Monetary Policy and the Great Recession

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Boston College

The Graduate School of Arts and Sciences

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MONETARY POLICY AND THE GREAT RECESSION

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Monetary Policy and the Great Recession

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Abstract

The Great Recession is arguably the most important macroeconomic event of the last three decades. Prior to the collapse of national output during 2008 & 2009, the United States experienced a sustained period of good economic outcomes with only two mild and short recessions.

In addition to the severity of the recession, several characteristics of this recession signify it as a unique event in the recent economic history of the United States. Some of these unique features include the following:
Large Increase in Uncertainty About the Future: The Great Recession and its subsequent slow recovery have been marked by a large increase in uncertainty about the future. Uncertainty, as measured by the VIX index of implied stock market volatility, peaked at the end of 2008 and has remained volatile over the past few years.

Many economists and the financial press believe the large increase in uncertainty may have played a role in the Great Recession and subsequent slow recovery. For example, Kocherlakota (2010) states, “I’ve been emphasizing uncertainties in the labor market. More generally, I believe that overall uncertainty is a large drag on the economic recovery.” In addition, Nobel laureate economist Peter Diamond argues, “What’s critical right now is not the functioning of the labor market, but the limits on the demand for labor coming from the great caution on the side of both consumers and firms because of the great uncertainty of what’s going to happen next.”
**Zero Bound on Nominal Interest Rates:** The Federal Reserve plays a key role in offsetting the negative impact of fluctuations in the economy. During normal times, the central bank typically lowers nominal short-term interest rates in response to declines in inflation and output. Since the end of 2008, however, the Federal Reserve has been unable to lower its nominal policy rate due to the zero lower bound on nominal interest rates.

Prior to the Great Recession, the Federal Reserve had not encountered the zero lower bound in the modern post-war period. The zero lower bound represents a significant constraint monetary policy’s ability to fully stabilize the economy.

**Unprecedented Use of Forward Guidance:** Even though the Federal Reserve remains constrained by the zero lower bound, the monetary authority can still affect the economy through expectations about future nominal policy rates. By providing agents in the economy with *forward guidance* on the future path of policy rates, monetary policy can
stimulate the economy even when current policy rates remain constrained. Throughout the Great Recession and the subsequent recovery, the Federal Reserve provided the economy with explicit statements about the future path of monetary policy. In particular, the central bank has discussed the timing and macroeconomic conditions necessary to begin raising its nominal policy rate. Using this policy tool, the Federal Reserve continues to respond to the state of the economy at the zero lower bound.

**Large Fiscal Expansion:** During the Great Recession, the United States engaged in a very large program of government spending and tax reductions.

The massive fiscal expansion was designed to raise national income and help mitigate the severe economic contraction. A common justification for the fiscal expansion is the reduced capacity of the monetary authority to stimulate the economy at the zero lower bound.
Many economists argue that the benefits of increasing government spending are significantly higher when the monetary authority is constrained by the zero lower bound.

The goal of this dissertation is to better understand how these various elements contributed to the macroeconomic outcomes during and after the Great Recession. In addition to understanding each of the elements above in isolation, a key component of this analysis focuses on the interaction between the above elements. A key unifying theme between all of the elements is the role in monetary policy. In modern models of the macroeconomy, the monetary authority is crucial in determining how a particular economic mechanism affects the macroeconomy. In the first and second chapters, I show that monetary policy plays a key role in offsetting the negative effects of increased uncertainty about the future. My third chapter highlights how assumptions about monetary policy can change the impact of various shocks and policy interventions. For example, suppose the fiscal authority wants to increase national output by increasing government spending. A key calculation in this situation is the fiscal multiplier, which is dollar increase in national income for each dollar of government spending. I show that fiscal multipliers are dramatically affected by the assumptions about monetary policy even if the monetary authority is constrained by the zero lower bound.

The unique nature of the elements discussed above makes analyzing their contribution difficult using standard macroeconomic tools. The most popular method for analyzing dy-
namic, stochastic general equilibrium models of the macroeconomy relies on linearizing the model around its deterministic steady state and examining the local dynamics around that approximation. However, the nature of the unique elements above make it impossible to fully capture dynamics using local linearization methods. For example, the zero lower bound on nominal interest rates often occurs far from the deterministic steady state of the model. Therefore, linearization around the steady state cannot capture the dynamics associated with the zero lower bound. The overall goal of this dissertation is to use and develop tools in computational macroeconomics to help better understand the Great Recession. Each of the chapters outlined below examine at least one of the topics listed above and its impact in explaining the macroeconomics of the Great Recession. In particular, the essays highlight the role of the monetary authority in generating the observed macroeconomic outcomes over the past several years.

Can increased uncertainty about the future cause a contraction in output and its components? In joint work with Susanto Basu, my first chapter examines the role of uncertainty shocks in a one-sector, representative-agent, dynamic, stochastic general-equilibrium model. When prices are flexible, uncertainty shocks are not capable of producing business-cycle comovements among key macroeconomic variables. With countercyclical markups through sticky prices, however, uncertainty shocks can generate fluctuations that are consistent with business cycles. Monetary policy usually plays a key role in offsetting the negative impact of uncertainty shocks. If the central bank is constrained by the zero lower bound, then
monetary policy can no longer perform its usual stabilizing function and higher uncertainty has even more negative effects on the economy. We calibrate the size of uncertainty shocks using fluctuations in the VIX and find that increased uncertainty about the future may indeed have played a significant role in worsening the Great Recession, which is consistent with statements by policymakers, economists, and the financial press.

In sole-authored work, the second chapter continues to explore the interactions between the zero lower bound and increased uncertainty about the future. From a positive perspective, the essay further shows why increased uncertainty about the future can reduce a central bank’s ability to stabilize the economy. The inability to offset contractionary shocks at the zero lower bound endogenously generates downside risk for the economy. This increase in risk induces precautionary saving by households, which causes larger contractions in output and inflation and prolongs the zero lower bound episode. The essay also examines the normative implications of uncertainty and shows how monetary policy can attenuate the negative effects of higher uncertainty. When the economy faces significant uncertainty, optimal monetary policy implies further lowering real rates by committing to a higher price-level target. Under optimal policy, the monetary authority accepts higher inflation risk in the future to minimize downside risk when the economy hits the zero lower bound. In the face of large shocks, raising the central bank’s inflation target can attenuate much of the downside risk posed by the zero lower bound.
In my third chapter, I examine how assumptions about monetary policy affect the economy at the zero lower bound. Even when current policy rates are zero, I argue that assumptions regarding the future conduct of monetary policy are crucial in determining the effects of real fluctuations at the zero lower bound. Under standard Taylor (1993)-type policy rules, government spending multipliers are large, improvements in technology cause large contractions in output, and structural reforms that decrease firm market power are bad for the economy. However, these policy rules imply that the central bank stops responding to the economy at the zero lower bound. This assumption is inconsistent with recent statements and actions by monetary policymakers. If monetary policy endogenously responds to current economic conditions using expectations about future policy, then spending multipliers are much smaller and increases in technology and firm competitiveness remain expansionary. Thus, the model-implied benefits of higher government spending are highly sensitive to the specification of monetary policy.
To Tara
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## Contents

1 Uncertainty Shocks in a Model of Effective Demand  

   1.1 Introduction ........................................... 1  
   1.2 Intuition ............................................. 5  
   1.3 Model ................................................ 9  
      1.3.1 Households .................................... 9  
      1.3.2 Intermediate Goods Producers ...................... 12  
      1.3.3 Final Goods Producers .............................. 14  
      1.3.4 Monetary Policy .................................. 15  
      1.3.5 Equilibrium ....................................... 16  
      1.3.6 Shock Processes ................................... 16  
      1.3.7 Solution Method .................................. 17  
   1.4 Calibration and Baseline Results .................... 18  
      1.4.1 Calibration ....................................... 18  
      1.4.2 Uncertainty Shocks & Business Cycle Comovements .... 19
## Contents

1.5 Discussion and Connections ........................................ 21
   1.5.1 Specific Example of General Principle .................... 21
   1.5.2 Extension to Sticky Nominal Wages ....................... 22
   1.5.3 Connections with Existing Literature .................... 25

1.6 Quantitative Results & Great Recession Application ........... 28
   1.6.1 Uncertainty Shock Calibration .............................. 28
   1.6.2 Quantitative Impact of Uncertainty Shocks ............... 31
   1.6.3 The Role of Uncertainty Shocks in the Great Recession .... 33

1.7 Uncertainty Shocks and the Zero Lower Bound ................. 36
   1.7.1 Solution Method and Calibration .......................... 37
   1.7.2 Interactions of Uncertainty and Monetary Policy .......... 37
   1.7.3 Impulse Response Analysis ................................ 39
   1.7.4 Revisiting the Role of Uncertainty Shocks in the Great Recession .... 41
   1.7.5 Complexity of Uncertainty at the Zero Lower Bound ....... 42

