Essays in Macroeconomics with Frictions and Uncertainty Shocks

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ESSAYS IN MACROECONOMICS
WITH FRICTIONS AND UNCERTAINTY SHOCKS

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Abstract

Title: Essays in Macroeconomics with Frictions and Uncertainty Shocks

C1. Financial Effects of Credit Constrained Workers’ Unemployment

C2. International Financial Business Cycles

C3. Uncertainty, Collateral Constrained Borrower, and the Business Cycle

Author: Taesu Kang
Advisor: Fabio Ghironi

This dissertation consists of three essays on macroeconomics with frictions and uncertainty shocks.

The first essay is “Collateral Constrained Workers’ Unemployment”. Financial market and labor market are closely interconnected each other in the sense that unemployed workers have difficulty not only in borrowing new loan but also in repaying outstanding loan. In addition, if unemployment entails loss from default and no new loan, credit constrained workers will accept lower wage to avoid the loss from losing job. In this paper, we try to investigate the role of the interaction between financial market and labor market over the business cycle. To do that, we assume credit constrained workers can borrow against their houses and repay outstanding loans only when they are employed. We also introduce labor search and matching framework into our model to consider unemployment and wage bargaining process explicitly. With this setup, we find that adverse housing preference shock leads to substantial negative impact on labor market by reducing the benefit from maintaining job. As a result, high unemployment significantly amplifies the business cycle by reducing supply of loan and increasing default. This result would be helpful to understand recent "Great Recession" which was originated from the collapse of housing market and accompanied by high unemployment and default rate.
The second essay is "International Financial Business Cycles". Recent international macroeconomics literature on global imbalances explains the U.S. persistent current account deficit and emerging countries’ surplus, i.e., the U.S. is the borrower. Little research has been done on the banking-sector level, where U.S. banks are lenders to banks in emerging countries. We build a two country framework where banks are explicitly modeled to investigate how lending in the banking sector can affect the international macroeconomy during the recent crisis. In steady state, banks in the developing country borrows from the U.S. banks. When the borrowers in the U.S. pay back less than contractually agreed and damage the balance sheet of the U.S. banks, with the presence of bank capital requirement constraint, U.S. banks raise lending rates and decrease the loans made to U.S. borrowers as well as banks in the developing country. The results are a sharp increase in the lending spread, a reduction in output and a depreciation in the real exchange rate of the developing country. They are the experience of many emerging Asian markets following the U.S. financial crisis starting in late 2007. Another feature of our model captures an empirical fact, documented by Devereux and Yetman (2010), that across different economies, countries with lower financial rating can suffer more when the lending country deleverages.

The third essay is "Uncertainty, Collateral Constrained Borrowers, and Business Cycle". Standard RBC model fails to generate the co-movement of key macro variables under uncertainty shock because precautionary saving motive decreases consumption but increases investment and labor. To fill this gap, we build a DSGE model with collateral constrained borrowers who can borrow against housing and capital. In the model with modest risk aversion, we can generate the desired co-movement of key macro variables under uncertainty shock and the co-movement comes from the collateral constraint channel through drop in housing price. Under uncertainty shock, highly indebted borrowers sell collaterals to avoid uncertainty in future consumption. As a result, housing price goes down and it makes credit crunch to borrowers through collateral constraint channel. The negative effect of uncertainty shock is strengthened in the economy with higher indebted borrowers.
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Chapter 1

Financial Effects of Credit Constrained Workers’ Unemployment

by Taesu Kang

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2Boston College, Email: kangta@bc.edu
1.1 Introduction

Financial market and labor market are not independent but highly interconnected each other. Why? First, financial intermediaries are reluctant to extend loans for unemployed workers since the unemployed have difficulty in repaying outstanding loan. For example, Federal Housing Administration (FHA), the largest mortgage insurer in the world, requires at least two years steady employment history and less than 30 percent of Debt-to-Income (DTI) ratio\(^3\) to be qualified for its mortgage insurance\(^4\). Second, unemployment as well as deep negative equity of house has been considered as one of main triggers of mortgage default during recent "Great Recession". Foote et al. (2009) [19] argues that 1 percentage point increase in unemployment rate raises the probability of a 90-day-delinquency by 10-20 percent. Elul et al. (2010) [15] also finds that county-level unemployment shocks are associated with higher default rate. Third, workers consider the financial effects of unemployment when they are in wage bargaining process. In other words, credit constrained workers know that they could not borrow or would go default if they lost job. If there is penalty from default or benefit from borrowing, credit constrained workers would accept lower wage to avoid the loss from losing job. On the contrary, decrease of penalty from default or benefit from borrowing would discourage credit constrained workers from maintaining job. Ellingsen and Holden (2002) [13] argues that indebted workers are more willing to accept a low wage in order to reduce the probability of unemployment.

<table>
<thead>
<tr>
<th>Table 1.1: Correlations between Key Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>House Price</td>
</tr>
<tr>
<td>House Price</td>
</tr>
<tr>
<td>GDP</td>
</tr>
<tr>
<td>Unemp. Rate</td>
</tr>
<tr>
<td>Mortg. Loan</td>
</tr>
<tr>
<td>Delinq. Rate</td>
</tr>
</tbody>
</table>

We could see the interaction between financial market and labor market indirectly by

\(^3\)The ratio of monthly mortgage-related (or all debt) payments to monthly gross income.

\(^4\)Unemployment does not necessarily imply no mortgage loan. The unemployed could get mortgage loan from risk taking financial intermediaries especially during housing boom if they pay high premium.
investigate empirical characteristics of related data. Figure 1.1 shows the trends of key variables of housing, labor, and financial market. House price$^5$ started to stagnate from 2006:I and then fall by around 30% during the 2007-2009. Unemployment rate and delinquency rate dramatically rose from 4.5% and 2% respectively to around 10% both during the same period. Outstanding mortgage loan hit the peak in April 2008, which was 2 years later than the house price did, and thereafter has been decreasing (around 7% drop from the peak by 2011:II). Interestingly, all those variables are still much far from the pre-crisis levels even long after the official end of the recession (June 2009). Figure 1.2 presents the trends of growth rates of gdp, consumption, and investment. The key macro variables also sharply fell during the recession but had been recovered to the pre-crisis levels, at least almost, by 2011:II. Table 1.1 shows correlations over the business cycles among key variables. We detrended all variables by using Hodrick-Prescott filter to extract business cycle components from the time series with trends$^6$. During 1987:1$^7$ - 2011:II, unemployment rate is positively correlated with default rate (0.75) but negatively correlated with outstanding mortgage loan (-0.49$^8$). We admit high correlation does not directly imply the connection between two variables. In addition, robust empirical exercise to see whether unemployment is an important factor to affect default and the size of outstanding loan is beyond the scope of this paper but in this paper we try to investigate the role of the interaction between financial market and labor market, if there exists, within our theoretical model and we will see if the theoretical model can generate the comovements among key variables better during the Great Recession.

To consider the interaction in a theoretical DSGE framework, we borrow basic setup from Iacovillo (2011) [30] and change the model in three directions: First, we assume collateral constrained workers can borrow only when they are employed. Second, we also assume indebted workers went default if they lost job. Third, to consider unemployment explicitly in the model, we introduce job search and matching framework into our model. Now credit constrained workers will consider the financial effects of unemployment,

$^5$House price and outstanding mortgage loan are indexed to 100 in 1999:IV
$^6$Unemployment rate and default rate have unit-root in the level.
$^7$We can get delinquency rate data just from 1987:I.
$^8$If we focused on the period of Great Recession, correlation became much larger.
Figure 1.1: Trends of Key Variables

Figure 1.2: Trends of Growth Rates of GDP, Consumption, and Investment
default and no more loan, when they bargain with firms for wage bill. Iacoviello (2011) [30] has many merits to investigate the interaction under adverse housing preference shock since it has credit constrained workers, housing market, and most of all capital requirement constrained banking sector which is crucial to identify the role of default.

Under adverse housing preference shock, the baseline model without the interaction shows that lowered housing price suppresses investment, labor, and output through the conventional collateral constraint channel. By introducing the interaction into the baseline model, we find that the negative effect of adverse housing preference shock is significantly amplified through the interaction channel. First, credit constrained workers would require higher wage bill to work since the surplus from borrowing and the penalty from defaulting decrease under adverse housing preference shock. It implies that credit constrained workers’ unemployment increases more than before. Second, the number of workers who can borrow decreases so total loan to workers diminishes. Third, the number of default increases and it undermines the loan making ability of banking sector for both entrepreneurs and workers through capital requirement constraint. As a result, investment and output fall more when we consider the interaction. Furthermore, we find that each channel, default channel and no more loan channel, plays an independent and important role under the shock.

Our paper is linked to three strands of literatures. First, our work is related to literatures on the role of financial friction over labor market. Acemoglu (2001) [1] argues that the change in technologies have a persistent adverse effect on unemployment in a country with inefficient financial market since agents who need cash to start a new business cannot borrow necessary funds. Chugh (2009) [10] studies the interaction between financial friction and labor friction by introducing external finance premium into a standard search and matching model. He finds that aggregate TFP shock leads to large labor market fluctuations if the external finance premium is counter-cyclical. Those two papers focus on demand side of labor market in which firms (or entrepreneurs) face financial friction but we are interested in supply side in which credit constrained workers interacts with financial market. In our model, financial market condition directly affects workers’ decision in wage bargaining process and the workers’ decision in labor
market also directly change financial market condition through the changes in default rate and aggregate loan size.

Second, our paper is connected to literatures on default. Forlati and Lambertini (2011) [20] investigates the role of uncertainty in idiosyncratic housing price and argues that uncertainty increase in idiosyncratic housing price can lead credit crunch by increasing expected default and interest rate. Iacoviello (2011) [30] considers banking sector with capital requirement constraint and shows that exogenous repayment shock from collateral constrained borrowers could generate credit crunch by reducing the loan making ability of banking sector through the bankers’ capital requirement constraint. In Goodhart et al. (2009) [26], liquidity constrained agents can choose how much fraction of outstanding debt will be repaid. They argue that a model without default cannot predict the negative effects of technological shocks on financial stability in the short run. Our model is different from those default models in the sense that default is endogenously determined by job status but not by idiosyncratic housing price shock, borrowers’ strategic choice, or pure exogenous shock.

Third, our paper is associated with recent debates on the shift of the Beveridge Curve after the end of Great Recession. A Beveridge curve presents the inverse relationship between unemployment rate and job vacancy rate. After the end of the recession, job vacancy rate has risen by about 20% but unemployment rate has remained almost steady at very high level. Did Beveridge curve shift after the recession? Kocherlakota (2010) [35] argues that the mismatch in geography, skills, and demography is the main source of the change in the relationship between job vacancies rate and unemployment rate. Sahin et al. (2012) [44] empirically shows that the mismatch across industries and occupations accounts for 0.6 to 1.7 percentage points of the recent rise in the U.S. unemployment rate, whereas geographical mismatch plays no role. On the contrary, Elsby et al. (2011) [14] argues that the lackluster labor market recovery can be traced in large part to weakness in aggregate demand; only a small part seems attributable to increases in labor market frictions. Furlanetto and Groshenny (2011) [21] also shows that the current very high rate of unemployment reflects not change in matching efficiency but

---

9Pay back less than contractually agreed.
10strategic default.
insufficient aggregate demand, mainly caused by adverse financial factors and nominal rigidities. In this paper, we try to suggest another answer to the question from the supply side of labor. Lackluster housing demand and credit crunch in financial market might discourage credit constrained workers from maintaining or seeking job.

We would like to acknowledge Giovannini and Santoro (2011) [25] as an independent work from ours. Although research motivations and some modeling assumptions are different each other, a key assumption is very similar. They also consider a real business cycle model with labor friction and credit market friction in which borrowing is conditional on employment status. Our model might be different from theirs in that we have additional default channel of unemployment.

The rest of the paper is organized as follows. Section 2 describes a simple Unrestricted Vector Autoregression (VARs) model to see the empirical impulse responses of key variables to house price shock. In section 3, we present the DSGE model considering the interaction between financial market and labor market. In section 4, we describe wage bargaining process where credit constrained workers consider financial channels of unemployment. Section 5 reports calibration of parameters and theoretical impulse responses to adverse housing preference shock. Section 6 is concluding remarks. An appendix provides some additional details.
1.2 Unrestricted VARs Model

Before describing the model with housing preference shock and the interaction between financial market and labor market, let’s see first what data tells us. Granger-Causality tests and unrestricted VARs analysis is useful to document the key relationships in data and to compare the data to the model we will present in following sections.

Table 1.2: VAR Granger Causality/Block Exogeneity Wald Tests

<table>
<thead>
<tr>
<th>Dependent variable: House Price</th>
<th>Excluded</th>
<th>Chi-sq</th>
<th>df</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>4.78666</td>
<td>2</td>
<td>0.0913</td>
<td></td>
</tr>
<tr>
<td>UNEMP RATE</td>
<td>1.54639</td>
<td>2</td>
<td>0.4615</td>
<td></td>
</tr>
<tr>
<td>DELINQ RATE</td>
<td>1.35616</td>
<td>2</td>
<td>0.5076</td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>8.81134</td>
<td>6</td>
<td>0.1845</td>
<td></td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Dependent variable: GDP</th>
<th>Excluded</th>
<th>Chi-sq</th>
<th>df</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>HOUSE PRICE</td>
<td>12.71037</td>
<td>2</td>
<td>0.0017</td>
<td></td>
</tr>
<tr>
<td>UNEMP RATE</td>
<td>1.24805</td>
<td>2</td>
<td>0.5358</td>
<td></td>
</tr>
<tr>
<td>DELINQ RATE</td>
<td>5.41655</td>
<td>2</td>
<td>0.0667</td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>21.28491</td>
<td>6</td>
<td>0.0016</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Dependent variable: UNEMP RATE</th>
<th>Excluded</th>
<th>Chi-sq</th>
<th>df</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>HOUSE PRICE</td>
<td>8.66305</td>
<td>2</td>
<td>0.0131</td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>21.43708</td>
<td>2</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>DELINQ RATE</td>
<td>16.62359</td>
<td>2</td>
<td>0.0002</td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>80.15348</td>
<td>6</td>
<td>0.0000</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dependent variable: DELINQ RATE</th>
<th>Excluded</th>
<th>Chi-sq</th>
<th>df</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>HOUSE PRICE</td>
<td>8.79092</td>
<td>2</td>
<td>0.0123</td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>0.70285</td>
<td>2</td>
<td>0.7037</td>
<td></td>
</tr>
<tr>
<td>UNEMP RATE</td>
<td>11.78717</td>
<td>2</td>
<td>0.0028</td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>24.38718</td>
<td>6</td>
<td>0.0004</td>
<td></td>
</tr>
</tbody>
</table>

Our sample consists of quarterly data of housing price, gdp, unemployment rate, and delinquency rate from 1987:I to 2011:II. In the sample, all variables, even unemployment rate and delinquency rate, cannot reject the null hypothesis, having unit root, in level. So we need to detrend the variables to get stable time series. To do that, we take logarithm of housing price and gdp, and detrended all variables by using Hodrick-Prescott filter\textsuperscript{11} to extract business cycle components. All detrended data passed Augmented Dickey-Change in log of housing price and outstanding mortgage loan have unit-root.
Fuller unit-root test at 1% significance level\textsuperscript{12}.

\textless Table 1.2\textgreater presents the result of Granger Causality/Block Exogeneity Wald Tests. Housing price Granger-cause gdp (at 1% significance level), unemployment rate and delinquency rate (at 5% significance level). Other variables, however, Granger non-cause housing price even jointly. Unemployment rate and default rate Granger-cause each other at 1% significance level but Granger non-cause housing price and gdp. GDP Granger-cause unemployment rate also at 1% significance level.

Based on our purpose of analysis, the unrestricted VARs model is ordered\textsuperscript{13} 1) housing price, 2) gdp, 3) unemployment rate, and 4) delinquency rate. We choose 2 lag length according to Schwartz Information Criterion. \textless Figure 1.3\textgreater shows generalized impulse responses of variables to one standard deviation innovations. The generalized impulse responses seem to be consistent with our intuition. Housing price shock has positive effect on gdp but negative on unemployment rate and default rate. Unemployment rate innovation leads increase in default rate during 5 quarters. We will describe theoretical model in next section.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.3.png}
\caption{Impulse Responses to One Standard Deviation Innovation of Housing Price}
\end{figure}

\textsuperscript{12}Outstanding mortgage loan cannot pass the test even at 10% significance level so we remove it from Granger-Causality test and unrestricted VARs analysis.

\textsuperscript{13}Granger non-causality is not necessary or sufficient condition for the exogeneity of a variable.
1.3 The Model

Our model is based on Iacoviello (2011) [30]. We will introduce labor market friction and financial channels of unemployment into the basic model. For labor market friction, we refer to Hertweck (2010) [28]. We have 5 agents in the model: patient households, subprimers, entrepreneurs, bankers, and firms. We will use subscript \( p, s, e, \) and \( b \) for patient households, subprimers, entrepreneurs, and bankers respectively.

