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Can Ignorance Make Central Banks Behave?:
Instrument Uncertainty and the Inflationary Bias Problem

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Abstract: This paper presents a model in which “instrument uncertainty”—that is, an uncertain mapping from monetary policy to macroeconomic outcomes—may mitigate the inflationary bias problem that arises when efficient monetary policy rules are time-inconsistent. If the relation between monetary policy and macroeconomic outcomes is uncertain, the private sector has an incentive to scrutinize the past for clues about this relationship. This learning creates a link between past government behavior and present inflation expectations that the government can exploit to enhance its credibility. The model implies that the two conventional arguments for simple rules in monetary policy—one stressing the central bank’s poor forecasting abilities and the other stressing the perils of discretion—may work at cross-purposes. Moreover, it provides an explanation of the cyclical behavior of inflation due to political cycles and of the correlation between the level and variance of inflation.

Keywords: instrument uncertainty, monetary policy, inflationary bias, time-inconsistency, learning.
JEL Classification: D83, E31, E52

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Introduction

Monetarists are typically associated with the view that the best monetary policy is based on a simple rule. Until the late 1970s, the case for simple rules stressed simplicity: monetary authorities cannot with precision forecast the consequences of their actions, so attempts to fine-tune the economy with activist monetary policy will rarely stabilize—and may destabilize—an economy. Since the late 1970s,\(^1\) the case for simple rules has instead pushed the importance of rules: discretion instead of rules gives monetary authorities a temptation to create surprise inflation; the private sector understands this and sets its inflation expectations high enough to remove this temptation, in the process relegating society to a Pareto-inferior equilibrium. In the appropriate jargon, optimal monetary policy suffers from a time-inconsistency problem that leads to an inflationary bias.

The traditional and modern arguments for simple rules are typically viewed as complementary, or at least mutually compatible. As the popular taxonomy would have it, monetary policy is two-dimensional;\(^2\) along one dimension it is passive or activist, and along the other it is run by a rule or discretion. Any combination of the two dimensions is possible so that, for example, someone worried about the government's forecasting abilities and its temptation to create surprise inflation would favor a simple rule and rail against discretionary activism.

This paper shows that an inability to forecast precisely the effects of monetary policy may actually mitigate the inflationary bias problem. Ignorance of the current state of the economy can lead the government to deliver inflation that is lower than in the benchmark one-shot game with full information. This means that the two cases for simple rules can work at cross purposes: if the traditional monetarists are correct to worry about the government's forecasting abilities, then the new monetarists should worry less about the perils of discretion.

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\(^1\) That is, with the appearance of Kydland and Prescott (1977), Barro and Gordon (1983a,b), and many of the papers cited below.

\(^2\) See, for example, the discussion of these issues at the undergraduate level in Mankiw (1994), chapter 12, and at the graduate level in Blanchard and Fischer (1989), chapter 11.
The logic is straightforward. If the relation between monetary policy and macroeconomic outcomes is uncertain, both the government and the private sector have an incentive to scrutinize the past for clues about this relationship. The private sector's impulse to learn creates a link between past monetary policy and present inflation expectations, converting a sequence of one-shot games between the private sector and the government into a *bona fide* dynamic game. The government can take advantage of this intertemporal link to enhance its credibility: the private sector's need to learn from the past creates a way in which the private sector can reward or punish the government for its past behavior.

The private sector's strategy follows directly from the assumption of rational expectations. If the government's behavior generates useful information about the economy, it would be inefficient for the private sector to throw this information away. Yet the mere fact that expectations are rational is enough to deliver an equilibrium that is more efficient than the benchmark one-shot Nash equilibrium. Therefore, although the private sector operates what looks like a trigger strategy (in which the private sector explicitly threatens to punish the government for bad behavior), the private sector does not make any explicit threats or promises (besides the promise to process information efficiently). This means that the equilibrium does not suffer from many of the shortcomings that plague monetary policy models based on trigger-punishment strategies.\(^3\) It does not require the private sector to surmount coordination problems to agree to a particular strategy; the equilibrium survives whether horizons are finite or infinite; the pure-strategy equilibrium is unique; and the equilibrium is renegotiation-proof.

This paper can be viewed as a bridge between two well-known lines of research in monetary economics. One is the study of the "instrument problem," an important theme in the debates of the 1960s and 1970s over monetary policy. The second is the more recent study of time-inconsistency in monetary policy originating with the influential papers of Kydland and Prescott (1977) and Barro and Gordon (1983b), and especially that branch of the

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\(^3\)For instance, Barro and Gordon (1983b) and Canzoneri (1985), to name just two of many.
time-inconsistency literature that investigates repeated interaction between the government and the private sector.

