( e \). As effort increases, more ex ante unprofitable projects -- some of them also ex post unprofitable -- are identified at time 0. In most cases, this enables the planner to start fewer ex post unprofitable projects, hence a smaller \( \hat{t} \). The only exception is when the planner has to start so

\(^{21}\)In the case in which \( s < 1 - eE(\alpha) \), the analysis is also standard, but very messy and tedious. Note that, in equilibrium, \( s < 1 - eE(\alpha) \) is more likely if investment costs \( c+i \) is close to \( E(\alpha) \), but very unlikely when \( c+i \) is small relative \( E(\alpha) \). The reason is that, the greater the difference between \( E(\alpha) \) and \( c+i \), the greater is the loss of profit due to starting fewer projects than \( 1 - eE(\alpha) \). When \( c+i \) is sufficiently small, the loss of profit due to starting fewer projects than \( 1 - eE(\alpha) \) is sufficiently large and will dominate the benefit of a higher effort, i.e., savings of \( c \) and \( i \) invested in unprofitable projects. The discussion and proofs for this case are available upon request.
many projects that even some of those known to be \textit{ex post} unprofitable have to be included. In such a case, \( \hat{t}(s,t,e) \) does not change with \( e \).

In summary, we have,

\textbf{Lemma 1}: (i) The number of projects that are not identified to be \textit{ex ante} unprofitable at time 0, \( \hat{s} \), decreases with \( e \).

(ii) The number of \textit{ex post} unprofitable projects at time 1, \( \hat{t} \), is non-increasing in \( e \). Furthermore,

\[ \frac{\partial \hat{t}}{\partial e} = \begin{cases} 0 & \text{when } s > 1 - ie; \\ < 0 & \text{when } s < 1 - ie. \end{cases} \]

We now study \( \frac{\partial y}{\partial s} \) and \( \frac{\partial y}{\partial t} \). At time 0, given the outcome of screening, the agent chooses to start \( s \) projects to maximize \( y \). When \( s < \hat{s} \), the agent foregoes some projects that are expected to be profitable after the review, and starting more projects increases the expected gross value-added, \( y \), i.e., \( \frac{\partial y}{\partial s} > 0 \). When \( s > \hat{s} \), he starts some projects that he knows to be unprofitable, and starting more projects has an ambiguous effect on \( y \). When \( t \) is very large, starting some projects that are known to be \textit{ex ante} unprofitable enables the planner to save some very profitable projects from being terminated. This benefit may dominate the cost of the wasted first-stage investment, \( c \). Therefore, \( \frac{\partial y}{\partial s} \) may be positive for some \( s > \hat{s} \). When \( t \leq \hat{t} \), however, no \textit{ex post} profitable projects are to be terminated at time 1. Consequently, starting more projects at time 0 that are known to be \textit{ex ante} unprofitable does not have any benefit and thus reduces \( y \), i.e., \( \frac{\partial y}{\partial s} < 0 \). \( y \) is maximized at \( s = \hat{s} \) and \( \frac{\partial y}{\partial s}(e,s(e),t) = 0 \) when \( t \leq \hat{t} \). Lemma 2(i) summarizes these results.

Similarly, at time 1, given the number of projects to terminate, \( t \), and knowing the profitability of all the ongoing projects, the agent will choose the least profitable projects to terminate. When \( t < \hat{t} \), the agent retains some \textit{ex post} unprofitable projects, and terminating more projects increases the expected gross value-added, \( y \), i.e., \( \frac{\partial y}{\partial t} > 0 \). When \( t > \hat{t} \), the agent terminates some \textit{ex post} profitable projects, and \( \frac{\partial y}{\partial t} < 0 \). \( y \) is maximized at \( t = \hat{t} \) and \( \frac{\partial y}{\partial t}(e,s,\hat{t}) = 0 \). Lemma 2(ii) summarizes the results in this paragraph.

\textbf{Lemma 2}: (i) When \( s < \hat{s} \), \( \frac{\partial y}{\partial s} > 0 \). When \( t \leq \hat{t} \), \( \frac{\partial y}{\partial s} = \begin{cases} < 0 & \text{when } s > \hat{s}; \\ 0 & \text{when } s = \hat{s}. \end{cases} \) (ii)
\[ \frac{\partial y}{\partial t} = \begin{cases} > 0 & \text{when } t < \hat{t}; \\ 0 & \text{when } t = \hat{t}; \\ < 0 & \text{when } t > \hat{t}. \end{cases} \]

Lemma 3 below describes the effect of an effort change on \( \frac{\partial y}{\partial t} \) and \( \frac{\partial y}{\partial s} \). Part (i) of the lemma says that the marginal cost of continuing (or the marginal benefit of terminating) *ex post* unprofitable projects, \( \frac{\partial y}{\partial t} \), decreases with the effort level. When the agent’s effort is higher, more bad projects are discovered by the agent before the initial investment and the quality of projects started is higher. As a result, the cost of continuing an *ex post* unprofitable project is lower. An alternative interpretation of the result is that restricting project terminations raises the marginal benefit of effort, \( \frac{\partial y}{\partial e} \). The reason is that an *ex post* unprofitable project that the agent fails to screen out costs more if it cannot later be terminated than if it can. Therefore, requiring the agent to continue some *ex post* inefficient projects has the effect of increasing the marginal benefit of effort.

Part (ii) of the lemma says that the marginal cost of reducing \( s \), \( \frac{\partial y}{\partial s} \), decreases with effort level. Given the effort level, only projects that are identified to be unprofitable should not be started. When effort is lower, fewer projects are identified to be unprofitable and thus not being able to start a project is more costly.

**Lemma 3:** (i) \( \frac{\partial y}{\partial t}(e, s, \hat{t}(s, e)) \leq \frac{\partial y}{\partial t}(e - \Delta e, s, \hat{t}(s, e)) \) for all \( e \) and \( s \) and the inequality is strict when \( s < 1 - ie \); (ii) \( \frac{\partial y}{\partial s}(e, \hat{s}(e), t) < \frac{\partial y}{\partial s}(e - \Delta e, \hat{s}(e), t) \) for all \( e \) and \( t \leq \hat{t} \). (iii) \( \frac{\partial^2 y}{\partial e \partial t} \leq 0 \).