1.3.1 Labor Market Friction

To consider unemployment explicitly, we introduce job search and matching model\(^{14}\) of Mortensen and Pissarides (1994) [37]. In the job search and matching model, it’s costly for workers and firms to be matched and wages are determined by bilateral bargaining. For simplicity, let’s normalize the number of workers be equal to 1. If \( n_t \) denote the number of employed workers, \( 1 - n_t \) would be the number of unemployed workers. In a standard job search and matching model, firms try to fill their vacant positions by posting job opportunity, \( v_t \), and unemployed workers, \( 1 - n_t \), try to find job by paying constant search cost, \( e \). The number of meetings between firms and unemployed workers is determined by the exogenously given Cobb-Douglas type matching function as follows:

\[
m_t(v_t, 1 - n_t) = \eta v_t^\theta [e(1 - n_t)]^{1-\theta}
\]

where \( \eta \) is matching technology and constant. Matching function, \( m_t(\cdot, \cdot) \), is generally assumed to be continuous, non-negative, increasing in both arguments and concave. By matching function, it takes time for firms and the unemployed to meet.

For convenience, let’s define labor market tightness, \( \gamma_t \), as follows:

\[
\gamma_t = \frac{v_t}{1 - n_t}.
\]

Then, we can define vacancy filling rate, \( q(\gamma_t) \), and job finding rate, \( f(\gamma_t) \), as a function

\(^{14}\) Rogerson, Shimer, and Wright (2004) [43] well surveys the job search and matching theory.
of the market tightness:

\[ q(\gamma_t) \equiv \frac{m_t}{v_t} = \eta e^{\theta \gamma_t \theta - 1}, \]

\[ f(\gamma_t) \equiv \frac{m_t}{1 - n_t} = \eta e^{1-\theta \gamma_t \theta}. \]

As standard job search and matching model does, we assume that vacancy filling rate is given for individual firm and job finding rate is given for individual worker. In other words, individual firms and workers are small enough not to change the market vacancy filling rate and job finding rate respectively.

Now we can rewrite the matching function as a function of vacancy filling rate or job finding rate as follows:

\[ m_t = q(\gamma_t)v_t, \]

\[ m_t = f(\gamma_t)(1 - n_t). \]

If we assume job separation rate, \( \varphi \), is constant, the law of motion for the number of employed workers is given by:

\[ n_{t+1} = (1 - \varphi)n_t + m_t, \]

\[ = (1 - \varphi)n_t + q(\gamma_t)v_t, \]

\[ = (1 - \varphi)n_t + f(\gamma_t)(1 - n_t). \]

Finally, for simplicity, we assume there are two different labor markets for patient households and subprimers respectively\(^{15}\).

1.3.2 Patient Households

Let’s normalize the number of patient households to 1. If \( n_{p,t} \) denote the share of the employed among patient households, \( 1 - n_{p,t} \) would be unemployment rate of patient workers and impatient workers (subprimers) have no difference in their skills. Furthermore, although they have different financial positions, firms don’t know whether an individual worker belongs to patient group or not. So patient workers and impatient workers are perfectly substitutable in real world.

\(^{15}\)Patient workers and impatient workers (subprimers) have no difference in their skills. Furthermore, although they have different financial positions, firms don’t know whether an individual worker belongs to patient group or not. So patient workers and impatient workers are perfectly substitutable in real world.
tient households. Patient households have logarithmic utility function over consumption goods, \( c_{p,t} \), housing, \( h_{p,t} \), and leisure when they are employed, \( 1 - l_{p,t} \). If they are unemployed, they have to pay constant job search cost \( e \). When they are employed, they supply labor, \( l_{p,t} \), to firms for real wage, \( w_{p,t} \). They save, \( b_{p,t} \), to bankers at interest rate, \( R_{p,t} \). For simplicity, we assume patient households do not accumulate capital, which is some deviation from Iacoviello (2011)\textsuperscript{16} [30]. As a standard job search and matching model does, we assume the family of patient households offers insurance against idiosyncratic shocks for individual patient households. Because of the insurance, every patient households enjoy same amount of consumption and housing regardless of individual household’s job status.

Patient households solve following utility maximization problem:

\[
\max_{c_{p,t}, h_{p,t}, b_{p,t}} E_0 \sum_{t=0}^{\infty} \beta_p^t [\ln c_{p,t} + j_t \ln h_{p,t} + n_{p,t} \tau \ln (1 - l_{p,t}) + (1 - n_{p,t}) \tau \ln (1 - e)]
\]

subject to

\[
\begin{align*}
c_{p,t} + q_t (h_{p,t} - h_{p,t-1}) + b_{p,t} &= R_{p,t-1} b_{p,t-1} + n_{p,t} w_{p,t} l_{p,t} \\
n_{p,t+1} &= (1 - \varphi)n_{p,t} + f(\gamma_{p,t})(1 - n_{p,t})
\end{align*}
\]

(1.1)

(1.2)

where \( \beta_p \) is discount factor of patient households, \( q_t \) is housing price, \( \tau \) is a parameter to control relative preference for leisure. \( j_t \) controls relative preference for housing against consumption and has AR(1) process as follows:

\[
\ln j_t = (1 - \rho_j) \ln j + \rho_j \ln j_{t-1} + \epsilon_{j,t}, \quad \epsilon_{j,t} \sim i.i.d. (0, \sigma_j^2)
\]

(1.3)

where \( j \) is steady state value of \( j_t \) and \( \rho_j \) is persistency parameter of the shock. \( \epsilon_{j,t} \) is i.i.d. stochastic error term. Equation (1.2) is the law of motion for the number of the employed patient households. Individual patient household takes market job finding rate, \( f(\gamma_{p,t}) \), and wage bill, \( w_{p,t} l_{p,t} \), as given. The wage bill will be determined by bilateral bargaining with firms.

\textsuperscript{16}Iacoviello (2011) assumes both patient households and entrepreneurs accumulate capital for production.
Patient households’ first order conditions are given by:

\[
\frac{q_t}{c_{p,t}} = \frac{j_t}{h_{p,t}} + \beta_p E_t \left( \frac{q_{t+1}}{c_{p,t+1}} \right), \tag{1.4}
\]

\[
\frac{1}{c_{p,t}} = \beta_p E_t \left( \frac{R_{p,t}}{c_{p,t+1}} \right). \tag{1.5}
\]

By the Envelope theorem, we can find patient households’ surplus from marginal matching as follows:

\[
W_1(n_{p,t}) = \frac{1}{c_{p,t}} w_{p,t} l_{p,t} + (1 - \varphi) \beta_p E_t W_1(n_{p,t+1}) - \left[ \tau \ln(1 - e) - \tau \ln(1 - l_{p,t}) + f(\gamma_{p,t}) \beta_p E_t W_1(n_{p,t+1}) \right]
\]

where \(W_1(n_{p,t})\) is the first differential of patient households’ value function, \(W(n_{p,t})\) with respect to \(n_{p,t}\) and denotes patient households’ surplus from marginal matching. The surplus from marginal matching is the difference between the value of employment, which is the sum of utility from wage bill and expected present value of future matching, and the value of unemployment, which is the sum of utility from leisure and expected present value of future matching.

### 1.3.3 Subprimers

Let’s normalize the number of subprimers to 1. If \(n_{s,t}\) denotes the share of the employed among subprimers, \(1 - n_{s,t}\) would be unemployment rate of subprimers. Subprimers also have logarithmic utility function over consumption goods, \(c_{s,t}\), housing, \(h_{s,t}\), and leisure when they are employed, \(1 - l_{s,t}\). They have to pay job searching cost \(e\) when they are unemployed. They supply labor, \(l_{s,t}\), to firms for real wage, \(w_{s,t}\), when they are employed. In contrast to patient households, they borrow, \(b_{s,t}\), at interest rate, \(R_{s,t}\) from bankers. Subprimers are collateral constrained so the amount of loan is determined by some fraction of expected future value of their own housing. Subprimers family also offers insurance against idiosyncratic shock for individual subprimers so all subprimers enjoy same amount of consumption and housing regardless of individual subprimer’s job status.
To consider financial channels of unemployment, we add two more assumptions. First, subprimers can borrow only when they are employed. Therefore, the total amount of loan depends on employment rate as well as the expected future value of collateral. If unemployment rate increases, total amount of loan which subprimers can borrow decreases. Second, the family insurance does not cover the loan repayment of the unemployed. So unemployed subprimers with outstanding loan go default. If they go default, they don’t need to repay outstanding loan but lose the collateral regardless of its value. Since subprimers lose collateral even with positive equity by going default, subprimers have incentive to avoid unemployment when they have debt.

Subprimers solve following utility maximization problem:

$$\max_{c_{s,t}, h_{s,t}, b_{s,t}} \sum_{t=0}^{\infty} \beta_s^t \left[ \ln c_{s,t} + j_t \ln h_{s,t} + n_{s,t} \tau \ln (1-l_{s,t}) + (1-n_{s,t}) \tau \ln (1-e) \right]$$

subject to

$$c_{s,t} + q_t (h_{s,t} - h_{s,t-1} + \left[ n_{s,t}^\phi + (1-n_{s,t}^\phi) \frac{1}{m_s} \frac{q_t}{q_t^e} \right] R_{s,t-1} n_{s,t-1}^\chi b_{s,t-1}$$

$$= n_{s,t}^\chi b_{s,t} + n_{s,t} w_{s,t} l_{s,t} + t_t$$

(1.6)

$$R_{s,t} b_{s,t} \leq m_s E_t (q_{t+1} h_{s,t})$$

(1.7)

$$n_{s,t+1} = (1-\phi) n_{s,t} + f(\gamma_{s,t})(1-n_{s,t})$$

(1.8)

where $\beta_s$ is discount factor of subprimers, $m_s$ is Loan-to-Value (LTV) ratio against housing, and $q_t^e = E_{t-1}(q_t)$. An individual subprimer takes market job finding rate, $q(\gamma_{s,t})$, and wage bill, $w_{s,t} l_{s,t}$, as given. Equation (1.8) is the law of motion for the number of the employed subprimers.

$\chi$ and $\phi$ controls the financial channels of unemployment. If $\chi = 0$, job status does not affect borrowing so even unemployed can borrow. If $\phi = 0$, job status does not affect repayment of debt so even unemployed pay back all outstanding debt. When $\chi = \phi = 0$, our model would be reduced to Iacoviello (2011) [30] with just unemployment. $t_t$ is lump sum transfer from bankers. We assume bankers pay recovery cost, $\omega$, to sell foreclosed collateral and subprimers receive the recovery cost in lump sum from bankers.

Equation (1.7) captures the idea of collateral constraint. Individual debt repayment, $R_{s,t} b_{s,t}$, at next period should be less ($0 < m_s < 1$) than the expected next period value of housing, $E_t (q_{t+1} h_{s,t})$. 

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Subprimers’ first order conditions are given by:

\[
\frac{q_t}{c_{s,t}} = \frac{\dot{n}_t}{n_{s,t}} + \lambda_{s,t} m_s E_t \left( q_{t+1} \right) + \beta_h E_t \left( \frac{q_{t+1}}{c_{s,t+1}} \right) , \quad (1.9)
\]

\[
\frac{1}{c_{s,t}} = \lambda_{s,t} \frac{R_{s,t}}{n_{s,t}} + \beta_i E_t \left\{ \frac{1}{c_{s,t+1}} \left[ \phi_{s,t} + (1 - n_{s,t+1}) \frac{1}{m_s} \right] R_{s,t} \right\} , \quad (1.10)
\]

where \(\lambda_{s,t}\) is a Lagrangian multiplier for collateral constraint.

By the Envelope theorem, we can find subprimers’ surplus from marginal matching as follows:

\[
W_1(n_{s,t}) = \frac{1}{c_{s,t}} w_{s,t} I_{s,t} + (1 - \phi) \beta_i E_t W_1(n_{s,t+1})
- \left[ \tau \ln(1 - e) - \tau \ln(1 - l_{s,t}) + f(\gamma_{s,t}) \beta_i E_t W_1(n_{s,t+1}) \right]
+ \frac{1}{c_{s,t}} \left[ \chi n_{s,t-1}^{\chi-1} b_{s,t} + \phi n_{s,t-1}^{\phi-1} \left( \frac{1}{m_s q_{t}^{e}} - 1 \right) R_{s,t-1} n_{s,t-1}^{\phi} b_{s,t-1} \right]
- \beta_i E_t \left\{ \frac{1}{c_{s,t+1}} \left[ \phi_{s,t} + (1 - n_{s,t+1}) \frac{1}{m_s} \right] R_{s,t} b_{s,t} \chi n_{s,t-1}^{\chi-1} \right\}
\]

where \(W_1(n_{s,t})\) is the first differential of subprimers’ value function, \(W(n_{s,t})\), with respect to \(n_{s,t}\) and denotes subprimers’ surplus from marginal matching. Subprimers’ surplus from marginal matching is also the difference between the value of employment and the value of unemployment. Interestingly, subprimers’ value of employment has additional term, the surplus from marginal borrowing. If a subprimer could get job, it could borrow and save house if it has outstanding loan. That’s the gain from marginal borrowing. In next period, it should repay outstanding loan if it would not be separated from the job or it would lose housing if it would lose the job. That’s the cost from marginal borrowing. The additional term comes from the financial channels of unemployment we assume. It implies that institutional setup of financial market could affect labor market. When \(\chi = \phi = 0\), the surplus from marginal borrowing disappears.

### 1.3.4 Entrepreneurs

Entrepreneurs have preferences over only consumption goods, \(c_{e,t}\). They do not work but accumulate capital, \(k_t\), and housing, \(h_{e,t}\), for rental profit \(r_t^k\) and \(q_t r_t^h\) respectively. Entrepreneurs also borrow, \(b_{e,t}\), from bankers at interest rate, \(R_{e,t}\), and they are
collateral constrained against their own housing and capital.

Entrepreneurs solve following utility maximization problem:

$$\max_{c_{e,t}, h_{e,t}, k_{e,t}, b_{e,t}} E_0 \sum_{t=0}^{\infty} \frac{1}{\beta_t} (\ln c_{e,t})$$

subject to

$$c_{e,t} + q_t (h_{e,t} - h_{e,t-1}) + k_t - (1 - \delta)k_{t-1} + R_{e,t}b_{e,t-1} = b_{e,t} + r^k_t k_{t-1} + r^h_t q_t h_{e,t-1},$$

$$R_{e,t}b_{e,t} \leq m^k_e k_t + m^h_e E_t (q_{t+1} h_{e,t}) ,$$

where $\beta_t$ is discount factor of entrepreneurs, $m^k_e$ and $m^h_e$ are LTV ratios against capital and housing respectively, and $\delta$ is depreciation rate for capital.

Equation (1.12) captures the idea of collateral constraint which entrepreneurs face. Total repayment of debt, $R_{e,t}b_{e,t}$, at next period should be less ($0 < m^k_e, m^h_e < 1$) than the expected next period value of collaterals, $E_t (q_{t+1} h_{e,t}) + k_t$.

Entrepreneurs’ first order conditions are given by:

$$\frac{1}{c_{e,t}} = \lambda_{e,t} m^k_e + \beta_e E_t \left( \frac{1}{c_{e,t+1}} \left( r^k_t + 1 - \delta \right) \right) ,$$

$$\frac{q_t}{c_{e,t}} = \lambda_{e,t} m^h_e E_t (q_{t+1}) + \beta_e E_t \left( \frac{q_{t+1}}{c_{e,t+1}} \right) ,$$

$$\frac{1}{c_{e,t}} = \lambda_{e,t} R_{e,t} + \beta_e E_t \left( \frac{R_{e,t+1}}{c_{e,t+1}} \right) ,$$

where $\lambda_{e,t}$ is a Lagrangian multiplier for collateral constraint.

1.3.5 Bankers

Bankers have preferences over only consumption goods, $c_{b,t}$. They do not work but make profit by receiving deposit from patient households and making loan to subprimers and entrepreneurs. Bankers are constrained by capital adequacy regulation. They also face recovery cost, $0 < \omega < 1$, to sell foreclosed collateral.

\[\text{The Basel Accords (Basel II), published by the Basel Committee on Banking Supervision housed at the Bank for International Settlements, requires that the percentage of a bank’s capital to risk-weighted assets must be no lower than 8%.}\]
Bankers solve following utility maximization problem:

$$\max_{c_{b,t},b_{p,t},b_{s,t},b_{e,t}} E_0 \sum_{t=0}^{\infty} \beta_t^t (\ln c_{b,t})$$

subject to

$$c_{b,t} + R_{p,t-1} b_{p,t-1} + n_{s,t}^s b_{s,t} + b_{e,t}$$

$$= b_{p,t} + \left[ n_{s,t}^s + (1-\omega)\left(1 - n_{s,t}^s\right) \frac{1}{m_s q_t} \right] R_{s,t-1} n_{s,t-1}^s b_{s,t-1} + R_{e,t} b_{e,t-1},$$

(1.16)

$$b_{p,t} \leq \gamma_e b_{e,t} + \gamma_s n_{s,t}^s b_{s,t},$$

(1.17)

where $\beta_b$ is discount factor of bankers.

Equation (1.17) captures the idea of capital adequacy regulation. Deposit, $b_{p,t}$, should be less than some fraction, $0 < \gamma_e, \gamma_s < 1$, of total assets, $b_{e,t} + n_{s,t}^s b_{s,t}$. In other words, bankers should accumulate its own capital to extend loan. $\gamma_e$ and $\gamma_s$ control riskiness of loans to subprimers and entrepreneurs respectively.