The literature on the "instrument problem" casts the question of optimal monetary policy in terms of instruments and targets. The former are variables over which monetary authorities are presumed to have direct control (e.g., short-term interest rates, the monetary base, non-borrowed bank reserves), whereas the latter are the goals of monetary policy (full employment output, low inflation, etc.). In a system with more instruments than targets, the equilibrium level of macroeconomic variables is indeterminate. But this problem is remedied once the government sets enough of the instruments (usually the monetary base or an interest rate when only one instrument need be set). If the system is deterministic, the question of which variable to anchor is immaterial; the government will carry out the same actions regardless of which variables it designates as the instruments.

The choice of instrument is not irrelevant if the economy is subject to unobservable stochastic disturbances—that is, there is instrument uncertainty—and governments care about the variability of targets. This insight led to an important debate between advocates of monetary activism, on one hand, and of passive rules, on the other. Those who favored using monetary policy to stabilize the business cycle set out to find which variables serve best as instruments and how they ought to be manipulated. Poole (1970) demonstrated how the analysis depends on the relative size of the structural shocks to the economy. Brainard (1967) showed that the government is better off manipulating all of its instruments, even if they far exceed the number of targets. Benjamin Friedman (1975) established optimal monetary policy rules when information about economic conditions comes to light slowly and the economy’s response to monetary policy is delayed.
Critics of activism—Milton Friedman chief among them—responded that activism could only lead to greater macroeconomic instability and, thus, that simple rules were to be preferred. In Friedman’s words,

We simply do not know enough to be able to recognize minor disturbances when they occur to be able to predict either what their effects will be with any precision or what monetary policy is required to offset their effects. (Friedman (1967), p. 420)

There are many reasons for our limited ability to predict the effects of policy. Even if the economy behaves predictably on average, economists make forecasting errors by mistaking the economy’s position in the business cycle. Moreover, the economy probably does not behave predictably on average. The structure of the economy often changes permanently—important industries rise and fall, the labor market evolves, the banking system changes, stable money-demand functions break down—and these changes undermine our ability to predict the future. Our inability to predict the impact of policy can be collected under the rubric of instrument uncertainty.

The point of this paper is to show how instrument uncertainty can mitigate inflationary bias. The government is presumed to use monetary instruments to affect targets such as inflation, employment, and output. However, the government lacks complete control over the economy in that there is a stochastic wedge between the instrument and the target, a wedge that the government cannot observe when it sets policy. This is the essence of instrument uncertainty. In the spirit of Brainard (1967), Benjamin Friedman (1975), and many other, I also assume that mapping from instrument to target, though it may be stochastic, has some serial correlation. The optimal monetary policy is therefore a feedback rule, since past realizations of disturbances contain useful information about the current unknown structure of the economy.

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4Holbrook (1972) added force to the Friedman criticism by demonstrating how an activist policy could lead to ever wider swings in monetary policy the central bank intervened not only to offset contemporaneous shocks but also the delayed effects of past interventions.

5See Meltzer (1991) for further discussion.
This is not the first paper to describe ways that economies deal with time-inconsistency. The most relevant part of the existing literature explores how repeated interaction between the government and the private sector offers an opportunity for the economy to reach an equilibrium that is more efficient than the Nash equilibrium of the one-shot game. Barro and Gordon (1983a) show how the public's operation of a trigger strategy can provide the government with a way to commit itself to low inflation. Canzoneri (1985) and Garfinkel and Oh (1993) extend the trigger-strategy approach to the case when the government has superior information about the current state of the economy. Several other papers explore the role of reputation when information is incomplete. Backus and Driffil (1985) show that when the public believes the government may be one of two types—a weak government with preferences as in Barro and Gordon (1983a,b), and a tough one who cares only about keeping inflation low—then some parameter constellations can support pooling equilibria in which the weak government keeps inflation low early in the game to lull the public into believing that a tough government is in power. The advantage to such a strategy is that it reduces inflation expectations late in the game, thus increasing the utility that the weak government gets from an inflation surprise. Vickers (1986) complicates the Backus and Driffil model by giving both central-bank types a temptation to create surprise inflation, although for one type the temptation is stronger. Barro (1986) considers the case in which governments differ in their ability to precommit.