Lemma 4 tells us how the expected gross value-added, \( y \), changes with the screening effort. When effort is high, the agent has better information about the profitability of the projects. Given \( s \) and \( t \), better information leads to at least equal, and possibly higher, expected gross value-added. When the planner has to start too many projects or terminate too many projects, the improved information does not bring about any benefit. Therefore, we have,

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22Unlike the intuition for Lemma 3(i), the intuition for Lemma 3(ii) depends on the assumption that \( E(\alpha) \) is large so that unexamined projects should be started. We will further discuss this in Section 5.
Lemma 4: (i) $\frac{\partial y}{\partial e} \geq 0$ for all $e$, $s$, and $t$; (ii) If $s \leq \hat{s}(e)$, then $y(e - \Delta e, s, \hat{t}(s,e)) < y(e, s, \hat{t}(s,e))$ for all $e$ and all positive $\Delta e$; (iii) If $t \leq \hat{t}$, then $y(e - \Delta e, \hat{s}(e), t) < y(e, \hat{s}(e), t)$ for all $e$ and all positive $\Delta e$.

4. Persistence of Ex Post Inefficient Projects

In this section, we show that it is sometimes optimal for the principal to impose restrictions on project termination. As a result, some ex post inefficient projects persist. To highlight the idea, we consider the case where there are two effort levels, $e_h$ and $e_l$. We show that when it is more efficient for the principal to induce the lower effort, $e_l$, it is also optimal for the principal to set $s = \hat{s}(e_l)$ and $t = \hat{t}(e_l, \hat{s}(e_l))$ so that project-starting and -termination decisions are all efficient. However, when it is optimal for the principal to induce the higher effort, $e_h$, the principal should not only choose a different wage schedule, but also mandate $s^* < \hat{s}(e_h)$ and $t^* < \hat{t}(e_h, s^*)$ so that the agent must forego some projects he thinks to be efficient in the first stage and continue some of the ex post unprofitable projects in the second stage.

Let us first consider the case where the lower effort is optimal. Since the principal is risk neutral and the agent is risk averse, the least costly way for the principal to induce the low effort is to offer the agent a fixed wage subject to the (IR) constraint. In this case, the agent will discover $(c + i)e_l$ ex ante inefficient projects at time 0. Therefore, the principal should require the agent to start  $s(e_l) = 1 - (c + i)e_l$ projects at time 0. If so, there are $i(1 - e_l) ex post$ inefficient projects at time 1. Then, the principal should mandate termination of $t(e_l) = i(1 - e_l)$ projects at time 1. Note that the choice of $s$ and $t$ does not affect the (IC) constraint in (2) when the wage is fixed.

23 We discuss the continuous-effort case in Section 5.

24 See the discussion after the Corollary at the end of this section.

25 We assume that, when indifferent among some actions himself, the agent will choose the action that is best for the principal.

26 Recall that we have assumed that $E\{\alpha\} \geq c + i$. 
Now consider the principal’s optimal incentive contract for inducing high effort, $e_h$. The principal’s problem is

$$\max_{s,t} \Pi(s,t)$$

(3)

where,

$$\Pi(s,t) = \max_{w(x)} \int [x - w(x)] f(x; e_h, s, t) dx$$

s.t. \quad \int v(w(x)) f(x; e_h, s, t) dx - d(e_h) \geq 0$$

(4) (IC) \quad (\mu)

and \quad \int v(w(x)) f(x; e_l, s, t) dx - d(e_l) \geq 0$$

(4) (IR) \quad (\lambda)

We denote the solution to (3) as $(s^*, t^*)$.

The main result of the paper is that the \textit{ex ante} optimal number of terminations, $t^*$, is less than the \textit{ex post} optimal number of terminations, $\hat{t}$; that is, to maximize expected net revenue, the principal makes the agent keep some \textit{ex post} unprofitable projects. The intuition of the result is as follows. In order to improve \textit{ex ante} efficiency, the principal wants to make it easier to induce the higher effort by relaxing the incentive compatibility constraint, (IC). This can be achieved by restricting project termination by the agent at time 1. Since the agent's income increases with the gross value-added, as we will show later by a standard exercise, he is hurt by not being able to terminate all \textit{ex post} inefficient projects, regardless of the effort level. However, when the agent’s effort is lower, the quality of ongoing projects is poorer and the cost of retaining \textit{ex post} unprofitable projects is higher, as is shown and discussed in Lemma 3(i). Therefore, restricting project termination hurts the agent more if he exerts the lower effort, making it easier to induce the higher effort by relaxing the incentive compatibility constraint, (IC). Of course, continuing \textit{ex post} inefficient projects has its costs. However, the envelope theorem implies that the costs of a marginal deviation from the \textit{ex post} optimum are negligible when compared to the benefit from relaxing constraint (IC). Consequently, it is optimal for the principal to make the agent terminate
fewer projects than that which is _ex post_ efficient. In the rest of this section, we formalize the above argument.27

To characterize the solution to the problem, we make the assumption that the density function of \( \theta \), \( g(x) \), satisfies the monotone likelihood ratio condition:

\[
\frac{g(x+z)}{g(x)} \text{ is decreasing in } x \text{ for all positive } z.
\]  

(MLRC) is a common assumption in the principal-agent literature and is satisfied by normal distributions and t-distributions, among others. Lemma 5 states some properties of optimization problem (4).

**Lemma 5:** (i) The feasible set of the principal’s optimization program (4) is non-empty if and only if \( y(e_l,s,t) < y(e_h,s,t) \). If the feasible set of program (4) is non-empty, then: (ii) the constraint (IC) has a positive Lagrange multiplier, \( \mu \), and thus is binding; (iii) the optimal wage function, \( w(x) \), is strictly increasing.

The proof of the lemma is given in the appendix. Intuitively, if \( y(e_l,s,t) = y(e_h,s,t) \), then the higher effort cannot be induced; constraint (IC) cannot be satisfied. If \( y(e_l,s,t) < y(e_h,s,t) \), however, (IC) can be satisfied by choosing \( w(x) \) so that its slope is large enough.

\( y(e_l,s,t) < y(e_h,s,t) \), together with the monotone likelihood ratio condition, implies that

\[
\frac{f(x;e_l,s,t)}{f(x;e_h,s,t)} = \frac{g(x-y(e_l,s,t))}{g(x-y(e_h,s,t))}
\]

is decreasing in \( x \). That is, the likelihood of \( e_h \) with respect to that of \( e_l \) increases with \( x \). It is then natural that under these conditions the agent should be paid more when the value of \( x \) is higher.

Now, we are ready to present the main result of the paper.