Bankers’ first order conditions are given by:

$$\frac{1}{c_{b,t}} = \lambda_{b,t} + \beta_b E_t \left( \frac{R_{p,t}}{c_{b,t+1}} \right),$$

(1.18)

$$\frac{1}{c_{b,t}} = \lambda_{b,t} \gamma_s + \beta_b E_t \left\{ \frac{1}{c_{b,t+1}} \left[ n_{s,t+1}^s + (1-\omega)(1 - n_{s,t+1}^s) \frac{1}{m_s q_t^t} \right] R_{s,t} \right\},$$

(1.19)

$$\frac{1}{c_{b,t}} = \lambda_{b,t} \gamma_e + \beta_b E_t \left( \frac{R_{e,t+1}}{c_{b,t+1}} \right),$$

(1.20)

where $\lambda_{b,t}$ is a Lagrangian multiplier for capital adequacy constraint.

In steady state, if there is no default (i.e., $\phi = 0$), loan rate for subprimers is equal to that for entrepreneurs. Although $\phi \neq 0$, two loan rates become same if $1 - \omega = m_s$. The recovery cost gives us some intuition for Loan-to-Value (LTV) ratio. By setting $m_s \leq 1 - \omega$, bankers could avoid loss from default at least in steady state. By assumption, the recovery cost is transferred to subprimers in lump sump.

1.3.6 Firms

Firms are perfectly competitive in goods market. They use capital, $k_t$, labor, $n_{p,t} l_{p,t}, n_{s,t} l_{s,t}$, and housing, $h_{e,t}$, as inputs to produce consumption goods, $y_t$, and pay
for the use of inputs respectively. We assume firms face quadratic labor adjustment cost\(^{18}\) and firms’ production technology is Cobb-Douglass. To normalize the number of patient households and subprimers to 1 each, we assume labors of patient households and subprimers are complements instead of perfect substitutes.\(^{19}\)

We will control the relative size of each type of workers by \(\sigma\) in production function. As standard job search and matching model does, we also assume that firms are large enough to eliminate all uncertainty in \(n_{p,t+1}\) and \(n_{s,t+1}\) but small enough to take market vacancy filling rates and wage bills as given.

A representative firm choose \(k_{t-1}, h_{e,t-1}, v_{p,t}, v_{s,t}, n_{p,t+1}\), and \(n_{s,t+1}\) to solve following profit maximization problem:

\[
\text{max } E_0 \sum_{t=0}^{\infty} \left[ y_t - \left( r^k k_{t-1} + r^h q_{t} h_{e,t-1} + n_{p,t} w_{p,t} l_{p,t} + n_{s,t} w_{s,t} l_{s,t} + \frac{\kappa m_{p,t}^2}{2 n_{p,t}} + \frac{\kappa m_{s,t}^2}{2 n_{s,t}} \right) \right]
\]

subject to

\[
y_t = z_t k_{t-1}^{\mu} h_{e,t-1}^{\nu} \left[ (n_{p,t} l_{p,t})^{1-\sigma} (n_{s,t} l_{s,t})^{\sigma} \right]^{1-\mu-\nu}
\]

\[
n_{p,t+1} = (1 - \phi) n_{p,t} + q(\gamma_{p,t}) v_{p,t}
\]

\[
n_{s,t+1} = (1 - \phi) n_{s,t} + q(\gamma_{s,t}) v_{s,t}
\]

where \(\mu\), \(\nu\), and \(1 - \mu - \nu\) denote capital, housing, and labor share of output respectively. Patient households take \(1 - \sigma\) and subprimers take \(\sigma\) from the labor share of output, \(1 - \mu - \nu\). So we can say that higher \(\sigma\) implies higher share of subprimers in total households. \(z_t\) is exogenous technology shock and has AR(1) process as follows:

\[
\ln z_t = (1 - \rho_z) \ln z + \rho_z \ln z_{t-1} + \epsilon_{z,t}, \quad \epsilon_{z,t} \sim i.i.d. (0, \sigma_z^2)
\]

where \(z\) is steady state value of \(z_t\) and \(\rho_z\) is a persistency parameter of the shock. \(\epsilon_{z,t}\) is i.i.d. stochastic error term.

\(^{18}\)Hertweck (2010) \([28]\) use \(\frac{\kappa m^2}{2 n_t}\) for the labor adjustment cost instead of standard \(\kappa v_t\). In this setup, firms’ labor adjustment cost is determined by the number of filled position instead of the number of posted position.

\(^{19}\)We could use \(y_t = z_t k_{t-1}^{\mu} h_{e,t-1}^{\nu} \left[ (1 - \sigma) n_{p,t} l_{p,t} + \sigma n_{s,t} l_{s,t} \right]^{1-\mu-\nu}\) as a production technology. In that case, we should keep track of relative size of patient households and subprimers.
For convenience, let’s define the gross hiring rate \( x_{p,t} \) and \( x_{s,t} \) as follows:

\[
x_{p,t} = \frac{m_{p,t}}{n_{p,t}},
\]
\[
x_{s,t} = \frac{m_{s,t}}{n_{s,t}}.
\]

A representative firm’s first order conditions are given by:

\[
\mu \frac{y_t}{k_{t-1}} = r^k_t,
\]
\[
\nu \frac{y_t}{h_{e,t-1}} = r^h_t q_t,
\]
\[
\kappa x_{p,t} = \beta_e E_t \left\{ \Lambda_{e,t} \left[ (1 - \sigma)(1 - \mu - \nu) \frac{y_{t+1}}{n_{p,t+1}} - w_{p,t+1} l_{p,t+1} + \frac{\kappa}{2} x_{p,t+1}^2 + \kappa x_{p,t+1} (1 - \varphi) \right] \right\},
\]
\[
\kappa x_{s,t} = \beta_e E_t \left\{ \Lambda_{e,t} \left[ \sigma (1 - \mu - \nu) \frac{y_{t+1}}{n_{s,t+1}} - w_{s,t+1} l_{s,t+1} + \frac{\kappa}{2} x_{s,t+1}^2 + \kappa x_{s,t+1} (1 - \varphi) \right] \right\},
\]

where \( \Lambda_{e,t} = c_{e,t}/c_{e,t+1} \).

By the Envelope theorem, we can derive firms’ surplus from marginal matching for each type of workers as follows:

\[
V_1(n_{p,t}) = (1 - \sigma)(1 - \mu - \nu) \frac{y_t}{n_{p,t}} - w_{p,t} l_{p,t} + \frac{\kappa}{2} x_{p,t}^2 + (1 - \varphi) \beta_e E_t [\Lambda_{e,t} V_1(n_{p,t+1})],
\]
\[
V_2(n_{s,t}) = \sigma (1 - \mu - \nu) \frac{y_t}{n_{s,t}} - w_{s,t} l_{s,t} + \frac{\kappa}{2} x_{s,t}^2 + (1 - \varphi) \beta_e E_t [\Lambda_{e,t} V_2(n_{s,t+1})],
\]

where \( V_1(n_{p,t}) \) and \( V_2(n_{s,t}) \) are the first differentials of firms’ value function \( V(n_{p,t}, n_{s,t}) \) with respect to \( n_{p,t} \) and \( n_{s,t} \) respectively and denote the surplus of marginal matching for each type of workers. From additional matching, firms can produce more and save labor adjustment cost but have to pay wage bill.
1.3.7 Market Clearing Conditions

We assume supply of housing is fixed and normalized to 1. Thus, housing market clearing condition will be:

$$h_{p,t} + h_{s,t} + h_{e,t} = 1.$$  
(1.31)

Aggregate resource constraint is as follows:

$$y_t = c_{p,t} + c_{s,t} + c_{e,t} + c_{b,t} + k_t - (1 - \delta)k_{t-1} + \frac{\kappa m_{p,t}^2}{2 n_{p,t}} + \frac{\kappa m_{s,t}^2}{2 n_{s,t}}.$$
1.4 Wage Bargaining

1.4.1 Optimal Wage Bill

In standard job search and matching model, optimal wage bill is generally determined by solving the Nash bargaining problem. Nash (1950) [33] shows that there is unique solution to a bargaining problem if the problem satisfies following 4 axioms: (i) Invariance to Equivalent Utility Representations, (ii) Symmetry, (iii) Independence of Irrelevant Alternatives, and (iv) Pareto Efficiency.

We have to solve the following Nash bargaining problems to get optimal wage bills for patient households and subprimers:

$$w_{p,t} = \arg \max_{w_{p,t}} \left[ c_{p,t} W_1(n_{p,t}) \right]^{1-\xi} V_1(n_{p,t})^\xi,$$

$$w_{s,t} = \arg \max_{w_{p,t}} \left[ c_{s,t} W_1(n_{s,t}) \right]^{1-\xi} V_2(n_{s,t})^\xi,$$

where $\xi \in (0, 1)$ is the firms’ bargaining power. The solutions to the maximization problems satisfy

$$(1 - \xi) V_1(n_{p,t}) = \xi c_{p,t} W_1(n_{p,t}),$$

$$(1 - \xi) V_1(n_{p,t}) = \xi c_{p,t} W_1(n_{p,t}).$$

From the equation (1.32) and (1.33), we can find optimal wage bills for patient households and subprimers.

$$w_{p,t} = \left[ (1 - \sigma) (1 - \mu - \nu) \frac{y_t}{n_{p,t}} + \frac{\kappa}{2} x_{p,t} \right] - \xi c_{p,t} \left[ \tau \ln(1 - l_{p,t}) - \tau \ln(1 - e) \right]$$

$$+ (1 - \xi)(1 - \varphi) \beta_p E_t \left[ \Lambda_{e,t} V_1(n_{p,t+1}) \right] - \xi c_{p,t} \beta_p E_t \left\{ W_1(n_{p,t+1}) [(1 - \varphi) - f(\gamma_{p,t})] \right\},$$

$$(1.34)$$

\[\text{Nash (1950) assumes two players have same bargaining power, i.e., } \xi = 1/2. \text{ If we drop (ii) symmetry axiom, i.e., assume } \xi \in (0, 1), \text{ we can show there is unique solution satisfying other 3 axioms. See Orsborne and Rubinstein (1990) [39] for details.}\]
where $\Lambda_{s,t} = c_{s,t}/c_{s,t+1}$. Optimal wage bills have positive relationship with marginal productivity of labor and the value of leisure. The patient households’ optimal wage bill is standard. However, subprimers’ optimal wage bill has additional terms, which are related to the financial channels of unemployment. Subprimers’ optimal wage bill has negative relationship with the surplus from borrowing. If the surplus from borrowing is positive, it implies that subprimers would accept lower wage bill to access to financial market and avoid default. Again when $\chi = \phi = 0$, the subprimers’ optimal wage bill is not affected by the status in financial market.

### 1.4.2 Optimal Labor Effort

Standard job search and matching model generally assumes fixed labor effort. In that case, the wage bargaining process can pin down the wage. In our model, however, labor effort is time varying so the wage bargaining process just determines the multiplication of wage and labor effort but not individual wage and labor effort. To determine wage and labor effort, we assume firms and workers choose optimal labor effort to maximize mutual surplus which is the sum of both parties’ surpluses from marginal matching. Following Hertweck (2010) [28], if the marginal product of labor effort is given for both parties, optimal labor efforts should satisfies following conditions:

\[
(1 - \sigma)(1 - \mu - \nu) \frac{y_t}{n_{s,t+1}} = c_{s,t}\frac{1}{1 - l_{s,t}},
\]

\[
\sigma(1 - \mu - \nu) \frac{y_t}{n_{s,t+1}} = c_{s,t}\frac{1}{1 - l_{s,t}}.
\]
1.5 Results

1.5.1 Calibration

Since our model is based on Iacoviello (2011) [30], we basically follow the calibration of it. <Table 1.3> lists calibrations for model parameters in detail.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_p$</td>
<td>0.9925</td>
<td>Discount factor of patient households</td>
</tr>
<tr>
<td>$\beta_s$</td>
<td>0.9600</td>
<td>Discount factor of subprimers</td>
</tr>
<tr>
<td>$\beta_e$</td>
<td>0.9600</td>
<td>Discount factor of entrepreneurs</td>
</tr>
<tr>
<td>$\beta_b$</td>
<td>0.9650</td>
<td>Discount factor of bankers</td>
</tr>
<tr>
<td>$\tau$</td>
<td>2.0000</td>
<td>Weight of labor in utility function</td>
</tr>
<tr>
<td>$j$</td>
<td>0.0800</td>
<td>Weight of housing in utility function</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.3000</td>
<td>Output share of capital</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.0500</td>
<td>Output share of housing</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.3000</td>
<td>Income share of subprimers</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.0350</td>
<td>Depreciation rate</td>
</tr>
<tr>
<td>$m_s$</td>
<td>0.9000</td>
<td>Loan to value ratio for subprimers’ housing</td>
</tr>
<tr>
<td>$m_e$</td>
<td>0.9000</td>
<td>Loan to value ratio for entrepreneurs’ capital</td>
</tr>
<tr>
<td>$m_e$</td>
<td>0.9000</td>
<td>Loan to value ratio for entrepreneurs’ housing</td>
</tr>
<tr>
<td>$\gamma_s$</td>
<td>0.9000</td>
<td>Parameter to control banker’s capital-asset ratio</td>
</tr>
<tr>
<td>$\gamma_e$</td>
<td>0.9000</td>
<td>Parameter to control banker’s capital-asset ratio</td>
</tr>
<tr>
<td>$\chi$</td>
<td>1.0000</td>
<td>Parameter to control access to financial market</td>
</tr>
<tr>
<td>$\phi$</td>
<td>1.0000</td>
<td>Parameter to control default</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.1000</td>
<td>Bankers’ recovery cost of foreclosed collateral</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>7.5000</td>
<td>Parameter for labor adjustment cost</td>
</tr>
<tr>
<td>$e$</td>
<td>0.0833</td>
<td>Job search cost</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.5000</td>
<td>Bargaining power of firms</td>
</tr>
<tr>
<td>$\eta$</td>
<td>1.1305</td>
<td>Matching technology</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.5000</td>
<td>Parameter for matching function</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>0.0250</td>
<td>Job separation rate</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>0.9500</td>
<td>Technology shock persistence</td>
</tr>
<tr>
<td>$\rho_j$</td>
<td>0.8500</td>
<td>Housing Preference shock persistence</td>
</tr>
</tbody>
</table>

To make collateral constraints always bind, we need different discount factors satisfying $\beta_p > \beta_b > \beta_s, \beta_e$. We set $\beta_p = 0.9925$, $\beta_b = 0.965$, and $\beta_s = \beta_e = 0.96$. The output shares of capital, $\mu$, and housing, $\nu$, are set by 0.30 and 0.05 respectively. Then, the output share of labor income is $1 - \mu - \nu$. Subprimers’ share of labor income, $\sigma$, is set by 0.30. The weights of labor and housing in utility function are 2.00 and 0.08 respectively. Depreciation rate, $\delta$, is assumed to be 0.035. For the parameters to control
overall leverage, we set \( m_s = m^k_e = m^h_e = \gamma_s = \gamma_e = 0.90 \).

To investigate the role of the financial channels of unemployment, we set \( \chi = \phi = 1 \) as baseline scenario and we compare the baseline scenario to the following two different scenarios: (i) \( \chi = 1 \) and \( \phi = 0 \) and (ii) \( \chi = \phi = 0 \). Under baseline scenario, unemployed subprimers cannot both borrow and repay. Under scenario (i), the unemployed cannot borrow but can repay since family insurance covers the repayment regardless of job status. Scenario (ii) considers the case of no financial channels of unemployment. In scenario (ii), the unemployed can both borrow and repay.

Just before the recent crisis (early 2007), the unemployment rate of US was around 4.5%. We set the parameter for labor adjustment cost, \( \kappa \), as 7.5 to make the unemployment rates of patient households and subprimers match with the data. Shimer (2005) [46] estimates the quarterly job separation rate as around 2.5% in mid 2000s. We set the job separation rate, \( \varphi \) as 0.025 to match the empirical result. We assume firms and workers have same bargaining power, i.e., \( \xi = 0.50 \). For other parameters of labor market condition, we follows Hertweck (2010) [28]. Job searching cost, \( e \), is 0.0833, matching technology, \( \eta \), is 1.1305, and parameter in matching function, \( \theta \), is 0.50.

We assume bankers’ recovery cost, \( \omega \), is 0.10 to make loan rates for subprimers and entrepreneurs same in steady state. In steady state, bankers has no loss from default since they can recover all cost from the default by selling collateral. In other words, default would not affect bankers’ optimization problem in steady state. However, subprimers would consider the loss from default since they lose collateral although the loss could be recovered by lump sum transfer from bankers. Therefore, even in steady state, default would affect labor market through wage bargaining process.

For exogenous housing preference shock, we follows Iacoviello (2005) [29]. The persistency parameter of housing preference shock, \( \rho_j \), is set by 0.85. For the persistency parameter of exogenous technology shock, \( \rho_z \), we use generally accepted value, 0.95.\(^{21}\)

\(^{21}\)Iacoviello (2011) [30] has only repayment shock. Iacoviello (2005) [29] use 0.03 for \( \rho_z \) which is estimated from the model.
1.5.2 Adverse Housing Preference Shock

To see the role of the interaction between financial market and labor market over the business cycle, we make 50 percentage point adverse shock in housing preference. The adverse shock generates around 1.5 percentage point fall in housing price. When the adverse housing preference shock hits the economy, housing demand of patient households and subprimers decreases. Given fixed supply of aggregate housing, lowered demand leads to sharp drop in housing price. Under lowered housing price, subprimers cannot roll over some fraction of outstanding loan because the value of collateral falls. It implies subprimers’ housing demand shifts down more than patient households’ does. As a result, subprimers decrease housing but patient households increase housing because of the lowered housing price. The distributional change in housing ownership affects the consumption of patient households and subprimers. Subprimers increase consumption by selling housing but patient households’ consumption would be suppressed by acquiring additional housing.