All of these papers require relatively strong assumptions about information or about private sector behavior. Models following Barro and Gordon (1983b) require the private sector to agree on a sometimes complicated punishment strategy. Models with incomplete information assume that information about government preferences is truly asymmetric, a condition that would not apply when, for example, the government maximizes social welfare that is common knowledge. Moreover, both sets of

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6 Another important strand of the literature investigates the role of institutions. Rogoff (1986) suggests that society appoints conservative central bankers as a way to mitigate inflationary bias. Lohmann (1992) extends Rogoff's model to allow for the possibility that the legislature can override the central bank, and Waller (1992) extends the Rogoff model to a multisector economy. Walsh (1995) investigates what the optimal contract for a central banker should look like.
models suffer from multiple equilibria. The virtue of the model presented below is that it imposes very weak restrictions on the benchmark repeated game analyzed by Barro and Gordon, Backus and Driffil and the others mentioned above. It simply assumes that some information about the economy is unknown when the government takes action, and that the private sector and the government can learn about the economy by reviewing past experience.

The Model

I present my argument using a common variation of the familiar set-up of Barro and Gordon (1983a,b). This makes it easy to compare the results from this paper to those from the vast literature that uses the same basic set-up.

The government has a single-period payoff function

\[
(1) \quad u_i = A(\pi_t - \pi_t') - \frac{\pi_t^2}{2}
\]

where \(\pi_t\) is inflation in period \(t\) and \(\pi_t'\) represents the private sector's expectations of inflation in period \(t\). Whether or not this payoff function is also the social welfare function is irrelevant to all but the welfare analysis below. The function is assumed to be a reduced-form relation summarizing a more complex economic system that delivers two results. First, the government (or society in general) benefits from surprise inflation. This may be due to distortions in the economy that lead to a natural rate of output (or employment) that is below the optimal rate, in which case the government can exploit a Phillips-curve relation to boost output. Alternatively, the government may wish to derive seigniorage from surprise inflation (either directly or by using surprise inflation to default on part of its debt). The second result is that inflation is costly. The microfoundations of this result are rarely presented, although it is meant to represent the costs of anticipated

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7See Rogoff (1987) for a discussion of these issues.
8What follows is inspired by Holmstrom (1982), who presents a model with similar informational constraints but in a very different setting.
9See Barro and Gordon (1983a,b) for a discussion of these aspects of the model.
inflation, such as the need to economize on real balances, change prices more frequently, and deal with greater relative price variability.

The parameter $A$ measures the marginal rate of substitution between inflation and output or employment. When $A$ is high, the government gains a great deal from surprise inflation (via higher output and lower unemployment, for example) compared to the associated cost of anticipated inflation. This creates a strong temptation to create surprise inflation and is the source of inflationary bias in this model. Indeed, this parameter is frequently referred to simply as the inflationary bias.

The government must operate under instrument uncertainty. Its instruments—the levers of monetary policy—do not completely control the government’s targets. In keeping with the spirit of the inflationary bias literature, inflation is the primary target the government tries to control. I assume that inflation is a function of two parameters, $\pi_i = \pi(\phi_i, a_i)$, where $a_i$ is a quantitative measure of the instrument’s setting in period $t$, and $\phi_i$ is a stochastic wedge between the instrument’s setting and inflation, which is the target outcome. To make this a true case of instrument uncertainty, I assume that the government sets its instrument $a_i$ before it observes the realization of $\phi_i$. Otherwise, the instrument problem is rendered immaterial. I also assume the simplest form of instrument uncertainty, namely that $\pi_i = \phi_i - a_i$. Inflation is linear in the instrument and the stochastic wedge, and the source of uncertainty is additive. The instrument can be interpreted as the government’s effort to reduce inflation. Complicating the relationship between instrument and target would make the mathematics more difficult but would not remove the kernel of the argument, which is that the need to learn from the past creates an intertemporal link that the government can exploit to enhance its credibility.

The private sector has rational expectations so that $\pi_t^e$ is the mathematical expectation of inflation in period $t$ given all the information available at the moment the private sector forms its expectations.

Equation (1) and the assumption of rational expectations give rise to the standard time-inconsistency result in a one-shot game between the
government and the private sector. The government wishes to maximize $Eu$, where

\begin{equation}
Eu = A(\bar{\phi} - a - \pi^*) - \frac{E\phi^2 + a^2 - 2a\bar{\phi}}{2}
\end{equation}

and $\bar{\phi}$ is the known mean of $\phi$. The first-order condition yields an optimal action for the government of $a^* = \bar{\phi} - A$. The Nash equilibrium inflation is $\pi^* = \phi - \bar{\phi} + A$, which on average is equal to $A$. It is easy to show that both the government and society in general would be better off with $\pi^* = \phi - \bar{\phi}$ (which on average is equal to zero). The time-consistent Nash equilibrium is Pareto-inferior. But the Pareto-optimal policy is time-inconsistent.

The same result would hold in a dynamic game without any informational problems, unless there existed institutional mechanisms to deal with time-inconsistency (e.g., trigger-strategy agreements as in Barro and Gordon (1983a) and Canzoneri (1985), appointment procedures as in Rogoff (1985) and Lohmann (1992), or explicit contracts as in Walsh (1995)).