27We should point out that this is not a sufficient statistic argument as in Holmstrom (1979), because here \( t \) is a choice variable, which is different from the additional signal in his model.
Proposition 3: Suppose it is optimal for the principal to induce the high effort, \( e_h \). (i) If \( s^* \leq \hat{s}(e_h) \), then there exists \( t' < \hat{t}(s^*, e_h) \equiv \hat{t} \) such that \( \Pi(s^*, t') > \Pi(s^*, \hat{t}(s^*, e_h)) \). (ii) If \( s^* > \hat{s}(e_h) \), then either \( t^* < \hat{t}(s^*, e_h) \) or \( \frac{\partial \Pi}{\partial t}(s^*, \hat{t}) \leq 0 \).

Proof: (i) \( \Pi(s,t) \) is the value of the expected net revenue when \( w(x) \) is chosen optimally, given \( s \) and \( t \) and subject to constraints (IR) and (IC). When the feasible set of program (4) is non-empty, by the Envelope Theorem,

\[
\frac{\partial \Pi}{\partial t} = \frac{\partial L}{\partial t}(s,t; w_{s,t}(x)),
\]

where

\[
L = \int \left[ x - w(x) \right] f(x; e_h, s, t) dx + \lambda \left\{ \int v(w(x)) f(x; e_h, s, t) dx - d(e_h) \right\}
+ \mu \left\{ \int v(w(x)) f(x; e_h, s, t) dx - d(e_h) - \int v(w(x)) f(x; e_l, s, t) dx + d(e_l) \right\}
\]

is the Lagrangian of program (4),

\[
\frac{\partial L}{\partial t} = \int \left[ x - w_{s,t}(x) \right] f_t(x; e_h, s, t) dx + \lambda \int v(w_{s,t}(x)) f_t(x; e_h, s, t) dx
+ \mu \left\{ \int v(w_{s,t}(x)) f_t(x; e_h, s, t) dx - \int v(w_{s,t}(x)) f_t(x; e_l, s, t) dx \right\},
\]

and \( w_{s,t}(x) \) is the optimal wage function given \( s \) and \( t \). By the definition of \( f(x; e, s, t) \),

\[
f_t(x; e, s, t) = -g'(x - y(e, s, t)) \frac{\partial y}{\partial t}(e, s, t).
\] (5)

By Lemma 4(ii) and \( s^* \leq \hat{s}(e_h) \),

\[
y(e_l, s^*, \hat{t}(s^*, e_h)) < y(e_h, s^*, \hat{t}(s^*, e_h)).
\] (6)

Therefore, the feasible set of program (4) is not empty. By Lemma 2(ii),

\[
\frac{\partial y}{\partial t}(e_h, s^*, \hat{t}) = 0.
\] (7)

Substituting equations (5) and (7) into \( \frac{\partial L}{\partial t} \) yields

\[
\frac{\partial L}{\partial t} \bigg|_{x=s^*,t=\hat{t}} = \mu \frac{\partial y}{\partial t}(e_l, s^*, \hat{t}) \int v(w_{s,t}(x)) g'(x - y(e_l, s^*, \hat{t})) dx.
\]

Integration by parts yields

\[
\frac{\partial L}{\partial t} \bigg|_{x=s^*,t=\hat{t}} = -\mu \frac{\partial y}{\partial t}(e_l, s^*, \hat{t}) \int v'(w_{s,t}(x)) w_{s,t} \cdot g(x - y(e_l, s^*, \hat{t})) dx,
\]
because $g$ is a probability density function. Recall that $\hat{t} = \hat{t}(s^*, e_h)$. Inequality (6) and Lemma 5 imply that $w'_{s^*}(x) > 0$. By Lemma 3(i) and (7), $\frac{\partial y}{\partial t}(e_i, s^*, \hat{t}) > 0$. Therefore,

$$\frac{\partial \Pi}{\partial t}(s^*, \hat{t}) = \frac{\partial L}{\partial t}(s^*, \hat{t}; w'_{s^*}(x))$$

$$= -\mu \frac{\partial y}{\partial t}(e_i, s^*, \hat{t}) \int v'(w'_{s^*}(x))w'_{s^*}(x)(x - y(e_i, s^*, \hat{t}))dx < 0.$$  

Consequently, there exists $t' < \hat{t}(s^*, e_h) = \hat{t}$ such that $\Pi(s^*, t') > \Pi(s^*, \hat{t}(s^*, e_h))$.

(ii) Lemma 4(i) implies that

$$y(e_i, s^*, \hat{t}(s^*, e_h)) < y(e_h, s^*, \hat{t}(s^*, e_h)), \quad (6)$$

or

$$y(e_i, s^*, \hat{t}(s^*, e_h)) = y(e_h, s^*, \hat{t}(s^*, e_h)). \quad (8)$$

In the former case, the proof in (i) shows that $\frac{\partial y}{\partial t}(s^*, \hat{t}) \leq 0$. Here, the equality is not necessarily strict because in the application of Lemma 3(i), the condition $s < 1 - ie$ may not be satisfied. In the latter case, $\hat{t}(s^*, e_h) = \hat{t}(s^*, e_i)$; Otherwise, Lemma 1(ii) implies $\hat{t}(s^*, e_i) < \hat{t}(s^*, e_i)$ and then

$$y(e_i, s^*, \hat{t}(s^*, e_h))$$

$$< y(e_i, s^*, \hat{t}(s^*, e_i)) \quad \text{By Lemma 2(ii)}$$

$$\leq y(e_h, s^*, \hat{t}(s^*, e_i)) \quad \text{By Lemma 4(i)}$$

$$< y(e_h, s^*, \hat{t}(s^*, e_h)) \quad \text{By Lemma 2(ii)},$$

which contradicts equation (8). Lemma 3(iii) says that $\frac{\partial y}{\partial t}(e_i, s^*, t) \geq \frac{\partial y}{\partial t}(e_h, s^*, t)$. This, together with equation (8), $\hat{t}(s^*, e_h) = \hat{t}(s^*, e_i)$, and Lemma 4(i), implies that $y(e_i, s^*, t) \geq y(e_h, s^*, t)$ for all $t > \hat{t}$. Therefore, Lemma 5 says that at the optimum, $y(e_i, s^*, t^*) < y(e_h, s^*, t^*)$. Therefore, $t^* < \hat{t}(s^*, e_h)$. Q.E.D

Proposition 4: If it is optimal for the principal to induce the high effort, $e_h$, and if $t^* \leq \hat{t}(s^*, e_h)$, then there exists an $s' < \hat{s}(e_h)$ such that $\Pi(s', t^*) > \Pi(\hat{s}(e_h), t^*)$, i.e., it is better for the principal to require the agent to forego some projects in the first stage that he expects to be profitable than not to.