The adverse housing preference shock also affects financial market though the collateral constraint channel. The lowered housing price and subprimers’ selling housing make loan to subprimers fall sharply which leads to decrease in total demand for loan. Saving and borrowing rate decreases simultaneously and the lowered saving rate leads to decrease in patient households’ saving, which supports the patient households’ consumption.

Entrepreneurs should reduce consumption since they should repay some fraction of outstanding loan because the value of collateral, housing price, falls. To smooth consumption, they reduce capital accumulation but increase cost effective housing. Bankers also reduce consumption because of credit crunch in financial market which deteriorates bankers’ profit.

In labor market, unemployment rates of both type of workers increase since lowered marginal productivity of labor suppresses firms’ surplus from marginal matching and increased workers’ consumption make people ask higher wage bills to work. In equilibrium, optimal wage bills from wage bargaining decreases.

On balance, adverse housing preference shock leads to fall in housing price, loans,
investment, employment rate, and output. The results are consistent with the data during recent recession. However, the effect of the adverse shock on individual consumption varies across agents. In aggregate, subprimers’ increased consumption dominates, as a result aggregate consumption increases\textsuperscript{22} at the impact of the adverse housing preference shock.

Now let’s focus on the role of the financial channels of unemployment under the adverse housing preference shock. With the financial channels of unemployment, credit constrained workers consider the surplus from borrowing when they are in wage bargaining process. The surplus from borrowing depends on loan rate, loan size, unemployment rate, and housing price. Since the adverse shock causes fall in loan size, employment rate, and housing price, the surplus from borrowing decreases under the adverse shock. It makes subprimers ask higher wage bill to work so subprimers’ job status would be exacerbated. As a result, subprimers’ unemployment rate jumps up and total loan size to subprimers falls significantly. Subprimers’ increased wage bill supports their consumption and housing but the increased wage bill suppresses entrepreneurs’ capital and housing. Higher subprimers’ unemployment also affects financial market. It reduces total loan demand and increases default. Bankers take loss from the default and it undermines banking capital which reduces bankers’ loan making ability through capital adequacy constraint. Loan to entrepreneurs also falls more. As a result, investment and output decrease more. In conclusion, the negative effect of adverse housing preference shock is significantly amplified by the financial channels of unemployment.

We think our result could shed light on two labor market puzzles regarding the “Great Recession” with respect to the supply side of labor: Why has unemployment rate risen that sharply? and Why has unemployment rate not been recovered even after the recession seems to end? In conventional job search and matching model, workers consider only the trade-off between wage bill and leisure. In our model, however, credit constrained workers consider housing and financial market as well as the trade-off. During recent recession, housing price fell sharply and it significantly discouraged people to buy housing. It implies credit constrained workers’ incentive to work significantly

\textsuperscript{22}It contradicts to Iacoviello (2005) [29], where positive housing preference shock increases consumption.
falls since lowered housing price decreases both the loss from default and surplus from borrowing through collateral constraint channel. As of late 2011, housing market is still stagnated and people are reluctant to buy new housing. It discourages credit constrained workers to find job.

Figure 1.4: Adverse Housing Preference Shock

Figure 1.5: Adverse Housing Preference Shock
Figure 1.6: Adverse Housing Preference Shock

Figure 1.7: Adverse Housing Preference Shock

\[ \chi = 1, \phi = 1 \]
\[ \chi = 1, \phi = 0 \]
\[ \chi = 0, \phi = 0 \]
Figure 1.8: Adverse Housing Preference Shock

Figure 1.9: Adverse Housing Preference Shock

\[ \chi = 1, \phi = 1 \quad \chi = 1, \phi = 0 \quad \chi = 0, \phi = 0 \]
1.6 Conclusion

In most macro models, people can borrow as much as they want and they never fail to repay the debt. Furthermore, people has no difficulty in finding job whenever they want to work. In real world, however, people should be qualified to borrow and some borrowers cannot repay outstanding loan. In addition, they might not be able to find job although they want to work.

To overcome the over-simplification of the real economy, some macro models consider financial frictions and some focuses on labor market friction. However, we think the two market frictions are closely related to each other in the sense that unemployed workers can not borrow new loan and not repay outstanding loan. In addition, credit constrained workers would consider that financial effects of job status when they are in wage bargaining with their potential employers.

In this paper, we consider the interaction between financial and labor market by introducing financial friction and labor market friction simultaneously. To link the two markets, we assume the unemployed cannot borrow new loan and not repay outstanding loan. We call these two financial channels of unemployment. By the assumption, workers would consider the financial channels of unemployment when they are in wage bargaining process.

With this setup, we find that adverse housing preference shock discourages credit constrained workers from maintaining or seeking job since the adverse shock reduces not only the benefit from borrowing but also the penalty from defaulting. As a result, credit constrained workers’ unemployment increases and it lowers the size of outstanding mortgage loan and pushes up the default rate through the financial channels of unemployment we assume. In a consequence, the adverse housing preference shock significantly amplifies the business cycles in this model.

The contribution of this paper is two-fold. First, it offers a theoretical framework to analyze the interaction between financial market and labor market. Second, it gives alternative answer to the question: why did the collapse in housing market lead so big fluctuation in financial market and labor market during the recent ”Great Recession”?
Chapter 2

International Financial Business Cycles\textsuperscript{1}

by Taesu Kang\textsuperscript{2} and Tuan Dao\textsuperscript{3}

\textsuperscript{1}We would like to thank Professor Fabio Ghironi, Professor Peter Ireland and Professor Georg Strasser for their guidance, support and critics of the paper
\textsuperscript{2}Department of Economics, Boston College
\textsuperscript{3}Department of Economics, Boston College
2.1 Introduction

International macroeconomics literature on global imbalances explains why the U.S. runs a persistent current account deficit. While the U.S. is the net borrower at the country level, at the banking-sector level, this is not necessary the case. U.S. banks, and banks in other developed economies, are net lenders to banks in emerging Asian markets (EAM). At around late 2007, beginning of 2008, when losses in the mortgage market begins to damage U.S. banks’ balance sheets (<Figure 2.1>), U.S. banks deleverage and reduce deposits and credits (<Figure 2.2>). Not only do they contract loans made to U.S. borrowers, they contract loans made to foreign borrowing banks as well. <Figure 2.3> documents external (cross-border) assets of banks in developed economies and <Figure 2.4> documents external liabilities of banks in EAM.

Figure 2.1: Delinquency Rate On Single-Family Residential Mortgages

![Figure 2.1](source: FRED)

Figure 2.2: U.S. Bank Credit and Deposit (Growth Rate)

![Figure 2.2](source: Board Governors of Federal Reserve System)
Figure 2.3: External Assets of Banks in Developed Economies

Source: Bank for International Settlement

Figure 2.4: External Liabilities of Banks in Selected Emerging Asia

Source: Bank for International Settlement
With the exception of Japan, who was little exposed to U.S. Mortgage Backed Securities (MBS), all major developed economies show significant contractions in banks’ external assets, which result in significant contractions in EAM banks’ external liabilities\(^1\). The documented contraction in international inter-bank lending was followed by the worldwide drop in GDP growth, both among the developed world (<Figure 2.5>) and the developing world (<Figure 2.6>). These empirical evidence highlight the importance of the banking system in international transmission of shocks.

\[\text{Figure 2.5: GDP Growth of Developed Economies}\]

![Figure 2.5: GDP Growth of Developed Economies](source: World Bank)

\[\text{Figure 2.6: GDP Growth of Emerging Asia}\]

![Figure 2.6: GDP Growth of Emerging Asia](source: World Bank)

The recent financial crisis in the U.S. was characterized by decline in asset prices, disruption in the loan market, sharp increase in interest rate spread and a large drop in GDP. One thing many scholars have agreed is the banking system plays a vital role in this crisis. There is a number of recent working papers that include bank in

\(^1\)Kamin and DeMarco (2010) [34] document that the majority of foreign exposure to U.S. MBS are of European Banking Centers.
We would like to build a two country model with the banking system that plays an important role in international transmission of shock, which has been largely agreed to be the main cause of the recent crisis. Our model is built upon the closed economy version in Iacoviello (2011) [30]. In steady state, banks in the developing country (EAM/domestic country) borrow from banks in the developed country (the U.S./foreign country). When some borrowers in the U.S. pay back less than contractually agreed, with the presence of capital requirement constraint, U.S. banks cut back on lending to U.S. borrowers as well as EAM banks and raise the inter-bank lending rate. Domestic banks now face more expensive and less availability of foreign credit, and will reduce loans made to domestic borrowers. The financial (repayment) shock in the U.S. is transmitted across country via the banking system.

In another exercise, we investigate the behavior of the model under permanent and temporary shocks to the weight of domestic bank loan in the foreign bank’s capital requirement constraint. The permanent shock can be interpreted as a change in bank
regulation, such as moving from Basel I to Basel II. A temporary shock can be interpreted as an exogenous drop in domestic banks’ credit ratings. The results for these shocks are reductions in home output, investment and consumption; and a depreciation of home real exchange rate.

Our paper is related to a number of empirical papers on global banking. Peek and Rosengren (1997) [40] study the behavior of Japanese banks in the U.S.. During the financial crisis in the late 1980s and early 1990s in Japan, Japanese banks in the U.S. substantially contract the amount of loans made to U.S. borrowers. Cetorelli and Goldberg (2008) [7] documents that "foreign lending activity of U.S. bank affiliates abroad can rely less on the overall strength of the home office in times of tighter monetary condition in the U.S.". Popov and Udell (2010) [42] finds that financial distress by West European and U.S. parent banks has a significant impact on the availability of business loans for East European firms. Most recently, Imai and Takarabe (2011) [31] use the data from nationwide and local banks in Japan to test whether banking integration plays an important role in transmitting financial shocks across geographical boundaries. They found that nation wide banks do indeed transmit financial shocks originated from major cities to smaller local economies. The results of our model under different weights of interbank loan in the capital requirement constraint suggests that across countries, lower rated economies will suffer more when U.S. banks deleverage. This is consistent with empirical evidence for the recent crisis, documented by Devereux and Yetman (2010) [11].
2.2 The Model

There are two countries: the domestic country (EAM) and the foreign country (U.S.). In each country, there are five types of agents: patient households, impatient households, entrepreneurs, firms and banks. There are two sectors in the economy: tradable good sector and non-tradable good sector.

Both patient and impatient households (HHs) work for firms in tradable and non-tradable sectors. They earn wage income and consume tradable goods, non-tradable goods and housing. Patient HHs supply deposits for banks and earn a return from the deposits. Impatient HHs, on the other hand, borrow from banks to consume. They can only borrow up to a fraction of the value of their collateral (house).

Domestic bankers take the deposit from domestic depositors and can also borrow in the international inter-bank market. They can only borrow up to a fraction of the value of their capital. They pay a return for the fund they borrow and lend it to domestic borrowers for a higher return. Foreign bankers take the deposit from foreign depositors. They lend out to foreign borrowers and domestic bankers. Domestic and foreign bankers face capital requirement constraint.

Entrepreneurs accumulate physical capital used in both tradable and nontradable sectors. They finance their investment with income from capital rental and bank loan, which is subject to a collateral debt constraint.

Firms in tradable and nontradable sectors use capital and labor to produce goods. They pay wages to HHs.

Consumers’ consumption aggregate is given by: 
\[ c_t = \left( (c_t^N)^{\omega - 1} + (c_t^T)^{\omega - 1} \right)^{\frac{1}{\omega - 1}}, \]
where \(c^T\) and \(c^N\) are tradable and non-tradable consumptions. The corresponding price index is 
\[ P_t = \left[ (P_t^N)^{1-\omega} + (P_t^T)^{1-\omega} \right]^{\frac{1}{1-\omega}}, \]
where \(P^T\) and \(P^N\) are tradable and non-tradable price indices. The consumption aggregate and price indices for the foreign economy are identical. We denote the price of non-tradable (tradable) relative to the price of consumption baskets as \(p_t^N\) (\(p_t^T\)).

\(^\text{1}\)Much of the model’s features are similar to those of Iacoviello (2011) [30] closed economy model.
2.2.1 Patient Households

A continuum of domestic patient HHs deposit $d_t$, consume composite good $c_{p,t}$ and housing $h_{p,t}$, and supply labor to tradable, $n^T_{p,t}$, and nontradable, $n^N_{p,t}$, sectors. They earn wage income and return from their deposits. They maximize the infinite sum of utilities:

$$\max_{c_{p,t}, h_{p,t}, n^N_{p,t}, n^T_{p,t}, d_t} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln c_{p,t} + \nu \ln h_{p,t} + \tau_p \ln (1 - n^N_{p,t} - n^T_{p,t}) \right]$$

subject to budget constraint:

$$c_{p,t} + d_t + q_t \Delta h_{p,t} = R_{d,t} d_{t-1} + w^N_{p,t} n^N_{p,t} + w^T_{p,t} n^T_{p,t}. \quad (2.1)$$

$R_{d,t}$ is the return from the deposits and $q_t$ is the price of house. $w^N_{p,t}$ and $w^T_{p,t}$ are wages from nontradable and tradable sectors respectively. Their first order conditions are:

$$\frac{1}{c_{p,t}} = \beta_t E_t \left( \frac{R_{d,t+1}}{c_{p,t+1}} \right) \quad (2.2)$$

$$\frac{q_t}{c_{p,t}} = \frac{\nu}{h_{p,t}} + \beta_t E_t \left( \frac{q_{t+1}}{c_{p,t+1}} \right) \quad (2.3)$$

$$\frac{w^N_{p,t}}{c_{p,t}} = \frac{\tau_p}{1 - n^N_{p,t} - n^N_{p,t}} \quad (2.4)$$

$$\frac{w^T_{p,t}}{c_{p,t}} = \frac{\tau_p}{1 - n^N_{p,t} - n^T_{p,t}} \quad (2.5)$$

Foreign patient HHs optimization problem are identical and indexed with *.

2.2.2 Impatient Households

Domestic impatient HHs also consume goods and housing, and supply labor. $c_{i,t}$, $h_{i,t}$, $n^N_{i,t}$, $n^T_{i,t}$ are impatient HHs’ consumptions, houses, labor supply to nontradable and tradable sectors. Unlike patient HHs, however, they borrow money from banks, $l_{i,t}$, to finance consumption. They pay interest $R_{i,t}$ on the loan and can only borrow up to the
value of their house. Their maximization problem is:

\[
\max_{c_{i,t}, h_{i,t}, n^N_{i,t}, n^T_{i,t}} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln c_{i,t} + \nu \ln h_{i,t} + \tau_i \ln (1 - n^N_{i,t} - n^T_{i,t}) \right]
\]

subject to budget constraint:

\[
c_{i,t} + q_t \Delta h_{i,t} + R_{i,t} l_{i,t-1} = l_{i,t} + w^N_{i,t} n^N_{i,t} + w^T_{i,t} n^T_{i,t},
\]  

(2.6)

and borrowing constraint:

\[
l_{i,t} \leq m_i E_t \left( \frac{q_{t+1} h_{i,t}}{R_{i,t}} \right).
\]  

(2.7)

Foreign impatient HHs problem is equivalent, except that in their budget constraint, there is a repayment shock. Their budget constraint is:

\[
c^*_{i,t} + q^*_t \Delta h^*_{i,t} + R^*_{i,t} l^*_{i,t-1} - \epsilon_t
\]

\[= l^*_{i,t} + w^{N*}_{i,t} n^{N*}_{i,t} + w^{T*}_{i,t} n^{T*}_{i,t}
\]

As in Iacoviello 2010, \(\epsilon_t\) is a mean zero, AR(1) shock that captures the exogenous repayment shock in the U.S.. When \(\epsilon_t\) is greater than 0, U.S. impatient HHs pays back less than their debt obligation.