1.8 Conclusion .......................................................... 43

2 Forward Guidance Under Uncertainty ............................. 53
   2.1 Introduction .................................................... 53
   2.2 Intuition ........................................................ 57
      2.2.1 Household Consumption Under Uncertainty ............. 58
      2.2.2 Consumption Uncertainty in General Equilibrium ....... 59
Contents

2.2.3 Zero Lower Bound and Downside Risk ............................................. 61
2.2.4 From Intuition to Model Simulations .............................................. 62

2.3 Model ......................................................................................... 63
2.3.1 Households ............................................................................. 63
2.3.2 Intermediate Goods Producers .................................................. 65
2.3.3 Final Goods Producers ............................................................... 66
2.3.4 Monetary Policy ....................................................................... 67
2.3.5 Shock Processes ........................................................................ 68
2.3.6 Equilibrium .............................................................................. 68
2.3.7 Solution Method ....................................................................... 69
2.3.8 Calibration ............................................................................... 69
2.3.9 Transmission of Precautionary Saving to Macroeconomy ............ 71

2.4 Quantitative Effects of Uncertainty on Forward Guidance ............... 72
2.4.1 Single Shock Model Responses .................................................. 72
2.4.2 Model Simulations and Downside Risk in the Economy ............... 74
2.4.3 Optimal Monetary Policy Under Commitment ............................ 76
2.4.4 A Calibration Check Using Recent Macroeconomic Data .......... 79

2.5 Discussion and Connections with Existing Literature ...................... 82
2.5.1 Monetary Policy at the Zero Lower Bound .................................... 82
2.5.2 Contractionary Bias in the Nominal Interest-Rate Distribution .... 84
2.5.3 Uncertainty and the Effectiveness of Monetary Policy ................. 87
Contents

2.6 Extensions ......................................................... 88
  2.6.1 Raising the Central Bank’s Inflation Target ................. 88
2.7 Conclusions ....................................................... 89

3  Real Fluctuations at the Zero Lower Bound .................. 103
  3.1 Introduction ..................................................... 103
  3.2 Model ............................................................. 107
    3.2.1 Households ............................................... 108
    3.2.2 Final Goods Producers .................................. 109
    3.2.3 Intermediate Goods Producers ......................... 110
    3.2.4 Monetary and Fiscal Policy .............................. 111
    3.2.5 Shock Processes ......................................... 112
    3.2.6 Equilibrium and Solution Method ...................... 113
    3.2.7 Calibration ................................................ 114
  3.3 Effects of Real Shocks ........................................ 115
    3.3.1 Aggregate Demand and Aggregate Supply ................. 115
    3.3.2 Technology Shocks ....................................... 118
    3.3.3 Markup Shocks ............................................. 120
    3.3.4 Government Spending Shocks ............................ 121
    3.3.5 Amount of History-Dependence .......................... 122
    3.3.6 Forms of History-Dependence ............................ 124
## Contents

3.3.7 Unemployment and History-Dependence ........................................ 125

3.3.8 Discussion and Connections with Existing Literature .................... 127

3.4 Conclusions ..................................................................................... 128

A Uncertainty Shocks in a Model of Effective Demand 139

A.1 Solving the Model with a Zero Lower Bound Constraint .................... 139

A.1.1 Numerical Solution Method .............................................................. 139

A.2 Uncertainty, the Zero Lower Bound, and the Contractionary Bias .......... 140


A.2.2 Contractionary Bias in the Average Nominal Interest Rate .............. 142

A.2.3 Impulse Response Analysis Under History-Dependent Policy Rule ..... 144

A.2.4 Uncertainty, Contractionary Bias, and Equilibrium Existence ........... 145

B Forward Guidance Under Uncertainty 150

B.1 Derivations of Approximated Model .................................................. 150

B.1.1 Approximation of Consumption Euler Equation .............................. 150

B.1.2 Derivation of Higher-Order New-Keynesian Model .......................... 152

B.2 Numerical Solution Method ............................................................... 155

B.3 Optimal Monetary Policy Under Commitment ................................... 156

C Real Fluctuations at the Zero Lower Bound 159

C.1 Numerical Solution Method ............................................................... 159
List of Tables

1.1 Baseline Calibration ................................................. 45

2.1 Calibration of Model Parameters .................................. 91

3.1 Calibration of Model Parameters ................................. 129
List of Figures

1.1 Flexible Price Model Intuition ............................................ 44
1.2 Sticky Price Model Intuition ............................................. 44
1.3 Impulse Responses of Quantities to Second Moment Technology Shock . . . 46
1.4 Impulse Responses of Prices to Second Moment Technology Shock ........ 47
1.5 Impulse Responses of Quantities to Second Moment Preference Shock . . . 48
1.6 Impulse Responses of Prices to Second Moment Preference Shock ....... 49
1.7 VIX and VIX-Implied Uncertainty Shocks .............................. 50
1.8 Model-Implied VIX and Uncertainty Shock Calibration ............... 51
1.9 Demand Uncertainty Shock Under History-Dependent Taylor Rule ....... 52

2.1 Precautionary Labor Supply Intuition .................................... 92
2.2 Model Responses to Zero Lower Bound Episode under Price-Level Targeting 93
2.3 Model Simulations After Hitting Zero Lower Bound under Price-Level Targeting 94
2.4 Expected Distributions at Zero Lower Bound under Price-Level Targeting . 95
List of Figures

2.5 Model Responses to Zero Lower Bound Episode under Optimal Policy . . . 96
2.6 Model Simulations After Hitting Zero Lower Bound under Optimal Policy . 97
2.7 Expected Distributions at Zero Lower Bound under Optimal Policy . . . . 98
2.8 Model Simulations and Recent Macroeconomic Data . . . . . . . . . . . 99
2.9 Model Simulations and Recent Macroeconomic Data Without Zero Lower
Bound . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 100
2.10 Nominal Interest Rate Distribution with Zero Lower Bound Constraint . . 101
2.11 Simple Monetary Policy Rules & Fisher Relation with Zero Lower Bound
Constraint . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 101
2.12 Model Simulations Under Two and Four Percent Inflation Targets . . . . 102

3.1 Aggregate Supply and Demand and the Zero Lower Bound . . . . . . . 130
3.2 Downward Shift in Aggregate Supply . . . . . . . . . . . . . . . . . . . . . 131
3.3 Model Responses to Positive Technology Shock . . . . . . . . . . . . . . . 132
3.4 Effects of Positive Technology Shock . . . . . . . . . . . . . . . . . . . . . 133
3.5 Model Responses to Decrease in Firm Desired Markups . . . . . . . . . . 134
3.6 Effects of Decrease in Firm Desired Markups . . . . . . . . . . . . . . . . 135
3.7 Model Responses to Increase in Government Spending . . . . . . . . . . 136
3.8 Effects of Increase in Government Spending . . . . . . . . . . . . . . . . . 137
3.9 Model Responses Under Alternative Calibrations of History-Dependence . . 138

A.1 Demand Uncertainty Shock Under Alternative Policy Rules . . . . . . . 148
List of Figures

A.2 Nominal Interest Rate Distribution with Zero Lower Bound Constraint . . . 149
A.3 Simple Monetary Policy Rules & Fisher Relation with Zero Lower Bound . 149
Chapter 1

Uncertainty Shocks in a Model of Effective Demand

1.1 Introduction

Economists and the financial press often discuss uncertainty about the future as an important driver of economic fluctuations, and a contributor in the Great Recession and subsequent slow recovery. For example, Diamond (2010) says, “What’s critical right now is not the functioning of the labor market, but the limits on the demand for labor coming from the great caution on the side of both consumers and firms because of the great uncertainty of what’s going to happen next.” Recent research by Bloom (2009), Bloom et al. (2011), Fernàndez-Villaverde et al. (2011), Born and Pfeifer (2011), and Gilchrist, Sim and Zakrajšek (2010) also suggests that uncertainty shocks can cause fluctuations in
Chapter 1 Uncertainty Shocks in a Model of Effective Demand

macroeconomic aggregates. However, most of these papers experience difficulty in generating business-cycle comovements among output, consumption, investment, and hours worked from changes in uncertainty. If uncertainty is a contributing factor in the Great Recession and persistently slow recovery, then increased uncertainty should reduce output and its components.

In this paper, we show why competitive, one-sector, closed-economy models generally cannot generate business-cycle comovements in response to changes in uncertainty. Under reasonable assumptions, an increase in uncertainty about the future induces precautionary saving and lower consumption. If households supply labor inelastically, then total output remains constant since the level of technology and capital stock remain unchanged in response to the uncertainty shock. Unchanged total output and reduced consumption together imply that investment must rise. If households can adjust their labor supply and consumption and leisure are both normal goods, an increase in uncertainty also induces “precautionary labor supply,” or a desire for the household to supply more labor for a given level of the real wage. As current technology and the capital stock remain unchanged, the competitive demand for labor remains unchanged as well. Thus, higher uncertainty reduces consumption but raises output, investment, and hours worked. This lack of comovement is a robust prediction of simple neoclassical models subject to uncertainty fluctuations.

