Optimal Procedures in Criminal Law: Five Essays

Author: Murat Can Mungan

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Abstract

Becker (1968) provides a formal framework for analyzing various policies in criminal law. Within this framework there are potential criminals, who have varying benefits from committing an illegal act. They are subject to sanctions when they are caught and are found guilty for committing such acts. Accordingly, increased expected sanctions lead to greater deterrence. There are also costs associated with achieving such deterrence. Hence, there are optimal policy variables which balance costs and gains associated with increased deterrence.

In my dissertation, in five independent but closely related essays, I address various issues related to criminal law by making use of optimal crime and deterrence models, which are similar to Becker (1968). First, I analyze the standard of proof in criminal trials and extend a justification as to why there are higher standards of proof in criminal trials versus civil trials. Next, I introduce the concept of mixed warning strategies, and justify the use of mixed as well as pure warning strategies in law enforcement. In a related essay, I show that it is optimal to punish repeat offenders more severely than first time offenders, provided that offenders gain experience in evading detection by committing offenses. In my fourth essay, I identify reasons as to why it is welfare improving to allow individuals to self-report conduct crimes. Finally, I propose a simple framework to incorporate the concept of remorse in the economic analysis of criminal law, and show that the Beckerian maximal fine result need not hold when some individuals feel remorse.
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Chapter 0
An Introductory Appendix: The Simple Beckerian Framework

This dissertation will frequently make references to Becker (1968). Accordingly, it is useful to provide a brief review of what I call the simple Beckerian framework, by which I mean the way Becker’s model has been interpreted and incorporated in later works in the optimal law enforcement literature.

In this framework, it is assumed that some or all individuals in society are potential criminals. That is to say, some or all individuals can benefit from engaging in an activity declared to be illegal, and have varying benefits ($b$ with density function $f(b)$) from the commission of this act. The activity in question results in harm to society ($h > 0$), and this is why it is illegalized. It is further assumed that potential criminals react to incentives and can be potentially deterred by the threat of punishment. A threat of punishment exists when offenders are convicted with a positive probability ($p > 0$) and once convicted are subject to sanctions ($s > 0$).

Sanctions associated with the crime in question are set by the government. In this simple framework, it is assumed that sanctions are monetary, and therefore transferable. Accordingly, the imposition of sanctions does not result in any direct social cost. It is also assumed that the collection of fines is costless. There is, however, a maximum fine ($w$), which the government may not exceed when choosing sanctions.

Similarly, the probability of detection is set by the government. However, achieving a positive probability is assumed to be costly and the probability of detection is increasing in costs incurred by the government. In other words, $p$ is a function of governmental expenditures in detection (e.g. number of police officers or radar guns). However, it is more convenient to express the cost of detection as a function of the probability of detection. Since, $p$ is strictly increasing in expenditures,
it is an invertible function of its argument. Its inverse can be conveniently captured by the function \( c(p) \), with \( c' > 0 \).

The list below summarizes the notation introduced so far:

- \( b \): benefit from crime.
- \( f(b) \): density of benefits, \( f(b) \) is positive in \([0, \infty)\).
- \( h \): expected social harm from crime.
- \( p \): probability of detection.
- \( c(p) \): enforcement costs, with \( c' > 0 \).
- \( s \): sanction imposed on criminals who are caught.
- \( w \): maximum fine.

As briefly stated earlier, individuals have varying benefits from crime, and decide whether to commit crime based on the probability of detection and the monetary fine. Assuming that individuals are risk-neutral, they commit crime iff:

\[
b > ps
\]  

(0.1.)

The normative analysis is conducted by making use of a utilitarian social welfare function. The common approach is to count every individual with equal weight in the social calculus. This implies that criminal benefits enter the social welfare function. Whether they should is a discussion which I will not address in this appendix, but discuss to some extent in the proceeding chapters.

To construct the purely utilitarian function, a few things are worth noting. First, since individuals are assumed to be risk-neutral, it does not matter who incurs the harm \( (h) \) inflicted by the offender through the commission of a crime, it is simply the sum of harms inflicted on every party involved. Second, again, since individuals are assumed to be risk neutral, it does not matter to whom monetary sanctions \( (s) \) are transferred. What matters is that their is no loss of wealth in the transferring process. Given that there are no such losses, sanctions will not appear directly in the social welfare function. What is lost by the convict is another person’s gain and of an equal amount.
Given these observations, one can construct the social welfare function relatively easily:

\[ V(p, s) = \int_{\frac{h}{w}}^{\infty} (b - h) f(b) db - c(p) \]  
\[ (0.2.) \]

The first term of the integrand (i.e. \((b - h)\)) is simply the net benefit to society from the commission of crime by an individual with benefit \(b\), the lower limit of the integral is obtained by making use of equation 0.1 and \(c(p)\) is the cost of detection. \(V\) can now be used to find the optimal values of \(p\) and \(s\). A couple of observations immediately follow.

First, the maximum fine \((w)\) is optimal. To see this, for any \(p^h > 0\) take any sanction \(s^l < w\). In this case \(p^h s^l\) is the lower limit of the integral in equation 0.2. Now consider a punishment scheme where the probability of detection is \(p^l = \frac{p^h s^l}{w} < p^h\), and the monetary fine is \(w\). It follows that \(p^l w = p^h s^l\), hence the lower limit of the integral in 0.2. is the same under both punishment schemes. But the cost of detection is lower in the punishment scheme where fines are maximal (since \(p^l < p^h\)). Hence, as long as the probability of detection is positive the optimum fine is the maximal one. Furthermore, when the probability of detection is 0, fines are irrelevant (technically, making \(w\) optimal again).

Second, the optimal level of deterrence, which can conveniently be described by \(p^* w\), where \(p^*\) is the optimal probability of detection, results in under-deterrence. Stated differently, \(p^* w < h\). To see this, first note that it trivially follows that \(p^l > \frac{h}{w}\) cannot be optimal. Furthermore, \(p^l = \frac{h}{w}\) cannot be optimal either. The latter point can be noted by making the following observations. When the expected punishment is \(h\), marginal losses from decreasing deterrence are negligible, but the gains from decreasing \(c(\cdot)\) are not. Formally:

\[ V_p = -s(ps - h) f(ps) - c'(p) \]  
\[ (0.3.) \]

where the first term describes marginal net gains from increased deterrence and the second term describes marginal net losses due to an increase in \(p\). But the first term equals zero when \(p = \frac{h}{w}\) and \(s = w\), whereas the second term is positive. Accordingly, under-deterrence is optimal.
The objective of this appendix was to familiarize readers (who are not acquainted with the optimal law enforcement literature) with the Beckerian framework, and in particular Becker’s frequently cited two results. In the chapters that follow, I will refer to the first result multiple times as the Beckerian maximal fine (or sanction) result. In particular, chapter five seeks to identify a condition under which the Beckerian maximal fine does not necessarily hold.
Chapter 1
A Utilitarian Justification for Heightened Standards of Proof in Criminal Trials

1.1. Introduction

Many legal systems employ a high standard of proof for convicting suspects in criminal trials. The standard of proof beyond a reasonable doubt reflects this approach. Existing literature focuses on "fairness" and the non-transferability of criminal sanctions as rationales for having high standards of proof in criminal trials.\(^1\) Absent these considerations, the implication of standard models would be incompatible with the current legal practice, which employs proof beyond a reasonable doubt as the standard of proof. The reason is straightforward. In the context of standard crime and deterrence models, if there were no considerations of fairness and if criminal sanctions were completely transferrable, deterrence would be the only objective of the legal system. Since each type of judicial error is presumed to negatively affect deterrence in the same way, the judicial system would value a decrease in the rate of false acquittals as much as a decrease in the rate of false convictions.\(^2\) Hence, the optimal standard of proof would minimize the sum of judicial errors, and would presumably be a much lower standard of proof compared to beyond a reasonable doubt.

This chapter first provides a utilitarian justification as to why there is an asymmetry between costs associated with false convictions and false acquittals, even when criminal sanctions are assumed to be completely transferrable.\(^3\) It uses a social welfare function that aggregates individuals’ pref-

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1. See Lando (2009) and Miceli (1990), (1991) providing justifications for high standards of proof in criminal trials by incorporating notions of fairness, and Posner (1998) for a rationale based on the non-transferability of criminal sanctions. These are briefly discussed in the next section.

2. This follows from the traditional view on the effect of legal errors on deterrence. See Png (1986). See also Posner (1998) at p. 605, arguing that in the civil setting, in which sanctions are assumed to be transferrable, the only effect of judicial errors becomes "a reduction in the deterrent effect of the legal rule in question. That is why the errors are weighed (approximately) equally." For an opposing view see Lando (2006).

3. This is not to suggest that notions of fairness or the non-transferability of criminal sanctions are irrelevant.
erences, as opposed to assuming certain social cost functions. In doing so, it relies on the fact that non-criminals may engage in costly and precautionary activities to decrease their likelihood of being falsely convicted. The frequency of such activities are increasing in the rate of false convictions. Hence, while lowering the rate of false convictions and acquittals has a deterrent effect, decreasing the rate of false convictions also results in gains due to decreased costs from less frequent precautionary activities. Accordingly, this chapter provides a utilitarian justification for the frequently employed assumption that there are costs associated with false conviction rates, which are separate from costs associated with lower deterrence or costs arising from the non transferability of criminal sanctions.

Next, it shows that this asymmetry between false conviction and acquittal rates plays a crucial role in the determination of the standard of proof, and implies that the optimal standard of proof should be higher than that which minimizes the sum of judicial errors. The rationale behind results obtained in this chapter are fairly intuitive. The primary function of the standard of proof in criminal trials is to allow the legal system to decrease (increase) the rate of false acquittals at the cost of increasing (decreasing) false convictions. The traditional view is that these two types of errors affect deterrence in the same way. Hence, if the only objective of the legal system was deterrence, the optimal standard of proof would minimize the sum of these two errors. However, when there is an asymmetry between costs associated with false acquittals and false convictions, such that the latter error is more costly, the optimal standard of proof would be higher than that which minimizes the sum of judicial errors.\textsuperscript{4}

An important question which has to be addressed is whether the implications of this model are limited to the standard of proof in criminal trials, or whether they extend to civil trials as well. It is unlikely that the implications of this chapter extend to the civil law framework. The reason is closely

\textsuperscript{4}The earliest work I am aware of, which mentions a similar idea, is Bentham’s \textit{A Treatise on Judicial Evidence} (1825), where Bentham states that false convictions would produce ‘alarm’ in public. See Stein (2005) Chapter 6 p. 174 footnote 7, for a short description of Bentham’s arguments and the literature discussing them.
related to the fact that in ordinary civil law cases, the plaintiff is entitled to some form of recovery upon a finding that the defendant is liable. Accordingly, in the civil law framework, potential plaintiffs are likely to engage in precautionary activities to minimize the probability of suffering from Type II errors, just like potential defendants are likely to engage in precautionary activities to minimize the probability of being falsely found liable. Therefore, in the civil law framework, precautionary activities will be observed on both sides. This symmetry is likely to imply that a change in the standard of proof decreases precautionary activities on one side, but only at the cost of increasing precautionary activities on the other side. Hence, the findings in this chapter should only be interpreted within the criminal law framework.

Another important question is why the standard of proof that minimizes the sum of judicial errors should be treated as a benchmark. To explain the reason for this approach, two things must be noted. First, as argued, at the absence of considerations of fairness and non-transferability of criminal sanctions, the implication of standard crime and deterrence models is that the optimal standard of proof is that which minimizes the sum of judicial errors. Second, minimizing the sum of judicial errors is equivalent to placing an equal value on both types of judicial errors. The standard of proof in ordinary civil cases, namely preponderance of the evidence, is interpreted by many to minimize the sum of judicial errors, and precisely because both types of judicial errors are assumed to carry equal social costs. Hence, showing that incorporating precautionary costs leads to an optimal standard of proof which is higher than this benchmark standard is important for at least three reasons: (i) Identifying a source, namely precautionary costs, which pushes the optimal standard of proof higher, (ii) justifying the use of standards of proof in criminal trials which are higher than those imposed in civil trials, and (iii) providing a rationale as to why false conviction rates should be evaluated as being more costly than false acquittal rates.

In the proceeding parts of the chapter I formalize these ideas. The structure of this chapter is

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5See note 1, supra, and Clermont and Sherwin (2002). For an opposing view, see Demougin and Fluet (2005) at p. 193 arguing "that the common law standard of proof, given the rules of evidence, does not minimize expected error as usually argued in the legal literature".
as follows. Section 1.2. gives a brief review of the existing law and economics literature related to the problem being analyzed. Section 1.3. introduces a model in which individuals may engage in precautionary activities. The same section provides a justification as to why there are asymmetric costs associated with false convictions and false acquittals. Section 1.4. derives optimality conditions for the standard of proof. Section 1.5. consists of a few remarks and extensions and section 1.6. briefly concludes. Appendix A contains proofs of propositions.

1.2. Literature Review

Miceli (1990), (1991) and Lando (2009) are very closely related to this chapter. They employ similar approaches in analyzing issues related to the optimal standard of proof in criminal trials. Miceli (1990) finds that when the incentives of prosecutors are consistent with social interests, the optimal standard of proof balances the costs of the two judicial errors. His analysis does not take deterrence as a social goal, and assumes certain social costs associated with both types of errors. Miceli (1991) incorporates deterrence as a social goal, but focuses mainly on optimal sanctions and effort expenditures by law enforcers as opposed to the standard of proof, and it analyzes a representative offender. Lando (2009) derives the optimal standard of proof and relies on notions of fairness by making use of cost functions reflecting "justice disutility" or "injustice cost" (Lando (2009, p. 38)). It "analyze[s] which standard of proof maximizes the preferences of the average (non-criminal) citizen, i.e. which standard would be chosen in a democratic vote" (Lando (2009, p. 38)). The present chapter is similar in its approach. However, it first provides a utilitarian justification as to why there are additional costs associated with false conviction rates, and then shows that high standards of proof can be optimal due to such asymmetric costs.

Posner, in Economic Analysis of Law (1998), makes important comments regarding costs associated with judicial errors. He suggests that in the criminal law context the cost of the sanction imposed upon the criminal does not generate benefits of equal quantum that show up elsewhere in

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6 Other assumptions made in this paper include: (i) that the benefit of criminals should not enter the social calculus and (ii) that false conviction rates do not effect deterrence.
the social calculus. Hence, there are net costs associated with the conviction of a criminal. Therefore, false acquittals lead to savings due to fewer convictions whereas false convictions lead to expenses due to increased convictions. Accordingly, there is an asymmetry between costs associated with false conviction and acquittal rates. This asymmetry provides a rationale for high standards of proof in criminal trials. In the civil law context, Posner argues, this asymmetry does not exist (presumably because damages are transferrable). Therefore, "the errors are weighed (approximately) equally".7 My approach is very similar to Posner’s. I identify a source, which is only present in the criminal law context and implies asymmetric costs associated with Type I and II errors. My approach differs from Posner’s in the source I identify and the methodology I use to verify my claims. While Posner identifies non-transferability of criminal sanctions as the source of asymmetry, I identify precautionary costs. Furthermore, I employ a simple optimal crime and deterrence model to verify my claims and identify specific sufficient conditions under which my claims hold.

Rubinfeld and Sappington (1987), Miceli (1990), and Yilankaya (2002) analyze an issue closely related to the optimal standard of proof. They analyze how defendants and prosecutors would increase or decrease their efforts to generate evidence as a response to changes in the standard of proof. While Yilankaya (2002) analyzes the effect of the standard of proof on both prosecutors’ and defendants’ efforts, Rubinfeld and Sappington (1987) is concerned only with defendants’ incentives and Miceli (1990) is concerned only with prosecutors’ incentives. This chapter abstracts from this issue, and takes the evidence generating process as exogenously given.

Andreoni (1991) claims that increases in sanctions do not necessarily increase deterrence. It relies on the fact that jurors will convict an individual by weighing the costs associated with false acquittals and false convictions. Since the cost of false convictions is increasing in sanctions, higher sanctions may lead jurors to convict less frequently and this may in certain cases decrease overall deterrence. In Andreoni (1991), jurors determine reasonable doubt, and therefore the standard of proof, on their own. In contrast, the instant chapter takes the standard of proof as a choice variable.

to be determined in the legal system and assumes that jurors will always apply it correctly. It then asks the question what ought to be the standard of proof.

There are a number of other articles that are in some way related to the optimal standard of proof in criminal trials. For instance, Demougin and Fluet (2005) and (2006), Kaplow and Shavell (1994a), and Polinsky and Shavell (1989) are concerned with the standard of proof in civil trials, the effects of accuracy in the determination of liability and the effects of judicial errors on the incentives of individuals, respectively. Reinganum (1988), Kaplow (1994), and Sanchirico (2001) are other articles related to this literature.

1.3. The Model

1.3.1. Description

Individuals first have the option of engaging in, or abstaining from an activity which benefits them. This activity does not cause harm and is referred to as "the valuable activity". An individual who abstains from such activity is said to be engaging in "precautionary" behavior. If an individual engages in valuable activity, he faces a second decision. He can either commit crime, or not. In this regard, the valuable activity can be interpreted as a legal activity which is a prerequisite for another illegal one. In short, an individual has three options: (i) engage in precautionary behavior, (ii) engage in valuable activity without committing crime afterwards, and (iii) commit crime after engaging in valuable activity. Committing crime leads to a benefit \( b \) which is separate and independent from the benefit a person receives from engaging in valuable activity \( v \). \( v \) and \( b \) respectively are assumed to be distributed with densities \( k(. ) \) and \( f(. ) \), which are both positive in \([0, \infty)\). Crime leads to a negative externality for society \( h \). Individuals are assumed to be continuously and independently distributed over benefits from crime \( b \) and engaging in valuable activity \( v \).\(^9\)

The government possesses an imperfect detection mechanism, which may produce two types of
errors: (i) it may acquit a guilty individual with a probability of \( q_a \), and (ii) it may convict an innocent individual who has engaged in valuable activity with a probability of \( q_c \). Hence, \( q = q_a + q_c \) denotes the sum of judicial errors.\(^{10}\) For the purposes of this section, these errors are assumed to be exogenously given. The next section describes how these errors are determined as a function of the standard of proof (\( \mu \)) chosen by the government. The government also imposes an exogenously determined sanction on individuals who are convicted. To simplify notation, this sanction is assumed to have a magnitude of 1.\(^{11}\) Note that individuals who engage in precautionary behavior are never convicted and therefore never fined.

Some other assumptions can be listed as follows: Individuals are expected value maximizers, and there are no direct costs associated with the imposition of sanctions.

### 1.3.2. Notation

The following notation is used in characterizing interactions:

- \( v > 0 \): individuals’ benefit from engaging in valuable activity.
- \( b > 0 \): individuals’ benefit from committing crime.
- \( k(v) \): density of benefits from valuable activity.
- \( f(b) \): density of benefits from crime.
- \( q_a \): false acquittal rate.
- \( q_c \): false conviction rate.
- \( q = q_a + q_c \leq 1 \).
- \( \mu \in [0, 1] \): Standard of proof.
- \( h \): harm caused by criminal activity.

\(^{10}\)To simplify notation and for expositional convenience, I am assuming that every individual who engages in valuable activity is being audited, i.e. that the audit probability is 1. Results are not affected by the inclusion of an exogenously determined audit rate.

\(^{11}\)Since the main topic of this paper is the determination of the optimal standard of proof, I abstract from the determination of sanctions. However, under a fairly general set of sufficient conditions the Beckerian maximal sanction result extends to the current framework, and results are preserved. I provide sufficient conditions and a proof sketch of this claim in section 1.5.
1.3.3. Individual’s Decision Making Process

I assume that $q_a$ and $q_c$ are common knowledge. In this section, these are treated as fixed values. The next section describes how $q_a$ and $q_c$ are determined as a function of the standard of proof ($\mu$) chosen by the government. Given the assumption of common knowledge, to analyze individuals’ decision making process it should first be noted that individuals have three choices. Some individuals will decide to engage in valuable activity, and some will decide not to. Then, individuals who have engaged in valuable activity will have to decide whether or not to commit crime.

Not engaging in valuable activity is equivalent to taking precautions, which guarantees that the individual will not be falsely convicted. In this case an individual does not benefit from the valuable activity, but will not suffer from conviction either. Hence, returns from precautionary activity are

$$\Pi_P = 0$$

An individual, who engages in valuable activity, but does not commit crime, receives a benefit of $v$ but has expected costs of $q_c$, where the latter term reflects the expected cost of false conviction. Hence, returns from non-criminal engagement in valuable activity are

$$\Pi_V = v - q_c$$

An individual, who commits crime subsequent to engaging in valuable activity, receives a benefit of $v + b$. But, he also faces expected sanctions of $(1 - q_a)$, where $q_a$ reflects a discount due to the possibility of being falsely acquitted. Hence, a criminal’s expected utility is

$$\Pi_C = v + b - (1 - q_a)$$

Following Becker (1968), I assume that any individual will compare returns from non-criminal and criminal options, and choose the one maximizing his expected utility. Individuals can achieve this simply by comparing $\Pi_P$, $\Pi_V$ and $\Pi_C$. Figure 1.1. summarizes individuals’ best responses which are obtained by comparing $\Pi_P$, $\Pi_V$ and $\Pi_C$, along with the net social benefit associated with each individual’s action.
Two key values, namely $b^C$ and $v^C$, as denoted in figure 1.1., are worth formally defining, because they will be referred to in the proceeding sections.

\[ b^C \equiv 1 - q \quad \text{and} \quad v^C \equiv q_c \quad (1.1.) \]

It should be noted that individuals’ best responses are completely determined by $b^C$ and $v^C$. That is to say, once $b^C$ and $v^C$ are determined, the $b$-$v$ space is uniquely divided into areas I, II and III, as defined in figure 1.1.. Knowing how individuals will react to judicial errors, the government may maximize social welfare by affecting these errors through its choice of standard of proof. However, first the utilitarian social welfare function, which the government uses to evaluate the desirability of outcomes, must be derived. This is done in the next sub-section.

1.3.4. Social Welfare

Since individuals are assumed to be risk-neutral, the pure utilitarian social welfare is the sum of all individuals’ benefits minus harms to society. Fines do not enter social welfare, because they are transfers. Figure 1.1. represents each individual’s benefit minus the harm he causes to society.
Given individuals’ independent distributions over benefits from the valuable activity and crime, social welfare can be expressed as:

\[
W(q_a, q_c) = V + \int_0^\infty \int_0^\infty \max\{(1-q_a)(1-q_c)-v\} (b-h)f(b)dbk(v)dv - \int_0^{q_c} \int_0^{(1-q_c)-v} v f(b)dbk(v)dv
\]

Where \(V = \int_0^\infty \int_0^\infty vk(v)dvf(b)db\) denotes the maximum aggregate benefit derivable from valuable activity, which is a constant. For purposes of optimization, I will drop this constant and save on notation. Furthermore, to separate losses or gains due to deterrence and losses due to precaution, the second and third terms can be respectively labeled as \(D(q_a, q_c)\) and \(\pi(q_a, q_c)\). Incorporating these changes, (1.2.) becomes:

\[
D(q_a, q_c) - \pi(q_a, q_c)
\]

An immediate implication that follows from the expression of \(W\) is that Type I and Type II errors affect social welfare asymmetrically. This is summarized by the following proposition.

**Proposition 1.1.:** (i) False conviction (acquittal) rates increase (decrease) costs associated with precaution (\(\pi_{q_c}(q_a, q_c) > 0, \pi_{q_a}(q_a, q_c) < 0\)). (ii) Both types of errors decrease the level of deterrence (i.e. increase the level of crime), where an increase in the rate of false acquittals leads to a higher reduction than that due to an increase in the rate of false convictions.