First order conditions of impatient HHs are:

\[
\frac{1}{c_{i,t}} = \lambda_{i,t} R_{i,t} + \beta_i E_t \left( \frac{R_{i,t+1}}{c_{i,t+1}} \right)
\]  

(2.8)

\[
\frac{q_t}{c_{i,t}} = \nu \frac{h_{i,t}}{h_{i,t}} + \lambda_{i,t} m_i E_t(q_{t+1}) + \beta_i E_t \left( \frac{q_{t+1}}{c_{i,t+1}} \right)
\]  

(2.9)

\[
\frac{w^{N}_{i,t}}{c_{i,t}} = \frac{\tau_i}{1 - n^N_{i,t} - n^T_{i,t}}
\]  

(2.10)

\[
\frac{w^{T}_{i,t}}{c_{i,t}} = \frac{\tau_i}{1 - n^N_{i,t} - n^T_{i,t}}
\]  

(2.11)

\(\lambda_{i,t}\) is the Lagrangian multiplier of impatient HHs borrowing constraint.
2.2.3 Entrepreneurs

Entrepreneurs’ optimization problem is:

$$\max_{c_{e,t}, k_t^N, k_t^T, l_{e,t}} \sum_{t=0}^{\infty} \beta^t E_0 \ln c_{e,t}$$

subject to budget constraint:

$$c_{e,t} + k_t^N + k_t^T + R_{e,t} l_{e,t-1} + \frac{\phi_k}{2} (\Delta k_t^N)^2 + \frac{\phi_k}{2} (\Delta k_t^T)^2$$

$$= l_{e,t} + (r_{k,t}^N + 1 - \delta) k_{t-1}^N + (r_{k,t}^T + 1 - \delta) k_{t-1}^T$$

(2.12)

and borrowing constraint:

$$l_{e,t} \leq m_c (k_t^N + k_t^T).$$

(2.13)

c_{e,t} is entrepreneurs’ consumption. k_t^N, k_t^T are entrepreneurs’ capital in the tradable and nontradable sectors. They finance investment with income from capital rental in the two sectors r_{k,t}^N + 1, r_{k,t}^T + 1 and bank loan l_{e,t}. The bank loan cannot exceed the value of their capital. Entrepreneurs pay banks a return R_{e,t} on the loan. Similar to Backus, Kehoe and Kydland (1994), we assume that investment uses the same goods composite as the consumption basket. \(\frac{\phi_k}{2} (\Delta k_t^N)^2\) and \(\frac{\phi_k}{2} (\Delta k_t^T)^2\) are convex capital adjustment cost that entrepreneurs face when they change their stock of capital in the tradable and non-tradable sectors. Entrepreneurs’ first order conditions are:

$$\frac{1}{c_{e,t}} (1 + \phi_k \Delta k_t^N) = \frac{\lambda_{e,t}}{c_{e,t}} m_c + \beta_c E_t \left\{ \frac{1}{c_{e,t+1}} [(r_{k,t+1}^N + 1 - \delta) + \phi_k \Delta k_{t+1}^N] \right\}$$

(2.14)

$$\frac{1}{c_{e,t}} (1 + \phi_k \Delta k_t^T) = \frac{\lambda_{e,t}}{c_{e,t}} m_c + \beta_c E_t \left\{ \frac{1}{c_{e,t+1}} [(r_{k,t+1}^T + 1 - \delta) + \phi_k \Delta k_{t+1}^T] \right\}$$

(2.15)

$$\frac{\lambda_{e,t}}{c_{e,t}} = \frac{\lambda_{e,t}}{c_{e,t}} + \beta_c E_t \left( \frac{R_{e,t+1}}{c_{e,t+1}} \right)$$

(2.16)

\(\frac{\lambda_{e,t}}{c_{e,t}}\) is the Lagrangian multiplier of entrepreneurs’ borrowing constraint. Foreign entrepreneurs problems and first order conditions are similar.
2.2.4 Bankers

**Domestic Bankers**: Domestic bankers borrow from domestic depositors and foreign banks, supply loans to impatient HHs and entrepreneurs. The fund they obtain from the foreign bank is in term of tradable good. They pay returns on the fund they borrow, $R_{d,t}$ and $R_{f,t}$, to depositors and foreign banks respectively. They charge higher interests to the loans they lend out: $R_{i,t}$ and $R_{e,t}$ to impatient HHs and entrepreneurs. They face a capital requirement constraint and a collateral debt constraint. The two constraint together pin down the level of foreign asset in the model. Their optimization problem is:

\[
\max_{c_{b,t},d_t,l_{i,t},l_{e,t},l_{f,t}} E_0 \sum_{t=0}^{\infty} \beta_t^t \ln c_{b,t}
\]

subject to budget constraint:

\[
c_{b,t} + R_{d,t}d_{t-1} + l_{e,t} + l_{i,t} + R_{f,t}p_t^T l_{f,t-1} = d_t + R_{e,t}l_{e,t-1} + R_{i,t}l_{i,t-1} + p_t^T l_{f,t} - \left\{ \frac{\phi_e}{2}(\Delta l_{e,t})^2 + \frac{\phi_i}{2}(\Delta l_{i,t})^2 + \frac{\phi_d}{2}(\Delta d_t)^2 + \frac{\phi_f}{2}(\Delta (p_t^T l_{f,t}))^2 \right\}
\]  

(2.17)

capital requirement constraint: $d_t + p_t^T l_{f,t} \leq \gamma_d l_{e,t} + \gamma_i l_{i,t}$  

(2.18)

and foreign debt constraint: $p_t^T l_{f,t} \leq m_f \left( \frac{l_{i,t} + l_{e,t} - d_t}{R_{f,t}} \right)$  

(2.19)

The international inter-bank loan $l_{f,t}$ is denominated in tradable good price. In domestic consumption good unit, its value is $p_t^T l_{f,t}$. Domestic bankers use their capital as collateral, which is equal to total asset $l_{i,t} + l_{e,t}$ minus liability $d_t$. Similar assumption on interbank lending constraint has been made by Ali Dib (2010). $m_f$ is the loan to value in the international financial market. $\frac{\phi_e}{2}(\Delta l_{e,t})^2, \frac{\phi_i}{2}(\Delta l_{i,t})^2, \frac{\phi_d}{2}(\Delta d_t)^2, \frac{\phi_f}{2}(\Delta (p_t^T l_{f,t}))^2$ are adjustment costs that banks face when they change their loans and deposit.
Their first order conditions are:

\[
\frac{1}{c_{b,t}} \left[ 1 - \phi_d \Delta d_t \right] = \frac{\lambda'_{b,t}}{c_{b,t}} + \frac{\lambda'_{f,t}}{c_{b,t}} m_f + \beta_b E_t \left\{ \frac{1}{c_{b,t+1}} (R_{d,t+1} - \phi_d \Delta d_{t+1}) \right\}
\]

\[
\frac{1}{c_{b,t}} \left[ 1 + \phi_i \Delta l_{i,t} \right] = \frac{\lambda'_{b,t}}{c_{b,t}} \gamma_i + \frac{\lambda'_{f,t}}{c_{b,t}} m_f + \beta_b E_t \left\{ \frac{1}{c_{b,t+1}} (R_{i,t+1} + \phi_i \Delta l_{i,t+1}) \right\}
\]

\[
\frac{1}{c_{b,t}} \left[ 1 + \phi_e \Delta l_{e,t} \right] = \frac{\lambda'_{b,t}}{c_{b,t}} \gamma_e + \frac{\lambda'_{f,t}}{c_{b,t}} m_f + \beta_b E_t \left\{ \frac{1}{c_{b,t+1}} (R_{e,t+1} + \phi_e \Delta l_{e,t+1}) \right\}
\]

\[
\frac{1}{c_{b,t}} \left[ 1 - \phi_f \Delta (p^T l_{f,t}) \right] = \frac{\lambda'_{b,t}}{c_{b,t}} + \frac{\lambda'_{f,t}}{c_{b,t}} R_{f,t}
\]

\[
+ \beta_b E_t \left\{ \frac{1}{c_{b,t+1}} \left[ (R_{f,t+1} p^T l_{f,t+1}) - \phi_f \Delta (p^T l_{f,t+1}) \right] \right\}.
\]

\(\lambda'_{b,t}\) and \(\lambda'_{f,t}\) are multipliers on the capital requirement and foreign debt constraints, multiplied by banker consumptions. The intuition here is similar to that of Iacoviello 2010, with one exception, the presence of \(\lambda'_{f,t}\). To increase one unit of consumption today, bankers can either increase one unit of today’s deposit or today’s inter-bank loan (today’s liabilities), or reduce one unit of today’s consumers’ loan or business loan (today’s assets). If he, for example, choose to increase \(d_t\), re-arranging the equations gives:

\[
1 - \lambda'_{b,t} - \lambda'_{f,t} m_f - \phi_d \Delta d_t = E_t \left\{ \beta_b \frac{c_{b,t}}{c_{b,t+1}} (R_{d,t+1} - \phi_d \Delta d_{t+1}) \right\}.
\]

The right hand side of the equation is the cost of increasing one unit of deposit this period, which is equal to the additional return tomorrow that bankers has to pay on the deposit, less the lower cost that bankers pay on adjustment cost tomorrow, discounted into today value by bankers’s stochastic discount factor \(\left\{ \beta_b \frac{c_{b,t}}{c_{b,t+1}} \right\}\). The left hand side is the marginal benefit of consuming one more unit today, minus the cost of tightening capital requirement constraint, \(\lambda'_{b,t}\), minus the cost of tightening foreign debt constraint, \(\lambda'_{f,t} m_f\), minus the adjustment cost in changing deposit that bankers face today. Similar argument holds if bankers choose, instead, to increase foreign loan or decrease loans made to domestic borrowers.
**Foreign Bankers:** Foreign bankers borrow the fund from foreign depositors and supply loan to foreign impatient HHs, entrepreneurs. Foreign banks also lend to domestic banks in the form of tradable goods. They only face budget constraint and capital requirement constraint. They are subject to the endowment shock $\epsilon_t$. Their maximization problem is:

$$
\max_{c^*_b, d^*_f} E_0 \sum_{t=0}^{\infty} (\beta^*_b)^t \ln c^*_b, t
$$

subject to budget constraint:

$$
c^*_b, t + R^*_d, d^*_t, l^*_e, t - 1 + l^*_i, t - 1 + p^*_t l^*_f, t
$$

$$
= d^*_t + R^*_d, d^*_t, l^*_e, t - 1 + R^*_i, l^*_i, t - 1 + R^*_f, p^*_t l^*_f, t - 1 - \epsilon^*_t
$$

$$
- \left\{ \frac{\phi^*_e}{2} (\Delta l^*_e, t)^2 + \frac{\phi^*_i}{2} (\Delta l^*_i, t)^2 + \frac{\phi^*_d}{2} (\Delta d^*_t)^2 + \frac{\phi^*_f}{2} (\Delta p^*_t l^*_f, t)^2 \right\}
$$

(2.24)

and capital requirement constraint:

$$
d^*_t \leq \gamma^*_e l^*_e, t + \gamma^*_i l^*_i, t + \gamma^*_f p^*_t l^*_f, t.
$$

(2.25)

Their first order conditions are similar to those of domestic banks without the multiplier on the foreign debt constraint $\lambda^*_f, t$. When foreign banks increase their consumption by increasing deposit or reducing loans, only their capital requirement constraint is tightened.

### 2.2.5 Firms

Firms in tradable and nontradable sectors use labor from HHs, capital from entrepreneurs to produce tradable and nontradable goods. They pay wages to HHs and capital rental fees to entrepreneurs.

**Non-tradable Sector:**

$$
\max_{k^N_{t-1}, n^N_{p,t}, n^N_{i,t}} \pi^N_t = p^N_t y^N_t - \pi^N_t k^N_{t-1} - w^N_{p,t} n^N_{p,t} - w^N_{i,t} n^N_{i,t}
$$

subject to $y^N_t = z^N_t (k^N_{t-1})^\alpha \left[ (n^N_{p,t})^{1-\sigma} (n^N_{i,t})^\sigma \right]^{1-\alpha}$. 

43
Tradable Sector:

\[
\max \quad \pi_t^T = p_t^T y_t^T - r_{k,t}^T k_{t-1}^T - w_{p,t}^T n_{p,t}^T - w_{i,t}^T n_{i,t}^T
\]

subject to \( y_t^T = z_t^T (k_{t-1}^T)^\alpha \left[ (n_{p,t}^T)^{1-\sigma} (n_{i,t}^T)^\sigma \right]^{1-\alpha} \).

The Cobb-Douglas aggregate of labor is to control for the economic size of patient and impatient HHs in the economy, as in Iacoviello (2005 [29], 2011 [30]). The higher \( \sigma \) is, the larger the size of impatient HHs vs. patient HHs. The Cobb-Douglas aggregate is used, instead of a simple linear combination, to pin down the steady state labor supply to each sector. In the model with two sectors and two agents, even though total labor demand in each sector and total labor supply of each type of agents are determined, a linear aggregate cannot determine what fraction of labor effort of each agent is allocated to each sector.

2.2.6 Market Clearing Conditions

The housing market clearing conditions are:

\[
h_{p,t} + h_{i,t} = 1,
\]

\[
h_{p,t}^* + h_{i,t}^* = 1.
\]

The good market clearing conditions for tradable good are:

\[
y_t^T + l_{f,t} = (p_t^T)^{1-\omega} [c_{p,t} + c_{i,t} + c_{e,t} + c_{b,t} + k_t^N + k_t^T - (1 - \delta) (k_{t-1}^N + k_{t-1}^T) + \text{adj}_t] + R_{f,t} l_{f,t-1}
\]

\[
y_t^{T*} + R_{f,t} l_{f,t-1} = (p_t^{T*})^{1-\omega} [c_{p,t}^* + c_{i,t}^* + c_{e,t}^* + c_{b,t}^* + k_t^{N*} + k_t^{T*} - (1 - \delta) (k_{t-1}^{N*} + k_{t-1}^{T*}) + \text{adj}_t^*] + l_{f,t}
\]

Here, \( \text{adj}_t \) (\( \text{adj}_t^* \)) is the sum of all adjustment costs the domestic (foreign) bankers and entrepreneurs face. The market clearing conditions for non-tradable are implied from the budget constraints of all agents and the above four market clearing conditions.
2.3 Key Assumptions and Calibration

2.3.1 Key Assumptions

The steady state deposit and lending rates are as followed:

<table>
<thead>
<tr>
<th>Table 2.1: Deposit and Lending Rates in Steady State</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Deposit Rates</strong></td>
</tr>
<tr>
<td><strong>Interbank Rate</strong></td>
</tr>
<tr>
<td><strong>Loan to IHs</strong></td>
</tr>
<tr>
<td><strong>Loan to Entrepreneur</strong></td>
</tr>
</tbody>
</table>

where $\delta_1 = \frac{\lambda_f (R_f - m_f)}{(1 - \gamma_e R_d - \lambda_f m_f)(1 - \gamma_e)}$ and $\delta_2 = \frac{\lambda_e (R_f - m_f)}{(1 - \gamma_e R_d - \lambda_f m_f)(1 - \gamma_e)}$. Detailed solutions can be found in the Appendix.

In steady state, foreign banks take the deposit from foreign savers (patient HHs) and lend out to foreign impatient HHs, foreign entrepreneurs and domestic banks. In order for foreign banks to accept the deposit, the return on deposits that foreign banks must pay should be "low enough" for foreign banks. Specifically, $\frac{1}{\beta_p} > R^*_d = \frac{1}{\beta p}$, or foreign bankers are more impatient than foreign depositors. In order for foreign impatient HHs and entrepreneurs to borrow from foreign banks, the interest rates the foreign banks charge must be "low enough" for them, or $\frac{1}{\beta_e} > R^*_e = (1 - \gamma_e)\frac{1}{\beta e} + \gamma_e \frac{1}{\beta p}$ and $\frac{1}{\beta_i} > R^*_i = (1 - \gamma_i)\frac{1}{\beta e} + \gamma_i \frac{1}{\beta p}$. Foreign entrepreneurs and impatient HHs are more impatient than the weighted average of foreign bankers and foreign depositors. The intuition here is similar to that of Iacoviello 2010.

In the interbank market, domestic banks borrow from foreign banks because the fund supplied from foreign banks is cheaper than the fund supplied from domestic depositors. From the Appendix solution for the multiplier on the interbank borrowing constraint, one can easily verify that the condition $R_f < R_d$ ensures the binding of the constraint in steady state. It is equivalent to: $(1 - \gamma_f)\frac{1}{\beta_p} + \gamma_f \frac{1}{\beta p} < \frac{1}{\beta p}$, or savers in domestic country are more impatient than the weighted average of savers and bankers in the foreign country. For domestic borrowers to accept these rates the domestic bank charge, they have to be "impatient enough", or $\frac{1}{\beta_e} > R_e$ and $\frac{1}{\beta_i} > R_i$.

Within the large literature on the global imbalance, to generate the observed cur-
rent account in the U.S. and other developing nations, especially China, the common assumption is the representative agent in the U.S. is more impatient than a representative in the developing country. To generate the flow of fund at the banking sector level from the U.S. to EAM, we only assume that the savers in EAM are more impatient than the weighted average of savers and bankers in the U.S. Other agents in the EAM can be more patient than the U.S. Thus, our assumption does not contradict with the assumption in the global imbalance literature.