We also show that non-competitive, one-sector models with countercyclical markups
Chapter 1 Uncertainty Shocks in a Model of Effective Demand

through sticky prices can easily overcome the comovement problem and generate simultaneous drops in output, consumption, investment, and hours worked in response to an uncertainty shock. An increase in uncertainty induces precautionary labor supply by the representative household, which reduces firm marginal costs of production. Falling marginal costs with slowly-adjusting prices imply an increase in firm markups over marginal cost. A higher markup reduces the demand for consumption, and especially, investment goods. Since output is demand-determined in these models, output and employment must fall when consumption and investment both decline. Thus, comovement is restored, and uncertainty shocks cause fluctuations that look qualitatively like a business cycle. Returning to Diamond’s (2010) intuition, simple competitive business-cycle models do not exhibit movements in “the demand for labor” as a result of an uncertainty shock. However, uncertainty shocks easily cause fluctuations in the demand for labor in non-competitive, sticky-price models with endogenously-varying markups. Thus, the non-competitive model captures the intuition articulated by Diamond. Understanding the dynamics of the demand for labor explains why the two models behave so differently in response to a change in uncertainty. Importantly, the non-competitive model is able to match the estimated effects of uncertainty shocks in the data by Bloom (2009) and Alexopoulos and Cohen (2009), while the competitive model cannot.

To analyze the quantitative impact of uncertainty shocks under flexible and sticky prices, we calibrate and solve a representative-agent, dynamic stochastic general equilibrium model
Chapter 1 Uncertainty Shocks in a Model of Effective Demand

with nominal price rigidity. We examine uncertainty shocks to both technology and household discount factors, which we interpret as cost and demand uncertainty. We calibrate our uncertainty shock processes using the Chicago Board Options Exchange Volatility Index (VIX), which measures the expected volatility of the Standard and Poor’s 500 stock index over the next thirty days. Using a third-order approximation to the policy functions of our calibrated model, we show that uncertainty shocks can produce contractions in output and all its components when prices adjust slowly. In particular, we find that increased uncertainty associated with future demand can produce significant declines in output, hours, consumption, and investment. Our model predicts that a one standard deviation increase in the uncertainty about future demand produces a peak decline in output of about 0.2 percent.

Finally, we examine the role of monetary policy in determining the equilibrium effects of uncertainty shocks. Standard monetary policy rules imply that the central bank usually offsets increases in uncertainty by lowering its nominal policy rate. We show that increases in uncertainty have larger negative impacts on the economy if the monetary authority is constrained by the zero lower bound on nominal interest rates. In these circumstances, our model predicts that an increase in uncertainty causes a much larger decline in output and its components. The sharp increase in uncertainty during the financial crisis in late 2008 corresponds to a period when the Federal Reserve had a policy rate near zero. Thus, we believe that greater uncertainty may have plausibly contributed significantly to the large and persistent output decline starting at that time. Our results suggest that about one-fourth
of the drop in output that occurred in late 2008 can plausibly be ascribed to increased uncertainty about the future.

Our emphasis on the effects of uncertainty in a one-sector model does not mean that we deprecate alternative modeling strategies. For example, Bloom et al. (2011) examine changes in uncertainty in a heterogeneous-firm model with convex and non-convex adjustment costs. However, this complex model is unable to generate positive comovement of the four key macro aggregates following an uncertainty shock. Furthermore, heterogeneous-agent models are challenging technically to extend along other dimensions. For example, adding nominal price rigidity for each firm and a zero lower bound constraint on nominal interest rates would be difficult in the model of Bloom et al. (2011). We view our work as a complementary approach to modeling the business-cycle effects of uncertainty. The simplicity of our underlying framework allows us to tackle additional issues that we think are important for understanding the Great Recession.

1.2 Intuition

This section formalizes the intuition from the introduction using a few key equations that characterize a large class of one-sector business cycle models. We show that the causal ordering of these equations plays an important role in understanding the impact of uncertainty shocks. These equations link total output $Y_t$, household consumption $C_t$, investment $I_t$, hours worked $N_t$, and the real wage $W_t/P_t$. The following key equations consist of a
“demand” equation, an aggregate production function, and a static first-order condition for a representative consumer to maximize utility:

\[ Y_t = C_t + I_t, \quad (1.1) \]
\[ Y_t = F(K_t, Z_t N_t), \quad (1.2) \]
\[ \frac{W_t}{P_t} U_1(C_t, 1 - N_t) = U_2(C_t, 1 - N_t). \quad (1.3) \]

Typical partial-equilibrium results suggest that an increase in uncertainty about the future decreases both consumption and investment. When consumers face a stochastic income stream, higher uncertainty about the future induces precautionary saving by risk-averse households. Recent work by Bloom (2009) argues that an increase in uncertainty also depresses investment, particularly in the presence of non-convex costs of adjustment. If an increase in uncertainty lowers consumption and investment in partial equilibrium, Equation (1.1) suggests that it should lower total output in a general-equilibrium model. In a setting where output is demand-determined, economic intuition suggests that higher uncertainty should depress total output and its components.

However, the previous intuition is incorrect in a general-equilibrium neoclassical model with a representative firm and a consumer with additively time-separable preferences. In this neoclassical setting, labor demand (the partial derivative of Equation (1.2) with respect to \( N_t \)) is determined by the current level of capital and technology, neither of which changes when uncertainty increases. The first-order conditions for firm labor demand derived from
Equation (1.2) and the labor supply condition in Equation (1.3) can be combined to yield:

\[ Z_t F_2(K_t, Z_t N_t) U_1(C_t, 1 - N_t) = U_2(C_t, 1 - N_t). \]  

(1.4)

Equation (1.4) defines a positively-sloped “income expansion path” for consumption and leisure for given levels of capital and technology. If higher uncertainty reduces consumption, then Equation (1.4) shows that increased uncertainty must increase labor supply. However, Equation (1.2) implies that total output must rise. A reduction in consumption and an increase in total output in Equation (1.1) means that investment and consumption must move in opposite directions.\(^1\)

In a non-neoclassical setting, especially one with a time-varying markup of price over marginal cost, Equations (1.1) and (1.3) continue to apply, but Equation (1.4) must be modified, and becomes:

\[ \frac{1}{\mu_t} Z_t F_2(K_t, Z_t N_t) U_1(C_t, 1 - N_t) = U_2(C_t, 1 - N_t) \]  

(1.5)

where \( \mu_t \) is the markup of price over marginal cost.

In such a setting, Equation (1.1) is causally prior to Equations (1.2) and (1.3). From Equation (1.1), output is determined by aggregate demand. Equation (1.2) then determines the necessary quantity of labor input for given values of \( K_t \) and \( Z_t \). Finally, given \( C_t \)

\(^1\)This argument follows Barro and King (1984). Jaimovich (2008) shows that this prediction may not hold for certain classes of preferences that are not additively time-separable.
Chapter 1 Uncertainty Shocks in a Model of Effective Demand

(determined by demand and other factors), the necessary supply of labor is made consistent with consumer optimization by having the markup taking on its required value. Alternatively, the wage moves to the level necessary for firms to hire the required quantity of labor, and the variable markup ensures that the wage can move independently of the marginal product of labor.

The previous intuition can also be represented graphically using simplified labor supply and labor demand curves in real wage and hours worked space. Figures 1.1 and 1.2 show the impact of an increase in uncertainty under both flexible prices with constant markups and sticky prices with endogenously-varying markups. An increase in uncertainty induces wealth effects on the representative household through the forward-looking marginal utility of wealth denoted by \( \lambda_t \). An increase in the marginal utility of wealth shifts the household labor supply curve outward. With flexible prices and constant markups, the labor demand curve remains fixed for a given level of the real wage. In the flexible-price equilibrium, the desire of households to supply more labor translates into higher equilibrium hours worked and a lower real wage. When prices adjust slowly to changing marginal costs, however, firm markups over marginal cost rise when the household increases their labor supply. For a given level of the real wage, an increase in markups decreases the demand for labor from firms. Figure 1.2 shows that equilibrium hours worked may fall as a result of the outward shift in the labor supply curve and the inward shift of the labor demand curve. The relative magnitudes of the changes in labor supply and labor demand depend on the specifics of
the macroeconomic model and its parameter values. The following section shows that in a reasonably calibrated New-Keynesian sticky price model, firm markups increase enough to produce a decrease in equilibrium hours worked in response to an increase in uncertainty.

1.3 Model

This section outlines the baseline dynamic stochastic general equilibrium model that we use in our analysis of uncertainty shocks. Our model provides a specific quantitative example of the intuition of the previous section. The baseline model shares many features with the models of Ireland (2003), Ireland (2011), and Jermann (1998). The model features optimizing households and firms and a central bank that systematically adjusts the nominal interest rate to offset adverse shocks in the economy. We allow for sticky prices using the quadratic-adjustment costs specification of Rotemberg (1982). Our baseline model considers both technology shocks and household discount rate shocks. Both shocks have time-varying second moments, which have the interpretation of cost uncertainty and demand uncertainty.