The main message of this paper is that if there are informational problems, in particular those associated with instrument uncertainty, then the Nash equilibrium may be Pareto-superior to that of the full-information game (and under some conditions may coincide with the Pareto-optimal equilibrium). The inflationary bias problem may not be as serious as previous literature would suggest, and the many mechanisms that have been proposed to deal with it may not be necessary.

Assume for simplicity that $\phi_t$ can be decomposed into two random components, $\phi_t = \eta_t + \varepsilon_t$. Let $\eta_{t+1} = \eta_t + \delta_t$, and assume that $\delta_t$ is an independently distributed normal random variable with zero mean and precision (i.e., inverse of its variance) $p_\delta$. Let $\varepsilon_t$ be an independently normal distribution with mean zero and precision $p_\varepsilon$. The variables $\varepsilon_t$ and $\delta_t$ are uncorrelated. This definition of $\phi_t$ is a stark representation of the more general assumption that some shocks have persistence (e.g., technological change) while others are purely transitory (e.g., business cycle fluctuations).
For convenience I assume that the persistent shocks are in fact non-stationary whereas the transitory shocks display no serial correlation.

The distinction between persistent and purely transitory shocks is important. The government and the private sector have the same information set, which includes knowledge of the processes generating $\varepsilon_t$, $\delta_t$, and $\eta_t$, as well as the history of the game. Since transitory shocks are independent through time, information about past disturbances yields no useful information about the current disturbance. The same is not true of persistent shocks. Any serial correlation means that recent realizations contain useful information about the current realization; the government and the private sector have an incentive to scrutinize the recent past to improve their inflation forecasts. This scrutiny is precisely what turns a sequence of one-shot games into a truly dynamic game.

The government and the private sector both observe inflation at the end of each period $t$, which for both players is equivalent to observing $\phi_t$. The government can observe $\phi_t$ because it knows the action it has taken in period $t$, which together with $\pi_t$ gives $\phi_t$. The private sector can observe $\phi_t$ even if it cannot observe the government’s action: since there is no private information about shocks, the private sector can solve the government’s optimization problem and derive an expectation about the government’s action, which in equilibrium will be equivalent to the action itself.

Let $\phi_{t-1}$ be the history of structural shocks up to the beginning of period $t$. As mentioned above, the government and the private sector have an incentive to use $\phi_{t-1}$ to draw inferences about the current value of $\eta_t$, and thus the current structure of the economy. That $\phi_t$ conflates $\eta_t$ and $\varepsilon_t$, means that the private sector and government face a signal-extraction problem.

Agents learn about $\eta$ using Bayes’s rule. Given the assumptions about the random variables, learning happens in a well-understood way.\(^\text{10}\) Let $\mu_t$ be the commonly held prior on the mean of $\eta_t$. The posterior mean, after agents have observed $\phi_t$, is

\(^\text{10}\)See DeGroot (1969) for a discussion of Bayesian updating with conjugate priors.
(3) \[ \mu_{t+1} = \rho_t \mu_t + (1 - \rho_t) \phi_t, \]

where \( \rho_t = p_t/(p_t + p_e) \) and \( p_t \) is the precision on \( \eta_t \). One can derive the evolution of \( p_t \) by noting that, absent a \( \delta \) shock to \( \eta_t \), precision would evolve by \( \hat{p}_{t+1} = p_t + p_e \). Put differently, the posterior variance would shrink to \( 1/(p_t + p_e) \). However, the \( \delta \) shock adds to this variance, so that the true posterior variance is

(4) \[ \frac{1}{p_{t+1}} = \frac{1}{\hat{p}_{t+1}} + \frac{1}{p_\delta} \]

which upon eliminating \( \hat{p}_{t+1} \) gives

(5) \[ p_{t+1} = \frac{(p_t + p_e)p_\delta}{p_t + p_e + p_\delta}. \]

Using the definition of \( \rho_t \) gives, in turn,

(6) \[ \rho_{t+1} = \frac{1}{2 + p_e/p_\delta - \rho_t} \]

Note that \( \mu_{t+1} \) is a random walk without drift:

(7) \[ E\mu_{t+1} = \rho_t \mu_t + (1 - \rho_t)E\phi_t, \]
\[ = \rho_t \mu_t + (1 - \rho_t)\mu_t = \mu_t \]

and it evolves according to

(8) \[ \mu_{t+i} = \mu_0 \prod_{i=0}^{t+i} \rho_i + \sum_{j=0}^{t-1} \phi_j(1 - \rho_j) \prod_{i=j+1}^{t+i} \rho_i + (1 - \rho_t)\phi_t \]