2.3.2 Calibration

Table 2.2: Agents Discount Factor

<table>
<thead>
<tr>
<th>Domestic Agent Discount Factor</th>
<th>Value</th>
<th>Foreign Agent Discount Factor</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_p )</td>
<td>0.9875</td>
<td>( \beta^*_p )</td>
<td>0.9925</td>
</tr>
<tr>
<td>( \beta_i )</td>
<td>0.95</td>
<td>( \beta^*_i )</td>
<td>0.94</td>
</tr>
<tr>
<td>( \beta_e )</td>
<td>0.95</td>
<td>( \beta^*_e )</td>
<td>0.94</td>
</tr>
<tr>
<td>( \beta_b )</td>
<td>0.96</td>
<td>( \beta^*_b )</td>
<td>0.975</td>
</tr>
</tbody>
</table>

The discount factors for each agent are given by Table 2.2. All these values are within the range of two standard deviation bands interval (0.91, 0.99) estimated by Carroll and Samwick (1997) [6]. They are chosen according to the key assumptions. The fraction of impatient HHs \( \sigma \) is 0.5. Campbell and Mankiw (1990) [5] estimated the fraction of liquidity constrained HHs to be 0.5. Iacoviello (2005 [29], 2010 [30]) set the fraction of impatient HHs to be 0.36 and 0.3 respectively. Setting \( \sigma \) to be 0.5 is at the upperbound of the values used in the literature. It gives the convenience in algebraically solving the model in closed form without changing its fundamentals. Elasticity of substitution between tradable and non-tradable good \( \omega \) is 0.44 as estimated by Stockman and Tesar (1995) [48]. \( \gamma_i, \gamma_e \) are 0.9 as in Iacoviello (2011) [30]. We choose \( \gamma_f \) to be 0.9. Parameters controlling bankers’ adjustment cost \( \phi_d, \phi_i, \phi_e, \phi_f \) are 0.25. Loan to values \( m_i, m_e, m_f \) are 0.9, 0.9 and 0.7 respectively. Capital depreciation rate \( \delta \) is 0.025. The rest of the model’s parameters are chosen from the closed economy model by Iacoviello (2011) [30].
2.4 Results

2.4.1 Repayment shock

The repayment shock is exogenous. Alternatively, one can endogenize the default shock as function of the underlying state of the economy. For example, in Forlati and Lambertini (2011) [20], borrowers default endogenously, when they find that the value of their collateral is lower than the value of the loan they borrow. Within the context of this paper, we treat the repayment shock as exogenous for simplicity and tractability. A further step, to describe how default can happen endogenously and depend on the fundamental of the lending country, and through banking sector, spread to the borrowing country, is worthwhile for future investigation.

Figure 2.7: Impulse Response: Foreign repayment shock

<Figure 2.7> plots the impulse response results of foreign macroeconomic variables for the foreign repayment shocks. Default coming from foreign impatient HHs forces the foreign banks to contract both loans and deposit to maintain their required capital-asset
ratio. The results are a fall in output, asset price, investment, employment, loan and an increase in lending interest rates. Similar results have been obtained in Iacoviello (2011) [30] closed economy version.

Figure 2.8: Impulse Response: Foreign repayment shock

![Graph of Impulse Response: Foreign repayment shock](image)

<Figure 2.8> plots the impulse response of international interbank loan and interest rate. When the lending banks from the developed country contract the loan for all of their borrowers, they do so for the borrowing banks as well.

Figure 2.9: Impulse Response: Foreign repayment shock

![Graphs of Impulse Response: Foreign repayment shock](image)

<Figure 2.9> plots the impulse response results of domestic macroeconomic vari-
ables. When foreign banks contract asset by raising lending rates to maintain their capital requirement ratio, domestic banks now face more expensive (as \( R_f \) increases) and less availability credits (international borrowing constraint tighten when \( R_f \) increases), they have to raise domestic lending rates and reduce the loans made to domestic borrowers. A domestic credit crunch, characterized by a decrease in loan and an increase in borrowing interest rates has occurred following the default from abroad.

Domestic output, investment and asset prices fall, which are the typical results following a credit crunch. What is interesting here is the movement of resources across sector and the dynamics of the real exchange rate. The international loan is denominated in tradable good. When the loan that foreign banks made to domestic banks suddenly decreases, in foreign country, the demand for tradable good decreases and the price of tradable relative to non-tradable decreases. In domestic country, the supply of tradable good suddenly decreases, which increases the price of tradable relative to non-tradable.

Figure 2.10: Impulse Response: Foreign repayment shock

As a result, the real exchange rate decreases on impact. Over time, in domestic country, labor and investment move from non-tradable to tradable sector to equalize the prices in two sectors, exchange rate appreciate toward its steady state value. Figure 2.10 plots the impulse responses of real exchange rate and price of tradable and non-tradable in foreign and domestic country.
Figure 2.11: Real Effective Exchange Rate of Selected Emerging Asia

Source: Bank for International Settlement

<Figure 2.11> documents the real exchange rate movement of Chinese Taipei, India and Korea. The sharp reduction in the real exchange rate of these country against the U.S. happened around the time when U.S. banks substantially deleveraged their balance sheet with respect to Asia.

Figure 2.12: Impulse Response: Foreign repayment shock

<Figure 2.12> plots the impulse responses of repayment shock under different values of $\gamma_f$. A lower value of $\gamma_f$ can be interpreted as banks’ strategy to contract foreign loan
and gives priority to long-term domestic borrowers. Peek and Rosengren (1997) [40] documented this behavior among Japanese banks. It can also be interpreted as a lower credit rating of the domestic economy. With a smaller $\gamma_f$, the repayment shock generates much larger volatilities of domestic variables while decreasing the volatilities of foreign variables. In other word, a lower $\gamma_f$, helps mitigate the effects of the financial shock in the developed country where it originates, while amplifying the effects on the developing country. The intuition for this comes from foreign banks’ capital requirement constraint:

$$d_i^* \leq \gamma_e l_{e,t}^* + \gamma_i l_{i,t}^* + \gamma_f p^T l_{f,t}^*.$$ 

Deposit equals asset minus equity:

$$l_{t}^* + e_{t}^* + p^T l_{f,t}^* - E^* \leq \gamma_e l_{e,t}^* + \gamma_i l_{i,t}^* + \gamma_f p^T l_{f,t}^*$$

$$(1 - \gamma_e) l_{e,t}^* + (1 - \gamma_i) l_{i,t}^* + (1 - \gamma_f) p^T l_{f,t}^* \leq E^*.$$  \hspace{1cm} (2.26)

When default happens and decreases foreign banks’ equity, these banks will have to decrease the left hand side of the above equation. When $\gamma_f$ is smaller than $\gamma_i$, $\gamma_e$, it is more beneficial for the foreign banks to contract international loan. One unit decrease in $l_{f,t}$ will loosen the capital requirement constraint by $1 - \gamma_f$, which is larger than $1 - \gamma_e (1 - \gamma_i)$ if banks contract business (consumer) loan. The adjustment costs banks face are convex and together with the $\gamma$ will determine how banks contract its portfolio. Without the convexity in costs, banks will find it most beneficial to contract foreign loan only when $\gamma_f$ is lower relative to $\gamma_i$ and $\gamma_e$.

Devereux and Yetman (2010) [11], using the data for the recent crisis, found that the magnitude of capital flow from one country to the U.S. depends on the country’s foreign currency credit rating. A lower rating results in a larger capital outflow of the country to the U.S., following the recent U.S. crisis. A lower rating asset will have a higher weight in banks’ risk weighted asset (RWA) portfolio in equation (1.26), or a lower $\gamma_f$ in our model. Thus, the empirical evidence is in line with our model prediction, that countries perceived as more risky will suffer more from the U.S. crisis than less risky countries.
2.4.2 $\gamma_f$ shock

Permanent Shock

A permanent shock to $\gamma_f$ can be interpreted as a change in regulation. A real world example of this is the change from the Basel I Accord to the Basel II Accord. Under the Basel I Accord, banks’ assets were classified into categories such as sovereign, banks, collateral, etc. All debts under the same category will carry the same weight in banks’ RWA and banks were required to hold capital equal to 8% of banks’ total RWA. For example, all corporate debts will have the weight of 100% and all government debts will have the weight of 0%. The Basel II Accord no longer gives the same weight to all assets in one category if they have different level of risks. Borrowing banks in developing country, if considered risky by Basel II’s new assessment of risk, will have a higher weight in the lending bank’s RWA.

Figure 2.13: Impulse Response: Permanent Shock to $\gamma_f$
Figure 2.14: Impulse Response: Permanent Shock to $\gamma_f$

Figure 2.15: Impulse Response: Permanent Shock to $\gamma_f$
<Figure 2.13> to <Figure 2.16> have impulse response for a 10% permanent negative shock to $\gamma_f$. As the international inter-bank loan have a higher weight in the lending banks’ RWA, lending banks permanently increase the lending rate, $R_f$, and decrease the amount of loan made to borrowing banks in the developing country, $l_f$. The steady state interbank lending rate is: $R_f = \beta_B - \left[ \frac{1}{\beta_B} - \frac{1}{\beta_H} \right] \gamma_f$. When $\gamma_f$ decreases, $R_f$ converges to a higher steady state. The steady state lending rates to domestic borrowers are weighted average of interbank lending rate and domestic deposit rate. Thus, they converge to a new higher steady state. As a result, domestic consumption, output and investment converge to a lower steady state.

As $\gamma_f$ permanently decreases, from equation 25, we see that foreign banks’ capital requirement constraint tightens, foreign banks can ”loosen” the constraint by either deleveraging, reducing the total size of its RWA and deposit, or restructuring its portfolio, hold less asset with high weight and more asset with low weight. The foreign banks’ adjustment cost helps pin the optimal path for their deposit demand and loan supply. Contrary to the repayment shock, when the only option is to deleverage, foreign banks in this case also restructure their portfolio and holds more assets with lower weight in its RWA. As a result, foreign deposit goes down (deleverage effect) and loans to foreign IHs and entrepreneurs go up (portfolio restructuring effect). The foreign investment, consumption and output go up. New steady state foreign domestic lending rates, which
only depends on foreign bank and patient HHs time preference, stay the same.

**Temporary Shock**

<Figure 2.17> to <Figure 2.20> have the impulse responses for the temporary negative shock to $\gamma_f$. The temporary shock can be interpreted as an exogenous temporary drop in domestic banks’ credit rating. A real world example for this is the drop in domestic bank credit rating of South Korean banks during the Asian financial crisis in 1997.

![Figure 2.17: Impulse Response: Temporary Shock to $\gamma_f$](image)
Figure 2.18: Impulse Response: Temporary Shock to $\gamma_f$

![Graph of Impulse Response: Temporary Shock to $\gamma_f$](image)

Figure 2.19: Impulse Response: Temporary Shock to $\gamma_f$

![Graphs of Impulse Response: Temporary Shock to $\gamma_f$](image)
Figure 2.20: Impulse Response: Temporary Shock to $\gamma_f$

Figure 2.21 has the graph of credit ratings of nationwide South Korean banks and the South Korean Won - US Dollar exchange rate. Credit Ratings of major banks in South Korea drop significantly before and right at the beginning of the crisis. The results of the impulse response shows a drop in domestic GDP, consumption and investment. The foreign loan given to domestic banks contracts and interest rate increases. The real exchange rate also depreciates as a result of tightening foreign credit. These were also the experience of South Korea during the financial crisis.

Figure 2.21: Korean won/USD Exchange Rate and Korean Banks’ Credit Rating

Source: Moody’s
2.5 Relation to empirical facts and existing literature

For the foreign repayment shock, our model generates a drop in output, consumption, investment, loans and housing prices and an increase in bank lending rates in both home and foreign countries. The borrowing country’s real exchange rate also depreciates. Qualitatively, our model matches the empirical facts. The lowest drop in the foreign and domestic consumption are $2 \times 10^{-3}$ and $2 \times 10^{-4}$, respectively. The drops of foreign and domestic investment are $5 \times 10^{-3}$ and $5 \times 10^{-4}$. The transmission of shock to the foreign country is just 10%. Quantitatively, our model does not match the magnitude of international transmission observed in data.

Devereux and Yetman (2010) [11] build an international portfolio model to describe the recent crisis. Leveraged investors in each country holds foreign equity in their portfolio. The total value of their portfolio have to be greater than a constant times their equities. When a shock hits the home country and decreases home asset prices, the value of portfolio of home and foreign investors decreases, forcing them to deleverage. Eric van Wincoop (2011) [50] build a model with leveraged financial institutions, who invest in both home and foreign asset. The default shock in his model is similar to the repayment shock in ours. Since foreign financial institutions hold domestic asset, the domestic default shock damages the foreign bank balance sheet and spread the crisis to the foreign country. The main difference between our model and theirs, is in our model, leveraged domestic bank does not hold foreign asset. In our model, shock is transmitted through a credit crunch in the interbank loan market. Their models fit well for the comovement between U.S. and Europe since European Banking Centers were the majority foreign holders of U.S. MBS. Our model fits the story between U.S. and the EAM. EAM were not directly exposed to the U.S. MBS as only 3% of U.S. ABS are held outside of U.S., Europe and Carribean.

Kollmann et al. (2011) [36], Ueda (2010) [49] and Kalemli-Ozcan et al. (2011) [45] also build international business cycle model with leveraged bank(s). In their models, borrowers in both countries share a common lender(s). When shock hits one country and damage the balance sheet of the common lender(s), the common lender(s) contract loans in both countries. In their model, borrowers in one country have direct access to
the credit of the foreign lenders. The story works in the developed world. For the EAM, this is not the case as few borrowers in EAM have direct access to U.S. bank credit. In our model, borrowers in EAM only borrow fund from the U.S. through domestic banks. Thus, in steady state, banks in EAM are net borrowers in our model, which is an empirical fact and cannot be generated with a model of two symmetric countries.

Another main difference between our model and previous models with leveraged financial investors (banks) is in our capital requirement (leverage) constraint, we separate the weights of different assets in the lending bank RWA. Thus, we are able to investigate the behavior of international transmission of shock when borrowing banks have different credit rating. We found that when the borrowing economy has lower rating, the magnitude of capital flows back to the U.S. in the crisis is higher. Our result is consistent with the empirical finding in Devereux and Yetman (2010) [11].

2.6 Conclusion

Recent financial crisis in the U.S. highlights the role that banking sector plays in the global macroeconomy. There has been substantial empirical evidence that suggests financial crisis can be transmitted across border through the contraction in cross-border loan in banking system. The very first empirical paper was by Peek and Rosengren (1997) [40] and later (2000 [41] who study Japanese financial crisis and the effects on the U.S.. More recent empirical paper study the U.S. financial crisis and the effects on lending in other countries. Such paper are Cetorelli and Goldberg (2008 [7], 2009 [8]), Popov and Udell (2010) [42]. Our model provide a theoretical framework to support the hypothesis. When financial shock hits one country, the cross border inter-bank loan contracts and transmits the shock to another country.

Our paper is also related to a number of papers that study the effects of shocks to international lending rate on a small open economy. These include Faia and Iliopoulos (2010) [17], Christensen et al. (2009) [9], Buyukkarabacak (2008) [4]. These papers treat the source of shock as exogenous. Our paper go one step further and point out the lending country’s financial shock may be what is behind the increase in the international lending rate. Our paper also differ from other recent papers with leveraged banks
(investors) in three dimensions. First, the shock from the source country is not directly transmitted by damaging the foreign banks balance sheet, but rather, from contracting the loan in the interbank market. This helps apply our model for the EAM, who was not directly exposed to the U.S. MBS. Second, the borrowers in one country does not borrow directly from foreign bank, but through domestic bank. Thus, in steady state, at banking level, EAM are net borrowers from the U.S.. Third, we separate the weight of international loans from weights for consumer and business loans in the capital requirement constraint. This helps us investigate the dynamics of the borrowing country when its banks have different credit rating levels and when there is a bank regulation change in the lending country.
Chapter 3

Uncertainty, Collateral
Constrained Borrower, and the
Business Cycle

by Taesu Kang

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3.1 Introduction

Uncertainty has been known (at least suspected) as having negative effects on the economy. Empirically, Bloom (2009) [3] finds that uncertainty shock produces a rapid drop and rebound in output and employment because higher uncertainty temporarily keeps firms from investing and hiring. However, in the sense of macro modeling, to make co-movement among economic variables is not trivial under uncertainty shock because there exists precautionary saving motive in generally accepted utility functions. In standard RBC model, the precautionary saving motive decreases consumption but increases investment and labor under uncertainty shock, which results in increase in output.

Figure 3.1: Uncertainty Shock in standard RBC model

<Figure 3.1> shows the impulse responses of key macro variables to technology volatility shock in standard RBC model. The technology volatility shock makes households reduce consumption (increase save) and work more for precautionary reason. Since saving is equal to investment in standard RBC model, more saving implies more investment so more output, given more labor. Higher risk aversion, higher precautionary motive. So households with higher risk aversion reduce more consumption and increase more labor. As a result, volatility shock increase more output under higher risk aversion. It contradicts to empirical results and generally accepted intuition. To fill this gap, re-
searchers are trying to deviate from standard RBC model to generate the co-movement under uncertainty shock\(^3\).

Bloom, Floetotto, and Jaimovich (2010) [38] introduces idiosyncratic technology shock to heterogeneous firms and non-convex adjustment cost for capital and labor change. They find that uncertainty shock makes firms become more cautious about investment and hiring and that adjustment cost makes slow the reallocation of capital and labors across firms. As a result, uncertainty shock leads to large falls in productivity growth. Gilchrist, Sim, and Zakrajsek (2010) [47] also considers idiosyncratic shock to heterogeneous firms and imperfect capital markets when issuing risky bonds and equity to finance investment projects. They shows that increase in dispersion of firm level returns pushes up the credit spread of firms which leads to drop in investment. Gourio (2010) [27] introduces time varying risk of disaster into a standard RBC model and shows that increase in disaster risk can generate simultaneous drop in investment, labor, and output by reducing return on capital.

Finally, Basu and Bundick (2011) [2] argues that BFJ (2010), GSZ (2010), and Gourio (2010) are limited in getting desired co-movement among variables. For example, Gourio (2010) has increase in consumption at the impact of risk shock and the consumption decreases slowly. In addition, his main results depend on the assumption about intertemporal elasticity of substitution larger than 1 which is controversial. BB (2011) finds that a New Keynesian model with Rotemberg type price stickiness and modest capital adjustment cost can produce the desired co-movement among key macro variables without any other complicated modeling devices. In their framework, precautionary motive increases supply of labor but counter-cyclical markup via price stickiness reduces demand of labor. The decrease in labor demand also lowers investment by firms. Thus, the desire by households to work more in precautionary reason leads to lower labor, investment, and as a result output under uncertainty shock.