1.3.1 Households

In our model, the representative household maximizes lifetime utility given Epstein-Zin preferences over streams of consumption, $C_t$, and leisure, $1 - N_t$. The household solves its optimization problem subject to its risk aversion over the consumption-leisure basket $\sigma$ and its intertemporal elasticity of substitution $\psi$. The parameter $\theta_V \triangleq (1 - \sigma)(1 - 1/\psi)^{-1}$
controls the household’s preference for the resolution of uncertainty. The household receives labor income $W_t$ for each unit of labor $N_t$ supplied in the representative intermediate goods-producing firm. The representative household also owns the intermediate goods firm and holds equity shares $S_t$ and one-period riskless bonds $B_t$ issued by representative intermediate goods firm. Equity shares pay dividends $D^E_t$ for each share $S_t$ owned, and the riskless bonds return the gross one-period risk-free interest rate $R^R_t$. The household divides its income from labor and its financial assets between consumption $C_t$ and the amount of financial assets $S_{t+1}$ and $B_{t+1}$ to carry into next period. The discount rate of the household $\beta$ is subject to shocks via the stochastic process $a_t$. Since our model is a standard dynamic general-equilibrium model without government, any non-technological source of shocks must come from changes in preferences. Therefore, we interpret changes in the household discount factor as demand shocks hitting the economy.

The representative household maximizes lifetime utility by choosing $C_{t+s}, N_{t+s}, B_{t+s+1},$ and $S_{t+s+1}$ for all $s = 0, 1, 2, \ldots$ by solving the following problem:

$$V_t = \max \left[ a_t \left( C_t^{\eta} (1 - N_t)^{1-\eta} \right)^{\frac{1-\sigma}{\sigma}} + \beta \mathbb{E}_t V_{t+1}^{1-\sigma} \right]^{\frac{\sigma}{1-\sigma}}$$

subject to its intertemporal household budget constraint each period,

$$C_t + \frac{P^E_t}{P_t} S_{t+1} + \frac{1}{R^R_t} B_{t+1} \leq \frac{W_t}{P_t} N_t + \left( \frac{D^E_t}{P_t} + \frac{P^E_t}{P_t} \right) S_t + B_t.$$ \footnote{Our main results are robust to using expected utility preferences over consumption and leisure. The use of Epstein-Zin preferences allows us to calibrate our model using stock market data. Section 1.6.1 explains the details of our calibration method.}
Chapter 1 Uncertainty Shocks in a Model of Effective Demand

Using a Lagrangian approach, household optimization implies the following first-order conditions:

\[ \frac{\partial V_t}{\partial C_t} = \lambda_t \]  
\[ (1.6) \]

\[ \frac{\partial V_t}{\partial N_t} = \lambda_t \frac{W_t}{P_t} \]  
\[ (1.7) \]

\[ \frac{P^E_t}{P_t} = \mathbb{E}_t \left\{ \left( \frac{\beta \lambda_{t+1}}{\lambda_t} \right) \left( \frac{D^E_{t+1}}{P_{t+1}} + \frac{P^E_{t+1}}{P_{t+1}} \right) \right\} \]  
\[ (1.8) \]

\[ 1 = R_t^R \mathbb{E}_t \left\{ \left( \frac{\beta \lambda_{t+1}}{\lambda_t} \right) \right\} \]  
\[ (1.9) \]

where \( \lambda_t \) denotes the Lagrange multiplier on the household budget constraint. The utility function specification implies the following stochastic discount factor \( M_{t+1} \):

\[ M_{t+1} \triangleq \left( \frac{\partial V_t}{\partial C_t} / \partial C_t \right) = \left( \frac{\beta a_{t+1}}{a_t} \right) \left( \frac{C^\eta_{t+1} (1 - N_{t+1})^{1-\eta}}{C^\eta_t (1 - N_t)^{1-\eta}} \right)^{\frac{1-\sigma}{\sigma}} \left( \frac{C_t}{C_{t+1}} \right) \left( \frac{V_{t+1}^{1-\sigma}}{E_t \left[ V_{t+1}^{1-\sigma} \right]} \right)^{\frac{1}{1-\sigma}} \]  
\[ (1.10) \]

Using the stochastic discount factor, we can eliminate \( \lambda \) and simplify Equations (1.7) - (1.9) as follows:

\[ \frac{1 - \eta}{\eta} \frac{C_t}{1 - N_t} = \frac{W_t}{P_t} \]  
\[ (1.10) \]

\[ \frac{P^E_t}{P_t} = \mathbb{E}_t \left\{ \left( \frac{\beta a_{t+1}}{a_t} \right) \left( \frac{C^\eta_{t+1} (1 - N_{t+1})^{1-\eta}}{C^\eta_t (1 - N_t)^{1-\eta}} \right)^{\frac{1-\sigma}{\sigma}} \left( \frac{C_t}{C_{t+1}} \right) \left( \frac{V_{t+1}^{1-\sigma}}{E_t \left[ V_{t+1}^{1-\sigma} \right]} \right)^{\frac{1}{1-\sigma}} \left( \frac{D^E_{t+1}}{P_{t+1}} + \frac{P^E_{t+1}}{P_{t+1}} \right) \right\} \]  
\[ (1.11) \]

\[ 1 = R_t^R \mathbb{E}_t \left\{ \left( \frac{\beta a_{t+1}}{a_t} \right) \left( \frac{C^\eta_{t+1} (1 - N_{t+1})^{1-\eta}}{C^\eta_t (1 - N_t)^{1-\eta}} \right)^{\frac{1-\sigma}{\sigma}} \left( \frac{C_t}{C_{t+1}} \right) \left( \frac{V_{t+1}^{1-\sigma}}{E_t \left[ V_{t+1}^{1-\sigma} \right]} \right)^{\frac{1}{1-\sigma}} \right\} \]  
\[ (1.12) \]

Equation (1.10) represents the household intratemporal optimality condition with respect to consumption and leisure, and Equations (1.11) and (1.12) represent the Euler equations for equity shares and one-period riskless firm bonds.
1.3.2 Intermediate Goods Producers

Each intermediate goods-producing firm $i$ rents labor $N_t(i)$ from the representative household to produce intermediate good $Y_t(i)$. Intermediate goods are produced in a monopolistically competitive market where producers face a quadratic cost of changing their nominal price $P_t(i)$ each period. The intermediate-goods firms own the capital stock $K_t(i)$ for the economy and face adjustment costs for adjusting its rate of investment. Each firm issues equity shares $S_t(i)$ and one-period risk-less bonds $B_t(i)$. Firm $i$ chooses $N_t(i)$, $I_t(i)$, and $P_t(i)$ to maximize firm cash flows $D_t(i)/P_t(i)$ given aggregate demand $Y_t$ and price $P_t$ of the finished goods sector. The intermediate goods firms all have the same constant returns-to-scale Cobb-Douglas production function, subject to a fixed cost of production $\Phi$.

Each intermediate goods-producing firm maximizes discounted cash flows using the household stochastic discount factor:

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} M_{t+s} \left[ \frac{D_{t+s}(i)}{P_{t+s}} \right]$$

subject to the production function:

$$\left[ \frac{P_t(i)}{P_t} \right]^{-\theta} Y_t \leq K_t(i)^{\alpha} [Z_t N_t(i)]^{1-\alpha} - \Phi,$$

and subject to the capital accumulation equation:

$$K_{t+1}(i) = \left( 1 - \delta - \frac{\delta K}{2} \left( \frac{I_t(i)}{K_t(i)} - \delta \right) \right)^2 K_t(i) + I_t(i)$$
Chapter 1 Uncertainty Shocks in a Model of Effective Demand

where

\[
\frac{D_t(i)}{P_t} = \left[ \frac{P_t(i)}{P_t} \right]^{1-\theta} Y_t - \frac{W_t}{P_t} N_t(i) - I_t(i) - \frac{\phi_P}{2} \left[ \frac{P_t(i)}{\Pi P_{t-1}(i)} - 1 \right] Y_t
\]

The behavior of each firm \(i\) satisfies the following first-order conditions:

\[
\frac{W_t}{P_t} N_t(i) = (1 - \alpha) \xi_t K_t(i)^\alpha [Z_t N_t(i)]^{1-\alpha} \tag{1.13}
\]

\[
\frac{R^K_t}{P_t} K_t(i) = \alpha \xi_t K_t(i)^\alpha [Z_t N_t(i)]^{1-\alpha} \tag{1.14}
\]

\[
\phi_P \left[ \frac{P_t(i)}{\Pi P_{t-1}(i)} - 1 \right] \left[ \frac{P_t}{\Pi P_{t-1}(i)} \right] = (1 - \theta_\mu) \left[ \frac{P_t(i)}{P_t} \right]^{-\theta_\mu} + \theta_\mu \xi_t \left[ \frac{P_t(i)}{P_t} \right]^{-\theta_\mu-1} + \phi_P E_t \left\{ M_{t+1} \left[ \frac{P_{t+1}(i)}{\Pi P_{t+1}(i)} - 1 \right] \left[ \frac{P_{t+1}(i)}{P_{t+1}(i)} \right] \right\} \tag{1.15}
\]

\[
q_t = \frac{1}{q_t} = 1 - \phi_K \left( \frac{I_t}{K_t} - \delta \right) \tag{1.16}
\]

where \(\xi_t\) is the marginal cost of producing one additional unit of intermediate good \(i\), and \(q_t\) is the price of a marginal unit of installed capital. \(R^K_t/P_t\) is the marginal revenue product of capital, which is paid to the owners of the capital stock. Our adjustment cost specification is similar to the specification used by Jermann (1998) and Ireland (2003), and allows Tobin’s \(q\) to vary over time.