The proof of Proposition 4 is omitted because it is parallel to that of Proposition 3(i) above; it is the same except that Lemma 2(i) and Lemma 3(ii) are used instead of Lemma 2(ii) and Lemma
The intuition for Proposition 4 is also similar to that of Proposition 3 discussed earlier. Here, restricting project initiation hurts the agent more when exerting the lower effort and hence makes it easier to induce the higher effort by relaxing the incentive compatibility constraint, (IC), as is shown in Lemma 3(ii).

Since we don’t know the global properties of function $\Pi(s,t)$, Propositions 3 and 4 do not directly mean that $s^* < s(e_t)$ and $t^* < \hat{t}(s^*,e_t)$. However, if $\Pi(s,t)$ is concave with respect to $t$, Proposition 3 does imply that $t^* \leq \hat{t}(s^*,e_t)$. Then Proposition 4 says that, if $\Pi(s,t)$ is concave with respect to $s$, $s^* < s(e_t)$, which by Proposition 3(i), implies that $t^* < \hat{t}(s^*,e_t)$. In summary, we have,

**Corollary:** If $\Pi(s,t)$ is concave with respect to both $s$ and $t$, then under the *ex ante* optimal contract, the number of projects terminated in the second stage is less than the number of *ex post* unprofitable projects existing at time 1, and the agent foregoes some projects in the first stage he expects to be profitable, i.e., $t^* < \hat{t}(s^*,e_t)$ and $s^* < s(e_t)$.

In our verbal discussion on the desirability for the principal to restrict project termination and its intuition, we implicitly assumed the concavity of $\Pi(s,t)$ with respect to both $s$ and $t$. The assumption simplified the discussion by a great deal. In the remainder of this paper, we will continue to make the assumption to facilitate the discussion.28

5. Discussion of the Model

A. The Number of Projects

In our model, we assume that there are infinite many projects. If there are only a finite number of projects under the administration of the planner, the law of large numbers no longer applies so that $\hat{s}$ and $\hat{t}$ become stochastic. Because of this, the *ex ante* optimal $t^*$ may not be always less than $\hat{t}$, but $t^*$ is still less than the expected value of $\hat{t}$. Another change is that the cost

28We realize that it is desirable to express the condition about $\Pi(s,t)$ in the Corollary in terms of primitive parameters. Unfortunately, the complexity of the problem makes it intractable to do so.
of retaining *ex post* unprofitable projects becomes discontinuous so that we cannot use the envelope theorem argument. Consequently, this cost may not be dominated by the benefit of retaining *ex post* unprofitable projects that it makes it easier to induce effort. Therefore, we can only conclude that $r^* \leq \hat{r}$. Whether or not the inequality is strict depends on the parameters of costs and revenue.

B. The Assumption about $E(\alpha)$

In sections 2-4, we analyzed the model under the assumption that $E(\alpha) \geq c + i$. The assumption simplifies the exposition, but is not critical for the main result of the model that it is desirable for the principal to make the agent keep some *ex post* unprofitable projects. To see this, let us first give the condition for starting an unexamined project when there is no restriction on its termination. The expected revenue of an unexamined project, $\alpha$, has a uniform distribution on $[0,1]$. If the project is started, an investment cost of $c$ is incurred. After the investment, the value of $\alpha$ becomes known to the planner. If $\alpha < i$, the project will be terminated. Otherwise, the project is completed after an additional investment of $i$, producing revenue $\alpha$. Therefore, the expected value-added of the project is

$$V \equiv -c + \int_i^1 (\alpha - i)d\alpha = \frac{1}{2}(1-i)^2 - c.$$  

The project should be started if and only if $V \geq 0$.

If $V \geq 0$, the analysis in sections 2-4 is still valid and all results continue to hold.

If $V < 0$, we can show that it is still desirable for the principal to make the agent keep some *ex post* unprofitable projects, if there are any. The result about $s$, however, is reversed; it is now desirable for the principal to make the agent start some projects in the first stage that he expects to be unprofitable. The reason is as follows. Refer to Figure 2 again. If the contract does not restrict the numbers of project starts and terminations, then the agent would only start projects in area BCDE in Figure 2, which are reviewed and known to be profitable at time 0, and there will be no *ex post* unprofitable projects to terminate. This, however, may not be optimal for the principal because of the loss associated with not starting projects that are potentially profitable (projects in
area EDKJ in Figure 2). The principal can induce the agent to make a greater effort in project screening by requiring him to start more projects than those known to be profitable at time 0 (projects in area BCDE). This will make a lower effort more costly to the agent and, again, relaxing the agent's incentive constraint. The agent will respond to this by making greater effort to expand area BCDE. In terms of earlier analysis, Lemma 1(i) is now reversed but Lemma 2(i) remains, and thus Lemma 3(ii) is reversed.

The agent will also start some projects with returns smaller than c+i, starting with those closest to c+i. As the number of projects required to start increases, projects with returns known to be further smaller than c+i would have to be started. Beyond a certain point, when s is sufficiently large and the worst projects of known quality that have to be started have very small returns, the agent would rather start projects of unknown quality (projects not examined) than those further left of the line BE. This is particularly likely to happen if c+i is not too large. When this happens, some ex post unprofitable projects will be started and, for the reason that should be clear by now, some of them would be required to continue, i.e., \( t^* < \hat{t} \). Of course, if only projects very close to c+i are started, then, none of them is ex post unprofitable. In this case, \( t^* = \hat{t} = 0 \).

C. Independent Monitor

We have assumed there is only one agent, the planner. One might wonder what happens if there are two or more agents? There are several possibilities here.

First, if the two agents have exactly the same contracts and work on the same block of projects, then the problem is also exactly the same as the one we have just studied. The same result would, of course, be obtained.

The principal could also divide the project proposals into many small portions and hire many agents to each work on one portion. If a portion is large and contains many projects, then

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\(^{29}\)Recall that when all unscreened projects would be started, the cost of a lower effort is that more unprofitable projects would be started and terminated after incurring the cost $c$. 