In this paper, we try to produce the co-movement among key macro variables under uncertainty shock by introducing collateral constrained borrowers into otherwise standard RBC model. Our model is different from BFJ (2010) and GSJ (2010) in that we

\(^3\)Basu and Bundick (2011) summarizes the co-movement issue and related literatures very well. So we explain the issue and related literatures briefly.
do not depend on idiosyncratic technology shock and from BB (2011) in that we have no price stickiness and capital adjustment cost. Gourio (2010) is also different from ours since we have time varying volatility shock in technology instead of disaster shock.

Our model is basically based on Iacoviello (2005 [29], 2011 [30]) in which patient households save but subprimers and entrepreneurs borrow against their collateral. By introducing time-varying volatility in technology shock into the model, we find that highly indebted borrowers sell collaterals to avoid uncertainty in future consumption. The deleveraging lowers housing price and it causes credit crunch through the well known collateral constraint channel. As a result, housing price, loan, consumption, investment, labor, and output decrease at the impact of uncertainty shock in the economy under modest risk aversion. We find that the negative effect of the uncertainty shock is strengthened in the economy with higher indebted borrowers especially through larger drop in consumption. However, the desired co-movement disappears at least in capital in the economy with high precautionary motive since the high precautionary motive dominates the collateral constraint channel. Even in the economy with high precautionary motive, the collateral constraint channel keeps investment from jumping and still causes larger drop in other macro variables such as consumption, housing price, and loan.

For solution method, we use third-order perturbation method which Fernandez-Villaverde et al. (2009) suggests. As they points, uncertainty shock has no independent role up to second-order approximation in our model. However, third-order approximation is sufficient to identify the distinct role of uncertainty shock.

The rest of the paper is organized as follows. In section 2, we present the DSGE model with collateral constrained borrowers and time varying volatility shock. In addition, we explain the solution method we use to solve the model. Section 3 reports calibration of parameters and impulse responses of volatility shock and section 4 conclusions. An appendix provides some additional details.
3.2 The Model

We have 4 agents in the model: patient households, subprimers, entrepreneurs, and firms. We will use subscript $p$, $s$, and $e$ for patient households, subprimers, and entrepreneurs respectively.

3.2.1 Patient Households

Patient households have CRRA (Constant Relative Risk Aversion) utility function\(^4\) over consumption goods, $c_{p,t}$, housing, $h_{p,t}$, and leisure, $1 - l_{p,t}$. They supply labor, $l_{p,t}$, to firms for real wage, $w_{p,t}$. They save, $b_{p,t}$, at interest rate, $R_t$, to lend it to subprimers and entrepreneurs.

Patient households solve following utility maximization problem:

$$\max_{c_{p,t}, l_{p,t}, h_{p,t}} E_0 \sum_{t=0}^{\infty} \beta_p^t \left\{ \left[ \frac{c_{p,t}^{1-\gamma} (1 - l_{p,t})^\gamma h_{p,t}^{\gamma}}{1 - \sigma} - 1 \right] \right\}$$

subject to \( c_{p,t} + q_t (h_{p,t} - h_{p,t-1}) + b_{p,t} = R_{t-1} b_{p,t-1} + w_{p,t} l_{p,t} \) \hspace{1cm} (3.1)

where $\beta_p$ is discount factor of patient households, $q_t$ is housing price, $\tau$ and $\gamma$ are parameters to control relative preferences for leisure and housing.

First order conditions are given by:

$$u_{p,c,t} = \beta_p E_t (R_t u_{p,c,t+1}) \hspace{1cm} (3.2)$$

$$w_{p,t} u_{p,c,t} = u_{p,l,t} \hspace{1cm} (3.3)$$

$$q_t u_{p,c,t} = u_{p,h,t} + \beta_p E_t (q_{t+1} u_{p,c,t+1}) \hspace{1cm} (3.4)$$

where $u_{p,c,t}$, $u_{p,l,t}$, and $u_{p,h,t}$ denote marginal utility of consumption, leisure, and housing respectively.

Patient households’ marginal utilities of consumption, leisure, and housing are given

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\(^4\)Iacoviello (2005, 2010) use separable logarithmic utility function for simplicity. To see the role of risk aversion, we use CRRA utility function here. As it is well known, logarithmic utility function is a special case (when $\sigma = 1$) of CRRA utility function. Since we couldn’t find any example of CRRA utility function having consumption, leisure, and housing in it simultaneously, we choose the simplest form.
by:

\[ u_{p,c,t} = (1 - \tau - \gamma) \left[ c_{p,t}^{1-\tau-\gamma}(1-l_{p,t})h_{p,t}^{\gamma} \right]^{-\sigma} c_{p,t}^{1-\tau-\gamma}(1-l_{p,t})^{l_{p,t}}h_{p,t}^{\gamma}, \]

\[ u_{p,l,t} = \tau \left[ c_{p,t}^{1-\tau-\gamma}(1-l_{p,t})h_{p,t}^{\gamma} \right]^{-\sigma} c_{p,t}^{1-\tau-\gamma}(1-l_{p,t})^{l_{p,t}}h_{p,t}^{\gamma} - \sigma c_{1-\tau-\gamma}(1-l_{p,t})^{l_{p,t}}h_{p,t}^{\gamma}, \]

\[ u_{p,\gamma,t} = \gamma \left[ c_{p,t}^{1-\tau-\gamma}(1-l_{p,t})h_{p,t}^{\gamma} \right]^{-\sigma} c_{p,t}^{1-\tau-\gamma}(1-l_{p,t})^{l_{p,t}}h_{p,t}^{\gamma}. \]

### 3.2.2 Subprimers

Subprimers also have CRRA utility function over consumption goods, \( c_{s,t} \), housing, \( h_{s,t} \), and leisure, \( 1-l_{s,t} \). They supply labor, \( l_{s,t} \), to firms for real wage, \( w_{s,t} \). In contrast to patient households, however, they borrow, \( b_{s,t} \), at interest rate, \( R_{t} \) from patient households. We assume subprimers are collateral constrained so the amount of loan is determined by some fraction of expected future value of their own housing.

Subprimers solve following utility maximization problem:

\[
\max_{c_{s,t},l_{s,t},h_{s,t}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_s^t \left\{ \left[ c_{s,t}^{1-\tau-\gamma}(1-l_{s,t})h_{s,t}^{\gamma} \right]^{1-\sigma} - 1 \right\} \]

subject to \( c_{s,t} + q_{t} (h_{s,t} - h_{s,t-1}) + R_{t-1} b_{s,t-1} = b_{s,t} + w_{s,t} l_{s,t}, \) \( R_{t} b_{s,t} \leq m_{s} E_{t} (q_{t+1} h_{s,t}) , \) \( (3.5) \)

Equation (3.6) captures the idea of collateral constraint. Total repayment of debt, \( R_{t} b_{s,t} \), at next period should be less \((0 < m_{s} < 1)\) than the expected next period value of housing, \( E_{t} (q_{t+1} h_{s,t}) \).

Subprimers’ first order conditions are given by:

\[ u_{s,c,t} = \lambda_{s,t} R_{t} + \beta_s E_t (R_t u_{s,c,t+1}), \] \( (3.7) \)

\[ q_t u_{s,c,t} = u_{s,h,t} + \lambda_{s,t} m_s E_t (q_{t+1}) + \beta_s E_t (q_{t+1} u_{s,c,t+1}), \] \( (3.8) \)

\[ w_{s,t} u_{s,c,t} = u_{s,l,t}, \] \( (3.9) \)
where $\lambda_{s,t}$ is a Lagrangian multiplier for collateral constraint, $u_{s,c,t}$, $u_{s,l,t}$, and $u_{s,h,t}$ are marginal utility of consumption, leisure, and housing respectively.

Subprimers’ marginal utility of consumption, leisure, and housing are given by:

$$u_{s,c,t} = (1 - \tau - \gamma) \left[ c_{s,t}^{1 - \tau - \gamma} (1 - l_{s,t})^\gamma h_{s,t}^\gamma \right]^{-\sigma} c_{s,t}^{-\tau - \gamma} (1 - l_{s,t})^\gamma h_{s,t}^\gamma,$$

$$u_{s,l,t} = \tau \left[ c_{s,t}^{1 - \tau - \gamma} (1 - l_{s,t})^\gamma h_{s,t}^\gamma \right]^{-\sigma} c_{s,t}^{1 - \tau - \gamma} (1 - l_{s,t})^{-1} h_{s,t}^\gamma,$$

$$u_{s,h,t} = \gamma \left[ c_{s,t}^{1 - \tau - \gamma} (1 - l_{s,t})^\gamma h_{s,t}^\gamma \right]^{-\sigma} c_{s,t}^{1 - \tau - \gamma} (1 - l_{s,t})^\gamma h_{s,t}^\gamma.$$

### 3.2.3 Entrepreneurs

Entrepreneurs have preferences over only consumption goods, $c_{e,t}$. They do not work but accumulate capital, $k_{e,t}$, and housing, $h_{e,t}$, for rental profit $r^k_t$ and $q_t h^h_t$ respectively. Entrepreneurs also borrow, $b_{e,t}$, from patient households at interest rate, $R_t$, and they are collateral constrained against their own housing and capital.

As Iacoviello(2005, 2010), housing supply is assumed to be fixed and normalized to 1. The housing can be used as either living space for households or production space for entrepreneurs. Households get utility from housing since they use it as living space. Entrepreneurs do not get any utility from housing but they earn rent from the housing since they lend it to firms as an input for production.

Entrepreneurs solve following utility maximization problem:

$$\max_{c_{e,t}, h_{e,t}, k_{e,t}} E_0 \sum_{t=0}^{\infty} \beta_e^t \left( \frac{c_{e,t}^{1 - \sigma} - 1}{1 - \sigma} \right)$$

subject to $c_{e,t} + q_t (h_{e,t} - h_{e,t-1}) + k_t - (1 - \delta) k_{t-1} + R_{t-1} h_{e,t-1}$

$$= b_{e,t} + r^k_t k_{t-1} + r^h_t q_t h_{e,t-1}, \quad (3.10)$$

$$R_t b_{e,t} \leq m^h_e E_t (q_{t+1} h_{e,t}) + m^k_e k_t, \quad (3.11)$$

where $\beta_e$ is discount factor of entrepreneurs, $m^k_e$ and $m^h_e$ are LTV ratios against capital and housing respectively, and $\delta$ is depreciation rate for capital.

Equation (3.11) captures the idea of collateral constraint which entrepreneurs face. Total repayment of debt, $R_t b_{e,t}$, at next period should be less ($0 < m^k_e, m^h_e < 1$) than the expected next period value of collaterals, $E_t (q_{t+1} h_{e,t}) + k_t$. 

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Entrepreneurs first order conditions are given by:

\[ c_{e,t}^{-\sigma} = \lambda_{e,t} R_t + \beta_e E_t \left( R_t e_{e,t+1}^{-\sigma} \right), \]  
(3.12)

\[ c_{e,t}^{-\sigma} = \lambda_{e,t} m_k^e + \beta_e E_t \left\{ \left[ r^k_{t+1} + (1 - \delta) \right] e_{e,t+1}^{-\sigma} \right\}, \]  
(3.13)

\[ q_t c_{e,t}^{-\sigma} = \lambda_{e,t} m_h^e E_t (q_{t+1}) + \beta_e E_t \left\{ \left[ q_{t+1} \left( 1 + r^h_{t+1} \right) \right] e_{e,t+1}^{-\sigma} \right\}, \]  
(3.14)

where \( \lambda_{e,t} \) denotes a Lagrangian multiplier for collateral constraint.

3.2.4 Firms

Firms are perfect competitive. They use capital, \( k_t \), labor, \( l_{p,t}, l_{s,t} \), and housing, \( h_{e,t} \) as inputs to produce consumption goods, \( y_t \), and pay \( r^k_t, r^h_t q_t, w_{p,t}, w_{s,t} \) for the use of inputs respectively. We assume firms’ production technology is Cobb-Douglass\(^5\).

A representative firm solves the following profit maximization problem:

\[
\max_{k_{t-1}, h_{e,t-1}, l_{p,t}, l_{s,t}} \quad y_t - \left( w_{p,t} l_{p,t} + w_{s,t} l_{s,t} + r^k_t k_{t-1} + r^h_t q_t h_{e,t-1} \right)
\]

subject to

\[ y_t = \alpha a_t k_{t-1}^{\mu} h_{e,t-1}^{\nu} l_{p,t}^{\alpha(1-\mu-\nu)} l_{s,t}^{(1-\alpha)(1-\mu-\nu)} \]  
(3.15)

where \( \mu, \nu, \) and \( 1 - \mu - \nu \) denote capital, housing, and labor share of output respectively, \( a_t \) is exogenous technology shock.

We assume patient households take \( \alpha \) and subprimers take \( 1 - \alpha \) from the labor share of output, \( 1 - \mu - \nu \). So we can say that higher \( \alpha \) implies higher share of patient households in total households.

A representative firm’s first order conditions are given by:

\[ \mu \frac{y_t}{k_{t-1}} = r^k_t, \]  
(3.16)

\[ \nu \frac{y_t}{h_{e,t-1}} = r^h_t q_t, \]  
(3.17)

\[ \alpha(1 - \mu - \nu) \frac{y_t}{l_{p,t}} = w_{p,t}, \]  
(3.18)

\[ (1 - \alpha)(1 - \mu - \nu) \frac{y_t}{l_{s,t}} = w_{s,t}. \]  
(3.19)

\(^5\)For simplicity, we assume labors of patient households and subprimers are not substitutes but complements.
3.2.5 Market Clearing Conditions

We assume the amount of housing is fixed and normalized to 1. It implies no housing production and no depreciation on housing. So we have following housing market clearing condition:

\[ h_{p,t} + h_{s,t} + h_{e,t} = 1. \]  
(3.20)

Patient households’ saving should be equal to the sum of loans to subprimers and entrepreneurs. So loan market clearing condition is given by:

\[ b_{p,t} = b_{a,t} + b_{e,t}. \]  
(3.21)

 Aggregate resource constraint is as follows:

\[ y_t = c_{p,t} + c_{s,t} + c_{e,t} + k_t - (1 - \delta)k_{t-1}. \]  
(3.22)

3.2.6 Shock Processes

Technology shock and volatility shock follow AR(1) process as follows.

\[ \ln a_t = (1 - \rho_a) \ln \bar{a} + \rho_a \ln a_{t-1} + \epsilon^a_t, \]  
(3.23)

\[ \ln v_t = (1 - \rho_v) \ln \bar{v} + \rho_v \ln v_{t-1} + \sigma_v \epsilon^v_t, \]  
(3.24)

where \( \rho_a \) and \( \rho_v \) are parameters of persistency for each shock, \( \bar{a} \) and \( \bar{v} \) are steady state value of \( a_t \) and \( v_t \), \( \epsilon^a_t \) and \( \epsilon^v_t \) are white noise with mean 0 and standard deviation 1, \( \sigma_v \) is standard deviation of volatility shock and constant. From the equation (3.23) and (3.24), we know that the standard deviation of \( \ln a_t \) is \( v_{t-1} \) which is time varying.
3.2.7 Solution Method

Introducing time varying volatility into DSGE model is not trivial. BFJ (2010) and GSJ (2010) use VFI (Value Function Iteration) to identify the role of time-varying volatility. However, as Fernandez-Villaverde and Rubio-Ramirez (2010) [18] pointed, VFI has "curse of dimensionality" problem and might not be appropriate for the models with imperfections and rigidities.

As an alternative, we can consider perturbation method. Fernandez-Villaverde et al. (2009) [32] finds that in third-order approximation, volatility shock plays an independent role. BB (2011) applies third-order approximation in their analysis using the Perturbation AIM Algorithm and software developed by Swanson, Anderson, and Levin (2006) [16]. Benigno, Benigno, and Nistico (2010 [23], 2011 [24]) shows that if exogenous state variables follow conditionally linear stochastic processes, the second-order approximation would be sufficient to get distinct role of time varying volatility shock.