**Proof:** See Appendix A.

An interesting implication of proposition 1.1. is that Type I and Type II errors do not affect deterrence in the same magnitude as is assumed in many previous articles dealing with judicial errors and deterrence. Another implication of proposition 1.1. is that the optimal standard of proof is not necessarily the one which minimizes the sum of judicial errors. Determining what the optimal standard of proof in fact is, requires further investigation.

**1.4. Optimal Standard of Proof**

In this section I describe how the standard of proof (\(\mu\)) affects judicial errors. Then I investigate what the optimal standard of proof is and identify conditions under which it is in fact higher than that which minimizes the sum of judicial errors.
By choosing the standard of proof, the social planner may affect judicial errors, and therefore social welfare. To capture this idea, I will assume that \( q_a \) and \( q_c \) are functions of the standard of proof and this will be reflected by denoting these errors as \( q_a(\mu) \) and \( q_c(\mu) \).\(^{12}\) The standard of proof is represented by \( \mu \), which lies in the interval \([0, 1] \). \( \mu = 0 \) implies that the standard of proof is very weak (every accused is convicted), and \( \mu = 1 \) implies that the standard of proof is very high (no one can be proven guilty). Accordingly, \( q_a(0) = q_c(1) = 0 \) and \( q_a(1) = q_c(0) = 1 \). An increase in the standard of proof implies that it is harder to convict a suspect. Therefore, it will be assumed that \( q'_a(\mu) > 0 \) and \( q'_c(\mu) < 0 \) for all \( \mu \). Moreover, it will be assumed that \( 1 - q(\mu) = 1 - [q_a(\mu) + q_c(\mu)] \) is single peaked in \( \mu \), and its unique maximizer will be denoted as \( \mu' \). These general properties are sufficient to analyze the optimal standard of proof.\(^{13}\)

Given this dependency of judicial errors on the standard of proof, it is worth noting that any standard of proof will produce a unique set of judicial errors. These in turn will generate pairs of \((v^C,b^C)\)'s as defined in (1.1.). Hence, there will be a feasible set of \((v^C,b^C)\)'s, that the social planner may achieve by choosing the standard of proof. Recall that once the pair \((v^C,b^C)\) is ascertained, areas I, II and III as represented in figure 1.1. are specified, and accordingly social welfare is determined. Therefore, the social planner's problem can be thought of as choosing a pair of \((v^C,b^C)\) among the feasible set of such pairs. Curve \( IO \) in figures 1.2. and 1.3. below represent the set of feasible \((v^C,b^C)\)'s.\(^{14}\)

---

\(^{12}\)This approach abstracts from parties’ incentives to produce evidence. In reality, the standard of proof will have an impact on parties’ efforts, which will affect judicial errors. Many articles focus on these and related issues. For a brief review of Rubinfeld and Sappington (1987), Miceli (1990), and Yilankaya (2002), see section 1.2., supra. But also see, Demougin and Fluet (2005) and (2006), Froeb and Kobayashi (1996), Sanchirico (2001), and Hay and Spier (1997).

\(^{13}\)These properties are derivable from exogenous evidence generating processes, but their inclusion does not add anything to the model.

\(^{14}\)The fact that \( IO \) is single peaked and passes through the points \( O = (0, 0) \) and \( I = (1, 0) \) is implied by the properties of \( q_a, q_c, \) and \( q \).
Points $I$ and $O$ respectively represent the pairs $(q_c(1), 1-q(1)) = (0,0)$ and $(q_c(0), 1-q(0)) = (1,0)$. Points lying in between these two extremes represent pairs obtained through interior choices of standards of proof, where a move along the curve $IO$ towards $I$ is achieved through an increase in
the standard of proof. Another important point on $IO$ is denoted by $M$. This point represents the pair $(q_c(\mu'), 1 - q(\mu'))$, where $\mu'$ is that standard which minimizes the sum of judicial errors.

I briefly described the relevance and properties of curve $IO$, and now it can be used to solve the social planner’s problem. However, the best approach to solve this problem will depend on the harm associated with crime. In this section, for analytic and expositional convenience, I will focus on crimes causing a specific range of harms, which I call intermediate (i.e. $1 - q(\mu') < h < 1 - q_a(\mu')$). I discuss the cases of low and high harms in the next section.

It should first be noted that increasing $\mu$ corresponds to moving from a point on $IO$ towards another one closer to $I$. And moves along $IO$ shift the locations of areas I, II and III. These shifts affect the groups of individuals who belong to each area and accordingly individuals’ contributions to social welfare. Such shifts can be analyzed under three categories: (i) Individuals moving between areas I and II, (ii) individuals moving between areas II and III, and (iii) individuals moving between areas I and III. When moves are infinitesimal, these individuals can be represented by the lines separating such areas. Next, I will demonstrate the effects of an increase in $\mu$ on social welfare due to each type of shift separately.

(i) Individuals moving between areas I and II

An increase in $\mu$ is represented by a move towards $I$. Accordingly, any such increase causes some individuals in area I to move to area II. That is to say, some individuals who were taking precautions decide to engage in valuable activity (and yet still abstain from committing crime). This results in an increase in social welfare of $v^C = q_c(\mu)$ per person moving from area I to area II. Accordingly, higher standards of proof will always increase social welfare through effects categorized under (i).\footnote{Excluding the exceptional case of $q_c(1) = 0$, in which case infinitesimally increasing $\mu$ leads to negligible category (i) effects.}

(ii) Individuals moving between areas II and III

Individuals’ direction of move among areas II and III depend on the initial standard of proof.
When $\mu < \mu'$, an increase in the standard of proof pushes the line separating areas II and III up. Accordingly, individuals move from area III to area II. In other words, some individuals who were committing crime, now decide to only engage in valuable activity. This results in a net benefit of $h - b^C = h - (1 - q(\mu)) > 0$ per person moving from area III to area II. The inequality follows from the fact that harms are intermediate. Hence, when $\mu < \mu'$ an increase in $\mu$ results in gains in social welfare due to shifts categorized under (ii).

When $\mu > \mu'$, the line separating areas II and III moves down in response to an increase in $\mu$. This generates the opposite effect of that which is observed when $\mu < \mu'$. Hence, whenever $\mu > \mu'$ an increase in $\mu$ results in social losses.

(iii) Individuals moving between areas I and III

An increase in the standard of proof always pushes the line separating areas I and III to the left. Accordingly, individuals move from area I to area III. This means that some individuals who were taking precautions decide to commit crime. This results in a net change in welfare of $b^C + v^C - h = (1 - q(\mu)) + q_c(\mu) - h$. Intermediate harms imply that this value is positive when $\mu < \mu'$. Therefore, when $\mu < \mu'$, increasing social welfare results in net social gains from effects categorized under (iii).

When $\mu > \mu'$ and the standard of proof is sufficiently increased, $b^C + v^C - h$ becomes negative. Therefore, there is a critical standard of proof, denoted $\mu''$, such that there are social loses if $\mu > \mu''$.

Table 1.1. below, summarizes these observations.

<table>
<thead>
<tr>
<th>The Effects of an increase in $\mu$ due to:</th>
<th>$\mu &lt; \mu'$</th>
<th>$\mu &gt; \mu'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Moves between areas I and II</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>(ii) Moves between areas II and III</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>(iii) Moves between areas I and III</td>
<td>+</td>
<td>+ if $\mu &lt; \mu''$</td>
</tr>
<tr>
<td></td>
<td>- if $\mu &gt; \mu''$</td>
<td></td>
</tr>
</tbody>
</table>
An implication of the above table is that the optimal standard of proof is equal to or above the level which minimizes judicial errors. Since, social welfare is increasing in $\mu$ whenever $\mu < \mu'$ it follows that higher standards of proof are required. The only remaining step to verify that the optimal standard of proof is higher than that which minimizes the sum of judicial errors is to confirm that welfare is increasing in $\mu$ also when $\mu = \mu'$. This is formally achieved by the following proposition.

**Proposition 1.2.:** The optimal standard of proof is higher than that which minimizes the sum of judicial errors.

**Proof:** See Appendix A.

Proposition 1.2. establishes the fact that asymmetric costs associated with false acquittal rates and false conviction rates can serve as a justification for high standards of proofs. However, this section focused on a special range of crimes, namely those which cause intermediate harms. The next section shows that results always extend to crimes causing low harms, and extend to crimes causing high harms under reasonable assumptions.

1.5. Remarks and Extensions

1.5.1. Low Harm Crimes

Low harm crimes can be defined as those causing harms to society such that $h < (1 - q(\mu'))$. In this case, the relation between curve $IO$ and $h$ is given by figure 1.4.
An easy observation in this case is that point A, as plotted in figure 1.4., is obtained by setting the standard of proof at some $\mu_A > \mu'$ and results in higher welfare than any other point on the AO portion of the curve. This follows, because compared to all such points, A results in lower precautionary costs. Furthermore, for all individuals with $v > q_c(\mu_A)$, point A results in efficient deterrence (only individuals with $b > h$ commit crime). Moreover, for any other $v$, any other point on the AO portion of the curve over-deters a super set of individuals being deterred by point A. Therefore, A dominates any other point along the OA portion of the curve. And since $\mu_A > \mu'$ it must be the case that the optimal standard of proof is also greater than $\mu'$. Hence, the result in the previous section immediately extends to low harm cases.

1.5.2. High Harm Crimes

High harm crimes can be defined as those causing harms to society such that $h > 1 - q_a(\mu')$

As can be inferred from the discussion in section 1.4., when crimes cause high harms, impacts on welfare due to individuals moving between areas I and III can change signs. In fact, they will always be negative for standards of proof which are sufficiently close to $\mu'$. Hence, the relevant portion of table 1.1. will become:

**Table 1.2.: Effects of a Change in $\mu$ on Social Welfare, High Harms**

<table>
<thead>
<tr>
<th>The Effects of an increase in $\mu$ on:</th>
<th>For $\mu$ close enough to $\mu'$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>with $\mu &lt; \mu'$</td>
</tr>
<tr>
<td>(i) Moves between areas I and II</td>
<td>+</td>
</tr>
<tr>
<td>(ii) Moves between areas II and III</td>
<td>+</td>
</tr>
<tr>
<td>(iii) Moves between areas I and III</td>
<td>-</td>
</tr>
</tbody>
</table>

In this case it cannot immediately be stated that the optimal standard of proof is greater than $\mu'$. However, under many circumstances this result would continue to hold. First, it should be noted that for $\mu < \mu'$, an increase in $\mu$ leads to positive returns through two channels ((i) and (ii)) which include reductions in precautionary activities and under-deterrence for individuals with $v > q_c$. Whereas an increase in $\mu$ leads to negative returns only through increased criminal activity
for individuals with $v < q_e$.

While gains from increased deterrence due to moves between areas II and III are quite large around $\mu = 0$, and are likely to offset negative returns through effects on deterrence due to moves between areas I and III, they are quite small around $\mu = \mu'$. Hence, when harms are relatively large, it must be the case that costs of precautions around $\mu'$ are sufficiently high, otherwise lower standards of proof can be optimal.

The next sub-section discusses potential reasons as to why losses due to moves between areas I and III may be relatively small or close to none, and therefore why the optimal standard of proof might be high, even when crimes result in great harms.

1.5.3. Correlated $b$ and $v$ and Bounded Benefits From Crime

As pointed out in the previous remark, when harms are high, the optimal burden of proof is dependent on distributions over costs of precautions ($v$) and benefits from crime ($b$). Throughout the main parts of this chapter, it was assumed that these distributions are independent. However, it would be intuitive if they were positively correlated. It may very well be the case that the person who enjoys higher benefits from the commission of the crime also enjoys higher benefits from engaging in the pre-requisite activity. Assuming that such correlation exists, it will likely be the case that there is a high density of "high $b$ - high $v$" individuals and "low $b$ - low $v$" individuals, whereas there would be a low density of "low $b$ -high $v$" and "high $b$ - low $v$" individuals. In this case effects stemming from moves between areas I and III will very likely be low, because they would mainly effect "high $b$ - low $v$" individuals. The proportion of such individuals would be relatively low. However, effects on precautionary activities would mainly effect "low $b$ - low $v$" individuals. The proportion of such individuals would be relatively high. Accordingly, the magnitude of the latter effect would be relatively high, and dominate the former one. This would again imply that the optimal standard of proof is higher than that which minimizes the sum of judicial errors.

A second assumption made throughout this chapter was that individuals are distributed over benefits from crime ($b$) with support $[0, \infty)$. If this assumption is relaxed, and it is assumed that
the highest benefit derivable from crime is \( \bar{b} < (1 - q(\mu')) \), then the analysis becomes very similar to that in the 'Low Harm Case' and again the result obtained in proposition 1.2. extends to high harm levels.

1.5.4. Endogenous Sanctions

In previous sections, I abstracted from the issue of the determination of sanctions. This was mainly to preserve the focus on standards of proof, but also because incorporating endogenous sanctions, under a fairly general set of assumptions, does not lead to a new result. The Beckerian maximal sanction result continues to hold. Accordingly, one may substitute the exogenously determined sanction (which was assumed to have a magnitude of 1) in the previous sections of the chapter with the maximal fine \((w)\), and repeat the entire analysis.

The sufficient condition I will rely on, in showing that maximal sanctions are optimal, is that curve \(IO\) is concave in the region \(IM\). I will now sketch a proof showing that maximal fines are optimal when sanctions are determined endogenously, by considering two separate cases: (i) low maximal sanctions, i.e. \(w \leq h/(1 - q(\mu'))\), and (ii) high maximal sanctions, i.e. \(w > h/(1 - q(\mu'))\).

(i) \(w \leq h/(1 - q(\mu'))\)

Consider any initial sanction \(s\) below the maximal one. This will generate curve \(S\) representing the feasible set of \((v^C, b^C)\)'s as demonstrated in figure 1.5. below.
Increasing the sanction from $s$ to $z$ transforms this curve by multiplying every point on it by a scalar of $z/s$. Hence, the new curve $Z$ formed by such transformation circumscribes the initial one as represented in figure 1.5. This follows from the fact that $IO$ is concave in the region $IM$ and is decreasing in the region $MO$.

Pick any point on curve $S$, and call this point $A$. Point $A$ divides the $b - v$ space into areas I, II, and III as described in section 1.3. Since curve $Z$ circumscribes curve $S$, there is a point which is found by intersecting the line separating areas I and III formed by point $A$, and curve $Z$. Call this point $A^Z$. It is now easy to verify that point $A^Z$ dominates point $A$, if $h > z(1 - q(\mu'))$. A move from $A$ to $A^Z$ leads some individuals engaging in precautionary behavior and some individuals committing crime to switch to engaging in valuable activity only. Hence, as long as those individuals who switch from committing crime to engaging in valuable activity were inefficiently committing crime, such switch must be beneficial. But when $h > z(1 - q(\mu'))$, all individuals who quit committing crime must have had benefits such that $h > z(1 - q(\mu')) \geq b$. Hence, they were committing crime inefficiently. Next, note that $z$ can be chosen infinitesimally close to $s$, hence, as long as $h > s(1 - q(\mu'))$, one can find $z > s$, such that any point $A$ on the curve $S$ is dominated by another point $A^Z$ on curve $Z$. Therefore, any $s < \frac{h}{1 - q(\mu')}$ is dominated by a greater sanction $z$. This implies that, if $w \leq h/(1 - q(\mu'))$, maximal sanctions dominate any other sanction, and are therefore optimal.

(ii) $w > h/(1 - q(\mu'))$

On the other hand, if $w > h/(1 - q(\mu'))$ it follows from the preceding analysis that all sanctions such that $s < h/(1 - q(\mu'))$ are dominated by the sanction $h/(1 - q(\mu'))$. The next question is whether any sanction $s'$ such that $h/(1 - q(\mu')) \leq s' < w$ can be optimal. Consider the curve representing the feasible set of $(b^C, v^C)$’s formed by the imposition of any such $s'$. This curve will be like the one sketched in figure 1.4.. On the other hand, if $w > h/(1 - q(\mu'))$ it follows from the preceding analysis that all sanctions such that $s < h/(1 - q(\mu'))$ are dominated by the sanction $h/(1 - q(\mu'))$. The next question is whether any sanction $s'$ such that $h/(1 - q(\mu')) \leq s' < w$ can be optimal. Consider the curve representing the feasible set of $(b^C, v^C)$’s formed by the imposition of any such $s'$. This curve will be like the one sketched in figure 1.4.. It follows from the remarks in sub-section 1.5.1., that any point along the $OA$ portion of the curve is dominated by point $A$. But point $A$ must also be dominated by some other point along the $IA$ portion of the curve. This follows from the fact that slightly moving

\[ s' = \frac{h}{1 - q(\mu')} \]  

In this case, the analysis is very similar.
from point $A$ along the $IA$ portion of the curve results in marginal gains from decreased precaution but no marginal losses due to increased under-deterrence. Hence, all points along the $OA$ portion of the curve are dominated by some point other than $A$ along the $IA$ portion of the curve. Now pick any point on the $IA$ portion of the curve, which is not $A$. It follows from the analysis in part (i) that there exists some sanction $z' > s'$, such that the curve formed by the imposition of $z'$ contains a point which dominates point $A$. Hence, $w$ is the optimal sanction.

Parts (i) and (ii) show that when sanctions are determined endogenously, maximal sanctions are optimal. In my opinion, this is an unrealistic and disturbing result, one that many other important models in the literature suffer from. On the other hand, there is a wide literature providing many different reasons as to why this result may not hold.\footnote{See Garoupa (1997) and Polinsky and Shavell (2000). See also Chapter 5, where I provide another reason as to why maximal sanctions may not be optimal.} However, I believe that incorporating further assumptions, which would perhaps generate below maximal sanctions, would over-complicate the instant model and that the model would lose its focus on the standard of proof. The main point of this sub-section is to show that the incorporation of endogenous sanctions does not add anything to the model and does not produce any new results.

1.6. Conclusion

Existing law and economics literature usually does not provide a justification as to why asymmetric costs associated with false conviction rates and false acquittal rates are assumed. The main objective of the third section of this chapter was to provide a utilitarian justification. A model of crime and deterrence was developed where individuals could take precautionary steps to reduce their likelihood of false convictions. I showed in a very general setting that once individuals may engage in such behavior, asymmetric costs associated with the two types of judicial errors emerge.

I analyzed the implications of asymmetric costs in the fourth section. I showed that under certain circumstances the optimal standard of proof is greater than that which minimizes the sum of judicial errors. Hence, the current model provides a justification for higher standards of proofs by relying
only on utilitarian principles.

The main problem encountered in the derivation of the final result was that it does not necessarily follow from the general set of assumptions employed, when the harm associated with the crime is sufficiently high. However, I have remarked that under a fairly reasonable set of assumptions this problem is likely to vanish. If the costs of precautionary activities are positively correlated with the benefits from crime or if the benefit derivable from crime is bounded from above, the final result in the chapter also extends to cases where the harm from crime is high.

Overall, this chapter provides a justification for high standards of proof by exploiting the asymmetric cost structure associated with false acquittal and conviction rates, which are derived through utilitarian principles alone.

1.7. Appendix A

It should be noted that harms are said to be intermediate when \(1 - q(\mu') < h < 1 - q_a(\mu')\). These inequalities will be referred to a few times. Furthermore, \(F\) and \(K\) refer to the cumulative distribution functions associated with \(f\) and \(k\). It is assumed, without loss of generality, that \(\int_0^\infty bf(b)db = \int_0^\infty vk(v)dv = 1\). To save on notation, \(q(\mu), q_a(\mu)\) and \(q_c(\mu)\) will in some instances be referred to as \(q, q_a\) and \(q_c\).

Proof of Proposition 1.1:

(i) Differentiating \(\pi\) with respect to \(q_a\) and \(q_c\) establishes the fact that:

\[
\pi_{q_a}(q_a, q_c) = -\int_0^{q_c} v f(1 - q_a - v) k(v) dv < 0
\]

and

\[
\pi_{q_c}(q_a, q_c) = q_c k(q_c) F(1 - q) > 0
\]

(ii) Note that

\[
D(q_a, q_c) = \int_0^\infty \int_{\max\{(1-q),(1-q_a)-v\}}^\infty (b - h) f(b) db k(v) dv
\]

Hence \(q_a\) and \(q_c\) only affect the lower limit of the second integral. A decrease in this lower limit would imply that there is more crime and hence less deterrence. For all \(v\), a unit change in \(q_a\) will
result in a unit of reduction in $\max\{(1 - q), (1 - q_a) - v\}$. But a unit change in $q_c$ will result in a unit of reduction in $\max\{(1 - q), (1 - q_a) - v\}$ only for all $v > q_c$, whereas it will result in no reduction in $\max\{(1 - q), (1 - q_a) - v\}$ for all $v < q_c$. Hence, both types of errors decrease the level of deterrence, but an increase in the rate of false acquittals leads to a higher reduction than that due to an increase in the rate of false convictions. ■

Proof or Proposition 1.2.: By employing Leibniz Rule, one can verify that the derivative of social welfare (as described in (1.2.) and (1.3.)) with respect to $\mu$ is given by:

$$
\frac{dD(q_a(\mu), q_c(\mu))}{d\mu} - \frac{d\pi(q_a(\mu), q_c(\mu))}{d\mu} =
$$

$$
-q'_c(\mu)q_cF(1 - q)k(q_c) + q'(\mu)(1 - q - h)f(1 - q)(1 - K(q_c)) + q'_a(\mu)(1 - q_a - h)\int_{1-q}^{1-q_a} k(1 - q_a - b)f(b)db
$$

(A.1.)

The three terms captured by (A.1) correspond to the three effects identified in section 1.4., and in the same order. That is to say the first, second and third terms respectively correspond to effects (i), (ii) and (iii) identified in section 1.4..

The first term is positive when $\mu' \geq \mu > 0$. This follows from the fact that $q'_c(\mu)$ is always negative and $F(1 - q) > 0$ whenever $\mu \notin \{0, 1\}$. However, $F(1 - q) = 0$ when $\mu = 0$, making the first term 0.

The second term is positive when $\mu < \mu'$. This follows from the fact that harms are intermediate, making $1 - q(\mu) < h$ for all $\mu$, and the fact that $q'(\mu) < 0$ whenever $\mu < \mu'$, which follows from the properties of $q$. However, when $\mu = \mu'$, by definition $q'(\mu') = 0$, making the second term 0.

The third term is positive when $\mu' \geq \mu$. To see this, note that intermediate harms imply $0 < 1 - q_a(\mu') - h$. But $q'_a(\mu) > 0$ for all $\mu$, hence $0 < 1 - q_a(\mu) - h$ for all $\mu \leq \mu'$.
These observations can be summarized by Table A.1.:

<table>
<thead>
<tr>
<th></th>
<th>First Term</th>
<th>Second Term</th>
<th>Third Term</th>
<th>d_\mu W</th>
</tr>
</thead>
<tbody>
<tr>
<td>\mu = 0</td>
<td>= 0</td>
<td>&gt; 0</td>
<td>&gt; 0</td>
<td>&gt; 0</td>
</tr>
<tr>
<td>\mu' &gt; \mu &gt; 0</td>
<td>&gt; 0</td>
<td>&gt; 0</td>
<td>&gt; 0</td>
<td>&gt; 0</td>
</tr>
<tr>
<td>\mu = \mu'</td>
<td>&gt; 0</td>
<td>= 0</td>
<td>&gt; 0</td>
<td>&gt; 0</td>
</tr>
</tbody>
</table>

Table A.1. implies that social welfare is always increasing in \mu when \mu \leq \mu'. Hence, the optimal standard of proof is not in the interval [0, \mu']. And since W is continuous in \mu it follows that it has at least one maximizer in (\mu', 1]. Therefore, the optimal standard(s) of proof is (are) greater than that which minimizes the sum of judicial errors (\mu'). □
Chapter 2

Optimal Warning Strategies in Law Enforcement

2.1. Introduction

Law enforcers often issue warnings to first time offenders. However, leaving aside a few exceptions, the existing law and economics literature dealing with optimal penalty schemes for repeat offenders suggests that issuing warnings is a sub-optimal practice.\textsuperscript{18} Furthermore, to the best of my knowledge, the existing literature has not yet touched upon the optimality of employing mixed warning strategies.