Each intermediate goods firm finances a percentage \(\nu\) of its capital stock each period with one-period riskless bonds. The bonds pay the one-period real risk-free interest rate. Thus,
the quantity of bonds $B_t(i) = \nu K_t(i)$. Total firm cash flows are divided between payments to bond holders and equity holders as follows:

$$\frac{DE_t(i)}{P_t} = \frac{D_t(i)}{P_t} - \nu \left( K_t(i) - \frac{1}{R^R_t} K_{t+1}(i) \right).$$

(1.18)

Since the Modigliani and Miller (1958) theorem holds in our model, leverage does not affect firm value or optimal firm decisions. Leverage makes the payouts and price of equity more volatile and allows us to define a concept of equity returns in the model. We use the volatility of equity returns implied by the model to calibrate our uncertainty shock processes in Section 1.6.

### 1.3.3 Final Goods Producers

The representative final goods producer uses $Y_t(i)$ units of each intermediate good produced by the intermediate goods-producing firm $i \in [0, 1]$. The intermediate output is transformed into final output $Y_t$ using the following constant returns to scale technology:

$$\left[ \int_0^1 Y_t(i) \frac{\theta_{\mu-1}}{\theta_{\mu}} \, di \right] \frac{\theta_{\mu}}{\theta_{\mu-1}} \geq Y_t$$

Each intermediate good $Y_t(i)$ sells at nominal price $P_t(i)$ and each final good sells at nominal price $P_t$. The finished goods producer chooses $Y_t$ and $Y_t(i)$ for all $i \in [0, 1]$ to maximize the following expression of firm profits:

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i) \, di$$
Chapter 1 Uncertainty Shocks in a Model of Effective Demand

subject to the constant returns to scale production function. Finished goods-producer optimization results in the following first-order condition:

\[ Y_t(i) = \left[ \frac{P_t(i)}{\bar{P}_t} \right]^{-\theta} Y_t \]

The market for final goods is perfectly competitive, and thus the final goods-producing firm earns zero profits in equilibrium. Using the zero-profit condition, the first-order condition for profit maximization, and the firm objective function, the aggregate price index \( P_t \) can be written as follows:

\[ P_t = \left[ \int_0^1 P_t(i)^{1-\theta} d_i \right]^{\frac{1}{1-\theta}} \]

1.3.4 Monetary Policy

We assume a cashless economy where the monetary authority sets the net nominal interest rate \( r_t \) to stabilize inflation and output growth. Monetary policy adjusts the nominal interest rate in accordance with the following rule:

\[ r_t = \rho_r r_{t-1} + (1 - \rho_r) (r + \rho_\pi (\pi_t - \pi) + \rho_y \Delta y_t), \quad (1.19) \]

where \( r_t = \ln(R_t) \), \( \pi_t = \ln(\Pi_t) \), and \( \Delta y_t = \ln(Y_t/Y_{t-1}) \). Changes in the nominal interest rate affect expected inflation and the real interest through the Fisher relation \( \ln(R_t) = \ln(E_t \Pi_{t+1}) + \ln(R_t^R) \). Thus, we include the following Euler equation for a zero net supply nominal bond in our equilibrium conditions:

\[ 1 = R_t E_t \left\{ \beta a_{t+1} \left( \frac{C_{t+1}^\eta (1 - N_t)^{\eta} (1-\eta)}{C_t^\eta (1 - N_t)^{\eta} (1-\eta)} \right)^{\frac{1-\sigma}{\nu}} \left( \frac{C_t}{C_{t+1}} \right) \left( \frac{V_{t+1}}{E_t \left[ V_{t+1}^{1-\sigma} \right]} \right)^{1-\frac{1}{\nu}} \left( \frac{1}{\Pi_{t+1}} \right) \right\} \quad (1.20) \]
Chapter 1 Uncertainty Shocks in a Model of Effective Demand

1.3.5 Equilibrium

The assumption of Rotemberg (1982) (as opposed to Calvo (1983)) pricing implies that we can model our production sector as a single representative intermediate goods-producing firm. In the symmetric equilibrium, all intermediate goods firms choose the same price $P_t(i) = P_t$, employ the same amount of labor $N_t(i) = N_t$, and choose to hold the same amount of capital $K_t(i) = K_t$. Thus, all firms have the same cash flows and payout structure between bonds and equity. With a representative firm, we can define the unique markup of price over marginal cost as $\mu_t = 1/\Xi_t$, and gross inflation as $\Pi_t = P_t/P_{t-1}$.

1.3.6 Shock Processes

In our baseline model, we are interested in capturing the effects of independent changes in the level and volatility of both the technology process and the preference shock process. The technology and preference shock processes are parameterized as follows:

$$Z_t = (1 - \rho_z) Z + \rho_z (Z_{t-1}) + \sigma^z_t \varepsilon^z_t$$

$$\sigma^z_t = (1 - \rho_{\sigma^z}) \sigma^z + \rho_{\sigma^z} \sigma_{t-1}^z + \sigma_\sigma^z \varepsilon^z_{\sigma^z}$$

$$a_t = (1 - \rho_a) a + \rho_a a_{t-1} + \sigma^a_t \varepsilon^a_t$$

$$\sigma^a_t = (1 - \rho_{\sigma^a}) \sigma^a + \rho_{\sigma^a} \sigma_{t-1}^a + \sigma_\sigma^a \varepsilon^a_{\sigma^a}$$

$\varepsilon^z_t$ and $\varepsilon^a_t$ are first moment shocks that capture innovations to the level of the stochastic processes for technology and household discount factors. We refer to $\varepsilon^z_t$ and $\varepsilon^a_t$ as second moment or “uncertainty” shocks since they capture innovations to the volatility of the
exogenous processes of the model. An increase in the volatility of the shock process increases the uncertainty about the future time path of the stochastic process. All four stochastic shocks are independent, standard normal random variables.

1.3.7 Solution Method

Our primary focus of this paper is to examine the effects of increases in the second moments of the shock processes. Using a standard first-order or log-linear approximation to the equilibrium conditions of our model would not allow us to examine second moment shocks, since the approximated policy functions are invariant to the volatility of the shock processes. Similarly, second moment shocks would only enter as cross-products with the other state variables in a second-order approximation to the policy functions, and thus we could not study the effects of shocks to the second moments alone. In a third-order approximation, however, second moment shocks enter independently in the approximated policy functions. Thus, a third-order approximation allows us to compute an impulse response to an increase in the volatility of technology or discount rate shocks, while holding constant the levels of those variables.

To solve the baseline model, we use the Perturbation AIM algorithm and software developed by Swanson, Anderson and Levin (2006). Perturbation AIM uses Mathematica to compute the rational expectations solution to the model using $n$th-order Taylor series approximation around the nonstochastic steady state of the model. We find that a third-
order approximation to the policy functions is sufficient to capture the dynamics of the baseline model. As discussed in Fernández-Villaverde et al. (2010), approximations higher than first-order move the ergodic distributions of the endogenous variables of the model away from their deterministic steady-state values. In the following analysis, we compute the impulse responses in percent deviation from the ergodic mean of each model variable.

1.4 Calibration and Baseline Results

1.4.1 Calibration

Table 1.1 lists the calibrated parameters of the model. We calibrate the model at a quarterly frequency, using standard parameters for one-sector models of fluctuations. Since our model shares many features with the estimated models of Ireland (2003) and Ireland (2011), we calibrate our model to match the estimated parameters reported in those papers. We use the estimates in these papers to calibrate the steady-state volatilities for the technology and preference shocks, $\sigma_z$ and $\sigma_a$. We calibrate the steady-state level of the discount factor and technology processes $a$ and $Z$ to both equal one. To assist in numerically calibrating and solving the model, we introduce constants into the period utility function and the production function to normalize the value function $V$ and output $Y$ to both equal one at the deterministic steady state. We choose steady-state hours worked $N$ and the model-implied value for $\eta$ such that our model has a Frisch labor supply elasticity of 1. Our calibration of $\phi_K$ implies an elasticity of the investment-capital ratio with respect to marginal $q$ of 2.0.
The household IES is calibrated to 0.50, which is consistent with the empirical estimates of Basu and Kimball (2002). The fixed cost of production for the intermediate-goods firm $\Phi$ is calibrated to eliminate pure profits in the deterministic steady state of the model. Risk aversion over the consumption and leisure basket $\sigma$ is set to 60, which is inline with the estimated values of van Binsbergen et al. (2010) and Swanson and Rudebusch (2012). We discuss our calibration of the uncertainty shock stochastic processes in depth in Section 1.6. In the following analysis, we compare the results from our baseline sticky-price calibration ($\phi_P = 160$) with a flexible-price calibration ($\phi_P = 0$).