25
the same problem that an agent would not spend enough effort to inspect all of them would arise. If the portions are each very small, then there would be a large number of agents, raising the issue of the principal's span of control. If for any reason (not modeled in this paper) the principal cannot directly monitor so many agents so that another tier of agents need to be hired, then the problem becomes one of optimal hierarchy. The problem is beyond the scope of this paper, but it suffices to point out that a supervisor's problem with her subordinates in an incentive hierarchy is essentially the same as the planner's problem with project proposals at time 0 in this paper. In these hierarchies, it is in general optimal to have imperfect monitoring and declining efforts down the hierarchies.\textsuperscript{30}

Still another possibility is that the principal could hire a second agent to monitor the planner. One way for the second agent to monitor the planner's effort is to inspect project quality at time 1. As Proposition 1 shows, one can perfectly infer the planner's effort level from project quality at time 1. If the second agent can get perfect information about project quality at time 1 at a very low cost and provide the information to the principal, i.e., he has access to information about project quality, does not have an agency problem himself so that he will not collude with the first agent or cheat in any other way, then no ex post unprofitable projects would be refinanced because the first best effort, project starting and termination decisions can all be achieved. However, we can probably more reasonably expect the second agent to have an incentive problem of his own so that the principal has to sign a separate incentive contract with him. We have discussed the case in which the second agent has the same incentive contract with the first one. If the second agent has a different incentive contract, he is unlikely to get the same information about project quality as the planner. This prevents the principal from obtaining perfect information about project quality at time 1. If this is the case, the asymmetric information problem between the principal and the planner will continue to exist and, thereby, the results of our model continue to hold.

\textsuperscript{30}See, among others, Calvo and Wellisz (1978) and Qian (1994) for studies of incentive hierarchies with endogenously determined degree of supervision and monitoring.
D. Continuous Effort

In Section 4, we assumed that there were two effort levels. We have also proved results similar to Propositions 3 and 4 for the continuous-effort case using the first-order condition approach.\textsuperscript{31} If we assume that the marginal disutility of effort is zero at $e = 0$, then it is optimal to induce a positive effort level ($e > 0$) and thus also optimal to set $t^* < \hat{t}$ rather than $t^* = \hat{t}$. In contrast, in the two-effort case we considered, it may be optimal to select the lower effort, in which case there is no benefit in setting $t^* < \hat{t}$ because the lower effort requires no incentive to implement.

E. Renegotiation

One might think that, after the planner has made the effort, a Pareto efficient new contract can be signed through renegotiation, as the original contract leads to \textit{ex post} inefficient project-starting and -termination decisions and does not provide full insurance to the planner. Presumably, a new contract that specifies a fixed wage for the agent would induce \textit{ex post} efficient project-starting and -termination decisions and also fully insure the planner against any risk associated with output fluctuations.\textsuperscript{32}

Several considerations suggest that our results are robust with respect to renegotiation. First, if the game is repeated, renegotiation in earlier stages damages the principal’s reputation to maintain incentives for screening effort in later stages. Such reputation concerns restrain the principal from renegotiating the wage contract with the agent. The idea can be formally modeled by considering an infinitely repeated game between the same principal and generations of different agents. When the principal’s discount factor is sufficiently large, she will not renegotiate with any

\textsuperscript{31} It is difficult to show the validity of the first-order condition approach for this model.

\textsuperscript{32} We assume that when the planner is indifferent between different actions, he takes the action that is best for the principal. Alternatively, the payoff to the planner should increase slightly with the total surplus.
agent in equilibrium. Such formalization is a standard exercise and thus is not elaborated in this paper.

Second, even for the one-shot game we model, adding a renegotiation stage does not change the equilibrium allocation if the renegotiation rule requires that the agent propose the new contract, as specified by Ma (1994). The key to this is that the principal has less information about project profitability and thus about the agent’s effort than the agent has. The agent’s proposal of removing restrictions on project-starting and -termination gives the principal reason to believe that the agent has chosen a lower effort initially. Given such a belief, the principal will adopt a different sharing rule; i.e., one that maximizes profit, given the lower effort level. Having made the higher effort, the agent is worse off and therefore should not propose to renegotiate the original contract.

Critical for this argument is the assumption that only the agent can propose to renegotiate the contract. The assumption, however, is not unreasonable in this context. Fudenberg and Tirole (1990) consider the alternative renegotiation rule that the principal proposes a new contract after the agent has taken the action. They find that the set of implementable actions is much smaller and as a result, the outcome of the game is less efficient than in the case where only the agent proposes to renegotiate. Thus, without modeling, we can think of a larger game played between the state and the planner to choose the renegotiation rule before they start the contracting and the renegotiation game. If the game leads to the result that the more efficient rule is chosen, the result should be that only the planner can propose to renegotiate the contract. An institution may also be established to safeguard the rule.

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33 In Dewatripont and Maskin (1995), renegotiation is not prevented even in a repeated game. The reason is that, in our model, the number of closed projects is public information, but in their model, the profitability of a project is not public information.

34 We have formally shown that Ma's results can be adapted to this model. The proof is not included in the paper but is available upon request.

35 One might further ask if the rule itself is subject to renegotiation and the institution to change. The question is out of the scope of this paper. Suffice to point out that, first, changing the rule and institution may be more
This paper offers an explanation of the soft budget constraint by focusing on the moral hazard problem in the agent's project screening effort. We conjecture that the agent's concern about the reputation regarding his ability also leads to continuations of *ex post* inefficient projects, as terminating projects sends an unfavorable signal about his ability. In the latter context, there is no ground for renegotiation.\textsuperscript{36}

7. Summary and Concluding Remarks

We have shown that, when there is agency in project screening and termination decisions, it is in general optimal for the principal in the *ex ante* incentive contract to set restrictions upon the agent's freedom to terminate *ex post* unprofitable projects. The result suggests that the widespread soft budget problem in socialist economies has a very profound informational reason. It is thus likely to persist even when a socialist state strives to maximize economic profit. The result of the model is driven by the idea that, by reducing an agent's flexibility in getting away from a problem, the agent will be induced to make a greater effort to avoid the problem. It is thus quite robust to alternative technical assumptions.

Some people may be concerned about whether the kind of complex contract between the state and the planner we studied can be found in real world socialist economies. One possible response to this concern is that a model should be judged in "as if" terms, i.e., by its predictions. More importantly, as already mentioned in the Introduction, the strategy of this paper is to study the cause of the soft budget problem by constructing an "ideal" socialist economy in which the state is only concerned with efficiency in terms of economic profit, while asymmetric information about project quality exists between the state and its agent. In doing so, we have necessarily abstracted from many observable real world institutions, e.g., the state may have other concerns than

\textsuperscript{36}The learning problem here is similar to that of Holmstrom and Ricart i Costa (1986). The problem of optimal incentive when moral hazard and adverse selection problems both exist is studied by McAfee and McMillan (1991) and Picard and Rey (1990). It is also a basic assumption in Laffont and Tirole's (1993) study of the government procurement problem.
economic profit, or it may not have the full bargaining power to dictate the optimal contract. The most important insight of our model is that the soft budget constraint in socialist economies has a very profound informational reason. It is hard to believe that adding specific institutions of socialist economies like those mentioned above to the "ideal" socialism modeled in this paper would alter the result of our model.