In reality, warnings and in particular mixed warning strategies are commonly employed. That is to say, person A may receive a sanction for a particular offense, where person B is only warned. This situation can be explained through the use of mixed warning strategies. If law enforcers issue warnings to first time offenders only 50\% of the time, then it is only natural that some first time offenders are warned where others are sanctioned.\textsuperscript{19} This chapter provides a rationale as to why it may be in fact optimal to make use of warnings and in particular mixed warnings.

Warnings in general, are meant to give notice to or caution individuals who are presumed to lack information regarding a certain issue.\textsuperscript{20} It is most likely the case that warnings were initially meant to fulfill the same function in law enforcement. This chapter incorporates the informative function of warnings by introducing the assumption that there may be a fraction of society which is


\textsuperscript{19}Police officers may use discretion in issuing warnings. Sub-section 2.4.3 contains a few comments regarding the use of discretion, but the theoretical part of this paper abstracts from the issue and shows that mixed warnings can be optimal even at the absence of discretion. For an economic analysis and discussion of the determination of optimal discretion, see Shavell (2007).

\textsuperscript{20}Black’s Law Dictionary’s (8th ed. 2004) definition of warning: ‘The pointing out of a danger, esp. to one who would not otherwise be aware of it.’
uninformed of the illegality of a certain act.

A widely employed assumption in the law and economics literature is that there are certain costs associated with the punishment of the innocent.\textsuperscript{21} When uninformed individuals are present and the punishment of the innocent is assumed to be costly, issuing warnings can be an optimal practice. Issuing actual fines for first time offenders has two primary functions. (i) They deter informed first time offenders, and (ii) inform a fraction of uninformed first time offenders. However, this comes at the cost of punishing uninformed and therefore innocent individuals, who have unknowingly broken the law. On the other hand, warnings only have the function of informing a fraction of uninformed first time offenders, lack the deterrent function of actual fines, but do not generate costs associated with the punishment of the innocent. Hence, if the cost of punishing innocent individuals outweighs the benefits from increased deterrence of informed first time offenders, then issuing warnings is an optimal practice. Moreover, in certain cases, it may be possible to employ mixed warning strategies to balance the expected costs associated with the punishment of uninformed individuals and the costs associated with under-deterrence. Hence, incorporating informational problems and costs of punishing innocent individuals leads to a justification for the use of warnings and in particular mixed warning strategies.

Results obtained in this chapter are relevant for designing guidelines as to when and how warnings should be used. Also, the optimal use of discretion is strongly related to the use of warnings. In particular, the possibility of mitigating losses due to the punishment of innocent individuals, without resorting to discretion, leads one to believe that the amount of discretion given to law enforcers should be reduced. Furthermore, the framework provided in this chapter allows a discussion as to how warning strategies should evolve as a function of how old a law is. Presumably, the older a law is, the more knowledge there is concerning that law. Hence, as a law gets older, the necessity of using warnings in its enforcement is reduced. A last and rather trivial implication that follows

\textsuperscript{21}For instance, see Stigler (1970), Posner (1973), Png (1986), Miceli (1990), Chu et al. (2000), and Lando (2009). Also, chapter 1 can be interpreted as providing a utilitarian justification for the assumption that there are costs associated with the punishment of the innocent.
from this chapter is the inefficiency of *ex post facto* laws. By definition, no individual has knowledge concerning an *ex post facto* law at the time the offense takes place. Accordingly, there are losses associated with the punishment of uninformed individuals, but no gains associated with increased deterrence. Therefore, *ex post facto* laws are inefficient. These issues are discussed in further detail in section 2.4.

This chapter also contributes to the interpretation of the ”puzzle” concerning escalating punishments for repeat offenders. Repeat offenders are by definition informed individuals, since they were previously detected committing the illegal act and have been informed of its illegality. Accordingly, the punishment for repeat offenders does not generate costs associated with the punishment of innocent individuals. Therefore, they can be set to induce efficient deterrence for repeat offenders. On the other hand, the punishment for first time offenders must be chosen to serve two objectives: (i) deterrence and (ii) reducing costs associated with the punishment of innocent individuals. Hence, the optimal expected punishment for first time offenders must be chosen to balance costs associated with under-deterrence and the punishment of innocent individuals. These two observations imply that increasing *expected sanctions* for repeat offenders are optimal. The optimality of increasing sanctions, on the other hand, is dependent on the proportion of informed individuals in society and the harm associated with the offense. In particular, as shown in section 2.3., non-escalating punishments can be optimal only if the use of mixed warning strategies is socially desirable. An immediate extension of this result is that when mixed warning strategies are not possible (as is assumed in the existing literature so far), escalating punishments for repeat offenders are always optimal. These results are formalized in section 2.3., and discussed in section 2.4.

2.1.1. Brief Literature Review

There is a broad literature dealing with optimal punishment schemes for repeat offenders.\(^\text{23}\) Dana (2001) and Emons (2003) refer to the problem of punishing repeat offenders as a puzzle. This is because escalating punishments are observed frequently, but there are many models implying that this is a sub-optimal practice. See Chapter 3 for a more detailed discussion of this puzzle.

\(^\text{22}\) Dana (2001) and Emons (2003) refer to the problem of punishing repeat offenders as a puzzle. This is because escalating punishments are observed frequently, but there are many models implying that this is a sub-optimal practice.

\(^\text{23}\) See note 18, *supra*.
However, most of them ignore costs associated with the punishment of the innocent and accordingly do not generate results which provide a justification for the use of warnings in law enforcement. On the other hand, there are articles which suggest that it may be optimal to not sanction first time offenders. However, not punishing offenders is not the same thing as warning them. Warnings have an informative function, which existing literature has not yet focused on. Accordingly, to the best of my knowledge, the instant chapter is the first attempt in incorporating this informative function, and the first attempt to justify the use of mixed warning strategies.

To exemplify how the instant chapter builds on the existing literature Emons (2007) and Chu et al. (2000) should be briefly reviewed. The former because it is an example of a model which suggests that first time offenders should not be punished, the latter because it incorporates costs associated with the punishment of innocent individuals.

Emons (2007) concludes that, under certain circumstances, first time offenders should not be sanctioned and only repeat offenders should be punished. However, this model does not attach an informative function to warnings. The non-punishment of first time offenders in this model is due to reasons associated with wealth constraints. Furthermore, this conclusion requires that individuals must be constrained to choose between either always committing crime or never committing crime.

In Chu et al. (2000) the authors incorporate costs associated with the punishment of innocent individuals, however their result is not concerned with warnings. They focus on whether increasing penalty schemes are better than uniform ones. Furthermore, since their article does not focus on warnings, they do not model the informative function of warnings as the instant chapter does. It is perhaps also worth mentioning that neither of these papers attempt to model mixed warning strategies.

24 See Harrington (1988) which deviates from the main framework for analyzing optimal punishments for repeat offenders. An implication of this paper is that warnings may be optimal. However, this result is not due to the informational function of warnings, but due to the design of a mechanism which allows offenders to move from one group to another where the regulator determines the monitoring rates of each group. Accordingly, Harrington (1988) does not highlight the informational functions of warnings as the instant paper does.

25 In other words, individuals’ strategies are constrained to be history independent.
strategies.

2.1.2. Brief Summary

In sum, the present chapter is an attempt to provide a simple framework which allows the incorporation of the informative function of warnings and mixed warning strategies. Within this framework, this chapter captures the trade-off between costs of punishing innocent individuals and reduced levels of deterrence, and thereby identifies optimal warning strategies. This approach generates policy implications concerning how warnings should be issued in general, how warning strategies should evolve over time and how much discretion should be given to law enforcers. A second contribution of this chapter is that it adds to the existing literature on optimal punishments for repeat offenders by identifying conditions under which escalating punishments are optimal.

The next section provides a formal model. Section 2.3. is devoted to the analysis of optimal warning strategies and the optimality of escalating punishments for repeat offenders. Results and policy implications are discussed in section 2.4.. Section 2.5. discusses a few assumptions. Appendix B contains proofs for a lemma and various propositions extended in the modelling section.

2.2. The Model

Society consists of two types of individuals: Informed, and uninformed. Informed individuals are aware that a certain act is illegal and know the expected sanction associated with the commission of that act, whereas uninformed individuals are unaware of the illegality of the same act. \( \alpha \) denotes the proportion of uninformed individuals in society. Individuals, regardless of their types, derive benefits (\( b \) distributed with density \( f(b) \)) from the commission of this act, which causes an expected harm of \( h \) to society. There are two periods. The government possesses a detection mechanism, which catches offenders with a certain probability. This probability (\( p \)) is assumed to be fixed and interior.\(^{26}\) A policy variable chosen by the government is \( q \), the mixed or pure warning strategy

\(^{26}\)See chapter 3, section 2, where I argue for the plausibility of this assumption: "This is a commonly employed assumption in the literature. Polinsky and Rubinfeld (1991), Burnovski and Safra (1994), Chu et. al. (2000), Nyborg and Telle (2004) and Miceli and Bucci (2005) are examples of models which impose this assumption. Furthermore, when general enforcement is possible, for low levels of harm, \( p \) can be treated as a fixed value although it is endogenously
employed by law enforcers. When \( q = 0 \) [\( q = 1 \)], all first time offenders are warned [sanctioned].

\( s_1 \) and \( s_2 \) respectively denote the endogenously determined sanction imposed on first time offenders who are not warned and repeat offenders. A repeat offender is a person who is caught committing a crime in the second period subsequent to being warned or sanctioned in the first period.

Uninformed individuals are unaware of the illegality of the act and accordingly they do not expect to be sanctioned. Since they derive benefits from the commission of the act, they commit it. However, once they are warned or sanctioned, they are informed of the law and accordingly the illegality of the act.

I assume costs associated with the punishment of the innocent. Since only uninformed individuals are incapable of forming the necessary criminal intent, only they are presumed to be innocent. Accordingly, innocent individuals can only be subject to a sanction for first time offenders, because if they have a criminal record, they must have been informed through the imposition of a fine or warning for their first offense. Let \( \theta \) denote the number of people being punished despite being innocent, and let \( \Psi(s_1) \) denote the cost associated with the punishment of an innocent individual. Using this notation, total costs associated with the punishment of innocent individuals is given by \( \theta \Psi(s_1) \). In general, \( \Psi(0) \geq 0 \) reflecting the fact that individuals may be exposed to fixed costs upon being punished if they are in fact non-culpable. In the modelling part of the chapter I consider the more specific case of \( \Psi(0) > 0 \) and consider the implications of allowing for \( \Psi(0) = 0 \) in the discussion section.

Other assumptions employed in the model can be listed as follows: Individuals are risk-neutral expected utility maximizers with non-discounted additive utility over time. Collection of fines is costless. The following list provides the notation.

### 2.2.1. Notation

- \( h > 0 \); harm generated by the act.
- \( b > 0 \); benefit received from the commission of the act.

---

\( q = 0 \) [\( q = 1 \)], all first time offenders are warned [sanctioned]. The last point is formalized in Shavell (1991) ...”
$f(b)$; density function describing distribution of benefits among individuals. $f(b)$ is positive over $[0, \infty)$.

$p$; interior detection probability.

$q$ with $0 \leq q \leq 1$; mixed or pure warning strategy employed by law enforcers, where $q$ and $1 - q$ respectively denote the proportion of first time offenders being sanctioned and warned.

$s_1, s_2 \geq 0$; finite monetary fine extracted from first time offenders and repeat offenders respectively.

$\theta(q)$; endogenously determined proportion of society being punished despite being innocent.

$\Psi(s_1)$; with $\Psi(s_1) = k + C(s_1)$ is the cost of punishing a single innocent individual and $C' > 0$, $C'' \geq a$ for all $s_1$ where $k$ and $a$ are both positive constants.\footnote{In section 2.5., I demonstrate the purpose of having $C'' \geq a$. This is mainly a simplifying assumption for expositional purposes and to ease the description of proofs. A weaker condition which would grant the same simplifying properties is $C''(x) \geq \frac{a}{2}$. I also discuss the implications of having $k = 0$.}

$I(q, s_1) = \theta(q)\Psi(s_1)$; total costs associated with the punishment of innocent individuals. These will be called 'information costs'.

2.3. Analysis and Optimality Conditions

To analyze optimal policy variables, I will start by identifying informed and uninformed individuals’ responses to any given set of policies. Then, I will derive a utilitarian social welfare function, and solve the social planner’s problem given individuals’ best responses to policy variables.

2.3.1. Informed Individuals’ Decision Making Process

Informed individuals know that a certain act is a crime and it is punishable by law. As in Becker (1968), they weigh the benefits versus expected costs from committing crime and decide accordingly. When the second period is reached, an individual will only consider second period expected payoffs associated with her decisions, which is dependent on whether she enters the second period as a first time or repeat offender. An immediate observation can be made as follows:

Observation 2.1. (i) A first time offender commits crime in the second period iff $b > qps_1$.

(ii) A repeat offender commits crime iff $b > ps_2$. 
These inequalities govern the decisions of all informed individuals in the second period. Individuals foresee the way they will behave in the second period, and this behavior will depend on whether or not they are caught in the first period. Hence, their decisions in the first period can be summarized as follows:

**Lemma 2.1.** Let \( q_1 > s_2 \) (Case I), \( q_1 = s_2 \) (Case II) and \( q_1 < s_2 \) (Case III) denote all possible cases. Individuals will act in the first period under these three different cases depending on their benefits from crime as follows:

Case I: \[
\begin{align*}
\text{don’t commit illegal act} & \quad \text{if } \frac{p}{1+p}[q_1 + p s_2] \geq b \\
\text{commit} & \quad \text{otherwise}
\end{align*}
\]

Cases II and III: \[
\begin{align*}
\text{don’t commit illegal act} & \quad \text{if } q s_1 \geq b \\
\text{commit} & \quad \text{otherwise}
\end{align*}
\]

**Proof:** See Appendix B.

Lemma 2.1. and Observation 2.1. respectively describe individuals’ first and second period best responses to sanctions and warning strategies chosen by the government. These best responses can be summarized by the following:

**Observation 2.2.** Let cases I, II and III denote the same situations as they do in Lemma 2.1. Informed individuals’ behavior as a function of sanctions and warning strategies chosen by the government are given by:

Case I: \[
\begin{align*}
\text{don’t commit the act in either period} & \quad \text{if } b \in [0, \frac{p}{1+p}[q_1 + p s_2]] \\
\text{commit in 1st period and commit in 2nd period iff caught in 1st period} & \quad \text{if } b \in (\frac{p}{1+p}[q_1 + p s_2], q s_1] \\
\text{commit in both periods} & \quad \text{otherwise}
\end{align*}
\]

Case II: \[
\begin{align*}
\text{don’t commit the act in either period} & \quad \text{if } b \in [0, q s_1] \\
\text{commit in both periods} & \quad \text{otherwise}
\end{align*}
\]

\(^{28}\) I assume that indifferent individuals do not commit crime.
2.3.2. Uninformed Individuals’ Decision Making Process

Uninformed individuals, by definition, do not know that a certain act is a crime. However, they are made aware of its illegality, if they receive a warning or a fine in the first period and become informed individuals in the second period. Since these individuals receive benefits from committing the illegal act, and they do not perceive an expected cost from engaging in that activity, they commit crime in the first period. A fraction \( p \) of these individuals are caught and sanctioned or warned. These individuals become informed of the illegality of the act and therefore are deterred from committing crime for a second time if the penalty for repeat offenders is sufficiently high. Hence, these individuals’ behavior can be summarized by the following observation.

**Observation 2.3.** Uninformed individuals’ behavior as a function of sanctions and warning strategies chosen by the government are given by:

\[
\begin{cases}
\text{1st period} & \text{commit the act} \\
\text{2nd Period} & \text{commit the act if } ps_2 < b \text{ or} \\
\text{if not detected in the first period} & \text{otherwise}
\end{cases}
\]

2.3.3. Social Welfare

The utilitarian objective of the government is mitigating total net-losses arising from non-deterrence and abstaining from punishing innocent individuals. The latter objective of the government is reflected through a cost function which is increasing in the number of uninformed individuals being sanctioned and the severity of sanctions imposed upon them:

\[
I(q, s_1) = \theta(q)\Psi(s_1) = \theta(q)[k + C(s_1)]
\]

The first objective is reflected through net aggregate benefits from crime. This is described by:

\[
\int_{O_1(s_1, s_2, q)} (b - h)f(b)db + \int_{O_2(s_1, s_2, q)} (b - h)f(b)db
\]
Where $O_1$ and $O_2$ denote the set of offenders in the first and second periods, which are dependent on the policy variables chosen by the government and determined via Observations 2.2. and 2.3.

Hence, the objective of the government is to maximize:

$$\int_{O_1(s_1,s_2,q)} (b-h) f(b) db + \int_{O_2(s_1,s_2,q)} (b-h) f(b) db - \theta(q)[k + C(s_1)]$$  \hspace{1cm} (2.3.)

Note that $\theta(q)$, the number of uninformed individuals being sanctioned, is a constant multiplied by $q$. To see this observe that in the first period all uninformed individuals commit crime and $p$ of them are caught. In the second period, $(1-p)$ of them are still uninformed and commit crime, and a $p$ fraction of these individuals are also caught. However, only a $q$ proportion of these individuals are sanctioned. Hence, $q[(p \times 1) + (1-p) \times p] = q(2-p)p$ is the proportion of uninformed individuals being sanctioned, amounting to a total of $q(2-p)p\alpha$ individuals. Which implies that:

$$\theta(q) = q(2-p)p\alpha$$  \hspace{1cm} (2.4.)

Hence, (2.3.) becomes:

$$W = \int_{O_1(s_1,s_2,q)} (b-h) f(b) db + \int_{O_2(s_1,s_2,q)} (b-h) f(b) db - q(2-p)p\alpha[k + C(s_1)]$$  \hspace{1cm} (2.5.)

### 2.3.4. Optimality

Maximizing $W$ requires choosing a sanction pair $(s_1, s_2)$ and the warning strategy $q$. It should first be noted that informational costs, namely $I(q, s_1) = \theta[k + C(s_1)]$, are independent of the choice of second period sanctions. Having made this observation, one can determine the optimal sanctions ($s_1^*$ and $s_2^*$) and the optimal warning strategy ($q^*$) in two steps. Let $S$ denote the expected sanction for first time offenders ($qps_1$). One can first answer the following question: Given any targeted level of $S$, how can one minimize informational costs ($I(q, s_1)$)? The answer to this question will provide us with the policy variables $q$ and $s_1$, which minimize informational costs given any $S$. After these are determined as a function of $S$, one can go into the second step of the analysis, and find
the optimal level of expected sanctions for first time offenders \((S^*)\) and repeat offenders \((ps^*_2)\).

Once optimal expected sanctions for first time and repeat offenders are determined, one can easily determine optimal policy variables \(s^*_1\) and \(q^*\) by finding the information cost minimizing \(s_1\) and \(q\) when the targeted expected sanction is \(S^*\) by using the results obtained in step one. Following this two-step approach, the next sub-section identifies policy variables minimizing informational costs given any level of \(S\).

Information Cost Minimization

Informational costs are captured by \(\theta \Psi\), where \(\theta\) is the number of individuals being punished despite being uninformed, and \(\Psi\) is the cost of punishing a non-culpable individual. In open form, informational costs are given by:

\[
I(q,s_1) = \theta(q)\Psi(s_1) = q(2 - p)\rho_o[k + C(s_1)]
\]

(2.6.)

The objective in this sub-section is to minimize these costs, given any expected sanction for first time offenders \((S = qps_1)\). This problem can conveniently be summarized as:

\[
\min_{q,s_1} \theta(q)\Psi(s_1) \text{ such that } S = pqs_1, \ s_1 \geq 0 \text{ and } 0 \leq q \leq 1
\]

(2.7.)

The following proposition summarizes the solution to this problem.

**Proposition 2.1.** There exists \(\bar{S}\) such that (i) to achieve an expected first period sanction \(S_h \geq \bar{S}\), the information cost minimizing strategy is to impose no warnings \((q^m = 1)\) along with a sanction of \(s^m_1 = \frac{S_h}{p}\). (ii) To achieve an expected first period sanction \(S_l\) such that \(0 < S_l < \bar{S}\) the information cost minimizing strategy is to impose a mixed strategy for warnings \((0 < q^m < 1)\) along with a sanction of \(s^m_1 = \frac{S_l}{pq^m}\). (iii) To achieve an expected sanction of zero, the information cost minimizing strategy is pure warnings \((q^m = 0)\), in which case sanctions for first time offenders is irrelevant.

**Proof:** See Appendix B.

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\(^{29}\)An important convenience of this approach is that it allows the analysis to be easily extended to cases where the informational cost structure is altered into one which always generates pure warning or no-warning strategies.
Proposition 2.1. states that if the desired level of expected sanctions for first time offenders is high, then warnings should not be employed. On the other extreme, if the desired expected sanction is zero, first time offenders should always be warned. These results are rather intuitive. What is perhaps less intuitive is the result concerning mixed warning strategies: Mixed warning strategies are desirable if intermediate or low expected sanctions are targeted. Although these results lead one to conjecture that $q^m$ is increasing in the targeted expected sanction, they do not allow us to reach this conclusion. The next proposition verifies this conjecture, and determines the precise relation between the targeted expected sanction, and the information cost minimizing policy variables.

**Proposition 2.2.** (i) For all $S < S^*$ the information cost minimizing warning strategy ($q^m$) is linearly increasing in $S$, with $q^m = \frac{S}{S^*}$ and (ii) for all positive $S < S^*$ the information cost minimizing sanction ($s^m_1$) for first time offenders is a constant.

**Proof:** See Appendix B.

Figures 2.1. and 2.2. below summarize results obtained in Propositions 2.1. and 2.2.
Having determined information cost minimizing policy variables as a function of the targeted level of expected sanction, it is now possible to determine optimal expected sanctions and accordingly optimal policy variables.

**Optimal Policy Variables**

To determine optimal policy variables, the optimal expected sanction for first time offenders must be determined, which in turn will allow us to identify optimal warning strategies and sanctions for first time offenders. However, there is a third policy variable which must be chosen, namely $s_2$, the sanction for repeat offenders. But, since $s_2$ does not affect information costs, its determination is relatively straightforward. The next proposition identifies the optimal sanction for repeat offenders and identifies a range for optimal expected sanctions for first time offenders.\(^{30}\)

**Proposition 2.3.** The optimal sanction and warning strategy result in under-deterrence for first time offenders and first-best deterrence for repeat offenders. This is achieved by a sanction pair $(s_1^*, s_2^*)$ and warning strategy $q^*$ such that $s_1^* < \frac{h}{q^*p}$ and $s_2^* = \frac{h}{p}$.