### 1.4.2 Uncertainty Shocks & Business Cycle Comovements

Holding the calibrated parameters fixed, we analyze the effects of an exogenous increase in uncertainty associated with technology or household demand. Figures 1.3-1.4 plot the impulse responses of the model to a technology uncertainty shock and Figures 1.5-1.6 plot the responses to a demand uncertainty shock. The results are consistent with the intuition of Section 1.2 and the labor market diagrams in Figures 1.1 and 1.2. Uncertainty from either technology or household demand both enter Equation (1.4) or Equation (1.5) through the forward-looking marginal utility of wealth. An uncertainty shock associated with either stochastic process induces wealth effects on the household which triggers precautionary labor supply. Thus, the responses and time paths for the endogenous variables look qualitatively similar for both types of uncertainty shocks.
Households want to consume less and save more when uncertainty increases in the economy. In order to save more, households optimally wish to both reduce consumption and increase hours worked. Under flexible prices and constant markups, equilibrium labor supply and consumption follow the path that households desire when they face higher uncertainty. On impact of the uncertainty shock, the level of capital is predetermined, the level of the shock process is held constant, and thus labor demand is unchanged for a given real wage. Under flexible prices, the outward shift in labor supply combined with unchanged labor demand increases hours worked and output. After the impact period, households continue to save, consume less, and work more hours. Since firms owns the capital stock, higher household saving translates into higher capital accumulation for firms. Throughout the life of the uncertainty shock, consumption and investment move in opposite directions, which is inconsistent with basic business-cycle comovements.

Under sticky prices, households also want to consume less and save more when the economy is hit by an uncertainty shock associated with technology or household demand. On impact, households increase their labor supply and reduce consumption to accumulate more assets. With sticky prices, however, increased labor supply decreases the marginal costs of production of the intermediate goods firms. A reduction in marginal cost with slowly-adjusting prices increases firm markups. An increase in markups lowers the demand for household labor and lowers the real wage earned by the representative household. The de-
crease in labor demand also lowers investment in the capital stock by firms. In equilibrium, these effects combine to produce significant falls in output, consumption, investment, hours worked, and the real wage, which are consistent with business-cycle facts. Thus, the desire by households to work more can actually lead to lower labor input and output in equilibrium.

1.5 Discussion and Connections

1.5.1 Specific Example of General Principle

The differential response of our economy under flexible and sticky prices to uncertainty fluctuations is a specific instance of the general proposition established by Basu and Kimball (2005). They show that “good” shocks that cause output to rise in a flexible-price model generally tend to have contractionary effects in a model with nominal price rigidity. Basu and Kimball (2005) also show that the response of monetary policy is critical for determining the equilibrium response of output and other variables. If monetary policy follows a sensible rule, for example the celebrated Taylor (1993) rule, then the monetary authority typically lowers the nominal interest rate to offset the negative short-run effects of the shock. Our results show, however, this effect is not strong enough for standard parameter values. Even though the monetary authority in our model lowers interest rates when uncertainty rises, it does not succeed in offsetting the contractionary effects of uncertainty with nominal rigidities. In keeping with the bulk of the literature, we do not model why the monetary
policy rule does not react more aggressively to uncertainty in normal times. However, we do investigate in depth one particular barrier to expansionary monetary policy that is critical for understanding the Great Recession: the zero lower bound constraint on nominal interest rates. If uncertainty increases when the monetary authority is unable to lower the nominal interest rate further because the policy rate is essentially zero, as was the case in late 2008 and early 2009, then the short-run contractionary effect of the “good” shock dominates, and the equilibrium response of output becomes robustly negative. We explore this issue in Section 1.7.

1.5.2 Extension to Sticky Nominal Wages

Our exposition so far suggests that the mechanism we have identified works only in the special case where nominal prices are sticky but wages are flexible. Indeed, our intuition for the channel through which an increase in uncertainty raises the markup has emphasized these two elements. We argued that higher uncertainty induces households to work at lower wages, the reduction in the wage reduces firms marginal costs, but since their output prices are fixed, lower marginal costs translate to higher markups, which are contractionary. However, various types of evidence suggests that nominal wages are sticky, not flexible, especially at high frequencies. At the macro level, Christiano, Eichenbaum and Evans (2005) find that nominal wage stickiness is actually more important than nominal price stickiness for explaining the observed impact of monetary policy shocks. At the micro level, Barattieri, Basu and Gottschalk (2014) find that the wages of individual workers are
often unchanged for long periods of time (with wages changed, on average, less than once a year).

In this subsection, we show that our results extend readily to the case where either or both nominal prices and wages are sticky. Rather than writing down an extended model with two nominal frictions, we make our point heuristically, using the graphical labor supply-labor demand apparatus of Section 1.2. As we argued above, if households act competitively in the labor market:

\[ U_2(C_t, 1 - N_t) = \lambda_t W_t, \quad (1.21) \]

where \( W \) is the nominal wage and \( \lambda \) is the shadow value of nominal wealth (the utility value of the marginal dollar). Assuming firms have market power, cost-minimization implies that

\[ W_t = \frac{P_t}{\mu^P_t} Z_t F_2(K_t, Z_t N_t). \quad (1.22) \]

Thus,

\[ \frac{U_2(C_t, 1 - N_t)}{\lambda_t P_t} = \frac{1}{\mu^P_t} Z_t F_2(K_t, Z_t N_t), \quad (1.23) \]

where \( \mu^P_t \) is the price-markup over marginal cost.

Now assume a new model, where households also have market power, and set wages with a markup over their marginal disutility of work:

\[ W_t = \mu^W_t \frac{U_2(C_t, 1 - N_t)}{\lambda_t} \quad (1.24) \]
Then,

\[
\frac{U_2(C_t, 1 - N_t)}{\lambda_t P_t} = \frac{1}{\mu_t^W} \frac{1}{\mu_t^P} Z_t F_2(K_t, Z_t N_t) \tag{1.25}
\]

In our labor market diagrams, suppose we replace the labor supply curve with \(U_2(C_t, 1 - N_t)/\lambda_t P_t\). This quantity has the interpretation of being the disutility faced by the household of supplying one more unit of labor, expressed in units of real goods (the real marginal cost of supplying labor). On the vertical axis, put the equilibrium level of the real marginal disutility of work. Note that this ‘supply curve’ is shifted in exactly the same way by uncertainty as the standard labor supply curve of Figures 1.1 and 1.2 – higher uncertainty raises \(\lambda\), which shifts the supply curve out. But now the ‘demand curve’ (the right-hand side of (25)) is shifted by both price and wage markups – only the product of the two matters.

Take the polar opposite of the case we have analyzed so far: Assume perfect competition in product markets, but Rotemberg wage setting by monopolistically competitive households in the labor market. Then the price markup is always fixed at 1, but the wage markup would jump up in response to an increase in uncertainty (since the marginal cost of supplying labor falls but the wage is sticky), making the qualitative outcome exactly the same as in our current case with only sticky prices and flexible wages. Thus, while introducing nominal wage stickiness would certainly affect quantitative magnitudes, it would not change our qualitative results.
1.5.3 Connections with Existing Literature

Our framework can be used to understand the economic mechanisms at work in some recent papers in the literature. Recent work by Bloom et al. (2011), Chugh (2010), and Gilchrist, Sim and Zakrajšek (2010) uses flexible-price models to show that shocks to uncertainty can lead to fluctuations that resemble business cycles. Their modeling approach is to drop Equation (1.2) and use multi-sector models of production. Follow the insight of Bloom (2009), the normal industry equilibrium in these models features resource reallocation from low- to high-productivity firms. Higher uncertainty impedes the reallocation process by reducing the necessary investment or disinvestment needed to move capital and labor to higher-productivity uses. These models use multi-sector production and costly factor adjustment to transform a change in the expected future dispersion of total factor productivity (TFP) into a change in the current mean of the TFP distribution. This approach may allow equilibrium real wages, consumption and labor supply to move in the same direction. However, all three papers experience difficulties in getting the desired comovements, at least for calibrations that are consistent with steady-state growth. We view these approaches are complementary to ours since both mechanisms (cyclical markups and cyclical

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3This intuition also helps understand the recent work of Bidder and Smith (2012), which embeds stochastic volatility and preferences for robustness in a business-cycle model. In their setting, an increase in volatility of technology shocks affects the expected mean of the technology distribution by changing the conditional worst case distribution of the robustness-seeking agent. In a related paper, Ilut and Schneider (2011) embed ambiguity-averse agents in the model of Smets and Wouters (2007). They show that exogenous changes in the agents’ beliefs about the worst-case scenario can produce business-cycle comovements.
reallocation) could be at work simultaneously. However, we view our approach as a realistic and tractable alternative, since non-linear heterogeneous-agent models are computationally difficult to analyze. Our model of time-varying markups allows us to analyze uncertainty in the same representative-agent DSGE framework used to study other real and monetary shocks.