In this paper, we use third-order approximation for the solution of the model to identify the distinct role of time varying volatility. To get third-order policy function, we use Dynare++ software, stand alone C++ based version of Dynare. In addition, we cross check the result from Dynare++ with that from the method which BBN (2010) suggests.
3.3 Results

3.3.1 Calibration

For calibration, we basically follow BB (2011) for the parameters in exogenous shocks and Iacoviello (2005, 2011) for other parameters. Table 3.1 presents calibrated parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_p$</td>
<td>0.990</td>
<td>Discount factor of patient households</td>
</tr>
<tr>
<td>$\beta_s$</td>
<td>0.980</td>
<td>Discount factor of subprimers</td>
</tr>
<tr>
<td>$\beta_e$</td>
<td>0.980</td>
<td>Discount factor of entrepreneurs</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.650</td>
<td>Weight of labor in utility function</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.025</td>
<td>Weight of housing in utility function</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.500</td>
<td>Parameter for risk aversion</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.300</td>
<td>Output share of capital</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.033</td>
<td>Output share of housing</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.640</td>
<td>Income share of patient households</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
<td>Depreciation rate</td>
</tr>
<tr>
<td>$m_s$</td>
<td>0.900</td>
<td>Loan to value ratio for subprimers’ housing</td>
</tr>
<tr>
<td>$m_h^e$</td>
<td>0.900</td>
<td>Loan to value ratio for entrepreneurs’ capital</td>
</tr>
<tr>
<td>$m_h^e$</td>
<td>0.900</td>
<td>Loan to value ratio for entrepreneurs’ housing</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>0.990</td>
<td>Technology shock persistence</td>
</tr>
<tr>
<td>$\rho_v$</td>
<td>0.830</td>
<td>Volatility shock persistence</td>
</tr>
<tr>
<td>$\bar{a}$</td>
<td>1.000</td>
<td>Steady state value of $a_t$</td>
</tr>
<tr>
<td>$\bar{v}$</td>
<td>0.010</td>
<td>Steady state value of $v_t$</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>0.420</td>
<td>Standard deviation of volatility shock</td>
</tr>
</tbody>
</table>

We set $\beta_p = 0.99$ and $\beta_s = \beta_e = 0.98$\(^6\) to satisfy the condition for binding constraint ($\beta_p > \beta_s, \beta_e$). For the parameters controlling leverage, we choose $m_s = m_h^e = m_h^e = 0.90$ following Iacoviello (2011) [30]. Since we have different functional form in utility function, we choose different parameters for weights on leisure and housing in utility function. The weights on leisure in utility function, $\tau$, is set at 0.65, this number yield 0.3 of weighted average of households labor in steady state. The weights of housing in utility function is set at 0.025 which yields 2.1 of ratio of real estate wealth to output in steady state as in Iacoviello (2011). For the share of patient households, $\alpha$, we choose 0.64 as in Iacoviello (2005)\(^7\) since our model considers highly indebted economy. The

\(^6\)Iacoviello (2005, 2011) have slightly different numbers each other. In 2005, $\beta_p = 0.99$, $\beta_s = 0.95$, and $\beta_e = 0.98$. In 2011, $\beta_p = 0.9925$, $\beta_s = 0.96$, and $\beta_e = 0.96$.

\(^7\)In Iacoviello (2011), $\alpha = 0.70$. 

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shares of capital and housing to output are set 0.30 and 0.033$^8$ to make generally used share of labor to output be 0.667. As baseline risk-aversion, we use $\sigma = 1.5$ since although it shortly disappears, we have slight increase in investment at the impact of uncertainty shock at $\sigma = 2.0$ which is generally used. Regarding calibration of exogenous shock, various numbers are suggested by various researchers because the volatility shock process is not well known yet. So we use the calibration which BB (2011) suggests: $\rho_a = 0.99$, $\rho_v = 0.83$, $\bar{v} = \sigma_a = 0.01$, $\sigma_v = 0.42$.

### 3.3.2 Impulse Response (Technology Volatility Shock Only)

From the impulse responses, <Figure 3.2> to <Figure 3.5>, to one standard deviation shock in technology volatility, we finds that uncertainty shock has negative effect on key macro variables. In contrast to the result of standard RBC model, investment, labor, and output goes down with consumption by uncertainty shock. Under higher risk aversion, investment slightly increases at the impact of the shock but shortly goes down below its steady state level. Our model also predicts negative effect on housing price and size of loan by uncertainty shock.

![Figure 3.2: Uncertainty Shock under Various Risk Aversion](image)

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$^8$In Iacoviello (2011) [30], $\mu = 0.35$ and $\nu = 0.05$.  

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Figure 3.3: Uncertainty Shock under Various Risk Aversion

Figure 3.4: Uncertainty Shock under Various Risk Aversion
The co-movement among key macro variables do not come from price stickiness, adjustment cost in production inputs, idiosyncratic shocks to heterogenous firms, or disaster shock. However, housing price plays an important role in the propagation of uncertainty shock through collateral constraint channel. Uncertainty shock forces borrowers to sell housing to avoid uncertainty in future consumption. It leads to drop in housing price. The lowered housing price causes credit crunch in subprimers and entrepreneurs through collateral constraint channel, which makes decreases in aggregate consumption and investment.

Patient households buy housing at lowered price by reducing loans to borrowers. Impatient households sell housing to reduce uncertainty in future consumption. Entrepreneurs also reduce housing but it is not sufficient to maintain previous level of consumption. Although both households still have precautionary motive to work more, lowered labor demand by firms and increased in housing (patient households) and consumption (impatient households) lowers hours to work in equilibrium.

We finds that the role of risk aversion under uncertainty shock is almost canceled out by collateral constraint channel. In RBC model, higher risk aversion leads more investment and labor so more output. However, by introducing collateral constraint channel, under higher risk aversion, consumption drops more but investment and labor increases less. As a result, output decreases more under higher risk aversion. This result
comes from the sensitivity of housing price to the risk aversion under uncertainty shock. Under higher risk aversion, borrowers sell more portion of their housing. It leads to more drop in housing price. Although stronger precautionary motive increases labor and investment by reducing consumption, collateral constraint channel significantly dampens the increase in investment and labor but strengthens decrease in consumption.

We also check the impulse responses of volatility shock under various LTVs. The result shows that the level of indebtedness of borrowers plays an important role in the propagation of uncertainty shock. Higher indebted borrowers sell more portion of their housing so housing price drops more up to mid term in the higher indebted economy. However, the effects on consumption and investment are mixed. Consumption decreases more but investment decreases less although the consumption channel dominates the investment channel. As a result, output decreases more in the economy with higher indebted borrowers under uncertainty shock.

Figure 3.6: Uncertainty Shock under Various LTVs
Figure 3.7: Uncertainty Shock under Various LTVs

Figure 3.8: Uncertainty Shock under Various LTVs
Regarding quantitative magnitude, we find that the effect of technology volatility shock is very small. In our base line calibration ($\sigma = 1.5$ and $LTV = 0.90$), one standard deviation technology uncertainty shock generates only 0.007 percentage points drop in GDP at a peak. When $LTV = 0.95$, we still have only 0.016 percentage points drop in GDP at a peak. BB (2011) also finds that the technology volatility shock generates a peak drop in output of 0.02 percentage points. Although the magnitude of the effect is very small, our model can generate opposite direction of impulse responses. In standard RBC model, same technology volatility shock produces 0.004 percentage points increase in output at a peak.

### 3.3.3 Impulse Response (Technology Level and Volatility Shocks)

Uncertainty tends to increase when external shock hits the economy regardless of the sign of the external shock. <figure 3.10> and <figure 3.11> present the impulse responses of variables to the simultaneous positive one standard deviation shocks of technology level and volatility in RBC model and our extended model respectively. When we consider the case in which technology volatility shock works together with technology level shock, we should note that the volatility shock affects the model not only in the third order term but also in the second order terms.

In RBC model, the positive technology volatility shock amplifies the effect of the
positive technology level shock since the volatility shock tends to have positive effect on GDP in RBC model.

In contrast, the effect of the positive technology level shock will be dampen by the positive technology volatility shock since the volatility shock has negative effect on GDP in our extended model. Interestingly, the statement will be opposite in the case of the negative technology level shock. In RBC model, the effect of negative technology level shock will be dampen because of same reasoning. In the contrary, the positive technology volatility shock will amplify the effect of the negative technology level shock in our extended model.
3.4 Conclusion

We introduce time varying volatility shock in technology into a DSGE model with collateral constrained borrowers. Under uncertainty shock, borrowers sell housing to avoid uncertainty in future consumption and the decrease in housing demand leads to drop in housing price. Lowered housing price plays an important role in the propagation of uncertainty shock through collateral constraint channel. By decreasing supply of liquidity, collateral constraint channel causes simultaneous drop in aggregate consumption, investment, and output. Under higher risk aversion or higher LTVs, the collateral constraint channel is strengthened and has more negative effects on consumption and output under volatility shock. Although investment increases at first under higher risk aversion or higher LTVs, it shortly goes down below its steady state level.
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Chapter 1. Appendix

Solving for Steady State Lending Rates:

The steady state equations of domestic and foreign patient HHs’ FOCs for deposits give the deposit rates: \( R_d = \frac{1}{\beta_p} \) and \( R_d^* = \frac{1}{\beta_p^*} \). Foreign banks’ steady state FOCs for deposits and loans can be written as:

\[
egin{align*}
1 - \beta_b^* R_d^* &= \lambda_b^* \\
1 - \beta_b^* R_i^* &= \lambda_b^* \gamma_i^* \\
1 - \beta_b^* R_e^* &= \lambda_b^* \gamma_e^* \\
1 - \beta_b^* R_f^* &= \lambda_b^* \gamma_f^* 
\end{align*}
\]

where \( \lambda_b^* \) is the multiplier on foreign banks capital requirement constraint, multiplied by foreign bankers’ consumption, i.e., \( \lambda_b^* = \lambda_b^* c_{b,t} \). Simple algebra, replacing \( \lambda_b^* \) with \( (1 - \beta_b^* R_d^*) \), then yield the foreign banks’ lending rates. Domestic banks steady state FOCs for deposits and loans are:

\[
egin{align*}
1 - \beta_b R_d &= \lambda_b' + \lambda_f' m_f \\
1 - \beta_b R_i &= \lambda_b' \gamma_i + \lambda_f' m_f \\
1 - \beta_b R_e &= \lambda_b' \gamma_e + \lambda_f' m_f \\
1 - \beta_b R_f &= \lambda_b' + \lambda_f' R_f 
\end{align*}
\]

From the first and the last equation of the above system of four equations, one can solve for the value of \( \lambda_f \):

\[
\lambda_f' = \frac{\beta_b (R_d - R_f)}{R_f - m_f}
\]

The bottom of the equation is greater than 0, since \( R_f > 1 > m_f \). Thus, \( \lambda_f > 0 \) when \( R_d > R_f \). Combine the above system of equations to solve for domestic lending
rates, we have

\[
\begin{align*}
\frac{R_f - R_d}{R_i - R_d} &= \frac{\lambda'_f (m_f - R_f)}{\lambda'_b (1 - \gamma_i)} \\
\frac{R_f - R_d}{R_i - R_d} &= \frac{\lambda'_f (m_f - R_f)}{(1 - \beta_b R_d - \lambda'_f m_f)(1 - \gamma_i)} = -\delta_1 \\
R_i &= R_d + \frac{1}{\delta_1} (R_d - R_f)
\end{align*}
\]

\[
\begin{align*}
\frac{R_f - R_d}{R_e - R_d} &= \frac{\lambda'_f (m_f - R_f)}{\lambda'_b (1 - \gamma_e)} \\
\frac{R_f - R_d}{R_e - R_d} &= \frac{\lambda'_f (m_f - R_f)}{(1 - \beta_b R_d - \lambda'_f m_f)(1 - \gamma_e)} = -\delta_2 \\
R_e &= R_d + \frac{1}{\delta_2} (R_d - R_f)
\end{align*}
\]

All deposit and lending rate in the table are now determined.
Chapter 2. Appendix

Subprimers’ Budget Constraint:

We assume subprimers with outstanding loan lose collateral if they lose job and go default. Then, we can write subprimers’ budget as follows:

\[ c_{s,t} + qt(h_{s,t} - h_{s,t-1}) + n_{s,t}^\phi R_{s,t-1}n_{s,t-1}^\chi b_{s,t-1} + (1 - n_{s,t}^\phi)n_{s,t-1}^\chi qt h_{s,t-1} \]

\[ = n_{s,t}^\chi b_{s,t} + n_{s,t} w_{s,t} l_{s,t} + t_1. \]

(*): losing collateral

Under this budget constraint, subprimers would consider the effect of unemployment when they buy house. To make subprimers consider the effect of unemployment when they borrow, we need to rewrite (*) in terms of borrowing instead of collateral.

Since collateral constraint always binds, we can write the current value of housing in terms of borrowing as follows:

\[ R_{s,t-1} b_{s,t-1} = m_s E_{t-1} (qt h_{s,t-1}) \]

\[ \Rightarrow E_{t-1} (qt h_{s,t-1}) = \frac{1}{m_s} R_{s,t-1} b_{s,t-1} \]

\[ \Rightarrow h_{s,t-1} = \frac{1}{m_s} \frac{1}{E_{t-1} (qt)} R_{s,t-1} b_{s,t-1} \]

\[ \Rightarrow qt h_{s,t-1} = \frac{1}{m_s} q_t R_{s,t-1} b_{s,t-1}. \]

(**)

Plug (**) into the above budget constraint, we get equation (4):

\[ c_{s,t} + qt(h_{s,t} - h_{s,t-1}) + n_{s,t}^\phi R_{s,t-1} n_{s,t-1}^\chi b_{s,t-1} + (1 - n_{s,t}^\phi)n_{s,t-1}^\chi qt h_{s,t-1} \]

\[ = n_{s,t}^\chi b_{s,t} + n_{s,t} w_{s,t} l_{s,t} + t_1 \]

\[ \Rightarrow c_{s,t} + qt(h_{s,t} - h_{s,t-1}) + n_{s,t}^\phi R_{s,t-1} n_{s,t-1}^\chi b_{s,t-1} + (1 - n_{s,t}^\phi)n_{s,t-1}^\chi 1 m_s q_t R_{s,t-1} b_{s,t-1} \]

\[ = n_{s,t}^\chi b_{s,t} + n_{s,t} w_{s,t} l_{s,t} + t_1 \]

\[ \Rightarrow c_{s,t} + qt(h_{s,t} - h_{s,t-1}) + \left[ n_{s,t}^\phi + (1 - n_{s,t}^\phi) \frac{1}{m_s} q_t \right] n_{s,t-1}^\chi R_{s,t-1} b_{s,t-1} \]

Repayment of Loan

\[ = n_{s,t}^\chi b_{s,t} + n_{s,t} w_{s,t} l_{s,t} + t_1. \]

Now subprimers consider the effect of unemployment when they borrow. If they are employed, they would successfully pay back loan. Otherwise, they would lose the value of collateral.
Subprimers’ Surplus from Marginal Matching:

Given \( n_{s,t+1} = (1 - \varphi)n_{s,t} + f(\gamma_{s,t})(1 - n_{s,t}) \), subprimers solve following recursive form utility maximization problem:

\[
W(n_{s,t}, n_{s,t-1}) = \arg \max \ln c_{s,t} + j_t \ln h_{s,t} + n_{s,t} \tau \ln (1 - l_{s,t}) + (1 - n_{s,t}) \ln (1 - e) \\
+ \mu_{s,t}(\ast) + \lambda_{s,t}(\ast\ast) + \beta_t E_t [W(n_{s,t+1}, n_{s,t})]
\]

where \( \mu_{s,t} \) and \( \lambda_{s,t} \) are the Lagrangian multipliers for budget constraint and collateral constraint, \( \ast \) and \( \ast\ast \) are budget constraint and collateral constraint as follows:

\[
\ast = n^x_{s,t} b_{s,t} + n_{s,t} w_{s,t} l_{s,t} - c_s (h_{s,t} - h_{s,t-1}) \\
- \left\{ n^\phi_{s,t} + (1 - n^\phi_{s,t}) \frac{q_t}{m_s q_t^e} \right\} R_{s,t-1} n^x_{s,t-1} b_{s,t-1}, \\
\ast\ast = m_s E_t (q_{t+1} h_{s,t}) - R_{s,t} b_{s,t}.
\]

Note that subprimers’ value function, \( W(n_{s,t}, n_{s,t-1}) \), depends on \( n_{s,t-1} \) as well as \( n_{s,t} \) since outstanding loan is determined by last period employment. We should consider that when we differentiate the value function with respect to \( n_{s,t} \).

\[
\frac{\partial W(n_{s,t}, n_{s,t-1})}{\partial n_{s,t}} = W_1(n_{s,t}, n_{s,t-1}), \\
\frac{\partial W(n_{s,t+1}, n_{s,t})}{\partial n_{s,t}} = E_t \left[ \frac{\partial W(n_{s,t+1}, n_{s,t})}{\partial n_{s,t+1}} \frac{\partial n_{s,t+1}}{\partial n_{s,t}} |_{n_{s,t+1}} \right] \\
= E_t \left\{ W_1(n_{s,t+1}, n_{s,t}) [(1 - \varphi) - f(\gamma_{s,t})] - \mu_{s,t+1} \left[ n^\phi_{s,t+1} + (1 - n^\phi_{s,t+1}) \frac{1}{m_s} \right] R_{s,t} b_{s,t} \chi n^x_{s,t} \right\}.
\]

By Envelope theorem, we could find subprimers surplus from marginal matching:

\[
W_1(n_{s,t}, n_{s,t-1}) = \tau \ln (1 - l_{s,t}) - \tau \ln (1 - e) \\
+ \mu_{s,t} \left[ \chi n^x_{s,t} b_{s,t} + w_{s,t} l_{s,t} - \phi n^\phi_{s,t-1} \left( 1 - \frac{1}{m_s q_t^e} \right) R_{s,t-1} n^x_{s,t-1} b_{s,t-1} \right] \\
+ E_t \left\{ W_1(n_{s,t+1}, n_{s,t}) [(1 - \varphi) - f(\gamma_{s,t})] - \mu_{s,t+1} \left[ n^\phi_{s,t+1} + (1 - n^\phi_{s,t+1}) \frac{1}{m_s} \right] R_{s,t} b_{s,t} \chi n^x_{s,t} \right\}.
\]