It is worthwhile to point out that the agency problem in projects requiring sequential decisions is also common in capitalist market economies. For example, in the problem of employment decisions, an academic department of a university, a plant of a business company, or an office of a government may, on behalf of the university, the company, or the government, respectively, review job applicants, make hiring decisions, train new employees, and then make retention and separation decisions about them. Since central planning is also a feature of internal governance in many organizations in market economies, our model suggests that restrictive termination rules should also be expected in these organizations. Consequently, subunits are forced to retain some of the unproductive projects (employees) and continue to finance them. This provides an explanation of tenure in universities, the "no-layoff" rule in large Japanese and also some American companies, and other restrictive layoff rules: they have the benefit of inducing higher effort in initial screening and thereby reducing costly initial investment in bad projects (unproductive employees).

Given agency in sequential decision problems and persistence of ex post unprofitable projects in both capitalist and socialist economies, an interesting and important question is: What differences exist between the two settings?

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37 Carmichael (1988) offers an explanation of the tenure rule in universities. He points out that, in the academic job market, universities have to rely upon incumbent professors for information about job candidates' quality. Tenure is needed to protect the job security of incumbent professors so that they would truthfully report the quality of job candidates to the university, enabling it to hire the best candidates. This explanation for tenure deals with the issue of incumbent professors' willingness to report true information, but not with the issue of optimizing effort needed to obtain that information.

38 Although comparative static results of our model are difficult to derive, it seems reasonable to predict more restrictive layoff rules and greater screening efforts in recruiting in companies that invest more in employee training.
One most important and obvious difference is the lack or underdevelopment of equity markets in socialist economies. In capitalist economies, the market for equity shares of a publicly owned firm generates information about its performance. In the extreme case of an equity market generating perfect information regarding project quality, the first best allocation can be achieved, as shown in Proposition 1. In the less extreme case in which the principal gets limited signals about the agent's effort, in addition to final output and the number of closed projects, the soft budget constraint problem continues to exist, but is less severe. It is true that in capitalist economies many firms are privately owned and their shares not publicly traded. However, in capitalist economies where equity markets play a critical role in resource allocation, a firm's ownership structure is a matter of choice rather than one of imperative as in socialist economies; While investors in capitalist economies have the freedom to choose either public or private ownership, in socialist economies, state ownership is decreed. The critical importance of this difference becomes rather apparent when we think of likely different types of information that need different types of institutions to utilize. Titman and Subrahmanyam (1996), for example, make the observation that some information is serendipitous in nature, i.e., it is obtained costlessly and purely by chance. They show that, when information is readily available through deliberate effort, e.g., research and auditing, there is an advantage associated with limiting the number of active investors. Concentrated ownership would presumably do well in this case. However, when the influence of serendipitous information on the firm's value is strong, information regarding project values can be best obtained when stocks are traded on a market with the largest possible number of active investors. Presumably, when investors have the freedom to choose the ownership structure, firms can be sorted into public or private ownership depending on which one can better generate information regarding its performance to help alleviate agency problems. In contrast, when a

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Holmstrom (1979) shows that an informative signal about effort can always improve the efficiency of the optimal principal-agent contract. Furthermore, if we use second-order stochastic dominance to rank the accuracy of signals, then the result about sufficient statistics in Holmstrom (1982) implies that a more accurate signal leads to a more efficient outcome.
certain type of ownership, e.g., state ownership, is decreed, it is unlikely to be optimal for all firms.

Recent discussion of market socialism has revealed an increased recognition of the importance of the equity market with some fairly detailed proposals as to how such a market might be organized. (See Bardhan and Roemer, 1993.) Hardening the budget constraint will depend upon the degree to which the capitalist system's equity market can be mimicked; the extent to which this can be done under market socialism, however, remains an open question. This paper does not directly address this question and interested readers are referred to Bardhan and Roemer (1992, 1993) and Shleifer and Vishny (1994) for opposing views. However, the result of this paper does suggest that, in order to understand the efficiency of market socialism relative to a capitalist economy, it is important to understand how equity markets under the two systems generate information on managers' efforts as well as on the performance of individual firms (projects). In socialist market economies, the state is the dominant shareholder, while in capitalist market economies, ownership tends to be more diffused. On the one hand, diffused ownership creates the problem of free-riding and discourages small shareholders from monitoring management, as argued by Shleifer and Vishny (1986). On the other hand, the existence of liquidity traders among firm owners gives speculators incentives to collect information, as pointed out by Holmstrom and Tirole (1993).
Appendix

Lemma 1: (i) The number of projects that are not identified to be *ex ante* unprofitable at time 0, $\hat{s}$, decreases with $e$.

(ii) The number of *ex post* unprofitable projects at time 1, $\hat{i}$, is non-increasing in $e$, and

$$\frac{d\hat{i}}{de} = \begin{cases} 0 & \text{when } s > 1 - ie; \\ < 0 & \text{when } s < 1 - ie. \end{cases}$$

Proof: (i) This is obvious because $\hat{s}(e) = 1 - (c + i)e$.

(ii) The number of *ex post* unprofitable projects at time 1, $\hat{i}(s, t, e)$, is the size of area $U_p^1$, $m(U_p^1)$, where $U_p^1$ is shown in Figures 4 and 5. As effort increases, more *ex ante* unprofitable projects are identified at time 0, and this, in most cases, reduces the number of *ex post* unprofitable projects at time 1. However, such an improvement does not occur if the planner is to start too many projects; he has to start some projects that he knows to be *ex post* unprofitable. Figure 4 (a) shows that, when $s > 1 - ie$, $\frac{d\hat{i}}{de} = 0$; the area of $U_p^1$ does not change with a marginal change of $e$ because of the size of the shaded area does not change. When $1 - eE(\alpha) < s < 1 - ie$, Figures 4 (b) and 5 (a) show that $\frac{d\hat{i}}{de} < 0$. When $s < 1 - eE(\alpha)$, it can be shown that there exists $s' < 1 - eE(\alpha)$ so that

$$\hat{i} = \begin{cases} i(1 - e) & \text{if } s > s', \\ i(s - \frac{1}{2} + ew') & \text{if } s < s', \end{cases}$$

where $w'$ is the largest solution to equation $t = \sqrt{2w(s - \frac{1}{2} + ew)}$ and is less than $\frac{1}{2}$. Furthermore, $\hat{i}$ decreases with $e$. The proof is standard but tedious, and is available upon request. Q.E.D.