A recent paper by Fernández-Villaverde et al. (2010) studies the effects of uncertainty in a small open economy setting, where they directly shock the exogenous process for the real interest rate. Since a small open economy analysis is effectively done in a partial-equilibrium framework, they experience no difficulties in getting business-cycle comovements from an uncertainty shock. As we show, the difficulties come when the real interest rate is endogenous in a general equilibrium framework. In this setting, our mechanism changes the qualitative predictions of baseline DSGE models, and makes the model predictions consistent with the empirical evidence.

Another recent paper by Gourio (2012) follows Rietz (1988) and Barro (2006) and introduces a time-varying “disaster risk” into an otherwise-standard real business cycle. This shock can be viewed as bad news about the future first moment of technology combined with an increase in the future dispersion of technology. Thus, a higher risk of disaster is a combination of a negative news shock and a shock that increases uncertainty about the future. However, a key difference between Gourio (2012) and our work is that a re-
alized disaster affects the level of both technology and the capital stock. In our model, a realized innovation does not affect the level of capital at the impact of the shock. The additional assumption in Gourio (2012) implies that an increase in the probability of disaster directly lowers the risk-adjusted rate of return on capital. In order for investment to fall when the probability of disaster increases, Gourio must assume an intertemporal elasticity of substitution (IES) greater than one. With an IES greater than one, the substitution effect dominates the wealth effect when the probability of disaster increases. The lower risk-adjusted rate of return on investment induces the household to decrease investment. Since the return on investment is low, households supply less labor which lowers total output. Since leisure and consumption are normal goods, an increase in risk results in lower equilibrium output, investment, and hours, but higher equilibrium consumption. For the reasons we discuss in Section 1.2, his competitive one-sector model is unable to match basic business-cycle comovements. A key difference is that our mechanism is able to generate business-cycle comovement with any calibrated value for the IES.

In independent and simultaneous work, papers by Fernàndez-Villaverde et al. (2011) and Born and Pfeifer (2011) examine the role of fiscal uncertainty shocks in a model with nominal wage and price rigidities. Fernàndez-Villaverde et al. (2011) shows that uncertainty regarding future fiscal policy is transmitted to the macroeconomy primarily through uncertainty about future taxes on income from capital. As we discuss in the introduction, an increase in uncertainty with nominal rigidities changes markups and creates macroeconomic comove-
Chapter 1 Uncertainty Shocks in a Model of Effective Demand

ment. We view this work as highly complementary to our paper. Our work emphasizes the basic mechanism in a stripped-down model and shows why fluctuations in uncertainty can create business cycle comovement. These two papers show that the mechanism we identify can have important economic effects in the benchmark medium-scale model of Smets and Wouters (2007). Other than sharing a mechanism for generating comovement, these two papers differ greatly from our work. We focus on technology and demand uncertainty, rather than policy uncertainty. In addition, we follow a very different calibration strategy, which we discuss in the next section. The object of our paper is to understand the role of increased uncertainty in generating the Great Recession and the subsequent slow recovery. We also analyze the interaction between the zero lower bound on nominal interest rates and uncertainty shocks, which we view as important for understanding the economics of this period.

1.6 Quantitative Results & Great Recession Application

1.6.1 Uncertainty Shock Calibration

The intuition laid out in Sections 1.1 and 1.2, and the previous qualitative results suggest that uncertainty shocks can produce declines in output and its components when prices adjust slowly. This section uses the previous sticky-price model to determine if uncertainty shocks are quantitatively important for business cycle fluctuations. A related issue is determining the proper calibration of our shock processes for the uncertainty shocks associated
Chapter 1 Uncertainty Shocks in a Model of Effective Demand

with technology and household demand. The transmission of uncertainty to the macroeconomy in our model crucially depends on the calibration of the size and persistence of the uncertainty shock processes. However, aggregate uncertainty shocks are an *ex ante* concept, which may be difficult to measure using *ex post* economic data. To ensure our calibration of an unobservable process is reasonable, we want our model and uncertainty shock processes to be consistent with a well-known and observable measure of aggregate uncertainty.

We choose the Chicago Board Options Exchange Volatility Index (VIX) as our observable measure of aggregate uncertainty due to its prevalence in financial markets, ease of observability, and the ability to generate a model counterpart. The VIX is a forward-looking indicator of the expected volatility of the Standard and Poor’s 500 stock index. To match the frequency of our model, we aggregate an end-of-month VIX series to quarterly frequency by averaging over the three months in each quarter. The top panel of Figure 1.7 plots our quarterly VIX series. Using our VIX data series, denoted $V_{D_t}$, we estimate the following simple reduced-form autoregressive time series model:

$$\ln(V_{D_t}) = (1 - \rho_V)\ln(V_{D_t}) + \rho_V\ln(V_{D_{t-1}}) + \sigma_{V_t} V_{D_t} \varepsilon_{V_t} V_{D_t}, \quad \varepsilon_{V_t} V_{D_t} \sim N(0, 1). \quad (1.26)$$

The ordinary least squares regression results are $V^D = 20.4\%$, $\rho_V = 0.83$, and $\sigma^D = 0.19$ with an $R^2 = 0.68$. Using the estimated parameters, we can also compute a series of VIX-implied uncertainty shocks as the regression residuals divided by the sample standard deviation. Compared to its sample average of 20.4%, a one standard deviation VIX-implied uncertainty shock raises the level of the VIX to 24.27%. The bottom plot of Figure 1.7
shows the time series of the VIX-implied uncertainty shocks. We use this reduced-form time-series model to ensure a reasonable calibration for our technology and demand uncertainty shocks processes.

We want to create a model concept that is the counterpart to our observable measure of aggregate uncertainty. Therefore, we compute a model-implied VIX index as the expected conditional volatility of the return on the equity of the representative intermediate-goods producing firm. Using the third-order approximation to the policy functions of the model, we define our model-implied VIX $V_t^M$ as follows:

$$V_t^M = 100 \times \sqrt{4 \times \text{VAR}_t (R_{t+1}^E)},$$

where $\text{VAR}_t (R_{t+1}^E)$ is the quarterly conditional variance of the equity return.\footnote{Technically, the VIX is the expected volatility of equity returns under the risk-neutral measure. In the model, the results are quantitatively unchanged if we compute the model-implied VIX using the risk-neutral expectation.} We annualize the quarterly conditional variance, and then transform the annual volatility units into percentage points.

Using our model-implied VIX, we calibrate leverage and the uncertainty shock parameters using a two-step process. Given the other parameters for the model and the unconditional shock variances $\sigma^a$ and $\sigma^z$, we first choose the level of firm leverage such that the unconditional level of the model-implied VIX at the ergodic mean matches the average level of the
After matching the unconditional level of the model-implied VIX, we then choose our uncertainty shock parameters such that a one standard deviation uncertainty shock in our model, to either technology or household demand, generates an impulse response that closely matches our reduced-form estimate for the actual VIX in the data. For example, in our calibrated model a one standard deviation uncertainty shock to technology or household demand produces a 19 percent increase in the model-implied VIX and has a first-order autoregressive term of 0.83. Conditional on the values of the endogenous state variables, our model-implied VIX has an AR(1) representation in each of the two types of uncertainty shocks. Therefore, we are able to closely match the impulse response of the simple reduced-form model.

1.6.2 Quantitative Impact of Uncertainty Shocks

Figure 1.8 shows the impact of our calibrated uncertainty shock process on the endogenous variables of the sticky-price model. Section 1.4.2 shows that the responses are qualitatively similar for both technology and household demand uncertainty shocks. In this section, we analyze the quantitative differences between technology and household demand uncertainty shocks. The bottom right plot of Figure 1.8 shows that both uncertainty shocks under sticky prices produce a similar law of motion in the model-implied VIX, which approximately matches the reduced-form VIX model. The bottom middle plot of each figure shows

---

5Since the Modigliani & Miller (1963) theorem holds in our model, the amount of leverage does not affect firm decisions or firm value.
that the percentage increase in the volatility of the exogenous shocks to generate the same movement in the model-implied VIX differs between technology and household demand shocks. Household preference shocks require a 96 percent increase in volatility to produce the same movement in the model-implied VIX as a 37 percent increase in the volatility of technology.