Furthermore, \( p_{t+1} \) and \( \rho_{t+1} \) evolve deterministically. Finally, \( \rho_{t+1} \) converges to a stable steady state given by
\[ \rho = 1 + \frac{p_{\varepsilon}}{2p_{\delta}} - \left( \frac{p_{\varepsilon}^2}{4p_{\delta}^2} + \frac{p_{\varepsilon}}{p_{\delta}} \right)^{1/2} \]

Its stability can be discerned by evaluating

\[ \rho_{t+1} - \rho_t = \frac{1-(2+p_{\varepsilon}/p_{\delta})\rho_t + \rho_t^2}{2+p_{\varepsilon}/p_{\delta} - \rho_t} \]

The numerator is always positive, so the sign of \( \rho_{t+1} - \rho_t \) is the same as the sign of the numerator. It is straightforward to show, therefore, that \( \rho_{t+1} \) is rising when \( \rho_t \) is below the steady state, that is, when

\[ \rho_t < 1 + \frac{p_{\varepsilon}}{2p_{\delta}} - \left( \frac{p_{\varepsilon}^2}{4p_{\delta}^2} + \frac{p_{\varepsilon}}{p_{\delta}} \right)^{1/2} \]

From (3) one sees that \( \rho_t \) is the weight that the prior mean gets in the posterior mean, and \( 1-\rho_t \) is the weight that the most recent observation of \( \phi \) gets. Partial differentiation of (9) shows that \( \rho \) is falling in \( p_{\varepsilon} \) and rising in \( p_{\delta} \).

If the variance of \( \varepsilon \) is very great, \textit{ceteris paribus}, then the private sector and the government attach very little weight to recent observations of \( \phi \). These signals of the current value of \( \eta \) are contaminated with a great deal of noise and are therefore of little use for inference about \( \eta \). If, on the other hand, the variance of \( \delta \) is very large, so that the economy frequently experiences significant persistent structural change, then agents give most of the weight to recent observations of \( \phi \). The information contained in the prior rapidly becomes outdated, so the government and private sector rely primarily on the information in the most recent realization of \( \phi \).

Note, also, that \( \rho \) goes to unity as \( p_{\delta} \) goes to infinity. When \( p_{\delta} \) is large, agents believe they know the current value of \( \eta \) with virtual certainty, so they have no need for current observations of \( \phi \).

The government’s problem in period 0 is to
where $\beta$ is the discount factor and $E$ is the expectations operator using information available at the beginning of period 0. Government action in any period $t$ has an immediate impact on contemporaneous payoffs through the standard channels of anticipated and surprise inflation. But it also affects future payoffs because of its influence on inflation expectations. The government must consider both effects when choosing $a_t$.

The private sector’s problem is to set

$$\pi^e_{t+1} = E(\pi_{t+1} | \phi^t) = E(\eta_{t+1} - a_{t+1} + e_{t+1} | \phi^t) = \mu_{t+1}(\phi^t) - a_{t+1}^e(\phi^t)$$

where $a_{t+1}^e$ is the private sector’s expectation of the government’s action in $t+1$. Government actions affect the information contained in $\phi^t$, hence they affect future expectations. The government must anticipate this effect. However, posteriors evolve stochastically because of instrument uncertainty. At best, the government can only derive an expected value of future inflation expectations.

Consider the choice of $a_0$. The government must evaluate

$$E_0\pi^e_{t+1} = \mu_0 \prod_{i=0}^t \rho_i + \sum_{j=0}^{i-1} E_0(\phi_j)(1-\rho_j) \prod_{i=j+1}^t \rho_i + (1-\rho_i)E_0(\phi_i) - E_0(a_{t+1}^e)$$

Note that $E_0(\phi_j) = E_0(\pi_j + a_j^e) = E_0(\eta_j - a_j + a_j^e) = \mu_0 - a_j + E_0(a_j^e)$. The first two equalities follow from the definition of $\phi_j$ and the assumption about the determinants of inflation, and the last equality follows from the fact that $E_0(\eta_j) = E_0(\mu_j) = \mu_0$ since the posterior mean is a random walk. Eliminating $E_0(\phi_j)$ in (***) and differentiating (**) with respect to $a_0$ reveals that the marginal impact of $a_0$ on expectations in $t+1$ is, in utility terms,
The total marginal effect of $a_0$ on the stream of utility beginning in period 1 is

\[ A(1 - \rho_0)\beta^{i+1} \prod_{i=1}^{\infty} \rho_i \]

which, in steady state, is

\[ A \frac{(1 - \rho)\beta}{1 - \beta \rho} \]