**Proof:** See Appendix B.

Proposition 2.3. verifies a claim extended in the introduction: The expected punishment for first time offenders must be chosen to balance costs associated with under-deterrence and the punishment of innocent individuals, whereas the expected punishment for repeat offenders must be chosen to induce first-best deterrence. This follows from the fact that all repeat offenders are informed individuals, but there are some first time offenders who are uninformed. Accordingly, high expected sanctions for first time offenders generate costs associated with the punishment of innocent individuals, but the same is not true for expected sanctions for repeat offenders. Therefore, the optimal expected sanction for first time offenders must be chosen to balance costs associated with under-deterrence and the punishment of innocent individuals. On the other hand, since the only effect of punishing repeat offenders is deterrence, optimal expected sanctions for repeat offenders must be

\(^{30}\)This proposition assumes $\alpha > 0$. When $\alpha = 0$, the optimal first period sanctions and warning strategies are such that $s_1^* = \frac{h}{q^*p}$. 

36
set to achieve first-best deterrence. This immediately leads to the following corollary.

**Corollary 2.1.:** Increasing expected sanctions for repeat offenders are optimal (i.e. \( s_1^* q^* p < s_2^* p \)).

There is a broad literature on the optimal punishment for repeat offenders.\(^{31}\) The main debate is centered around whether there is an economic rationale as to why repeat offenders should be punished more severely. Corollary 2.1. points out that under the assumptions of this model, it is always optimal to subject repeat offenders to higher expected sanctions. The optimality of increasing actual sanctions (\( s_1^* \) and \( s_2^* \)), on the other hand, is dependent on what type of warning strategies are optimal and the harm associated with the offense. Therefore, the identification of conditions for optimal increasing sanctions are delayed until after the determination of optimal warning strategies.

To proceed, note that since \( s_2^* \) is pinned down (by proposition 2.3.) and the information cost minimization problem is solved (by propositions 2.1. and 2.2.), one can easily express the maximum value for social welfare as a function only of the expected sanction for first time offenders:

\[
V(S) = K + (2 - p)(1 - \alpha) \int_{S}^{h} (b - h)f(b)db - I^*(S)
\]

(2.8.)

Where \( I^*(S) \) denotes informational costs evaluated when \( q \) and \( s_1 \) are chosen to minimize such costs and \( K \) is simply a constant term capturing benefits/losses due to crime committed by uninformed individuals and repeat offenders.\(^{32}\) In the remaining parts of the chapter, it will be assumed that \( V \) is concave in \( S \). Now all that remains to be done is to maximize \( V \) with respect to expected sanctions for first-time offenders. Once this is done, optimal warning strategies can be determined by making use of propositions 2.1. and 2.2.. The next proposition summarizes results obtained once these steps are followed.\(^{33}\)

\(^{31}\)See note 18, supra.

\(^{32}\)For the explicit expression for \( K \) and a brief explanation of the derivation of \( V(S) \) see the proof of Proposition 2.3. in Appendix B.

\(^{33}\)This proposition reports results when \( \alpha > 0 \). When \( \alpha = 0 \) any warning strategy accompanied by proper sanctions for first time offenders is optimal. However, for expositional convenience, critical values \( h' \) and \( h'' \) in Proposition 2.4. and Figure 2.3., are reported as functions with domain \([0, 1]\).
Proposition 2.4.: For all $\alpha$, there are harm levels $h'(\alpha)$ and $h''(\alpha)$ such that, (i) for crimes resulting in harm $h \leq h'$ the optimal warning strategy is pure warnings ($q^* = 0$), (ii) for crimes resulting in harm $h$ with $h' < h < h''$ the optimal warning strategy is mixed warnings ($0 < q^* < 1$), where $q^*$ is increasing in $h$, and (iii) for crimes resulting in harm $h \geq h''$ the optimal warning strategy is no-warning ($q^* = 1$). Furthermore, $h'(\alpha)$ and $h''(\alpha)$ are both convex and increasing in $\alpha$, approach infinity as $\alpha$ approaches 1, and $h'(0) = 0$ and $h''(0) = s$.

Proof: See Appendix B.
This proposition describes what the optimal warning strategy is as a function of the proportion of uninformed individuals and the harm associated with crime. By using this proposition, one can plot the optimal warning strategies in $h - \alpha$ space, as is done in figure 2.3. above. The full interpretation of proposition 2.4. and figure 2.3. is provided in the next section. However, an interesting implication of propositions 2.3. and 2.4., which is related to the optimal punishment of repeat offenders can be briefly summarized by the following result.

**Proposition 2.5.**: Escalating penalties for repeat offenders are optimal (i.e. $s_2^* > s_1^*$), unless the harm associated with the offense is sufficiently high (i.e. $h > S$) and mixed warning strategies are optimal ($0 < q^* < 1$).

**Proof**: See Appendix B.

Proposition 2.5. contributes to the interpretation of the escalating punishments puzzle, by identifying conditions (highlighted in figure 2.3.), under which this practice is optimal. It states that escalating punishments are usually optimal, and that a necessary condition for their non-optimality is the possibility of imposing mixed warning strategies. This leads to the conjecture that in standard settings, where warnings are not possible, increasing punishments for repeat offenders are always optimal. In other words, if the current model was constrained such that $q = 1$ always holds, escalating punishments would always be optimal. This conjecture is verified by the following corollary.

**Corollary 2.2.**: In a setting where warnings are not possible, escalating punishments are always optimal.

**Proof**: Follows from the proof of proposition 2.3..

As explained in the preceding parts, this chapter introduces two new assumptions: the possibility of using mixed warning strategies and costs associated with the punishment of innocent individuals. Corollary 2.2. examines optimal punishment schemes when the former assumption is withdrawn from the model, and thereby identifies the net effects of the latter assumption on optimal punishment schemes for repeatable offenses. Corollary 2.2. is best interpreted as verifying the intuitive conjecture.

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34See note 22, supra.
that costs associated with the punishment of uninformed individuals provides a justification for the
use of escalating penalties.

2.4. Policy Implications

In section 2.3., I derived optimal warning strategies and conditions for optimal escalating punishment
schemes. This section is devoted to interpreting the implications of those optimality conditions.

Figure 2.3. illustrates most of the main findings concerning optimal warning strategies. Before pro-
ceeding, a couple of general and technical properties of figure 2.3. are worth highlighting, because
they provide nice shortcuts when interpreting results.

The first observation is rather trivial: high harm - high information offenses are best addressed
via a no-warning strategy, and low harm - low information offenses are best addressed via a pure
warning strategy. The next observation highlights a rather non-trivial asymmetry between harm
and information levels. For any information level, there are corresponding harm levels which make
all three (pure, mixed an no-warning) types of warning strategies optimal. However, the same is
not true for harm levels. If the harm is sufficiently low, (i.e. \( h < \bar{s} \)), then no-warning strategies can
never be optimal (regardless of the level of information).

Next, I will proceed by considering whether results are consistent with warning strategies em-
ployed in reality. Then, I will consider policy implications concerning how warning strategies shall
evolve over time and how the availability of mixed warning strategies affects the way we view dis-
cretion in the context of law enforcement. Finally, I will interpret results concerning the optimality
of escalating punishment schemes, and describe how my findings relate to the existing literature on
the punishment of repeat offenders.

2.4.1. Consistency of Results

Figure 2.3. describes optimal warning strategies as a function of the proportion of informed
individuals and the level of harm associated with the offense. One can assign certain offenses on
this graph to see whether results obtained are consistent with warning strategies employed by law
enforcers in reality.
Assigning extreme offenses, such as murder, on the graph is relatively easy. Almost all individuals are informed of the illegality of the crime and harms associated with the crime are very high. Hence, murder would go on the northwest portion of the graph. This suggests that the optimal warning strategy in the enforcement of such crimes is no-warning. This is consistent with the warning strategies employed in reality.

On the other hand, making an illegal U-turn or illegally switching lanes are associated with lower levels of expected harm. Assuming that expected harms from these offenses fall below the threshold level of $S$ is sufficient to suggest that some type of warnings (pure or mixed) should be employed for these offenses. Furthermore, in certain cases (e.g. when signs are not posted properly or the lanes are not drawn properly) the proportion of individuals who are informed of the law may also be quite low. This would imply that these offenses would go into the southeast corner of the graph, which would suggest pure warnings. Although I am unable to cite empirical data concerning the frequency with which warnings are issued in such cases, personal experience leads me to believe that something quite close to pure warnings are being employed in many places.

For intermediate offenses, such as speeding and reckless driving, the optimal warning strategy may depend on the severity of the offenses. For instance, driving 100 m.p.h. in a residential area is presumably associated with high expected harms and most individuals are likely to guess that such an act is illegal. In this case, it will be optimal to employ a no-warning strategy. On the other hand, on a country road on the border of two states individuals may lack information about speed limits and may have different presumptions. In this case, if the harm associated with this act is sufficiently small, mixed strategies might be optimal. In reality, it is not unheard of that police officers issue warnings upon catching first time offenders who are speeding. However, presumably, an officer will not let a person doing 100 m.p.h. in a residential area walk away with a warning.

**Inefficiency of Ex Post Facto Laws**

A separate and rather trivial implication of the model is that *ex post facto* laws should never be passed. The proportion of individuals, who have knowledge of an *ex post facto* law, at the time of...
the commission of the offense, is by definition 0. This implies that \( \alpha = 1 \), which suggests that no individual should be sanctioned. This is a rather trivial implication of the model. However, it is one worth pointing out to show that the results are consistent with our intuitions.

Overall, I believe that results presented in this chapter are quite consistent with warning strategies employed in reality. If, however, there are offenses which are outliers, it may be an interesting task to see if these offenses have some rare properties which are not captured in this chapter. One can then study these rare properties which may allow us to make observations concerning their impact on social welfare.

2.4.2. Policy Implications in a Dynamic Setting

The model was constructed in a static setting. Accordingly, it abstracted from issues such as the level of information varying across time. Nevertheless, results obtained through this static model allow us to draw inferences regarding the effects of various policies in a dynamic setting.

Presumably the level of information in society will be low for laws which have been recently passed. Accordingly, one can think of new laws as being associated with a high level of \( \alpha \). As time goes by, and the law gets older, a higher proportion of society familiarizes itself with the law due to information dissemination. Hence, the proportion of uninformed individuals \( (\alpha) \) decreases as the law ages. This observation leads to the conjecture that newly introduced laws should make use of warnings often, in other words, they ought to be enforced via low-\( q \) warning strategies. And as time goes by, and \( \alpha \) decreases, warnings should be relied on less heavily. That is to say, law enforcers should move from low-\( q \) warning strategies to high-\( q \) warning strategies, as the law gets older. This seems like a rather intuitive result.

There can, however, be exceptions. Consider the case where a technological advance makes it possible for individuals to engage in highly harmful conduct, which is not yet illegalized (in this regard, one can consider the use of internet for copyright infringement purposes). The policy recommendation of the instant model would be passing a law illegalizing such conduct and enforcing it without resorting to warnings at all or using high-\( q \) warning strategies (in this regard, one can
consider the *Digital Millennium Copyright Act* being implemented strictly).

Accordingly, the instant model can be extended to a dynamic framework to identify how warning strategies should evolve over time, to accommodate changes in the level of information in society.

### 2.4.3. Discretion

In reality police officers often use discretion in issuing warnings as opposed to sanctions. There may be problems associated with giving police officers discretion, such as increased opportunities for framing and bribery. On the other hand, presumably when police officers are acting honestly, on average they make correct rather than wrong guesses in determining whether or not an offender was informed of the law. Hence, there are costs and benefits associated with the use of discretion by police officers, and without further investigation it is not clear whether police discretion should be allowed.\(^{35}\)

This chapter derived optimal warning strategies, at the absence of discretionary behavior. But the implications of the model can aid in determining optimal discretion policies in two ways. First, implementing mixed warning strategies as provided in this model, without resorting to discretion, increases the benefits associated with a no-discretion regime. In a regular law enforcement framework, one would consider the non-discretionary framework as consisting of no-warning regimes. However, mixed warning-strategies as provided in this model, can be implemented without resorting to discretion.\(^{36}\) Accordingly, when comparing the costs and benefits of discretionary versus non-discretionary regimes, the implications of this model would suggest the use of non-discretionary regimes more often than otherwise. Second, it is plausible to think that the implications of this model extend to cases where police officers can use discretion. When a police officer is acting dishonestly the optimal warning strategies derived in this model will not affect him. When, however, he is acting honestly there will be cases in which he is not confident about his guess regarding the innocence of an individual. In these cases, discretionary behavior can be supplemented by the use of warnings as suggested in this chapter. Accordingly, police officers can be instructed to issue warnings a certain proportion

\(^{35}\)See Shavell (2007), for an extensive analysis as to when and how discretion should be allowed.

\(^{36}\)For instance by requiring police officers to issue warnings \(x\)% of the time and in a particular order.
of the time, when they are not confident about their assessments concerning the suspects’ guilt.

2.4.4. Escalating Punishment Schemes

Determining how repeat offenders ought to be punished has been regarded as a puzzling task in the existing literature.\(^{37}\) This is mainly because many models\(^{38}\) imply results, which are contrary to our intuition that repeat offenders ought to be punished more severely. Although the instant chapter was mainly concerned with optimal warning strategies in law enforcement, it also contributes to the literature on the optimal punishment for repeat offenders in several ways.

In general, my results overlap with the intuitive conjecture that repeat offenders ought to be punished more severely. These results were previously summarized by corollaries 2.1., 2.2. and proposition 2.5., and were briefly interpreted in section 2.3.. The driving assumption behind these results is that there are uninformed individuals and their punishment is costly. Repeat offenders are by definition informed individuals, since they were previously detected committing the illegal act and have been informed of its illegality. On the other hand, a proportion of first time offenders are uninformed individuals. Hence, repeat offenders can be subjected to sanctions which result in first-best deterrence. On the contrary, such punishments would be undesirably high for first time offenders due to costs associated with the punishment of uninformed individuals. Therefore, under many conditions, escalating punishments are optimal.

This observation demonstrates a reason as to why we are confronted with a puzzle in the repeat offender literature. We are abstracting from issues related to lack of knowledge concerning laws and the existence of mistakes. This leads to a second implication, which is related to the optimal increase in the punishment of repeat offenders. If lack of information is (at least partially) responsible for escalating punishments for repeat offenders, then the increase in punishment should be related to how little information there is concerning a law. In particular, \textit{ceteris paribus}, the more uninformed individuals in society, the higher should be the increase in the punishment for repeat offenders.

In sum, incorporating informational problems in repeat offender models will most likely produce

\(^{37}\)See note 22, \textit{supra}.

\(^{38}\)Emons (2003), (2004) and (2007) are examples for such models.
results which are consistent with our intuition that repeat offenders should be punished more severely. Furthermore, such inclusion is likely to provide general guidelines as to how punishments for repeat offenders should be increased.

2.5. Assumptions Used

In this model, $I(q, s_1)$ denotes informational costs. A few functional assumptions have been imposed on informational costs. To be specific, it was assumed that $I(q, s_1) = \theta(q)[k+C(s_1)]$. $\theta(q)$ is the number of individuals being punished despite being uninformed. $k > 0$ is a constant reflecting the fact that there are fixed costs associated with the punishment of uninformed individuals. And, $C(s_1)$ is the second component reflecting the fact that costs are increasing with the severity of punishment.

More specific assumptions involved the following: $C$ is convex and its second derivative is bounded from below. This means that costs are always accelerating in the severity of punishment with or above a certain rate.

2.5.1. Multiplicative Structure

I believe the multiplicative nature of this cost function is a product of utilitarianism. $I = \theta[k+C]$ simply states that costs are equal to the sum of per individual costs times the number of individuals incurring such costs. Accordingly, I believe that this particular form is justifiable on utilitarian grounds. The more problematic assumptions are associated with the specific functional form of $C$ and there being fixed costs associated with the sanctioning of innocent individuals.

2.5.2. $k > 0$

First, I would like to address the assumption that $k > 0$, which I believe is less problematic. This assumption simply states that, once individuals are punished despite being innocent, costs are incurred, which are independent of the severity of the punishment. This can be related to the creation of distrust among individuals towards the legal system and its enforcement or simply a feeling of being wronged which may trigger an adverse feeling towards law enforcement in general.

It may be argued that in the special case where $s_1 = 0$ and $q > 0$, the functional form makes no sense. In this case, some individuals are not being warned, but are not being sanctioned either.
(since \( s_1 = 0 \)). I will not discuss whether \( \theta \)-sanctions can trigger a feeling of being wronged or whether they would create distrust. I will rather point out the fact that in this model this case is never observed in equilibrium, hence the question of whether there should be costs in this special case is moot. An alternative reaction might be to alter the structure of informational costs so that \( I(q, s_1) = 0 \) whenever \( q = 0 \) or \( s_1 = 0 \). This would not alter any of the results.

Having identified the reasons as to why I think this assumption is justifiable, I would like to comment on what the model implies when \( k = 0 \). When \( k = 0 \), information costs are minimized whenever \( q = 1 \), because informational costs are always decreasing in \( q \) (see proof of proposition 1 and set \( k = 0 \)). Hence, no-warning would always be optimal. From a positive standpoint, the inconsistency of this result with what is observed in reality is perhaps another reason as to why \( k > 0 \) is a plausible assumption.

2.5.3. \( C'' \geq a > 0 \)

This assumption basically states that costs are always accelerating in the severity of punishment with or above a certain rate. Admittedly, it is not an intuitive assumption. However, I would like to note that this is not a necessary condition. It is rather a sufficient condition which I have used to ease the exposition and derivation of results. The only purpose this assumption serves is to guarantee the existence of the critical expected sanction (\( S \)) which was referred to in a few propositions. In the model’s current form, for any \( S > \hat{S} \), variable costs \( (qC(\frac{S}{P})) \) decrease faster than fixed costs \( (kq) \) increase, in response to an increase in \( q \), regardless of the choice of \( q \). Hence, for such \( S \), it is optimal to choose \( q = 1 \), which minimizes \( C \) but maximizes \( kq \). I believe that it is plausible to assume that such a critical level exists. Otherwise, it would follow that no-warnings can never be optimal. Even in cases such as murder, where presumably almost all individuals are informed of the law.

A weaker sufficient condition would be \( C''(x) \geq \frac{a}{x} \) for some positive \( a \). However, this assumption is no more intuitive than the assumption I have used, and requires slightly more complicated notation.
2.6. Appendix B

Proof of Lemma 2.1:

One can define the second period payoffs of informed individuals who have been detected and those who have not been detected in the first period, respectively, as follows:

\[
\Pi_d^2 = \max\{b - ps_2, 0\} \quad \text{and} \quad \Pi_n^2 = \max\{b - qps_1, 0\}
\]

(H.1.)

Hence, an informed individual’s expected utility from committing crime in the first period is:

\[
U^C = [b - qps_1] + \left[p\Pi_d^2 + (1 - p) \cdot \Pi_n^2\right]
\]

(B.2.)

and an informed individual’s expected utility from not committing crime in the first period is:

\[
U^L = \Pi_n^2
\]

(B.3.)

Now let \( \Sigma = \max\{ps_2, qps_1\} \) and \( \Sigma = \min\{ps_2, qps_1\} \).

\( b > \Sigma \) implies that \( \Pi_d^2 > 0 \) and \( \Pi_n^2 > 0 \), therefore

\[
U^L = \Pi_n^2 = [b - qps_1] < [b - qps_1] + \left[p\Pi_d^2 + (1 - p) \cdot \Pi_n^2\right] = U^C \quad \text{whenever} \quad b > \Sigma
\]

(B.4.)

Hence, when \( b > \Sigma \) individuals will commit the crime in the first period.

\( b \leq \Sigma \) implies that \( \Pi_d^2 = 0 \) and \( \Pi_n^2 = 0 \) and \( [b - qps_1] \leq 0 \), therefore

\[
U^L = \Pi_n^2 = 0 \geq [b - qps_1] + \left[p\Pi_d^2 + (1 - p) \cdot \Pi_n^2\right] = U^C \quad \text{whenever} \quad b \leq \Sigma
\]

(B.5.)

Hence, when \( b \leq \Sigma \) individuals will not commit crime in the first period.

Now it is only necessary to check the behavior of informed individuals when \( s_1 \) and \( s_2 \) are chosen such that such that \( \Sigma < b \leq \Sigma \).

In Case II, \( \Sigma = \Sigma \), therefore we already have the requested result for this case, namely that individuals will commit crime only if \( b > qps_1 = ps_2 \).

In Case I, informed individuals with benefits such that \( ps_2 < b \leq qps_1 \) will commit crime in the second period only if they are caught in the first one. Hence, if the relation below holds, informed
individuals will commit crime.

\[ U^C = [b - qps] + [p(b - ps)] > 0 = U^L \] (B.6.)

Therefore, informed individuals will commit crime in the first period if

\[ b > \frac{p}{1 + p}qs + ps = b^* \] (B.7.)

Note that \( ps < b^* \leq qps \).

In Case III, individuals with benefits such that \( qps < b \leq ps \) will commit crime in the second period only if they are not caught in the first period. Hence, if the relation below holds, informed individuals will commit crime.

\[ U^C = [b - qps] + [(1 - p)(b - qps)] > b - qps = U^L \] (B.8.)

Hence, individuals commit crime in the first period if \( b > qps \). \( \blacksquare \)

Proof of Proposition 2.1:

Part (iii): \( S = 0 \) requires either \( s_1 = 0, \ q = 0 \) or both. To prove the claim it is sufficient to note that for any \( q' > 0 \) it follows that \( I(q',0) = q(2 - p)\alpha k > 0 = I(0,s') \) for all \( s' \).

Parts (i) and (ii): Consider the following constrained minimization problem:

\[ \min_{Q,s_1} I(q,s_1) \text{ such that } S = pq, \ s_1 \geq 0 \text{ and } 0 < q \leq 1 \] (B.9.)

The solution to this problem describes the optimal policy variables to achieve a given level of expected sanctions for first time offenders. An equivalent and more convenient formulation of the same problem, utilizing (2.4.), is:

\[ \min_{Q,s_1} q(2 - p)\alpha [k + C(\frac{S}{pq})] = \min_{Q,s_1} \Omega(q,S) \] (B.10.)

Differentiating \( \Omega(q,S) \) with respect to \( q \) results in the following F.O.C.:

\[ \Omega_q(q,S) = (2 - p)\alpha k + C(\frac{S}{pq}) - C'(\frac{S}{pq})\frac{s}{pq} \] (B.11.)
and the following S.O.C:

\[ \Omega_{qq}(q, S) = (2 - p)p\alpha C''(s) \frac{S^2}{pq^2 q^3} > 0 \quad (B.12. \) }

Let \( R(x) \equiv C(x) - xC'(x) \). Since \( C(0) = 0, C' > 0 \) and \( C'' > 0 \), it follows that \( R(x) < 0 \) for all \( x > 0 \) and \( R' = -xC''(x) < 0 \). Accordingly, when \( S \) is fixed at a particular value, \( R \) and \( \Omega_q(q, S) \) are increasing in \( q \). This implies that \( \Omega_q(q, S) \) is maximized with respect to \( q \) when \( q = 1 \). Hence, if for a given \( S \), \( \Omega_q(1, S) < 0 \), then \( \Omega_q(q', S) < 0 \) for all \( q' \). And if there exists \( S \) such that \( k = -R(\frac{S}{p}) \), then \( \Omega_q(q, S) < 0 \) for all \( S > S \) regardless of the level of \( q \). But such \( S \) exists since \( C''(s_1) \geq a \), where \( a \) is a positive constant. To see this note that \( R' = -xC''(x) < 0 \) for all \( x \), which implies that \( R \) is divergent. Hence, there exists \( S \) such that \( k = -R(\frac{S}{p}) \). Therefore, whenever \( S \geq S \), information costs are minimized when \( q^m = 1 \), because, per the above observation, \( \Omega_q(q, S) < 0 \) for all \( S > S \) and for all \( q \).