40The proof of Lemmas 1-4 are given graphically. Algebraic proof is available upon request.
The shaded area, NS, consists of projects that are not started in the first stage. The striped area consists of projects that, in the first stage, are known to be unprofitable but are started. $U_p^1$ is the set of projects that are ex post unprofitable at time 1.

Figure 4: $s > \hat{s}$

The (whole) shaded area consists of projects that are not started in the first stage. $U_p^1$ is the set of projects that are ex post unprofitable at time 1. The heavily shaded area consists of projects that, in the first stage, are expected to be profitable but are not started.

Figure 5: $s < \hat{s}$
Lemma 2: (i) When \( s < \hat{s}, \frac{\partial y}{\partial s} > 0 \). When \( t \leq \hat{t}, \frac{\partial y}{\partial s} = \begin{cases} < 0 & \text{when } s > \hat{s}; \\ 0 & \text{when } s = \hat{s}. \end{cases} \) (ii) 

\[
\frac{\partial y}{\partial t} = \begin{cases} > 0 & \text{when } t < \hat{t}; \\ 0 & \text{when } t = \hat{t}; \\ < 0 & \text{when } t > \hat{t}. \end{cases}
\]

Proof: (i) At time 0, given the outcome of screening, the agent chooses to start \( s \) projects with the highest expected returns. When \( s < \hat{s} \), as Figure 5 shows, the planner foregoes some projects that are expected to be profitable after the review. The heavily shaded area represents such lost opportunities. As \( s \) increases, the heavily shaded area shrinks and the expected gross value-added, \( y \), increases, i.e., \( \frac{\partial y}{\partial s} > 0 \). When \( s > \hat{s} \), as shown in Figure 4, the agent starts some projects that he knows to be unprofitable (the striped area), and starting more projects (widening the striped area) has an ambiguous effect on \( y \). When \( t \) is very large, starting some projects that are known to be \emph{ex ante} unprofitable enables the planner to save some very profitable projects from being terminated. This benefit may dominate the cost of the wasted first-stage investment, \( c \). Therefore, \( \frac{\partial y}{\partial s} \) may be positive for some \( s > \hat{s} \). When \( t \leq \hat{t} \), however, no \emph{ex post} profitable projects are to be terminated at time 1. Consequently, starting more projects at time 0 that are known to be \emph{ex ante} unprofitable does not have any benefit and thus reduces \( y \), i.e., \( \frac{\partial y}{\partial s} < 0 \). \( y \) is maximized at \( s = \hat{s} \) and \( \frac{\partial y}{\partial s}(e,s,e,t) = 0 \) when \( t \leq \hat{t} \).

(ii) At time 1, given the number of projects to terminate, \( t \), and knowing the profitability of all the ongoing projects, the agent will choose the least profitable projects to terminate. When \( t < \hat{t} \), the agent retains some \emph{ex post} unprofitable projects, and terminating more projects increases the expected gross value-added, \( y \), i.e., \( \frac{\partial y}{\partial t} > 0 \). When \( t > \hat{t} \), the agent terminates some \emph{ex post} profitable projects, and \( \frac{\partial y}{\partial t} < 0 \). At \( t = \hat{t} \), \( y \) is maximized and \( \frac{\partial y}{\partial t} = 0 \). Q.E.D.

Lemma 3: (i) \( \frac{\partial y}{\partial t}(e,s,\hat{t}(s,e)) \leq \frac{\partial y}{\partial t}(e - \Delta e, s, \hat{t}(s,e)) \) for all \( e \) and \( s \) and the inequality is strict when \( s < 1 - ie \); (ii) \( \frac{\partial y}{\partial s}(e,\hat{s}(e),t) < \frac{\partial y}{\partial s}(e - \Delta e, \hat{s}(e),t) \) for all \( e \) and \( t \leq \hat{t} \). (iii) \( \frac{\partial^2 y}{\partial e \partial t} \leq 0 \).
Proof: (i) By Lemma 2(ii), \( \frac{\partial y}{\partial t} (e, s, \hat{t}(s, e)) = 0 \). By Lemma 1 (ii), \( \hat{t}(s, e) \leq \hat{t}(s, e - \Delta e) \). By Lemma 2(ii), \( \frac{\partial y}{\partial t} (e - \Delta e, s, \hat{t}(s, e)) \geq 0 \). Therefore, \( \frac{\partial y}{\partial t} (e, s, \hat{t}(s, e)) \leq \frac{\partial y}{\partial t} (e - \Delta e, s, \hat{t}(s, e)) \). When \( s < 1 - ie \), again by Lemma 1 (ii) and Lemma 2(ii), all inequalities in this paragraph become strict.

(ii) By Lemma 2(i), \( \frac{\partial y}{\partial s} (e, \hat{s}(e), t) = 0 \). When the effort decreases to \( e - \Delta e \), fewer low quality projects are found at time 0. Therefore, \( \hat{s}(e - \Delta e) > \hat{s}(e) \). Then, by applying Lemma 2(i) to the case in which effort is \( e - \Delta e \), \( \frac{\partial y}{\partial s} (e - \Delta e, \hat{s}(e), t) > 0 \). Therefore,

\[
\frac{\partial y}{\partial s} (e, \hat{s}(e), t) < \frac{\partial y}{\partial s} (e - \Delta e, \hat{s}(e), t).
\]

(iii) As the effort level increases from \( e \) to \( e + \Delta e \), more low quality projects are screened out at time 0 and the quality of projects at time 1 becomes (weakly) higher. (1) When \( t < \hat{t} \), \( \frac{\partial y}{\partial t} \) is the benefit of terminating an additional \textit{ex post} unprofitable project and decreases (weakly) with \( e \); (2) when \( t > \hat{t} \), \( -\frac{\partial y}{\partial t} \) is the cost of terminating an additional \textit{ex post} profitable project and increases (weakly) with \( e \); and (3) when \( t = \hat{t}(e) \), \( t \geq \hat{t}(e + \Delta e) \) and \( \frac{\partial y}{\partial t} \) remains the same or changes from 0 to negative as the effort level increases from \( e \) to \( e + \Delta e \). Therefore, \( \frac{\partial^2 y}{\partial e \partial t} \leq 0 \). Q.E.D.