The government's first-order condition for optimality combines the contemporaneous marginal effect of $a_0$, given by $\mu_0 - A - a_0$ and the future effects captured in (*****) to yield

\[ a_0^* = \mu_0 - A \frac{1 - \beta}{1 - \beta \rho} \]

and, in general,

\[ \pi_i^* = \eta_i - \mu_i + A \frac{1 - \beta}{1 - \beta \rho} + \varepsilon_i \]

When the government follows its optimal policy, average inflation is

\[ \pi^* = A \frac{1 - \beta}{1 - \beta \rho} \]

Equation (20) clearly shows the effect of ignorance on the steady state equilibrium to this game: to the extent that instrument uncertainty reduces the coefficient on $A$ below unity, it lowers inflation below that of the benchmark one-shot game. Indeed, since both $\beta$ and $\rho$ are between zero and one, average inflation is between zero, which is the Pareto optimum, and $A$, the Nash equilibrium to the one-shot game. The presence of $\beta$ underscores the importance of repeated interaction. The payoff to keeping inflation low in
the present accrues only in the future, so the government must care about the future if it is to rely on instrument uncertainty to enhance its credibility. The presence of $\rho$ captures the way uncertainty influences equilibrium inflation.

Three important consequences of instrument uncertainty are evident from (18). The first is that average inflation is falling in the variance of the increments of $\eta$ (i.e., it is rising in $p_\delta$), tending towards a limit of $A(1 - \beta)$. If $\eta$ tends to change by large amounts, causing a great deal of persistent instrument uncertainty, the information contained in the private sector's priors about $\eta$ becomes outdated very quickly. The private sector must rely heavily on very recent observations of $\phi$, which in turn means that the government's current actions figure prominently in the private sector's inflation expectations in the near future; the link between one period and the next is strong. This enhances the government's ability to commit itself to lower inflation, for the private sector can react to a government defection almost immediately. Keep in mind that this reaction is not due to any special strategy employed by the private sector but follows simply from the fact that rational agents want to learn from the past.

A second implication of the model is that average inflation is rising in the variance of $\varepsilon$. A high variance of $\varepsilon$ renders the private sector's signal extraction problem more difficult, because it makes $\phi$ a very noisy signal of the persistent change in the relationship between instrument and target. The private sector, in forming a posterior judgment about $\eta$, places little weight on recent observations of $\phi$. This weakens the link between one period and the next and makes it more difficult for the government to promise credibly that it will maintain low inflation. Because the private sector reacts sluggishly to new information, the benefits of monetary restraint accrue slowly and over many periods. With discounting, the present value of these future benefits is small compared to the immediate payoff to creating surprise inflation. The private sector understands this, and therefore places little stock in government promises to resist temptation and keep inflation low. As the variance of $\varepsilon$ tends to infinity, equilibrium average inflation tends to $A$, the Nash equilibrium inflation rate in the one-shot game.
A third implication is that average inflation is inversely related to the discount factor. When the government cares very little about the future (i.e., $\beta$ is low), average inflation tends towards $A$, the Nash equilibrium to the one-shot game. Returns to keeping inflation low accrue only in the future, so a low discount factor reduces the discounted present value of the gains to monetary restraint relative to the immediate gains from surprise inflation. The private sector recognizes the strong temptation the government’s has to create surprise inflation, and thus is skeptical of government promises of monetary restraint. The reverse is true when the discount factor is close to unity. A higher discount factor increases the relative importance of future returns and makes it easier for the government to promise credibly that it will keep inflation low. In the limit when $\beta$ is one, average inflation falls to zero, the Pareto-optimum.

The preceding three points can be summarized: the need to learn from the past can create an intertemporal link that the government can use to enhance its credibility. The link is stronger the more the private sector cares about the past. From the vantage point of the private sector, the recent past must contain useful information (i.e., persistent shocks must be quite variable) that is relatively easy to extract (i.e., the white noise variance must be small). The intertemporal link is also stronger the more the government cares about the future (i.e., the higher is its discount factor).

Several other qualities of equilibrium inflation deserve note. One is that average inflation will in general be lower than that of the one-shot Nash equilibrium even if the government’s horizon is finite. This is in contrast to the trigger-strategy equilibria of Barro and Gordon (1983a), Canzoneri (1985), Garfinkel and Oh (1993) and others, in which anything short of an infinite horizon causes the equilibrium to revert to that of the one-shot game.\footnote{See Rogoff (1987) for a discussion of models in which the equilibrium in a one-shot game is not unique so that a punishment strategy can support low inflation even under a finite horizon.} When the horizon only goes to period $T$, the total marginal benefit of $a_0$ on the stream of utility beginning in period 1 is

$$A(1 - \rho_0)\beta \sum_{t=0}^{T-1} \beta^t \prod_{i=1}^{t} \rho_i$$
Equilibrium inflation is less than that of the one-shot Nash game so long as this benefit is positive, which occurs when $\rho_0 < 1$ and $\beta > 0$. Indeed, instrument uncertainty reduces equilibrium inflation even in a two-period game (i.e., when $T=1$).