For any \( S' \) such that \( 0 < S' < S \), there exists a unique and positive \( q^m < 1 \) which satisfies \( k = -R(\frac{S'}{pq^m}) \). To see this first note that \( -R(\frac{S'}{p}) < -R(\frac{S}{p}) = k \). Where the inequality follows from the fact that \( R' < 0 \), and the equality follows from the definition of \( S \). Next, note that since \( R \) is divergent, there exists \( q' > 0 \) which is small enough such that \( -R(\frac{S'}{p}) > k \). Combining these two results we have that \( -R(\frac{S'}{p}) < k < -R(\frac{S}{pq^m}) \). By utilizing the intermediate value theorem, and the fact that \( R' < 0 \), it follows that there exists a unique \( q^m \) such that \( 0 < q' < q^m < 1 \) and \( k = -R(\frac{S'}{q^m}) \). Hence, for any \( S' \) such that \( 0 < S' < S \), there exists a unique and positive \( q^m < 1 \) which minimizes information costs.

**Proof of Proposition 2.2.:**

Proposition 2.1. states that for all positive \( S < S \) there is a mixed warning strategy that minimizes information costs. Hence, there is an interior solution to the minimization problem in (B.10.). Let \( t(q^m, S) = (2 - p)p\alpha[k + C(\frac{S}{pq^m}) - C'(\frac{S}{pq^m}) \frac{S}{pq^m}] \). Applying the implicit function theorem we have that:

\[ -\frac{t_S}{t_{q^m}} = \frac{q^m}{S} = \frac{dq^m}{dS} \quad (B.13. \) }
This is a simple first order differential equation, whose solution is \( q^m = cS \). Where \( c \) is the unknown slope, describing \( q^m \) as a function of \( S \). But \( c \) must be such that \( 1 = cS \), since by definition \( \Omega_q(1, S) = 0 \).

**Proof of Proposition 2.3.:**

It should first be noted that the choice of \( s_2 \) does not effect information costs. Keeping this in mind, the proof consists of four steps. (i) First, I will show that \( ps_2 = h \) gives uninformed individuals proper incentives. (ii) Next, I will show that whenever \( s_1 \) is chosen such that \( qps_1 \leq h \), it is welfare maximizing to set \( ps_2 = h \). (iii) Then I will show that any punishment scheme where \( qps_1 > h \) is sub-optimal. These observations together imply that optimal sanction pairs and warning strategies are such that \( s_1^* \leq \frac{h}{q'p} \), \( s_2^* = \frac{h}{p} \). (iv) Finally, I will show that given \( s_2^* = \frac{h}{p} \), expected sanctions for first time offenders (\( q^*ps_1^* \)) must be set to balance information costs and benefits from deterrence, which requires under-deterrence.

(i) It is trivial to show \( s_2 = \frac{h}{p} \) provides uninformed individuals with proper incentives. \( p \) proportion of uninformed individuals are either warned or sanctioned in the first period. These are transformed into informed individuals who face expected punishments of \( ps_2 \) in case they commit crime in the second period. Hence, setting \( ps_2 = h \), gives them proper incentives. \((1 - p)\) proportion of uninformed individuals remain uninformed, and the choice of \( s_2 \) does not influence their behavior.

(ii) Whenever, \( qps_1 = h \), it trivially follows that \( s_2^* = \frac{h}{p} \) results in first best deterrence, and that there is no other \( s_2 \) that achieves the same result. Whenever \( qps_1 < h \), by making use of Observation 2.2., Figures 2.4. a-d below represent the behavior of informed individuals. These figures imply the following statements:

\( a- \) any sanction scheme where \( ps_2 < qps_1 \) is dominated by the sanctions scheme with \( ps_2' = qps_1 \).

\( b- \) the sanction scheme where \( ps_2 = qps_1 \) is dominated by the sanction scheme with \( ps_2' = h \).

\( c- \) any sanction scheme where \( qps_1 < ps_2 < h \) is dominated by the sanction scheme with \( ps_2' = h \).

\( d- \) any sanction scheme where \( qps_1 < h < ps_2 \) is dominated by the sanction scheme with \( ps_2' = h \).
Figures 2.4.a-d: Efficiency of $s_2 = h/p$

(a) First Period:
- Path 1: $0 \rightarrow p, s_2 \rightarrow h \rightarrow b$
  - Don't commit crime
- Path 2: $0 \rightarrow q, s_2 \rightarrow h \rightarrow b$
  - Commit crime
- Efficiency gains from decreased under-deterrence (by a switch from $s_1$ to $s_2$)

(b) Second Period:
- Path 1: $0 \rightarrow p, s_2 \rightarrow h \rightarrow b$
  - Don't commit crime
- Path 2: $0 \rightarrow q, s_2 \rightarrow h \rightarrow b$
  - Commit crime
- Efficiency gains from decreased under-deterrence (by a switch from $s_1$ to $s_2$)

(c) First Period:
- Path 1: $0 \rightarrow q, s_2 \rightarrow h \rightarrow b$
  - Don't commit crime
- Path 2: $0 \rightarrow q, s_2 \rightarrow h \rightarrow b$
  - Commit crime
- Efficiency gains from decreased under-deterrence (by a switch from $s_1$ to $s_2$)

(d) Second Period:
- Path 1: $0 \rightarrow p, s_2 \rightarrow h \rightarrow b$
  - Don't commit crime
- Path 2: $0 \rightarrow q, s_2 \rightarrow h \rightarrow b$
  - Commit crime
- Efficiency gains from decreased over-deterrence (by a switch from $s_1$ to $s_2$)
These observations together imply that whenever $s_1$ and $q$ are chosen such that $qps_1 \leq h$, it is optimal to set $s_2 = h/p$.

(iii) It follows that any sanction pair such that $qps_1 > h$ is dominated by the sanction pair $ps_2' = qps_1'$, since the latter sanction pair results in first-best deterrence in both periods and less informational costs. Hence, any sanction pair where $qps_1 > h$ is sub-optimal.

(iv) The above observations show that the optimal sanction pair is such that $s_2 = h/p$ and $qps_1 \leq h$.

Given this observation, and the results from Propositions 2.1. and 2.2., the claim can be proven as follows:

Let

$$I^*(S) = \min_{q, s_1} \theta(q) \Psi(s_1) \text{ such that } S = qps_1$$  \hfill (B.14.)

Then employing Observation 2.2. and the fact that $ps_2 = h$, social welfare can be expressed as:

$$V(S) = (1 - \alpha) \left[ 2 \int_{h}^{\infty} (b - h)f(b)db + (2 - p) \int_{s}^{h} (b - h)f(b)db \right]$$

$$+ \alpha \left[ (2 - p) \int_{0}^{\infty} (b - h)f(b)db + p \int_{h}^{\infty} (b - h)f(b)db \right] - I^*(S)$$  \hfill (B.15.)

Differentiating $V$ with respect to $S$, we have:

$$V_S(S) = \begin{cases} -(2 - p)(1 - \alpha)(S - h)f(S) - (2 - p)\alpha \frac{1}{2} \left( k + C\left(\frac{S}{p}\right) \right) & \text{if } S \geq S \geq 0 \\ -(2 - p)(1 - \alpha)(S - h)f(S) - (2 - p)\alpha C'\left(\frac{S}{p}\right) & \text{if } S > S \end{cases}$$  \hfill (B.16.)

Note that this expression is negative when $S = h$ (regardless of whether $h > S$). Hence, it follows that the $S$ maximizing this expression is such that $S < h$. In other words, $q^*ps_1^* < h$, which was the claim to be proven.

Observations (i)-(iv) together imply that the optimal sanction pair is such that $s_1^* < \frac{h}{q^*p}$ and $s_2^* = \frac{h}{p}$.

Proof of Proposition 2.4.:

\[ \text{It should be noted that } V(S) \text{ is differentiable. To see this, evaluate both expressions for } V_S \text{ at } S \text{ and verify that they are equal by using the definition for } S. \]
(i) Since $V$ is concave in $S$, it is always decreasing if $V_S(0) \leq 0$. But

$$V_S(0) = (2 - p)(1 - \alpha)h f(0) - (2 - p)\frac{p\alpha}{k + C(\frac{S}{p})} \leq 0 \text{ iff } h \leq \frac{\alpha p}{(1 - \alpha)f(0)} \equiv h' \text{. (B.17.)}$$

Hence, for all $h \leq h'$, $S = 0$ maximizes $V$, in which case the optimal warning strategy is $q^* = 0$. (See proposition 2.1.)

(iii) Similarly,

$$V_S(S) \geq 0 \text{ if } h'' \equiv S + \frac{\alpha p}{k + C(S)} \leq h$$

Hence, for all $h \geq h''$, $S^*$, the maximizer of $V$ is such that $S^* > S$, in which case the optimal warning strategy is $q^* = 1$. (See proposition 2.1.)

(ii) When $h' < h < h''$, than it follows that $V_S(0) > 0 > V_S(S)$. Hence, $V$ has a maximizer $S^*$, such that $S > S^* > 0$, in which case the optimal warning strategy is mixed. (See proposition 2.1.)

To see that $q^*$ is increasing in $h$, note that $\frac{dq^*}{dh} = \frac{1}{S} \frac{dS^*}{dh} = -\frac{1}{S} \left(\frac{V_{kS}}{V_{SS}}\right) = -\frac{1}{S} \left(\frac{2 - p}{(1 - \alpha)f(S)}\right) > 0$ since $V$ is concave in $S$. It is easy to verify that $h'$ and $h''$ are both convex and increasing in $\alpha$, approach infinity as $\alpha$ approaches 1, and $h'(0) = 0$ and $h''(0) = S$.

**Proof of Proposition 2.5.:**

I will prove the claim by verifying the following four observations: Escalating penalties are optimal when (i) pure warning strategies are optimal ($q^* = 0$), (ii) no-warning strategies are optimal ($q^* = 1$), and (iii) when mixed strategies are optimal ($0 < q^* < 1$) and $h < S$. But (iv) decreasing penalties are optimal when $h > S$ and $0 < q^* < 1$.

(i) When pure warnings are optimal the choice of $s_1$ is irrelevant (see proposition 2.1. part (iii)), in other words any $s_1$ is optimal. Therefore, it trivially follows that there are optimal escalating sanctions.

(ii) It follows from proposition 2.3. that $s_1^* p q^* < h = s_2^* p$. But when no-warnings are optimal, it follows that $q^* = 1$, hence $s_1^* p < h = s_2^* p$, which implies that $s_1^* < s_2^*$. (ii) and (iv) When mixed warning strategies are optimal, it follows that $s_1^* = \frac{s}{p}$ (see proposition 2.2.) and that $s_2^* = \frac{h}{p}$ (see proposition 2.3.). Hence, $s_1^* \leq s_2^*$, $\text{iff } S \leq h$. ■
Chapter 3
Repeat Offenders: If They Learn, We Punish Them More Severely

3.1. Introduction and Literature Review

After Becker (1968) provided the framework to analyze crime and deterrence in the context of modern economic theory, many have engaged in the analysis of different dimensions of the problem. The existing literature on repeat offenders has generated different results, some of which directly contradict each other. The qualified nature of most results in the literature and the fact that there have been multiple answers to the question of how repeat offenders should be punished, have lead some scholars to regard this problem as a "puzzle".

This chapter contributes to the interpretation of this puzzle, by identifying a source which may produce a marginal effect in the direction of punishing repeat offenders more severely. It introduces and formalizes the idea that repeat offenders may gain experience and learn from their offenses and convictions. This may lead offenders to be detected with a lower probability in their subsequent offenses. However, as pointed out in the existing literature, it is possible that law enforcers also

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40 See Garoupa (1997) and Polinsky and Shavell (2000) for surveys of economic literature related to crime and deterrence.
41 See Emons (2003) and (2004) and Polinsky and Shavell (1998) for brief reviews on existing literature in economics dealing with the issue of repeat offenders.
43 To the best of my knowledge, Polinsky and Rubinfeld (1991) is the first article which mentions a similar idea. See Polinsky and Rubinfeld (1991) Section 4.d. Dana (2001) also mentions this possibility.
learn. Once an individual gets a first conviction, he is known to the enforcers and this usually increases his probability of detection for a later crime, since law enforcers acquire information about that individual and may simply look for the ‘usual suspects’ first. The main result of this chapter is that, if the net learning effect favors offenders rather than law enforcers, then there are optimal escalating punishment schemes.

Whether or not offenders learn more than law enforcers is an empirical question, and presumably it’s answer will depend on the type of offense. However, most likely there are offenses where the learning effect is greater on the offenders’ side. Consider the offenses of speeding, littering, riding on public transportation without a ticket or smoking in a non-smoking area. Most probably the learning effect on the law enforcers’ side is close to none, whereas repeat offenders may very well learn about the mechanism employed by the law enforcers in detecting offenders. Hence, this chapter provides a justification for the widely accepted practice of punishing repeat offenders more severely, at least for some types of offenses.

The plausibility of this learning assumption will become more clear after a brief review of the existing literature, which demonstrates the strict conditions and specialized assumptions required for escalating penalties to be optimal. Polinsky and Rubinfeld (1991), Polinsky and Shavell (1998), Chu et al. (2000), Miceli and Bucci (2005), and Emons (2007) are perhaps the most important articles which produce the result that some form of increasing penalties are optimal.

To demonstrate the restrictive conditions required for the derivation of optimal escalating penalties, quoting Miceli and Bucci (2005) is perhaps the ideal way to begin: “... this justification for escalating penalties, like earlier theories, seems to apply to a fairly restrictive set of circumstances—specifically, crimes that should definitely be deterred.” Similar problems are encountered in other important articles as well.

44 See Garoupa and Jellal (2004) and Dana (2001) for a discussion on the impacts of learning by the government which Garoupa and Jellal refer to as the “demand side”.

45 To justify escalating penalties, they also assume that within society there are irrational people who commit crime regardless of the severity of punishments.
Emons (2007) concludes that punishing repeat offenders more severely is optimal if the benefit from crime is sufficiently high. Even this qualified conclusion requires that individuals must be constrained to choose between either always intentionally committing crime or never intentionally committing crime.⁴⁶ Hence, this result is dependent on a specialized assumption, one requiring individuals to commit to a certain path of actions.⁴⁷ Chu et al. (2000) derive a limited result, namely that increasing penalty schemes are better than uniform ones. They do not compare increasing penalty schemes to decreasing ones. Furthermore, social welfare is not the sum of individuals' utilities and it is assumed that gains from crime are illicit. Polinsky and Shavell (1998) is perhaps less restrictive. However, their result is not consistent with the widely accepted legal practice of punishing repeat offenders more severely. It states that, for his second offense, a person should be punished the same way he was punished for his first offense.⁴⁸ This does not describe the widely employed legal practice of punishing repeat offenders more severely. Polinsky and Rubinfeld (1991) assume that individuals’ benefits are composed of acceptable and illicit gains. This assumptions is referred to as 'nonstandard' in the literature and is interpreted as being problematic.⁴⁹ Furthermore, their main result states that escalating penalties can be optimal only within some parametric range.⁵⁰

In contrast to the articles which I have briefly reviewed, the instant chapter does not assume (i) that individuals may have illicit benefits, (ii) that there are irrational individuals, or (iii) that individuals must commit to a certain path of actions. Furthermore, it derives the objective of the legal system by aggregating individuals’ utilities as opposed to simply assuming certain cost

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⁴⁶In other words, the individuals’ strategies are constrained to be history independent.
⁴⁷This specialized assumption is not employed in Emons (2003) or (2004), and those papers imply that escalating penalties are never optimal. This should also demonstrate that results in Emons (2007) are dependant on this specialized assumption. Another non-traditional assumption employed in Emons (2007) is that gains from crime are illicit.
⁴⁸See Proposition 1 in Polinsky and Shavell (1998).
⁵⁰It has been noted in Chu et al. (2000) Section 1.2., that this result could be more useful, if it could be shown that this particular parametric range is consistent with reality.
functions and it does not eliminate decreasing penalty schemes as available policy tools. It could therefore be stated that the only nonstandard assumption in this model is the one on learning and accordingly that the model’s deviation from the standard set of assumptions in the crime and deterrence literature is quite small. Finally, this chapter is different from Polinsky and Shavell (1998) in that it justifies the current legal practice, at least for a certain class of crimes.

It should also be noted that the learning assumption in this chapter is distinct from those introduced in Sah (1991), Ben-Shahar (1997), and At and Chappe (2008). These models assume individuals lack information about the frequency with which they will be caught if they commit an offense, whereas the instant model assumes that individuals gain experience in breaking the law, and therefore evade detection more often. Furthermore, the focus of and the results obtained in these papers are not related to optimal punishment schemes when repeated offenses are possible.

To distinguish the learning assumption employed in the instant chapter from earlier learning assumptions, consider Ben-Shahar (1997). Assume the actual probability of detection is $\frac{1}{2}$ and the actual sanction is $100, making the expected sanction $50. Some individuals have inaccurate beliefs about the expected sanction. In particular, there can be an individual, call him ‘Schopenhauer’, who thinks that the expected sanction is $150. If Schopenhauer is convicted of a crime, he ‘learns’ that the actual probability of detection is $\frac{1}{2}$ and the actual sanction is $100. In contrast, in my model, Schopenhauer knows that the actual probability of detection is $\frac{1}{2}$ and that the actual sanction is $100, from the beginning. He does not need to learn this through the commission of a crime. However, if he commits crime, his probability of detection will be different in the second period, for instance $\frac{1}{4}$. In this case, Schopenhauer has learned how to evade detection as opposed to learning about how frequently detection takes place.

In sum, this chapter formalizes a particular type of learning for the first time in the law and economics literature. It shows that this innovative assumption is sufficient to justify the practice of punishing repeat offenders more severely, when the net learning effect favors offenders. Accordingly, reliance on other types of nonstandard assumptions, which were used in previous important articles
deriving optimality conditions for escalating penalties, are not necessary.

The rest of this chapter is organized as follows: Section 3.2, provides a short description of the model, derives individuals’ best responses to sanctions and a pure utilitarian social welfare function, and states the general result, section 3.3. is devoted to remarks and conclusions. Appendix C at the end contains the proof of a proposition.

### 3.2. The Model

Society consists of individuals who are continuously distributed over benefits \( b \) from committing an act which causes an expected harm of \( h \) to society. There are two periods in which individuals decide whether or not to commit this act, which henceforth will be referred to as an offense. Individuals’ benefits are not observable by the government. The government invests in a detection mechanism, which catches offenders with a certain probability. This probability \( p \) is assumed to be fixed and is interior.\(^{51}\)

There are three types of individuals in this model: First time offenders (Type \( F \)), Offenders with a clean record (Type \( O \)), and ex-Convicts (Type \( C \)). In the first period all individuals are Type \( F \), and choose whether or not to commit the offense. Those who choose not to commit the offense, remain Type \( F \) individuals. Those who commit the offense but are not detected become Type \( O \) individuals, whereas those who are detected become Type \( C \) individuals.

Since the government may only observe the detection history of individuals, it cannot distinguish between Type \( F \) and \( O \) individuals. Therefore, Type \( F \) and \( O \) individuals will collectively be referred to as non-detected individuals (Type \( N \)). The government announces (possibly different) sanctions for ex-convicts (Type \( C \)) and individuals with a clean record (Type \( N \)).

This model allows for learning: Offenders (Types \( O \) and \( C \)) learn about the way in which the

\(^{51}\)This is a commonly employed assumption in the literature. Polinsky and Rubinfeld (1991), Burnovski and Safra (1994), Chu et. al. (2000), Nyborg and Telle (2004) and Miceli and Bucci (2005) are examples of models which impose this assumption. Furthermore, when general enforcement is possible, for low levels of harm, \( p \) can be treated as a fixed value although it is endogenously determined. The last point is formalized in Shavell (1991) and it’s implications are discussed in Section 3.3.
detection mechanism works. The model also allows for additional learning from conviction: Type C individuals may learn more than Type O individuals. Formally, let \( q_O \) and \( q'_C \) refer respectively to the probabilities with which non-detected offenders and ex-convicts are detected in the second period if they decide to commit an offense. The probability of detection for first time offenders is still \( p \). These probabilities describe learning effects and it follows that at the absence of learning on the law enforcer’s side they have the following relation: \( q'_C \leq q_O < p \). The learning effect from the commission of an offense, which occurs purely due to experience is captured by \( p - q_O = E \). The learning effect from being convicted, is captured by \( q_O - q'_C = X \).

However, law enforcers may also learn. It is assumed that law enforcers will be able to detect ex-convicts easier, because they have information about these individuals in their records. This advantage, or learning effect will be captured by the parameter \( L \), which will increase the probability of detection of ex-convicts \( (q'_C) \). Hence, the adjusted probability of detection for ex-convicts \( (q_C) \), which is the relevant one for the analysis, will be higher than \( q'_C \) (since \( q_C = q'_C + L > q'_C \)). Furthermore, it does not follow that \( q_C \leq q_O < p \). The next sections will derive optimality conditions depending on the relationship between \( L \), \( X \) and \( E \).

Other assumptions employed in the model can be listed as follows: Individuals are risk-neutral expected utility maximizers with additive utility over time and they do not have an inability to pay high fines.\(^{52}\) Sanctions and probabilities announced by the government are common knowledge and are credible. Collection of fines is costless.

The following list provides the notation, and formalizes some assumptions.

### 3.2.1. Notation

- \( b \); benefit an individual obtains from committing the offense;
- \( f(b) \); density function describing benefits among individuals, which is positive in \([0, \infty)\).
- \( h > 0 \); harm generated by the offense.

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\(^{52}\)The no-maximal fine assumption is mainly a simplifying one. Remarks concerning the no-maximal fine and fixed probability of detection assumptions are provided in section 3.3..
\( p, q_O, q_C \): probabilities that Type \( F, O \) and \( C \) individuals are detected respectively.

\( q'_C \): probability that an ex-convict would be detected if there was no learning on the law enforcer's side.

\( X, E \): magnitudes of learning from being convicted and the commission of an offense respectively.

\( L \): magnitude of learning by law enforcers.

\( s_N > 0 \): finite monetary fine extracted from non-detected individuals.

\( s_C > 0 \): finite monetary fine extracted from ex-convicts.

\((.;.;)\); defines a sanction pair, where the first and second components refer to \( s_N \) and \( s_C \) respectively.

### 3.2.2. Individuals’ Decision Making Process

In this sub-section, individuals’ best responses to sanctions chosen by the government are analyzed.

When the second period is reached, individuals need not consider anything but the second period payoffs associated with their decisions, hence an immediate observation can be made as follows:

**Observation 3.1.** (i) A type \( F \) individual, commits offense in the second period iff \( b > ps_N \). (ii) A type \( O \) individual, commits offense in the second period iff \( b > q_O s_N \). (iii) A type \( C \) individual, commits offense in the second period iff \( b > q_C s_C \).

These inequalities govern the decisions of all individuals in the second period. For \( i \in \{F, O, C\} \), let \( \Pi_{i,2} \) denote the second period expected benefit of a Type \( i \) individual in the second period.

Utilizing observation 3.1. leads to a next observation which will be useful in describing individuals’ behavior in the first period:

**Observation 3.2.** Individuals’ payoff in the second period, depending on their types will be given by: \( \Pi_{F,2} = \max\{b - ps_N, 0\} \), \( \Pi_{O,2} = \max\{b - q_O s_N, 0\} \), \( \Pi_{C,2} = \max\{b - q_C s_C, 0\} \).

Individuals foresee the way they will behave in the second period, and that this behavior will depend on whether or not they are caught in the first period. Accordingly, let \( \Pi_{c,1} \) and \( \Pi_{n,1} \) respectively denote the expected utilities from committing the offense and abstaining from it in the
first period. These are given by:

\[ \Pi_{c,1}(b) = [b - ps_N] + [p\Pi_{C,2}(b) + (1 - p)\Pi_{O,2}(b)] \] and \( \Pi_{n,1}(b) = \Pi_{F,2}(b) \) (3.1.)