Lemma 4: (i) \( \frac{\partial y}{\partial e} \geq 0 \) for all \( e, s, \) and \( t \); (ii) If \( s \leq \hat{s}(e) \), then \( y(e - \Delta e, s, \hat{t}(s, e)) < y(e, s, \hat{t}(s, e)) \) for all \( e \) and all positive \( \Delta e \); (iii) If \( t \leq \hat{t} \), then \( y(e - \Delta e, \hat{s}(e), t) < y(e, \hat{s}(e), t) \) for all \( e \) and all positive \( \Delta e \).

Proof: (i) When effort is high, the agent has better information about the profitability of the projects. Given \( s \) and \( t \), better information leads to at least equal, and possibly higher, expected gross value-added. \( \frac{\partial y}{\partial e} = 0 \) when \( e, s, \) and \( t \) are all large enough.

(ii) Since \( s \) and \( t \) are the same in \( y(e - \Delta e, s, \hat{t}(s, e)) \) and \( y(e, s, \hat{t}(s, e)) \), the total investment costs are the same and the numbers of completed projects are also the same in the two cases. We only need to compare the quality of completed projects in the two cases. \( s \leq \hat{s}(e) < 1 - ie \). Then by Lemma 1(ii), \( \hat{t}(s, e - \Delta e) > \hat{t}(s, e) \). Therefore, the overall quality of completed projects is strictly poorer when the effort is \( e - \Delta e \) than when the effort is \( e \); some \textit{ex post} unprofitable projects are
completed in the former case but not in the latter case. Consequently, \( y(e - \Delta e, s, \hat{t}(s,e)) \) is strictly less than \( y(e, s, \hat{t}(s,e)) \).

This argument can be illustrated graphically. Let us first consider the case in which \( \hat{s} - s \leq e[E(\alpha) - c - i] \) that was illustrated by Figure 5 (a). When the effort is \( e \), since \( t = \hat{t}(s,e) \), the projects that are completed are those in the union of the shaded and the dotted areas in Figure 6; the set of terminated projects, \( T(e, s, \hat{t}(s,e)) \), is the same as \( U_p^1 \) in Figure 5 (a). When the effort decreases to \( e - \Delta e \), the projects that are completed are those in the union of the striped and the dotted areas in Figure 6; \( T(e - \Delta e, s, \hat{t}(s,e)) \) (the area to the left of the striped area) is narrower and taller than \( T(e, s, \hat{t}(s,e)) \). Since \( s \) and \( t \) are the same in both cases, the dotted area and the striped area are of the same size. Therefore, the total expected revenue is strictly lower in the lower effort case than in the higher effort case because projects in the dotted area are more profitable than those in the striped area. Consequently, \( y(e - \Delta e, s, \hat{t}(s,e)) \) is strictly less than \( y(e, s, \hat{t}(s,e)) \).

(iii) As the effort decreases from \( e \) to \( e - \Delta e \), the number of projects found to be \textit{ex ante} unprofitable at time 0 becomes smaller, i.e., \( \hat{s}(e - \Delta e) > \hat{s}(e) \). In \( y(e - \Delta e, \hat{s}(e), t) \), some efficient projects are not started and the overall quality of projects started is lower than in \( y(e, \hat{s}(e), t) \). Therefore, \( y(e - \Delta e, \hat{s}(e), t) < y(e, \hat{s}(e), t) \). Graphically, in Figure 7, the union of the striped area
and the dotted area is the set of completed projects in the higher effort case while the union of the shaded area and the dotted area is the corresponding set in the lower effort case. The striped area is of the same size as, and contains lower quality projects than, the shaded area.

Q.E.D.

**Lemma 5:** (i) The feasible set of the principal’s optimization program (4) is non-empty if and only if \( y(e_l,s,t) < y(e_h,s,t) \). If the feasible set of program (4) is non-empty, then: (ii) the constraint (IC) has a positive Lagrange multiplier, \( \mu \), and thus is binding; (iii) the optimal wage function, \( w(x) \), is strictly increasing.

**Proof:** (i) By Lemma 4 (i), \( y(e_l,s,t) \leq y(e_h,s,t) \). If \( y(e_l,s,t) = y(e_h,s,t) \), then the definition of \( f(x;e,s,t) \) implies that \( f(x;e_l,s,t) = f(x;e_h,s,t) \). With this equality, constraint (IC) becomes \(-d(e_h) \geq -d(e_l)\), which contradicts \( d(e_h) > d(e_l) \). Therefore, the feasible set of program (4) is empty. If \( y(e_l,s,t) < y(e_h,s,t) \), however, (IC) can be satisfied by choosing \( w(x) \) so that its slope is large enough.

(ii) The Lagrangian of program (4) is

\[
L = \int [x - w(x)] f(x;e_h,s,t) dx + \lambda \left\{ \int v(w(x)) f(x;e_h,s,t) dx - d(e_h) \right\} \\
+ \mu \left\{ \int v(w(x)) f(x;e_l,s,t) dx - d(e_l) - \int v(w(x)) f(x;e_l,s,t) dx + d(e_l) \right\}
\]

Pointwise optimization of the Lagrangian with respect to the sharing rule, \( w(x) \), and rearrangement yield,
\[
\frac{1}{v'(w(x))} = \lambda + \mu \left[ 1 - \frac{f(x; e_l, s, t)}{f(x; e_h, s, t)} \right].
\] (FOC-w)

Both \(\lambda\) and \(\mu\) are non-negative.

If \(\mu = 0\), then \(w(x)\) is constant. Let \(w(x) = w_0\). Then constraint (IC) becomes

\[v(w_0) - d(e_h) \geq v(w_0) - d(e_l),\]

which contradicts \(d(e_h) > d(e_l)\). Therefore, \(\mu\) must be positive.

(iii) By the definition of \(f(x; e, s, t)\),

\[\frac{f(x; e_l, s, t)}{f(x; e_h, s, t)} = \frac{g(x - y(e_l, s, t))}{g(x - y(e_h, s, t))}.\]

Since \(y(e_l, s, t) < y(e_h, s, t)\), the monotone likelihood ratio condition implies that the right hand side of the above equation is a decreasing function of \(x\). Since \(\mu > 0\), \(v'' < 0\), and by (FOC-w), the optimal wage function, \(w(x)\), increases with \(x\). Q.E.D.
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