It is also the case that inflation will rise, on average, as the government's horizon becomes shorter. In steady state, inflation given a horizon of $T$ is

$$\pi_t = \eta_t - \mu_t + A \left[ 1 - (1 - \rho)\beta \frac{1 - \beta^T \rho^+}{1 - \beta \rho} \right] + \epsilon_t$$

By inspection it is clear that this is decreasing in the length of the horizon, $T$. Thus, this model gives a rationale for so-called "political business cycles," since one can interpret a shortening time horizon as the approaching end of a government's term in office (assuming that any given government cares about welfare only when it is in office).

This explanation differs from that offered by, for instance, Rogoff and Sibert (1988), who assume that information is incomplete. When information is incomplete (that is, when the public does not know the government's true preferences), the government has an incentive to signal that it is of the "good" type (e.g., favors low inflation, fiscal restraint, etc.) in order to boost its re-election hopes. The need to send signals, which arises because of the incompleteness of information, distorts the economy, and relatively high inflation late in a government's term reflects one possible distortion.

This model demonstrates how inflation can display a political business cycle even when information is complete. As the end of a government's term nears, the gains to monetary restraint fall and government promises to resist the temptation of surprise inflation lose credibility, raising equilibrium inflation. The same pattern holds even if there is some exogenous probability that the government will win re-election. If this probability is less than one, the discount factor is smaller after the election than before. As the election nears, the same stream of future gains is discounted more heavily, reducing
the discounted present value of future gains and raising equilibrium inflation.

This model also offers an explanation of the relationship between the mean and variance of inflation, a relationship that many studies have found to be positive. Using the expressions for actual and mean inflation, one can show that in steady state $V(\pi) = E\left[\left(\eta_t - \mu_t\right)^2\right] + E(e_t^2) = (p_\epsilon + p_\delta) / p_\epsilon p_\delta$. From the definition of $p_i$ one can further show that $V(\pi) = 1 / \rho p_\epsilon$. It is straightforward to show that the variance of inflation is unambiguously decreasing in both $p_\epsilon$ and $p_\delta$. Recalling that average inflation is rising in $p_\delta$ and falling in $p_\epsilon$ one arrives at the conclusion that the relationship between the mean and variance of inflation is in general ambiguous. If greater inflation variability is due to more variable white noise (i.e., lower $p_\epsilon$), then it will be accompanied by higher inflation. If, on the other hand, greater inflation variability is due to greater persistent uncertainty (i.e., lower $p_\delta$), then it will be accompanied by lower inflation. This may explain why the correlation between inflation and its variance is much weaker for low-inflation countries compared to high-inflation countries (Logue and Willet (1976)). In countries with a history of low inflation, a rise in inflation variability may be largely due to a rise in the variance of persistent shocks, which also diminishes average inflation.

Steady-state inflation is lower the greater is the uncertainty about persistent changes in the economy. Yet it is not clear whether this improves social welfare. If one consider the government’s objective function as a measure of social welfare, it is clear that society benefits from lower average inflation but suffers from more variable inflation. Thus, instrument uncertainty has an ambiguous effect on welfare, since it may reduce average inflation while it raises variability.

To assess the marginal effect on welfare of more instrument uncertainty one must calculate expected steady-state utility:

\[
E_0 V = \sum_{t=0}^{\infty} \beta^t \left\{ A\left(E_0 \pi_t^* - E_0 \pi_t^2\right) - E_0 \frac{\pi_t^2}{2}\right\} = -\frac{1}{1-\beta} E_0 \frac{\pi_t^2}{2}
\]
The second equality follows from the assumption of rational expectations. Note also that

\[(24) \quad E_0 \pi_t^2 = \frac{1}{\rho \rho^\prime} + A^2 \frac{(1-\beta)^2}{(1-\beta \rho)^2} \]

Differentiating (20) with respect to \(p_\delta\) (noting that \(\rho\) is a function of \(p_\delta\)) yields the following condition under which greater uncertainty about persistent structural change raises welfare:

\[(25) \quad A^2 > \frac{(1-\beta \rho)^3}{2(1-\beta)^2 \beta \rho^2 p_\epsilon} \]

Whether or not this condition holds depends on parameter values. However, the marginal welfare gain from extra uncertainty about permanent structural change is positive when \(A\) is large, that is, when the inflationary bias problem is serious. In other words, instrument uncertainty reduces the time-inconsistency problem precisely when we would like it to: when the credibility problem that leads to a large inflationary bias is serious.