An individual with benefit \( b \) will commit the offense in the first period iff \( \Pi_{c,1}(b) \geq \Pi_{n,1}(b) \). By utilizing (3.1.), this inequality becomes:

\[ [b - ps_N] + [p\Pi_{C,2}(b) + (1 - p)\Pi_{O,2}(b)] \geq \Pi_{F,2}(b) \] (3.2.)

Observation 3.1. and inequality (3.2.) describe how individuals will react to sanction pairs announced by the government. Knowing how individuals will react to these, the government may announce sanction pairs to achieve optimality.

### 3.2.3. Social Welfare and Policy Implications

A pure utilitarian social welfare function is used to evaluate the desirability of outcomes. Since individuals are assumed to be risk neutral expected utility maximizers, and since revenues and harms will be distributed back to society, the utilitarian social welfare function is simply the sum of all individuals’ benefits from violating the law minus aggregate harms:

\[ \int_{\mu_1(s_N, s_C)} (b - h)f(b)db + \int_{\mu_2(s_N, s_C)} (b - h)f(b)db \] (3.3.)

where \( \mu_1(s_N, s_C) \) and \( \mu_2(s_N, s_C) \) denote respectively the set of individuals who decide to commit the offense in the first and second periods as a function of the sanctions announced by the government.

Social welfare is obviously maximized if \( s_N \) and \( s_C \) can be chosen such that the set of individuals who commit the offense are those individuals with \( b > h \). This condition will be referred to as first-best deterrence. Proposition 3.1. identifies the necessary and sufficient conditions to achieve first-best deterrence.

**Proposition 3.1.:** First-best deterrence is achieved iff the following constraints simultaneously hold:

\[ q_Os_N \leq h \] (3.4.)

\[ q_Cs_C \leq h \] (3.5.)
Proof: Assuming the sanction pair announced by the government induces first-best deterrence, there will be individuals of Type O and C in the second period with benefits $b > h$. First-best deterrence requires that these individuals are not deterred from committing the offense in the second period, hence by Observation 3.1., $s_N$ and $s_C$ must be such that (3.4.) and (3.5.) hold. Next, observe that $p_sN \geq h$ must hold for first-best deterrence, otherwise there will be individuals with $b < h$ who commit the offense in the first period. This implies that $\Pi_n, 1(h) = 0$. Furthermore, note that $\Pi_{c, 1}$ is strictly increasing in $b$ and at least as fast as $\Pi_n, 1$. Hence, unless $\Pi_{c, 1}(h) = 0$ there will either be individuals with $b > h$ who do not commit the offense or individuals with $b < h$ who commit the offense in the first period. Given that (3.4.) and (3.5.) hold, $\Pi_{c, 1}(h) = [h - psN] + [p(h - qCsC) + (1 - p)(h - qOsN)]$. Setting this expression equal to zero, we have (3.6.). Therefore, (3.4.), (3.5.) and (3.6.) are necessary conditions for first-best deterrence. One can verify that they are also sufficient by checking how individuals behave when these constraints are satisfied.

Proposition 3.1. identifies the conditions under which first-best deterrence is possible. Depending on the learning parameters ($E$, $X$ and $L$), first-best deterrence may or may not be achievable through increasing penalty schemes. This is summarized by the following:

**Proposition 3.2.:** (i) If offenders learn more in comparison to law enforcers ($\frac{E}{(2-p)} + X > L$), then there is always an optimal increasing penalty scheme. (ii) Otherwise ($\frac{E}{(2-p)} + X \leq L$), there are no increasing penalty schemes which induce first-best deterrence.

**Proof:** See Appendix C.

Proposition 3.2. summarizes when it is optimal to punish repeat offenders more severely. The main statement of the proposition is self explanatory. Perhaps a short remark should be made to highlight a property associated with this result. The condition for optimal increasing penalty schemes, namely $\frac{E}{(2-p)} + X > L$, is an asymmetric one. This is so, because offenders have two separate ways from which they can learn to evade the detection mechanism, whereas law enforcers have two separate ways from which they can learn to evade the detection mechanism, whereas law enforcers have two separate ways from which they can learn to evade the detection mechanism, whereas law enforcers...
have a single way to gather information about individuals. Hence, even if offenders are put in a
disadvantage by being convicted (if \( L - X > 0 \)), increasing penalty schemes may still be optimal
(i.e. when \( \frac{E}{(1-p)} > L - X > 0 \)).

The next section provides a few concluding remarks concerning some assumptions employed in
the derivation of proposition 3.2. and a few remarks concerning types of crimes for which the learning
condition in proposition 3.2. is likely to be satisfied.

### 3.3. Remarks and Conclusion

I have shown that the practice of punishing repeat offenders more severely is justifiable, when
there is a learning effect on the offenders’ side which outweighs the learning effect on the law enforcer’s
side. The main idea driving this result is simple. When offenders are talented in evading the law,
they have to be punished more severely so that optimal deterrence can be achieved. Interpreting
this result, requires a closer look at the simplifying assumptions invoked.

The two main simplifying assumptions were that the probability of detection, \( p \), is fixed, and
that individuals do not have a binding wealth constraint. These two assumptions make first-best
deterrence desirable. The types of crimes for which these assumptions are less problematic can
be identified by taking a closer look at Shavell (1991). In this article, the author shows that for
crimes which do not result in great harms, general enforcement effort by law enforcers should not
be complemented by specific enforcement, and that individuals’ wealth constraints are non-binding.
Hence, for crimes which do not result in great harms, these assumptions seem to be plausible.

More importantly, the same types of crimes seem to be those in which the learning effect on the
offenders’ side are more likely to outweigh the learning effect on the law enforcers’ side. This follows
from a similar reasoning as in Shavell (1991). Although not explicitly modelled in this chapter or in
Shavell (1991), learning on the enforcers’ side is presumably costly, and therefore can be interpreted
as a special type of specific enforcement effort. If it is not worthwhile for law enforcers’ to complement
general enforcement effort by specific enforcement, it is most likely not worthwhile to complement
it by engaging in efforts which would lead to learning on the law enforcers’ side either.
This conjecture seems to be verified in reality. Offenses such as speeding, double parking, smoking in a non-smoking area, littering and riding on public transportation without a ticket, result in harms which would not justify the cost of learning on the law enforcers’ side and are usually enforced through general enforcement efforts only. This being the case, the probability of detection, \( p \), is fixed at a certain optimal level for all types of crimes which are enforced through the same general enforcement mechanism. Furthermore, since the relative level of harms for such crimes are low, wealth constraints do not become an issue. Due to these reasons, it would be safe to assert that the results obtained in this chapter provide a justification for severer punishments for repeat offenders when the crime in question results in small harms to society.

The analysis provided in this chapter may also guide us in interpreting the relative importance of learning by offenders versus law enforcers. Repeat offenders are punished more severely in most legal systems. Absent of other considerations, and assuming that this practice is in-fact an efficient one, one can conclude by using a positive approach that learning by offenders is perhaps more important than learning by law enforcers in many cases. Given that this is the case, the next question is whether different types of crimes should be treated differently in terms of increasing penalties for recidivism. Based on the analysis provided in this chapter, this question is answered positively. The increase in the punishment for a recidivist should depend on the typical learning effects associated with the crime in question. In particular, the higher the net learning effect is on the offenders’ side, the higher should be the increase in the punishment of a recidivist. Therefore, a deeper understanding and a rigorous analysis of issues related to learning are of great importance.

The ideas formalized in this chapter also have important implications concerning procedural issues in criminal law, in particular issues concerning the standard of proof. If repeat offenders become more sophisticated, through the commission of offenses, it becomes exceedingly harder to prove a non-detected offender’s guilt beyond a reasonable doubt. In this case, one might argue that the optimal standard of proof should be lower than it ordinarily would be, or that aggregating
probabilities of guilt across criminal cases should be possible. This is another reason as to why issues related to learning should be investigated with care in future research.

In sum, the contributions of this chapter can be listed as follows: First, it identifies and formalizes a source which produces a marginal effect in the direction of punishing repeat offenders more severely. Second, under specific assumptions, it justifies the tendency in legal systems to punish recidivists more severely. Overall, the findings in this chapter suggest that the traditional variables identified so far in the literature (harms, benefit distributions, costs of detection, etc.) are not the only relevant ones in deciding how repeat offenders should be punished. The degree of learning by offenders and law enforcers is also a very important factor which should influence the way punishment schemes and perhaps criminal procedures are designed.

3.4. Appendix C

Proof of Proposition 3.2.:

Part (i): Choose \( s_C = \frac{h}{q_C} \). This satisfies (3.5.). To satisfy (3.6.) simultaneously, (C.1.) below must hold.

\[
2h - s_N[(1-p)q_O + p] - hp = 0 \iff s_N = \frac{(2-p)h}{(1-p)q_O + p} \quad (C.1.)
\]

This value of \( s_N \) satisfies (3.4.). Hence, \( s^* = \left( \frac{(2-p)h}{(1-p)q_O + p} ; \frac{h}{q_C} \right) \) satisfies (3.4.), (3.5.) and (3.6.) making it an optimal sanction pair. Furthermore,

\[
s_C = \frac{h}{q_C} > \frac{(2-p)h}{(1-p)q_O + p} = s_N \text{ iff } \frac{E}{(2-p)} + X > L \quad (C.2.)
\]

This implies that \( \left( \frac{(2-p)h}{(1-p)q_O + p} ; \frac{h}{q_C} \right) \) is an increasing sanction scheme when \( \frac{E}{(2-p)} + X > L \).

Part (ii): (3.6.) implicitly defines \( s_C \) as a decreasing function in \( s_N \). Furthermore, \( s^* \) is that sanction pair with the highest \( s_C \) satisfying (3.5.). Hence, \( s^* \) as defined in Part (i) is the sanction pair with the smallest \( \frac{s_N}{q_C} \) ratio, which satisfies (3.6.) and (3.5.) together. Therefore, if \( s^* \) is not an increasing sanction pair, there are no increasing sanction pairs satisfying (3.5.) and (3.6.) together.

\textsuperscript{54}The latter point is made by Harel and Porat (2008) even at the absence of learning, but as they acknowledge, their argument seems to be stronger when offenders learn as in this paper.
But when \( \frac{E}{(2-p)} + X \leq L \), (C.2.) implies that \( s^* \) is not an increasing sanction pair and accordingly there is no increasing sanction pair which achieves first-best deterrence.
Chapter 4

Benefits of Ex-Ante Self-Reporting of Conduct Crimes

4.1. Introduction

Through criminal law legislators often declare certain conduct illegal. Individuals engaging in such conduct are sanctioned, regardless of whether or not the conduct results in harm. Hence, it is the conduct, and not the harmful result that is being punished. Very common examples include speeding and driving under the influence of alcohol. I will refer to such illegalized conduct as conduct crimes.\textsuperscript{55} What conduct crimes share in common is that they create or increase the likelihood of harm, and this is the main reason as to why they are illegalized.

The Beckerian approach to analyzing crime suggests that laws should be designed to incentivize individuals to internalize the additional expected harm they generate through their acts.\textsuperscript{56} How such internalization can be achieved in the case of conduct crimes has not been frequently investigated.\textsuperscript{57} Shavell (1990) provides the primary framework for analyzing optimal punishments for attempts and is perfectly suitable for an analysis of conduct crimes.\textsuperscript{58} In fact, Shavell’s definition of attempts is

\textsuperscript{55}See Dressler (2007 pp. 126, 145) for references to the ’result crime’ vs. ’conduct crime’ dichotomy.

\textsuperscript{56}See Becker (1968). See also Garoupa (1997) and Polinsky and Shavell (2000) for surveys of existing literature on the economic analysis of law enforcement.

\textsuperscript{57}For existing literature employing economic analysis on a closely related topic, namely the punishments of attempts, see Shavell (1990) and Friedman (1991). See also Shavell (1990, p. 435) footnote 2 for a summary of pre-existing literature on the punishment of attempts and related issues.

\textsuperscript{58}See Shavell (1990). Shavell’s model seems to be a perfect fit for analyzing conduct crimes. In Shavell (1990), an implicit assumption is that individuals gain benefits from committing the conduct even if it results in no harm. That is to say the infliction of harm upon the victim is not necessary for the offender to benefit from his act. This assumption may be problematic when it comes to typical attempts. A person ordinarily derives a benefit from succeeding in crimes such as murder or rape, and not from simply attempting to commit these crimes. Shavell (1990, p. 449) also points to a variant of this observation: ”First, those who intend a result presumably obtain greater benefits from its occurrence than those who do not...”. This is perhaps why Friedman (1991), which is a note on the punishment of attempts written subsequent to Shavell (1990), uses a model in which individuals obtain benefits only upon successful
one which includes conduct crimes.\textsuperscript{59}

The main idea in Shavell (1990) as applied to conduct crimes can be summarized as follows. In some instances we would like to increase the level of deterrence without spending more resources for detection of crimes. One way to achieve higher deterrence is to punish harmful results more severely, however in certain cases it is impossible or not desirable to increase the punishment for harmful results.\textsuperscript{60} In such cases, deterrence can be increased by complementing punishment for results with punishments for conduct. In fact, the main finding in Shavell (1990) is that such complementation is indeed optimal.\textsuperscript{61} I will refer to the optimal punishment scheme in this framework, which consists of punishing conduct when the punishment for results can no longer be increased, as the \textit{Best No-Reporting Solution} in the remaining parts of this chapter.

The \textit{Best No-Reporting Solution} comes at the cost of deterring individuals who are less likely to cause harm when they engage in the forbidden and dangerous conduct and derive low but positive benefits from that act.\textsuperscript{62} The main objective of this chapter is to explore whether and how such social losses associated with over-deterrence can be mitigated. I find that the number of individuals being over-deterred can be reduced by allowing individuals to self-report before committing conduct-crimes, that this can be achieved without generating additional social costs, and that it is accordingly completion of the crime. A similar problem does not exist with conduct crimes, because in typical conduct crimes a person’s objective is to commit the conduct, not to cause harm. Consider for instance speeding. See Section 3 for a more detailed discussion about what distinguishes conduct crimes from ordinary attempts.

\textsuperscript{59}The opening sentence of Shavell (1990) is as follows: ”By an ”attempt” I shall simply mean a potentially harmful act that does not happen to result in harm, such as shooting at someone but missing.”.

\textsuperscript{60}See Shavell (1990, pp. 448-9) discussing two main reasons as to why it may not be desirable to increase the punishment for harmful results: Concerns related to marginal deterrence and proportionality of punishments.

\textsuperscript{61}See Proposition 2 in Shavell (1990, p. 464).

\textsuperscript{62}To demonstrate the validity of this claim, consider an individual who may speed as much as he desires, but will still never get into an accident or harm anybody because he is extremely talented. If speeding is punished, this individual will never speed given that his benefit from speeding is sufficiently low. In this case, this individual is over-deterred, because he has positive benefits from speeding and does not generate expected social harms by speeding. See Shavell (1990) for a statement of this result.
socially beneficial to allow self-reporting. A secondary goal of this chapter is to consider whether allowing self-reporting of conduct crimes can result in mitigating costs arising from the under-deterrence of individuals who are more likely to cause harm. I answer this question affirmatively, but argue that there may be technological considerations that require policy makers to limit the use of self-reporting to address only over-deterrence.

This chapter is organized as follows. The next section consists of a mainly verbal description of the optimal deterrence model employed to analyze punishments for conduct crimes and demonstrates that there are gains associated with allowing self-reporting. In section 4.3., I discuss the implications and scope of the model along with some potential extensions. In the same section, I give a real life example, namely the use of radar detectors, which is functionally quite similar to the practice of allowing self-reporting. In appendix D I formalize and prove propositions extended in section 4.2..

4.2. Benefits of Ex-Ante Self-Reporting of Conduct Crimes

By ex-ante self-reporting I am referring to a future offender's act of informing law enforcers of a conduct crime that he will be committing. The future offender, by informing law enforcers, accepts a sanction for his conduct. In the remaining parts of the chapter, I will frequently refer to this act as simply self-reporting.

In this section, I demonstrate the advantages of allowing ex-ante self-reporting of conduct crimes. To achieve this purpose, I first describe an optimal deterrence model for conduct crimes, in which individuals are not allowed to self-report. In this framework, I describe optimal punishment

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63 See Kaplow and Shavell (1994a) for a general analysis of self-reporting in law enforcement. The authors are mainly concerned with self-reporting in the context of result-crimes and show that it "offers two advantages over schemes without self-reporting: enforcement resources are saved because individuals who report their harmful acts need not be detected, and risk is reduced because individuals who report their behavior bear certain rather than uncertain sanctions." Kaplow and Shavell (1994a, p. 583). The instant paper can be interpreted as identifying a third and independent advantage associated with self-reporting schemes in the context of conduct crimes as opposed to result crimes.

64 See sub-section 4.3.4.

65 Equivalently, a model in which individuals are not penalized less severely for the commission of a conduct crime when they self-report. This model corresponds to that described in Shavell (1990), where 'attempts' are interpreted
schemes and losses due to over-deterrence of talented individuals and under-deterrence of untalented
individuals. Then I extend this model to allow self-reporting and show that self-reporting can reduce
over-deterrence. Finally, I identify and describe optimal punishment schemes when self-reporting is
possible, and show that these schemes address under-deterrence problems as well.

4.2.1. Main Framework

An individual has the option of engaging in certain conduct which may cause harm \((h)\). The like-
lihood of conduct being harmful depends on the \('talent' level of the individual, which is represented
by \(q \in [0, 1]\). Once an individual with \(q = 1\) commits the conduct, he will certainly cause harm,
whereas an individual with \(q = 0\) will certainly not cause harm. Individuals obtain a personal benefit
\((b)\) for committing the illegalized conduct. Personal benefits from the commission of the conduct
and talent levels are assumed to vary across individuals and are not observable by the court.\(^{66}\)

An individual who commits the conduct can be caught and sanctioned. The probability an
offender is caught may depend on whether or not the conduct results in harm. \(p_1\) and \(p_2\) respectively
denote the probability with which non-harm and harm will be sanctioned.\(^{67}\) I am assuming that
\(p_2 \geq p_1\), which reflects the fact that the observation of harm increases the likelihood of detection.\(^{68}\)

How offenders are punished depends on whether or not self-reporting is possible. Sub-sections
4.2.2. and 4.2.3. respectively describe sanction schemes when self-reporting is not possible and when
it is.

4.2.2. Law Enforcement without Self-Reporting

When self reporting is not possible, an offender is punished with a sanction of \(s_1\) for harmless

\(^{66}\)Over-deterrence is not a problem when courts have perfect information about individuals’ benefits from the
conduct and the ex-ante likelihood with which individuals will cause harm. For the analysis of this case, refer to
Shavell (1990). In this case, there is no room for improvement through a reduction in the number of individuals who
are being over-deterred.

\(^{67}\)I am assuming that \(p_1\) and \(p_2\) are determined exogenously. This assumption can be justified due to the use of
general as opposed to specific law enforcement as defined in Shavell (1991). See also chapter 3 section 2.

\(^{68}\)This assumption also simplifies the derivation of various propositions by eliminating a number of cases. It is
harmless in the sense that it does not affect results obtained.
conduct. On the other hand, if the conduct results in harm, the offender can be punished more severely at $s_2 \geq s_1$. This case is referred to as the No Self-Reporting Regime. The incentives an individual faces under the No Self-Reporting Regime is conveniently represented by the following decision tree.

This case corresponds to Shavell (1990). The results obtained in this case can be summarized as follows. First, note that since individuals differ in their likelihood of causing harm, they are associated with different levels of expected harm to society from the commission of the conduct. Next, it should be noted that it is desirable to incentivize individuals to internalize the expected harm they generate through the commission of the conduct. Hence, it would be preferable to impose sanctions that equate the expected punishment for committing the conduct to the expected social harm to society. One way to do this would be to punish individuals only when they cause harm with an expected sanction equaling the harm ($s_2 = \frac{b}{p_2}$). However, when fines necessary to achieve this goal are too high (higher than the maximal fine ($w$)), punishment for harmful results are inadequate and result in under-deterrence for individuals of all talent levels.\(^6\) In this case, punishment for results, which is equal to the maximal fine ($s_2^* = w$), must be complemented by punishment for conduct to reduce losses due to under-deterrence. The optimal sanction for conduct crimes ($s_1^*$) is one that results in over-deterrence some talented (low $q$) individuals, and under-deterrence some untalented (high $q$) individuals.\(^7\) This punishment scheme is called the Best No-Reporting Solution.

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\(^6\) Except for individuals with $q = 0$, who are assumed to have a negligible measure.

\(^7\) See Proposition 2 in Shavell (1990, p. 464).
4.2.3. Law Enforcement with Self-Reporting and Mitigating Over-Deterrence

Under the Best No-Reporting Solution, under-deterrence of untalented individuals as well as over-deterrence of talented individuals result in social losses. Accordingly, reducing the number of individuals being over-deterred without increasing the number of individuals being under-deterred would lead to an increase in overall welfare. A potential way to achieve this is by designing an offer, that would be accepted only by some individuals who are being over-deterred, and would allow them to commit the conduct. Allowing self-reporting, and properly reducing the punishment of individuals who self-report, works exactly in this way. This case, which is called the Self-Reporting Regime, can be simplified and described as follows. Under the Self-Reporting Regime individuals can choose from two punishment schemes: The Default Scheme and the Reporting-Scheme. When an individual chooses the Reporting-Scheme, he makes an up-front payment of \( y_r \) and is not sanctioned.
for harmless conduct but is subject to an additional fine of $y_a$ for harm. Hence, when he causes harm and is caught, he is subject to a total sanction of $y_h = y_r + y_a$. An individual who chooses the Default Scheme faces punishments of the type described in the No Self-Reporting Regime. However, to distinguish (optimal) sanctions imposed under the two regimes, the sanctions under the Default Scheme will be denoted as $z_1$ and $z_2$. The incentives an individual faces under the Self-Reporting Regime is depicted in figure 4.2.

Talented individuals are more frequently subjected to sanctions for harmless conduct, because their conduct seldom results in harm. Since self-reporting provides a discount for harmless conduct, intuitively, it is the more advantageous option for talented individuals. Hence, by self-reporting, talented individuals can engage in conduct, which they know is unlikely to result in harm. On the other hand, untalented individuals will not find this option beneficial if its cost is sufficiently high. These individuals are more afraid of being punished for results rather than harmless conduct. Hence, if self-reporting relieves them only from punishments associated with harmless conduct, they will find the price too high for the option. Therefore, by exploiting the difference in talented and untalented individuals’ preferences over self-reporting, one can design a Self-Reporting Regime which mitigates the costs associated with over-deterrence and leaving costs due to under-deterrence unchanged. This naturally leads to an increase in social welfare. This finding is summarized by proposition 4.1., which is best interpreted through the illustration of a Self-Reporting Regime that dominates the Best No-Reporting Solution. Figures 4.3.a-d below, represent such a regime, and demonstrate gains from allowing self-reporting.

**Proposition 4.1.** The Best No-Reporting Solution can be improved upon by allowing ex-ante self-reporting.

**Proof:** See Appendix D.
Figure 4.3.a: Best No-Reporting Solution

Best No-Reporting Solution

\[ C_1(q) = p_1(1-q)s_1^* + p_2qw \]

\[ H(q) = qh \]

= Individuals engaging in conduct.

Figure 4.3.b: Self-Reporting Regime (Over-Deterrence Mitigating)

Self-Reporting Behavior

= Individuals who self-report.

Figure 4.3.c: Self-Reporting Regime (Over-Deterrence Mitigating)

Commission of Conduct

= Reduction in Over-Deterrence

Figure 4.3.d: Self-Reporting Regime (Over-Deterrence Mitigating)

Reduction in Over-Deterrence

= Reduction in Over-Deterrence
Figure 4.3.a represents the *Best No-Reporting Solution*, where $C_S(q)$ is the expected punishment associated with the commission of illegal conduct, and $H(q) = qh$ is the expected social harm. Figures 4.3.b and 4.3.c illustrate a *Self-Reporting Regime* which results in higher welfare, compared to the *Best No-Reporting Solution*, where $C_R(q)$ and $C_D(q)$ respectively represent the expected costs associated with the commission of illegal conduct under the *Reporting-Scheme* and the *Default Scheme*. The *Self-Reporting Regime* depicted in these figures is a special one, because its *Default Scheme* is identical to the *Best No-Reporting Solution*. Stated differently, $z_1 = s_1^*$ and $z_2 = s_2^* = w$, or $C_D(q) = C_S(q)$ for all $q$. I have deliberately chosen this special regime, because it has some nice properties, which I discuss in the final section. To ease reference in the discussion section, I call this particular regime the *Over-Deterrence Mitigating Regime*.

Most of the implications of these figures are self-explanatory. However, the reduction in over-deterrence, which is represented in Figure 4.3.d, is worth describing. When self-reporting is not possible, there is a problem of over-deterrence for some individuals who fall in the specific talent range of $q < q_O$. The *Reporting-Scheme* is chosen to target such over-deterrence. To achieve this purpose, it offers a reduction in the sanction for harmless conduct for those who self-report, but only enough to incentivize some individuals who are sufficiently talented ($q < q_O$). Hence, a portion of individuals (triangle $ABC$ in figure 4.3.d) switch from not committing the conduct (in the *Best No-Reporting Solution*) to self-reporting and committing. This leads to a reduction in over-deterrence, and therefore an increase in welfare.

### 4.2.4. Optimal Punishment Schemes with Self-Reporting

So far I have only asked whether one can improve on the *Best No-Reporting Solution* by enabling self-reporting, and answered this question affirmatively. This was useful for establishing the important result that when self-reporting is possible, not enabling it is socially undesirable. In deriving this result, I relied on a *Self-Reporting Regime* where the *Default Scheme* was identical to the *Best No-Reporting Solution*. I have done this with a particular goal in mind, related to self-reporting

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71Refer to Appendix D for the expression of $y_o^*$ and a formal derivation of the qualitative properties of the graphs.
technologies, which I discuss in sub-section 4.3.4. However, in general, one need not constrain the
set of possible punishment schemes in this manner. In particular, when self-reporting is possible,
one can provide proper incentives for talented individuals through appropriate Reporting-Schemes
and deter untalented individuals through rigid Default Schemes. This would result in savings due
to reductions in over-deterrence as well as under-deterrence. This sub-section describes the way to
achieve this desirable result and characterizes optimal punishment schemes with self-reporting.

First, it should be noted that the optimal Self-Reporting Regime has to provide a discount for the
punishment of harmless conduct for individuals who self-report. Otherwise, individuals would never
prefer to self-report, and therefore the Self-Reporting Regime would be functionally no different than
a No Self-Reporting Regime. But as discussed in the previous section, and formalized by proposition
4.1., there is always a Self-Reporting Regime that dominates any No Self-Reporting Regime. Hence,
under the optimal Self-Reporting Regime there must be a discount for self-reporting. In this case,
some individuals who would suffer from over-deterrence in the Best No-Reporting Solution will choose
to self-report and commit the conduct (which will likely be harmless). Accordingly, for purposes of
mitigating over-deterrence of talented individuals, the relevant scheme will be the Reporting-Scheme.

Second, it should be noted that some level of under-deterrence is, by definition, unavoidable.
This follows from the fact that the maximal punishment for harm is insufficient to deter the least
talented individual \( q = 1 \) with a sufficiently high benefit from committing the conduct. Hence,
there is always an under-deterrence problem. Such under-deterrence is minimized when individuals
who do not self-report are punished with maximal fines for harmful as well as harmless conduct.
Furthermore, this use of maximal fines does not generate over-deterrence for talented individuals
(as would be the case in a No Self-Reporting Regime), since those individuals choose to self-report
and are not affected by an increase in the punishment for not self-reporting.

These two observations reveal the fact that the optimal Self-Reporting Regime consists of punish-
ing those who do not self-report with maximal fines for harmful as well as harmless conduct, and a
less severe punishment for individuals who self-report. This derivation is incomplete, and intends to
convey the main rationale behind the optimal punishment scheme with self-reporting. A complete and formal derivation of this result, which is summarized by proposition 4.2. below, is provided in appendix D. An optimal Self-Reporting Regime, as derived in proposition 4.2. is represented in Figure 4.4., below.

**Proposition 4.2.:** Under the optimal Self-Reporting Regime:

(i) there is a cut-off talent level $q^C$ such that only (talented) individuals with $q < q^C$, who intend to commit the conduct, self-report, and (untalented) individuals with $q > q^C$ do not self report,

(ii) individuals with $q = q^C$ are under-deterrred.

(iii) those who do not self-report are punished with the maximal fine for harmful as well as harmless conduct, i.e. $z_1^* = z_2^* = w$

(iv) those who self-report and commit harmful conduct are punished with the maximal fine, i.e. $y_h = w$

(v) those who self-report and commit harmless conduct are punished, but less severely than a person who does not self-report, i.e. $0 < y_r < w$

(vi) $y_r$ is that punishment which optimally trades-off under-deterrrence with over-deterrence.

There is an interesting implication of proposition 4.2.. Although one would suspect that allowing
self-reporting would likely increase the level of crime by providing a discounted punishments to those who self report, this need not be the case. It is true that under the optimal solution, talented individuals are provided with greater incentives to commit the conduct. But it is also true that untalented individuals are deterred more through the use of maximal fines for non-reported conduct. Hence, the overall effect on the level of crime will depend on the talent distribution. Another important thing to point out is that the imposition of maximal fines for non-reported conduct may change the way we view self-reporting. In this framework, self-reporting is almost a duty imposed on potential offenders. However, as I have noted before, the use of these punishment regimes may not be desirable due to technological constraints. I consider this possibility in the next section, and argue that in such cases it is more desirable to use self-reporting for over-deterrence mitigating purposes only.

4.3. Discussion

In this section, I give a real life example, namely the use of radar detectors, where individuals engage in behavior which is functionally similar to ex-ante self-reporting of conduct crimes. Then, I consider potential extensions and the scope of the analysis.

4.3.1. Radar Detectors

The use of radar detectors (in states where they are legal) is a good example where we observe individuals engaging in behavior which is functionally similar to ex-ante self-reporting of conduct crimes. By purchasing a radar detector, an individual pays a cost which allows him to identify portions of roads that are being monitored. Therefore, he can speed in the unmonitored portions of the road without having to worry about paying a speeding ticket. Hence, he is not deterred (at least in the unmonitored portions of the road). If the price for radar detectors is sufficiently high only talented individuals will buy them. Accordingly, legalizing and correctly pricing radar detectors (perhaps through taxes) can mitigate losses due to over-deterrence.

There is, however, one important difference between ex-ante self-reporting as described in the preceding parts of this chapter and the use of radar detectors. After self-reporting, an individual is
punished for a single conduct crime that he will be committing in the future, whereas by purchasing a radar detector an individual acquires the technology to avoid multiple punishments for conduct in the future. If every individual was endowed with the same number of opportunities to commit conduct crimes, absent further considerations, the purchase of a radar detector would be the functional equivalent of self-reporting of all future conduct. However, since this condition is not met, the legalization of radar detectors can at most be an imperfect substitute for enabling ex-ante self reporting.

4.3.2. Level of Generality: Regulations

I have limited the analysis in this chapter to conduct crimes, however the scope of the analysis seems to extend easily to regulations. There are types of regulations in which a regulatory agency demands compliance with a pre-determined standard.\textsuperscript{72} My analysis is applicable to such regulations as long as entities are heterogenous in the likelihood with which they will cause harm by violating the pre-determined standard.

Environmental regulations are good examples to demonstrate the validity of this claim. Consider a waste disposal regulation requiring entities to dispose of potentially dangerous substances in a specific manner. Here, the regulation makes certain conduct illegal: Waste disposal in a manner inconsistent with the requirements set forth by the regulation. In this regard entities not complying with the requirements of regulations could be interpreted as committing illegal conduct. These entities will be punished regardless of whether or not their conduct results in actual harm. Accordingly, regulations of this type can be analyzed under the same framework provided in this chapter, given that entities are heterogenous in the likelihood they will cause harm by not complying with regulatory requirements.

4.3.3. Level of Generality: Attempts

I have intentionally excluded criminal attempts from the scope of this chapter. This is mainly because I believe that criminal attempts are distinguishable from conduct crimes in a significant

\textsuperscript{72}For a short discussion on safety regulations, see the discussion section in Shavell (1990).
manner. As argued in the introduction, conducts and attempts share a lot in common. However, they are distinguished from each other when one considers the goal of potential offenders. In order to analyze ordinary attempts, one should focus on offenders whose benefits are conditional on inflicting harm. That is to say, it should be assumed that a person ordinarily shoots at her victim hoping that the bullet hits him, and not hoping that she misses her target. On the other hand, a person who is speeding ordinarily wishes to drive from point A to point B as fast as she can. In other words, she is not increasing her speed to increase the likelihood of hitting a pedestrian on her way.\(^7\)

This distinction is crucial in determining optimal-deterrence policies. If potential offenders are assumed to derive benefits that are smaller than the harm they will cause by hurting their victims, then there will be no such thing as over-deterrence in the context of criminal attempts. To formalize this idea and show the validity of this claim, consider the following simple notation:

\(b^-\) is the benefit of that individual who derives the most benefit from crime.

\(h\) is the harm associated with the completed crime, with \(h > b^-\).

\(q\) is the likelihood that an offender will succeed in committing crime.

In this framework, an individual with benefit \(b\) from crime is over-deterred if the following simple inequality holds:

\[
\text{Individual’s Expected Benefit} = qb > qh = \text{Expected Social Cost}
\]

However, as noted \(h > b^-\) and by definition \(b^- \geq b\). Accordingly, in this framework, over-deterrence is not possible.

It could be argued that the same result is likely to hold for conduct crimes if one is to assume \(b^- < h\). This argument is flawed. As pointed out, when analyzing conduct crimes it should be assumed that individuals derive benefits upon completing the conduct, not upon inflicting harm. Accordingly, for conduct crimes, an individual who derives a benefit \(b\) from the conduct will be over-deterred if:

\(^7\)See also footnote 58.
Individual’s Benefit from the Conduct = $b > qh =$ Expected Social Cost

Which holds if $q < \frac{b}{h}$. Therefore, individuals who are highly talented will be over-deterred under a punishment scheme that punishes conducts. This should demonstrate that the main rationale provided in this chapter extends to cases where the benefit derivable from the conduct is bounded from above.

4.3.4. Considerations Related to Self-Reporting Technologies

I previously stated that it may be desirable to use self-reporting to target over-deterrence only, as opposed to implementing an optimal Self-Reporting Regime, which would target over-deterrence as well as under-deterrence. This is likely to be the case, when self-reporting technologies are poor, and individuals may realize their benefits from committing the conduct immediately before the opportunity occurs. In these cases, an individual who realizes that he is better off self-reporting and committing the conduct, may nevertheless lack the means of self-reporting, simply because self-reporting may require time. Therefore, compared to the Best No-Reporting Solution the optimal Self-Reporting Regime may result in more over-deterrence for individuals who do not have the opportunity to self-report, since the former imposes harsher punishments for harmless conduct without self-reporting (i.e. $z^*_1 = w \geq s^*_1$). Accordingly, if the proportion of such individuals is sufficiently high, the optimal Self-Reporting Regime may actually be inferior to the Best No-Reporting Solution. Accordingly, the use of maximal fines for non-reported conduct may be undesirable.

On the other hand, the Over-Deterrence Mitigating Regime, discussed in section 1.C. and illustrated in figures 3b-d, does not share this caveat. Under the Over-Deterrence Mitigating Regime an individual who does not self-report is punished as severely as he would be under the Best No-Reporting Solution. Therefore, this regime would lead to less over-deterrence for individuals who have the opportunity to self-report and would leave other individuals’ incentives unaltered. Therefore, it would always lead to an improvement over the Best No-Reporting Solution.
4.4. Appendix D

Notation:

\( s_1, s_2 \geq 0 \): Sanction, under the No-Reporting Regime, for harmless and harmful conduct respectively.

\( z_1, z_2 \geq 0 \): Sanction, under the Default Scheme of a Self-Reporting Regime, for harmless and harmful conduct respectively.

\( y_r \geq 0 \): Sanction, under the Reporting Scheme of a Self-Reporting Regime, for self-reporting.

\( y_a \geq 0 \): Additional sanction, under the Reporting Scheme of a Self-Reporting Regime, for self-reported and harmful conduct.

\( y_h \equiv y_r + y_a \).

\((;.;.;.;.):\): List describing a No Self-Reporting Regime, where the components of the list describe values of \( s_1 \) and \( s_2 \), and in that order.

\((.;.;.;.;.;.;.;.):\): List describing a Self-Reporting Regime, where the components of the list describe values of \( z_1, z_2, y_r, \) and \( y_a \), and in that order.

\( p_1 \): Probability of detection of a harmless conduct.

\( p_2 \): Probability of detection of a harmful conduct.

\( h \): Harm associated with harmful conduct.

\( q \): An individual’s likelihood of causing harm upon committing conduct.

\( H(q) = qh \): Expected harm associated with the commission of the conduct by a type \( q \) individual.

\( b \): Benefit an individual enjoys upon commission of conduct.

\( w < h \): Maximal fine.

**Proposition 4.1.** The Best No-Reporting Solution can be improved upon by allowing ex-ante self-reporting.

**Proof:** \((s_1^*; w)\) corresponds to the Best No-Reporting Solution, where \( s_1^* \leq w \). This result is derived in Shavell (1990), and individuals’ incentives under this solution is represented in figure 4.3.a. Next, define regime \( O \), the Over-Deterrence Mitigating Regime:
\[ O \equiv (s_1^*; w; y_r^O; y_a^O) \] (D.1.)

where

\[ y_r^O = \frac{p_1 s_1^*[h - p_2 w]}{[1 - p_2][p_1 s_1^* + [h - p_2 w]]} \] (D.2.)

and

\[ y_a^O = [w - y_r] \] (D.3.)

In \( O \), the Default Scheme is identical to the Best No-Reporting Solution. Accordingly, the expected cost of choosing the Default Scheme and engaging in the illegalized act \( (C_D(q)) \) equals the expected cost of engaging in the illegalized act under the Best No-Reporting Solution \( (C_S(q)) \):

\[ C_D(q) = C_S(q) = (1 - q)p_1 s_1^* + q p_2 w \] (D.4.)

On the other hand, the expected cost of self-reporting and engaging in the illegalized conduct is

\[ C_R(q) = q p_2[w - y_r^O] + y_r^O \] (D.5.)

The cost of not engaging in the illegal act is 0 and \( y_r^O \) under the Default Scheme and Reporting Scheme respectively.

Figures 4.3.b-d represent expected social harm \( (H(q)) \), \( C_D \) and \( C_R \) as a function of \( q \), and the talent level \( q_O \) at which these functions intersect. Through simple algebra one can verify that the figures accurately represent the intersection point of all three curves and their slopes. More precisely, one can verify that (i) \( q_O = \frac{p_1 s_1^*}{h + p_1 s_1^* - p_2 w} \), (ii) \( C_D(q_O) = C_R(q_O) = H(q_O) \), and (iii) \( C_R \) has a greater slope than \( C_D \). To see this one can follow these steps:

(i)

\[ C_D(q) = (1 - q)p_1 s_1^* + q p_2 w = q p_2(w - y_r^O) + y_r^O = C_R(q) \] (D.6.)

iff \( q = \frac{p_1 s_1^* - y_r^O}{p_1 s_1^* - p_2 y_r^O} \equiv q_O \) (D.7.)
plugging in $y^O_r$ as defined in (D.2.) into (D.7.) reveals that:

$$q_O = \frac{p_1 s^*_1}{h + p_1 s^*_1 - p_2 w}$$  \hspace{1cm} (D.8.)

(ii) The observation in (i) reveals that $C_D(q_O) = C_R(q_O)$, hence showing that $C_D(q_O) = q_O h$, should suffice to verify the claim. Using (D.4.), we have that

$$C_D(q_O) = (1 - q_O)p_1 s^*_1 + q_O p_2 w = q_O h \iff p_1 s^*_1 = q_O (p_2 w + h - p_1 s^*_1)$$  \hspace{1cm} (D.9.)

Plugging in the value for $q_O$ as provided in (D.8.) reveals that (D.9.) holds. Therefore, $C_D(q_O) = C_R(q_O) = H(q_O)$.

(iii) (D.4.) and (D.5.) imply that the slopes of $C_D$ and $C_R$ are respectively given by $[p_2 w - p_1 s^*_1]$ and $p_2[w - y^O_r]$. Hence, the claim holds iff $p_1 s^*_1 > p_2 y^O_r$. But the expression for $y^O_r$ as provided in (D.2.) reveals that $p_1 s^*_1 > y^O_r$. Hence, $C_R$ has a greater slope than $C_D$.

Hence, the cost curves in figures 4.3.b-d are accurate representations. Therefore, they can be used to determine the behavior of individuals with various talent levels and benefits. This is achieved through the next three observations.

**Observation 4.1.**: An individual with a $b$ and $q$ such that $\min\{C_R(q), C_D(q)\} \geq b$ does not profit from engaging in the illegalized activity under either scheme, hence he will choose to abstain from engaging in the illegalized activity and choose the Default Scheme which allows him to do this in the cheapest way (since $0 < y^O_r$). These individuals are represented by the white area in figure 4.3.c.

**Observation 4.2.**: For individuals with $q < q_O$, it follows that $C_R < C_D$, hence they will either choose to self-report and engage in the illegal activity or choose the Default Scheme and not engage in the valuable activity. For individuals with $b > C_R$ the former option is more profitable, hence these individuals will choose to self-report and commit the conduct. These individuals are represented by the shaded area in figure 4.3.b.

**Observation 4.3.**: $C_R > C_D$ for individuals with $q > q_O$, hence these individuals will always choose the Default Scheme, and those with $b > C_D$ will engage in the activity.
These three observations are reflected in figures 4.3.b and 4.3.c. It follows from figures 4.3.a-d that the incentives provided in the Best No-Reporting Solution and regime O differ from each other only for individuals in the triangular region ABC. These individuals do not commit the conduct in the Best No-Reporting Solution but commit it in regime O. Since, \( b > qh \) for these individuals, O improves upon the Best No-Reporting Solution by decreasing over-deterrence.

The following Lemmas are useful in proving proposition 4.2.

**Lemma 4.1.** A Self-Reporting Regime \((z'_1; z'_2; y'_r; y'_a)\) such that \( p_1z'_1 \leq y'_r \) and \( p_2z'_2 \leq p_2y'_a + y'_r \) is sub-optimal.

**Proof:** In this case, \( C_R \geq C_D \) for all \( q \), hence nobody chooses to self-report. Therefore, the No Self-Reporting Regime \((z'_1; z'_2)\) achieves the same social welfare as \((z'_1; z'_2; y'_r; y'_a)\). By definition the Best No-Reporting Solution produces at least as much social welfare as \((z'_1; z'_2)\). But regime O, as defined in proposition 4.1., dominates the Best No-Reporting Solution, and is not a regime that satisfies the condition of lemma 4.1. Accordingly, \((z'_1; z'_2; y'_r; y'_a)\) such that \( p_1z'_1 \leq y'_r \) and \( p_2z'_2 \leq p_2y'_a + y'_r \) is sub-optimal.

**Lemma 4.2.** A Self-Reporting Regime \((z'_1; z'_2; y'_r; y'_a)\) such that \( p_1z'_1 \geq y'_r \) and \( p_2z'_2 \geq p_2y'_a + y'_r \) is sub-optimal.

**Proof:** In case both constraints hold with equality lemma 4.1. implies that \((z'_1; z'_2; y'_r; y'_a)\) is sub-optimal. When at least one constraint holds with inequality, it must be the case that \( C_D > C_R \) for almost all \( q \), hence almost every individual who commits the conduct also self-reports. Denote by \( I \) the point where \( C_R(q) \) intersects \( H(q) \). Next, observe that \( y'_h = y'_r + y'_a < w \) must hold. Otherwise, \( p_2y'_a + y'_r > p_2w \geq p_2z'_1 \), which is a violation of the condition stated in lemma 4.2. But when \( y'_h < w \), one can rotate \( C_R(q) \) counter-clockwise around point \( I \), by appropriately decreasing \( y'_r \) to some \( y''_r \) and increasing \( y'_a \) to \( y''_a = w - y''_r \). The new expected cost curve obtained by this rotation is denoted as \( C''_R(q) \) in figure D.1. As reflected in figure D.1, \((z'_1; z'_2; y''_r; y''_a)\) results in lower under-deterrence.
but the same amount of over-deterrence as \((z_1'; z_2'; y_r'; y_a')\), and \(p_2z' > p_2y_a'' + y_r''\). Hence, any Self-Reporting Regime \((z_1'; z_2'; y_r'; y_a')\) such that \(p_1z_1' \geq y_r'\) and \(p_2z_2' \geq p_2y_a' + y_r'\) is sub-optimal. ■

**Lemma 4.3.** An optimal Self-Reporting Regime \((z_1^*; z_2^*; y_r^*; y_a^*)\) must be such that \(p_1z_1^* > y_r^*\) and \(p_2z_2^* < p_2y_a^* + y_r^*\).

**Proof:** Lemma’s 4.1. and 4.2. together imply that any optimal Self-Reporting Regime must satisfy either

\[
p_1z_1 < y_r \quad \text{and} \quad p_2z_2 > p_2y_a + y_r \tag{D.10}\]

or

\[
p_1z_1 > y_r \quad \text{and} \quad p_2z_2 < p_2y_a + y_r \tag{D.11}\]

Assume \((z_1'; z_2'; y_r'; y_a')\) satisfies (D.10.). In this case there are two possibilities. Either (i) \(C_R(q)\) and \(C_D(q)\) intersect at some point below (or at) the curve \(H(q)\) or (ii) they intersect above it.

(i) If \(C_R(q)\) and \(C_D(q)\) intersect below (or at) \(H(q)\), another Self-Reporting Regime described as \((z_1'; z_2'; w; 0)\) dominates \((z_1'; z_2'; y_r'; y_a')\), since it results in less under-deterrence but the same level of over-deterrence. Hence, \((z_1'; z_2'; y_r'; y_a')\) cannot be optimal if it satisfies (D.10.) and the intersection of

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*Figure D.1.* illustrates a case where both conditions in Lemma 4.2. hold strictly. It is easy to verify that the analysis is not affected when either condition holds with equality.
$C_R(q)$ and $C_D(q)$ occurs below (or at) $H(q)$. These results are reflected in figure D.2, where $C^*_R(q)$ denotes the expected cost of self-reporting and committing the conduct under regime $(z'_1; z'_2; w; 0)$.