This result arises because of the ambiguous effect of an increase in the variance of \(\delta\). Greater variance makes inflation more variable but reduces average inflation. For governments with low values of \(A\), (i.e., mild inflationary bias problems), the marginal gain to lower inflation is small compared to the cost of more variable inflation. For governments with high values of \(A\), the opposite is true.

As for uncertainty about transitory structural change, it stands to reason that a lower variance of \(\epsilon\) (i.e., greater precision) always improves expected welfare, since it reduces average inflation \emph{and} the variance of inflation (since \(\pi_t^* = \eta_t - \mu_t + A(1-\beta)/(1-\beta \rho) + \epsilon_t\)). Differentiating (20) with respect to \(p_\epsilon\) gives the condition

\[(26) \quad \frac{\partial \rho}{\partial p_\epsilon} \frac{2A^2(1-\beta)^2 \beta \rho^2 p_\epsilon^2 - (1-\beta \rho)^3 p_\epsilon}{(1-\beta \rho)^2 \rho^3} < 1 \]
that must hold for lower transitory variance to improve expected welfare. Since \( \frac{\partial \rho}{\partial p_\epsilon} < 0 \), the left-hand side is falling in \( A \). Therefore, expected utility is increasing in \( p_\epsilon \) when \( A > A^{\text{max}} \), where \( A^{\text{max}} \) is the value of \( A \) that makes (22) an equality. It follows that if (22) holds for \( A = 0 \) then it must hold for all positive values of \( A \). Let \( r \equiv p_\epsilon / p_\delta \). After setting \( A = 0 \) and noting that

\[
(27) \quad \frac{\partial \rho}{\partial p_\epsilon} = -\frac{\rho}{2p_\delta(r^2/4 + r)^{1/2}}
\]

one arrives at the condition \( r < 2(r^2/4 + r)^{1/2} \) which is always true. Thus, a lower variance of \( \epsilon \) improves expected welfare.

**Discussion and Conclusions**

Economists conventionally believe that society is better off the more governments understand how policy affects the economy. This is a clear implication of the literature on instrument uncertainty: absent any unobservable shocks, the instrument problem goes away. But if governments are uncertain of the effects of their actions, economists, especially monetarists, advocate simple policies according to simple rules; attempts to fine-tune the economy rarely improve it and more often than not harm it.

This paper casts doubt on the wholesale conclusion that ignorance is harmful. It presents model that shows how ignorance in the form of instrument uncertainty can be beneficial, at least for economies prone to inflationary bias. Ignorance compels the private sector to learn from the past, which creates an intertemporal link that the government can use to enhance its credibility and deliver lower average inflation. The result rests on two important assumptions, however. One is that the government must care about its reputation, so that it selects policy taking into account the policy’s present and future impact on the economy. The second is that instrument uncertainty must have some persistence. Otherwise, past information contains no useful information about the present and there is no need for learning.
One accomplishment of this model is to link research on instrument uncertainty and inflationary bias. Moreover, it shows how proposals for simple monetary policy rules based on worries about instrument uncertainty can undermine the case for simple rules based on the inflationary bias problem. The model also offers an explanation for two phenomena economists have often pondered: the tendency of inflation to rise towards the end of a policymaker’s term in office, even if this policymaker has some chance of winning re-election; and the correlation between the level and variance of inflation, which appears to be stronger for high-inflation countries than for low-inflation countries.

The model can be extended in several ways. Obviously many of the assumptions made along the way can be relaxed. In particular, it would be worth exploring a setting in which the persistent disturbance is mean-reverting rather than a random walk. The private sector will still find it optimal to learn from the past; any serial correlation ensures that this is so. However, the degree of serial correlation will play a role in determining equilibrium inflation. One would suspect that weaker serial correlation, which reduces the amount of useful information contained in the history of the game, would lead to higher average inflation. Another extension could explore different mappings from $\phi$ and $a$ to inflation.

The model could also be used to study the choice of different kinds of monetary policy. For example, it is commonly believed that governments in developing countries pay a price for targeting the nominal exchange rate (e.g., real exchange rate overvaluation) that they could avoid by targeting the money supply or real exchange rate. Yet nominal exchange rate pegs are quite common. If the advantage of a nominal exchange rate peg is its visibility, the model in this paper offers a reasonable justification for such pegs: they are less noisy signals of monetary policy than other targets, and hence can help bolster the government’s credibility. A similar argument might help explain why many governments prefer to target interest rates rather than monetary aggregates.

Finally, the model can also be embellished to include shocks observed by the government prior to setting its policy (as in Canzoneri (1985) and
others), so that the government has a greater need to employ stabilization policy. I prefer not to conjecture how the results would change in this case.
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