(ii) If $C_R(q)$ and $C_D(q)$ intersect above $H(q)$, then one can design a new Self-Reporting Regime, by keeping $z'_2$, $y'_r$, and $y'_a$ constant, but decreasing $z'_1$ to $z''_1$, such that the new expected cost curve (denoted in figure D.3 as $C^*_D(q)$) intersects $C_R(q)$ at $H(q)$. In this case $(z''_1; z'_2; y'_r; y'_a)$ dominates $(z'_1; z'_2; y'_r; y'_a)$, since it results in less over-deterrence but the same level of under-deterrence. Hence, $(z'_1; z'_2; y'_r; y'_a)$ cannot be optimal if it satisfies (D.10.) and the intersection of $C_R(q)$ and $C_D(q)$ occurs above $H(q)$. These results are reflected in figure D.3.
These two observations reveal the fact that a Self-Reporting Regime satisfying (D.10.) cannot be optimal. Therefore, the optimal Self-Reporting Regime must satisfy (D.11.).

**Proposition 4.2.** Under the optimal Self-Reporting Regime:

(i) there is a cut-off talent level $q^C$ such that only (talented) individuals with $q < q^C$, who intend to commit the conduct, self-report, and (untalented) individuals with $q > q^C$ do not self-report,

(ii) individuals with $q = q^C$ are under-deterred.

(iii) those who do not self-report are punished with the maximal fine for harmful as well as harmless conduct, i.e. $z^*_1 = z^*_2 = w$

(iv) those who self-report and commit harmful conduct are punished with the maximal fine, i.e. $y_h = w$

(v) those who self-report and commit harmless conduct are punished, but less severely than a person who does not self-report, i.e. $0 < y_r < w$

(vi) $y_r$ is that punishment which optimally trades-off under-deterrence with over-deterrence.

**Proof:**

(i) Lemma 4.3. implies that the optimal Self-Reporting Regime, must satisfy (D.11.). In this case, $C_D$ and $C_R$ intersect at some talent level $q_C$. It follows that $C_D(q) > C_R(q)$ for all individuals with $q < q_C$. Hence, it pays off for an individual with $q < q_C$, who intends to commit the conduct, to self report. On the other hand, an individual who does not intend to commit the conduct to not self-report since self reporting costs $y_r$ and not self-reporting costs $0$. It also follows that $C_D(q) < C_R(q)$ for all individuals with $q > q_C$. Hence, in this range it is less costly for any individual to not self-report.

(ii) Lemma 4.3. implies that the optimal Self-Reporting Regime must satisfy (D.11.). In this case, $C_D$ and $C_R$ can intersect below (or at) the curve $H(q)$ or above it.

Consider a regime $(z'_1; z'_2; y'_r; y'_a)$, where $C_D$ and $C_R$ intersect above $H(q)$. A new Self-Reporting
Regime can be designed by decreasing $y'_r$ to $y''_r$ such that the new expected cost (denoted in the figure as $C'_R(q)$) intersects $C_D(q)$ at $H(q)$. In this case $(z'_1; z'_2; y''_r; y'_a)$ dominates $(z'_1; z'_2; y'_r; y'_a)$, because it results in less under-deterrence. Hence, the optimal Self-Reporting Regime must be such that $C_D$ and $C_R$ intersect below (or at) $H(q)$. This finding is illustrated in figure D.4. Furthermore, as the proof of part (v) and (vi) will reveal, a Self-Reporting Regime where $C_D$ and $C_R$ intersect at $H(q)$ is sub-optimal. Therefore, under the optimal Self-Reporting Regime individuals with $q = q^C$ are under-deterrned.

(iii) Part (ii) establishes the fact that $C_D$ and $C_R$ intersect below (or at) $H(q)$. Any such Self-Reporting Regime $(z'_1; z'_2; y'_r; y'_a)$, where $z'_1 < w$ or $z'_2 < w$, is sub-optimal. To see this, note that increasing $z_1$ and $z_2$ leads to less under-deterrence at no additional cost (i.e. additional over-deterrence). This can be verified by figure D.5, which shows that an increase in $z'_1$ or $z'_2$ leads to a reduction in under-deterrence.
(iv) Denote by $I$ the point at which $C_D(q)$ and $H(q)$ intersect. Next, note that whenever $y_a + y_r < w$, the expected cost curve $C_R(q)$ can be rotated counter-clockwise around point $I$ by increasing $y_a$ and decreasing $y_r$. Next, note that such rotation is socially desirable, since leads to less under-deterrence and over-deterrence. This is illustrated via figure D.6. Hence, the optimal Self-Reporting Regime requires that $y_a + y_r = y_h = w$.

![Figure D.6: Demonstration of Proposition 4.2.(iv)](image)

(v) and (vi) Given properties (i)-(iv), all that remains to be determined is $y_r^*$, since the optimal Self-Reporting Regime is of the form $(w; w; y_r^*; w - y_r^*)$. There is another condition that properties (i)-(iv) imply: Curves $C_D$ and $C_R$ must intersect at some point below (or at) $H(q)$. This condition places an upper bound on the choice of $y_r^*$. To see this note that $q_C$, the talent level at which the intersection of $C_D$ and $C_R$ occur, is a function of $y_r$. In particular, in the following way:

$$q_C(y_r) = \frac{[wp_1 - y_r]}{[wp_1 - p_2 y_r]} \quad \text{(D.12.)}$$

(D.12.) implies that $\frac{dq_C}{dy_r} < 0$, $q_C(p_1 w) = 0$, and $q_C(0) = 1$. Next, denote by $q_D$, the talent level at which curves $H$ and $C_D$ intersect. It is clear that $q_D \in (0, 1)$. Now, the intermediate value theorem can be invoked to state that there exists some $y_r^C$, an interior and critical $y_r$, such that $q_C(y_r^C) = q_D$. Since $\frac{dq_C}{dy_r} < 0$, it must be the case that $y_r^* \leq y_r^C$, otherwise curves $C_D$ and $C_R$ will intersect above...
the curve $H$, which is a requirement for optimality. Therefore, the welfare maximization problem can be expressed as\textsuperscript{75}:

$$\max_{y_r \in (0, y^{C}_r]} W(y_r) = \max_{y_r \in (0, y^{C}_r]} \left[ \int_0^{q_C(y_r)} \int_{C_R(q,y_r)}^{\infty} (b - qh)f(b)dbg(q) dq + \int_{q_C(y_r)}^{1} \int_{C_D(q)}^{\infty} (b - qh)f(b)dbg(q) dq \right]$$ \hspace{1cm} (D.13.)

where

$$C_R(q, y_r) = y_r + qp_2[w - y_r] = qp_2w + [1 - qp_2]y_r$$ \hspace{1cm} (D.14.)

$$C_D(q) = w[(1 - q)p_1 + qp_2]$$ \hspace{1cm} (D.15.)

$f$ and $g$ respectively denote the density functions associated with benefits and talent levels, and $C_R = C_R(q, y_r)$ to reflect the dependency of $C_R$ on the choice of $y_r$.

Differentiating $W$ with respect to $y_r$, yields:

$$W_{y_r}(y_r) = -\int_0^{q_C(y_r)} (1 - qp_2)(C_R(q, y_r) - qh) f(C_P(q, y_r)) g(q) dq$$ \hspace{1cm} (D.16.)

Next, note that

$$W_{y_r}(y^{C}_r) < 0$$ \hspace{1cm} (D.17.)

since the integrand is positive for all $q < q_C(y^{C}_r)$. And similarly,

$$W_{y_r}(0) > 0$$ \hspace{1cm} (D.18.)

since the integrand is negative for all $q < q_C(0)$.

Hence, $y^*_r$, must satisfy:

$$w > y^*_r > 0$$ \hspace{1cm} (D.19.)

and

$$W_{y_r}(y^{*}_r) = 0$$ \hspace{1cm} (D.20.)

which implies that the optimal sanction for self reporting is chosen to optimally trade-off under-deterrence with over-deterrence.\footnote{$W$ is maximized over $(0, y^{C}_r]$, because $y_r = 0$ would be a violation of the requirement identified by Lemma 4.3.}
Chapter 5
Effects of State-Dependent Benefits on Optimal Law Enforcement

" 'Conscience! What is conscience? I make it up myself. Why do I suffer then? Out of habit. Out of universal human habit over seven thousand years. ...' "76

_Ivan Fyodorovich Karamazov_

"On the other hand, what about my conscience? I'll be running away from suffering! I was shown a path –and I rejected the path; there was a way of purification—..."77

_Dmitri Fyodorovich Karamazov_

5.1. Introduction

In the Beckerian model of crime and deterrence maximal fines are optimal.78 Many models have identified conditions under which this conclusion no longer holds.79 This note adds to the existing literature by presenting an extension to the standard crime and deterrence model. It introduces individuals who feel guilty upon committing crime but are partially relieved from such guilt on being punished. I demonstrate that the existence of such individuals implies that the maximal fine result in Becker (1968) does not necessarily hold.

The assumption that a proportion of individuals possess state dependent benefits as defined in this note is a non-standard one and needs to be supported. The type of individual I am referring to in this note is one who feels guilty after committing crime. On being sanctioned, however, he relieves part of such guilt. This assumption is a plausible explanation for the behavior of certain

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76Ivan paraphrasing his imaginary visitor. Dostoevsky (1880) at p. 653.
77Mitya considering an escape plan. Dostoevsky (1880) at p. 595.
78This result holds in any model, in which (i) the level of deterrence is determined by ps, (ii) increasing p always results in net costs, and (iii) s alone does not effect social welfare, where p is the probability of detection and s is the fine imposed.
79For a review of existing literature on law enforcement, see Garoupa (1997) and Polinsky and Shavell (2000).
criminals who turn themselves in years after committing crime, despite being confident that nobody has sufficient evidence to trace the crime back to them.\textsuperscript{80} This and related behavior has been observed and analyzed by various scholars of law\textsuperscript{81} and has been the subject of very influential literary pieces.\textsuperscript{82} Scholars who have been interested in this topic have concluded that guilt is attributable to a variety of reasons including religious and social feelings and have noted that religious practices, such as confessions, are associated with the same guilt related process.\textsuperscript{83} Furthermore, it has been documented that individuals, after being sanctioned, experienced "a great burden being lifted".\textsuperscript{84} Some scholars have argued that relief of a guilty conscience is associated with observing punishment as a method for undergoing penance, asking for forgiveness, or experiencing atonement.\textsuperscript{85} Accordingly, they have concluded that the law, through enabling punishment, offers a method to achieve these goals.\textsuperscript{86} Therefore, it is plausible to think that an individual, upon committing an offense, feels guilty. If he is punished, however, he relieves his guilt, perhaps by assuming that he is forgiven or that he ought to be forgiven.

In sum it has been documented that some individuals, if not all, feel a sense of guilt after committing a crime which they at least partially recover from on being punished. I am not making the stronger assumption that every individual feels guilty about committing crime and is relieved from guilt on being punished. I am simply assuming that some individuals act according to such feelings.

In the next section, I analyze the implications of guilt relief for optimal punishment through a standard deterrence model. In particular, I show that the Beckerian result of maximal fines does not necessarily hold. In section 5.3., I discuss the implications of the model, propose directions for future

\textsuperscript{80}See Adler (1992) at p. 47 and Bernick (1982).
\textsuperscript{81}For instance, see Adler (1992), Bader (2003), Bernick (1982), Bibas (2007), Bush (2003), Cochran (1998), Tasioulas (2007), and Winick (1997).
\textsuperscript{82}For instance, \textit{Brothers Karamazov} and \textit{Crime and Punishment}, both by Dostoevsky.
\textsuperscript{84}See Bernick (1982) at p. 308.
\textsuperscript{85}See for instance, Adler (1992) at p.91-94 and Tasioulas (2007) at p. 496.
\textsuperscript{86}See Adler (1992) and Tasioulas (2007).
research, and discuss ways in which the model presented can be embedded in more sophisticated models. In appendix E I prove a proposition extended in the modelling section.

5.2. The Model and Optimal Sanctions

Consider the ‘individual with a conscience’, who feels guilt after committing a crime, but is partially relieved of such on being punished. Since some individuals do not behave this way, I consider two types: (Type $K$) ‘individuals with a conscience’ and (Type $\tilde{K}$) ‘standard individuals’.\footnote{Labeled $K$ after Karamazov.} Standard individuals are those who have state independent benefits from crime; they receive the same benefits from crime, regardless of whether they are punished.

5.2.1. Notation

I employ the following notation:

- $b$: benefit from crime.
- $f(b)$: density of benefits, $f(b)$ is positive in $[0, \infty)$.
- $F(b)$: cumulative distribution function associated with $f(b)$.
- $g$: disutility due to feeling of guilt.
- $r$: partial relief of guilt upon being punished.
- $h$: expected social harm from crime.
- $p$: probability of detection.
- $c(p)$: enforcement costs, with $c' > 0$, and $c'' > 0$.
- $s$: sanction imposed on criminals who are caught.
- $m$: maximal sanction.
- $\alpha$: proportion of Type $K$ individuals in society.

5.2.2. Social Welfare and Policy Implications

Using this notation, a standard individual’s utility is given by:
\( U_{\tilde{K}} = \begin{cases} 
0 & \text{if he does not commit crime} \\
 b & \text{if he commits crime and is not punished} \\
 b - s & \text{if he commits crime and is punished} 
\end{cases} \) 

Here, the benefit from crime is \( b \) regardless of whether or not the Type \( \tilde{K} \) criminal is caught. However, a Type \( K \) individual has state dependent benefits:

\( U_K = \begin{cases} 
0 & \text{if he does not commit crime} \\
 (b - g) & \text{if he commits crime and is not punished} \\
 (b - g + r) - s & \text{if he commits crime and is punished} 
\end{cases} \) 

Here, the benefit from committing crime is only \((b - g)\) if the criminal is not punished but \((b - g + r)\) if he is.

When benefits are state dependent in this sense, holding the expected punishment from committing crime \((ps)\) constant and increasing the probability of detection \((p)\) has two effects: (i) increasing enforcement costs \((c(p))\), and (ii) increasing the expected utility of Type \( K \) individuals. This is the main trade-off, which determines the optimal probability of detection and the optimal sanction. Depending on the structure of this trade-off, increasing \( p \) may result in net marginal benefits, even for probabilities of detection high enough that maximal sanctions are no longer binding. Next, I derive the pure utilitarian social welfare function, which I use to evaluate the desirability of outcomes.\(^{88}\) Then, I extend a proposition that identifies a sufficient condition under which the Beckerian result of maximal fines no longer holds. I provide a formal proof for this proposition in the appendix E.

To achieve this purpose, it is necessary to consider both Types’ incentives to commit crime.

A Type \( \tilde{K} \) individual will commit crime iff:

\[ b > ps \equiv b_{\tilde{K}} \]  \( (5.3.) \)

And a Type \( K \) individual will commit crime iff:

\[ b > g + p(s - r) \equiv b_K \]  \( (5.4.) \)

\(^{88}\)In the derivations that follow I assume that indifferent individuals do not commit crime.
I use a utilitarian social welfare function to evaluate the desirability of outcomes, which is simply the sum of all individuals’ utilities and revenues collected, minus aggregate harms and costs. Using the cut-off benefits $b_K$ and $b_{\tilde{K}}$ defined in (5.3.) and (5.4.), the utilitarian social welfare is expressed by:

$$W = \alpha \int_{b_{\tilde{K}}}^{\infty} [(b - g + pr) - h] f(b) \, db + (1 - \alpha) \int_{b_K}^{\infty} (b - h) f(b) \, db - c(p)$$  \hspace*{1cm} (5.5.)

It is useful to note that whenever $p$ is sufficiently large ($p > \frac{h}{m}$), it is optimal to set $s = \frac{h}{p}$. Another useful observation is that holding $ps$ constant and increasing $p$ leads to gains due to increases in the first term, leaves the second term unchanged and leads to losses due to increased enforcement costs in the last term. Combining these main observations with a few other minor ones leads to the following result.

**Proposition 5.1.**: Maximal fines are sub-optimal when $\alpha r [1 - F(h + g)] > c'(\frac{h}{m})$.

**Proof:** See Appendix E.

Proposition 5.1. states that the Beckerian maximal fine result does not necessarily hold when state dependent benefits are present, and provides a sufficient condition for optimal fines that are below maximal. This sufficient condition embodies a variant of a standard marginal cost-benefit comparison. $c'(\frac{h}{m})$ is the maximum marginal enforcement cost arising from an increase in the probability of detection for probabilities that could trigger a maximal fine. On the other hand, $\alpha r [1 - F(h + g)]$ is the marginal benefit arising from an increase in the expected utilities of Type $K$ individuals in response to an increase in the probability of detection. Hence, when the latter is greater than the former, it is beneficial to increase the probability of detection up to levels at which the imposition of maximal fines is not desirable. This condition is interpreted by the following corollary.

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89 I am following the standard Polinsky-Shavell framework, where every benefit enters social welfare. If, however, criminals’ benefits are excluded, then the implications of the model would be no different than a standard model where all individuals have state independent benefits.

90 The formal derivation of these observations is presented in appendix E.
Corollary 5.1.: Maximal fines are likely to be suboptimal when (i) the proportion of Type K individuals is high, (ii) the harm associated with crime is low, (iii) the maximal fine is high, and (iv) the relief of guilt is strong.

Corollary 5.1. summarizes the intuitive conditions which give rise to optimal fines which are below maximal. When the proportion of individuals who suffer from guilt is high, supplying the means to relieve guilt becomes more important in comparison to reducing enforcement costs. The same is true when relief of guilt is strong. When the maximal fine is high (equivalently, when the harm associated with crime is low) it becomes less likely for this fine to be binding, since individuals come closer to having no wealth constraints.

5.3. Conclusion and Extensions

Within the standard crime and deterrence framework, I have shown that introducing state dependent benefits alters the standard Beckerian conclusion that maximal fines are optimal. This result itself is an interesting one. The approach I have taken can prove more useful when embedded in more sophisticated frameworks, however. Consider, for instance, restorative justice and therapeutic jurisprudence approaches. These approaches have been under-analyzed in the law and economics literature. This is perhaps because we have failed to acknowledge that law, as an institution, has the power to alter variables we have always taken as constants. In this chapter, I relaxed the assumption that benefits are constant across states, and have shown how this assumption on its own suffices to alter our previous conclusions. In this regard, this chapter is also an attempt to suggest that research focusing on restorative justice and therapeutic jurisprudence can prove to be quite productive. Next, I propose a few extensions in which my approach can be used to analyze more sophisticated issues.

There are many important questions related to my analysis. Are there different methods of punishment that might lead to higher levels of relief? If so, should one always prefer those methods

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91 See Bush (2003) and Winick (1997) for definitions and explanations of restorative justice and therapeutic jurisprudence respectively.

92 See Bush (2003) and Winick (1997) proposing that law can play a role in changing benefit and harm structures.
of punishment in light of the suggestions of this model? Or, are there ways in which such punishment methods are inferior to standard punishment methods? Although these questions are interesting on their own, I believe they form pieces of a larger picture.

In the standard law and economics framework, we usually consider only a couple of punishment methods: punishment through fines and imprisonment. We do not go into the detail about how these punishments are structured because their designs do not matter for purposes of deterrence. There are many ways of punishing individuals, however, and their design may affect things other than deterrence that enter the social calculus. Within the framework of the model I have just presented, consider civil service and monetary sanctions paid to the government as two alternative methods of punishment. Which one is more likely to achieve a higher level of relief on the part of the offender? Can one method mitigate the harm caused to the victim more than the other?

Presumably, a punishment in the form of civil service in an area closely related to the original offense will produce a greater relief on the offender’s side. A person feeling guilty can seek for forgiveness much more effectively if he is given the opportunity to serve those people who he has harmed in the first place. On the other hand, non-pecuniary damages stemming from a distorted relationship between the victim and the offender can be cured if the offender personally seeks to mitigate the harm done to the victim. This does not necessarily imply that punishment in the form of civil service dominates monetary sanctions. It merely suggests that different methods of punishment can serve different societal needs and affect social welfare through distinct channels. Accordingly, further research focusing on such issues can clarify under which circumstances civil service or other forms of restorative justice may be preferred to traditional methods of punishment, or whether a combination of the two can serve societal needs better. The framework I have provided can be used to evaluate the gains (or losses) associated with different methods of punishment stemming from guilt related issues on the offenders' side.
5.4. Appendix E

Proof of Proposition 5.1:

First, I consider the unconstrained maximization problem (that is without maximal fines) that the social planner faces. The solution to this problem provides the tools to derive the sufficient condition presented in proposition 5.1:

Differentiating $W$ with respect to $s$ yields:

$$W_s = -p(ps-h)[\alpha f(b_K) + (1-\alpha)f(b_{\tilde{K}})]$$ (E.1.)

This implies that whenever $p > 0$, $W$ is quasi-concave in $s$, and is maximized only when $s = \frac{h}{p}$.

Hence, $W$ is maximized either when $p = 0$ or when $p > 0$ together with a fine of $s^* = \frac{h}{p}$. Accordingly, the maximum value function associated with social welfare in the unconstrained problem is given by:

$$V(p) = \max_s W(p, s)$$

$$= \begin{cases} 
V^1(p) = \alpha \int_{h+g-pr}^{\infty} [b - g + pr - h]f(b)db + (1-\alpha)\int_{h}^{\infty} [b - h]f(b)db - c(p) & \text{if } p > 0 \\
V^0 = \alpha \int_{g}^{\infty} [b - g - h]f(b)db + (1-\alpha)\int_{0}^{\infty} [b - h]f(b)db - c(0) & \text{if } p = 0
\end{cases}$$ (E.2.)

An initial observation is that $V$ makes a jump up at $p = 0$. To see this note that

$$V^1(0) = \alpha \int_{h+g}^{\infty} [b - g - h]f(b)db + (1-\alpha)\int_{h}^{\infty} [b - h]f(b)db - c(0) > $$

$$= \alpha \left[ \int_{g}^{h+g} [b - g - h]f(b)db + \int_{h+g}^{\infty} [b - g - h]f(b)db \right]$$

$$+ (1-\alpha) \left[ \int_{0}^{h} [b - h]f(b)db + \int_{h}^{\infty} [b - h]f(b)db \right] - c(0) =$$

$$= \alpha \int_{g}^{\infty} [b - g - h]f(b)db + (1-\alpha)\int_{0}^{\infty} [b - h]f(b)db - c(0) = V_0$$ (E.3.)

Hence, $p = 0$ is never optimal.

Furthermore,

$$V_p^1 = \alpha \int_{h+g-pr}^{\infty} r f(b)db - c'(p)$$ (E.4.)
Assuming, without loss of generality, that \( \int_0^\infty f(b)db = 1 \), (9) becomes:

\[
V_p^1 = \alpha r[1 - F(h + g - pr)] - c'(p)
\]  

(E.5.)

Hence, social welfare is increasing in \( p \), in the unconstrained maximization problem iff:

\[
\alpha r[1 - F(h + g - pr)] > c'(p)
\]

(E.6.)

But \( \alpha r[1 - F(h + g - pr)] \geq \alpha r[1 - F(h + g)] \) and \( c'(\frac{h}{m}) \geq c'(p) \) for all \( p \) such that \( p \leq \frac{h}{m} \). Where the first inequality follows from the fact that \( F \) is increasing in its argument, and the second inequality follows from the fact that \( c \) is convex in \( p \). Therefore, whenever \( \alpha r[1 - F(h + g)] > c'(\frac{h}{m}) \), it follows that \( V^1 \) is increasing in \( p \) for all \( p \in [0, \frac{h}{m}] \). This implies that any maximizer of \( V^1 \) is in the interval \( p \in (\frac{h}{m}, 1] \). But, in this interval the maximal fine is not binding, because \( m > \frac{h}{p^*} = s^* \). Where \( p^* \) denotes any maximizer of \( V^1 \). Accordingly, the unconstrained and the constrained problems have the same solutions.

Therefore, the maximal fine is sub-optimal whenever

\[
\alpha r[1 - F(h + g)] > c'(\frac{h}{m})
\]

(E.7.)

\[\blacksquare\]
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