In addition, the quantitative transmission of uncertainty to the macroeconomy differs greatly between the technology and household demand shocks. A one standard deviation technology uncertainty shock generates a peak drop in output of less than 0.05 percent. However, a one standard deviation household demand uncertainty shock produces a peak drop in output of about 0.17 percent. Much of the quantitative difference in the output fluctuations originates from the behavior of investment. When the uncertainty about future technology increases, higher capital provides a hedge against possible negative shocks to future marginal costs. This additional substitution effect, which is not present under a demand uncertainty shock, provides an incentive for the firm to not disinvest in the capital stock when uncertainty about future technology increases. Accordingly, investment falls by only a few basis points after a technology uncertainty shock but falls by over 20 basis points after a demand uncertainty shock. Since capital and labor are complements in production, the time path of investment implies that equilibrium hours worked also falls by less after a technology uncertainty shock. Overall, our results suggest that household demand uncertainty shocks can cause quantitatively significant fluctuations in output and its components.
Our calibration strategy produces general-equilibrium results which are consistent with the empirical literature on the macroeconomic effects of stock market volatility. Alexopoulos and Cohen (2009) analyze the effects of stock market volatility on industrial production using a vector autoregression with a recursive identification scheme. They show that a one standard deviation increase in the VIX produces a statistically significant decline of output with a peak decline of approximately 0.25 percent. Our calibrated impulse responses of demand uncertainty shocks are close to this point estimate and well within its confidence interval, which provides additional evidence that our calibration strategy is reasonable.

1.6.3 The Role of Uncertainty Shocks in the Great Recession

The previous section shows that uncertainty shocks associated with household demand have quantitatively significant effects on output and its components. Many economists and the financial press believe the large increase in uncertainty in the fall of 2008 may have played a role in the Great Recession and subsequent slow recovery.\textsuperscript{6} The plot of the VIX in Figure 1.7 shows a large increase in expected stock market volatility around the collapse of Lehman Brothers in September of 2008. In particular, the bottom plot shows a three and a half standard deviation VIX-implied uncertainty shock during the end of 2008. In calibrating our model, one standard deviation uncertainty shocks to either household

\textsuperscript{6}For example, Kocherlakota (2010) states, “I’ve been emphasizing uncertainties in the labor market. More generally, I believe that overall uncertainty is a large drag on the economic recovery.”
demand or technology generate one standard deviation movements in the model-implied VIX. Thus, we cannot easily identify or partition the contribution of demand or technology uncertainty shocks in our model in generating the large change in the VIX in the fall of 2008. However, the utilization-adjusted total factor productivity series of Fernald (2011) shows very little evidence of stochastic volatility, either during the Great Recession or over the entire postwar period. Thus, if we assume demand uncertainty shocks explain the bulk of the movement in the VIX during the fall of 2008, our baseline model predicts that the increase in uncertainty in the Fall of 2008 should have lowered output by about 0.6 percent.\footnote{Given the AR(1) law of motion for volatility shocks in our third-order approximation to the policy functions, the impulse responses for the model scale approximately linearly in the size of the uncertainty shock.}

This decline in output may seem a small number relative to the size of the output drop in 2008-2009.\footnote{The CBO estimates that the output gap was -4.6 percent in 2008Q4.} However, as we show in the next section, the assumptions regarding monetary policy are crucial in determining the effects of changes in uncertainty on the macroeconomy. The Fed Funds target rate hit the zero lower bound on December 16, 2008. From then on, the Fed could no longer offset the contractionary effects of higher uncertainty on the economy. Under these circumstances, the predicted macroeconomic effects of uncertainty are substantially larger.

One potential criticism of using our model to determine the role of uncertainty shocks in
the Great Recession is that our model lacks a realistic financial sector and abstracts from financial frictions. Thus, one might argue that what we term an exogenous uncertainty shock is actually due to a financial crisis. We are quite sympathetic to the idea that a financial crisis can raise uncertainty, but we believe that it is important to investigate the full set of channels through which financial market disruptions can affect the macroeconomy. A financial market disruption, such as the failure of Lehman Brothers in the Fall of 2008, is a single event which can have multiple effects, just as a war might increase government expenditure, raise distortionary taxes, and lead to rationing, each of which has different macroeconomic effects. Recent work by Iacoviello (2011), Gertler and Karadi (2011), and many others focuses on the first-moment effects of the financial market disruption, such as a higher cost of capital and tighter borrowing constraints for households and firms. In this paper, we analyze the likely effects of the concurrent rise in uncertainty and its effect on the economy during the Great Recession, which are second-moment effects. To analyze this independent mechanism and the effects of the increase in uncertainty, we choose to model uncertainty in a simple but reasonable macroeconomic model that abstracts from financial frictions. Our paper complements other work on the Great Recession, since one could easily combine the first-moment and second-moment analyses to obtain a complete picture of the effects of the financial crisis. Adding a detailed financial sector to our model would obscure the transmission mechanism of uncertainty to the macroeconomy, and we eschew this course of action for the sake of clarity.
1.7 Uncertainty Shocks and the Zero Lower Bound

Finally, we examine the role of monetary policy in determining the general-equilibrium effects of uncertainty shocks. In our model, the monetary authority follows a standard interest-rate rule that responds to inflation and output growth. The impulse responses in Figure 1.6 show that the monetary authority aggressively lowers the nominal interest rate in response to a demand uncertainty shock. However, the calibrated interest rate rule does not decrease the policy rate enough to offset the negative impact on output and the other model variables. If the interest rate rule allowed the monetary authority to conduct policy optimally and replicate the flexible-price equilibrium allocations, then monetary policy could undo the negative effects of the uncertainty shock. However, if the monetary authority is constrained by the zero lower bound on nominal interest rates, then monetary policy cannot replicate the flexible-price outcome. The sharp increase in uncertainty during the financial crisis in late 2008 corresponds to a period when the Federal Reserve had a policy rate near zero. Thus, we believe that the zero lower bound may have plausibly contributed significantly to the large and persistent output decline starting at that time. We show in this section that increases in uncertainty have much larger effects on output when monetary policy is constrained by the zero lower bound. Our results suggest that the second-moment effects of the financial crisis may be important for understanding the large declines in output and employment in late 2008 and 2009.
1.7.1 Solution Method and Calibration

To analyze the impact of the zero lower bound, we solve a modified version of our baseline model using the policy function iteration method of Coleman (1990). This global approximation method allows us to model the occasionally-binding zero lower bound constraint. This method discretizes the state variables and solves for the policy functions which satisfy all the equilibrium conditions of the model. Appendix A.1 contains the details of the policy function iteration algorithm. To make the model computationally feasible using policy function iteration, we simplify our baseline model by reducing the number of state variables and Euler equations. We remove technology shocks and examine only the impact of shocks associated with household demand. Also, we eliminate two Euler equations by removing leverage and assuming that households receive firm dividends as a lump-sum payment.

1.7.2 Interactions of Uncertainty and Monetary Policy

In addition to the difficulty of modeling changes in uncertainty at the zero lower bound, increases in uncertainty can produce an additional source of fluctuations beyond the precautionary working and saving channel. This additional amplification mechanism, which we refer to as the contractionary bias, can dramatically affect the economy when uncertainty increases at the zero lower bound. The contractionary bias emerges from the interaction of uncertainty and the zero lower bound when monetary policy follows a standard Taylor (1993)-type policy rule. In this situation, an increase in uncertainty causes an increase in the average nominal interest rate since the distribution of the nominal interest rate is
left-truncated by the zero lower bound. For any given level of inflation, a higher nominal interest rate raises the real interest rate, which discourages consumption and investment and depress output in economy. In Appendix A.2, we discuss this issue in detail and show this contractionary bias in the average nominal interest rate can dramatically affect the economy when uncertainty increases at the zero lower bound. In the main text, however, we choose to eliminate the contractionary bias mechanism from our results. We view the contractionary bias channel as a technical consequence of examining changes in uncertainty at the zero lower bound under a particular simple monetary policy rule, which probably does not represent the actual conduct of Fed policy at the zero lower bound. Note, however, that since we are removing an amplification mechanism, our results represent a lower bound on the effects of changes in uncertainty at the zero lower bound. Indeed, if we assumed that Fed policy follows the same simple Taylor rule at the zero lower bound that it does during normal times, then we could explain the entire output drop in the Great Recession as being due to increased uncertainty!

To remove the contractionary bias, we follow the conjecture of Mendes (2011) and assume that the monetary authority implements policy using the following history-dependent

\footnote{Our specific model is along the lines of the Fed announcing a loose path of future policy even after the economy emerges from the zero lower bound, which is something that it has arguably done. We assume that the expected future path of policy offsets the higher-than-desired nominal interest rates caused by the zero lower bound. Thus, the average expected nominal interest rate remains unchanged when uncertainty increases at the zero lower bound.}
monetary policy rule:

\[ r_t^d = r + \rho \pi_t (\pi_t - \pi) + (r_{t-1}^d - r_{t-1}) \]  \hspace{1cm} (1.28)

\[ r_t = \max(0, r_t^d) \]  \hspace{1cm} (1.29)

where \( r_t^d \) is the desired policy rate of the monetary authority, and \( r_t \) is the actual policy rate subject to the zero lower bound. When the monetary authority is unconstrained by the zero lower bound, the policy rule in Equation (1.28) responds exactly as a simple Taylor (1993)-type policy rule. However, when the monetary authority encounters the zero lower bound, the history-dependent monetary policy rule lowers future desired policy rates to offset the previous higher-than-desired nominal rates that obtained due to the zero lower bound. Since deviations from the desired path of the policy rate are offset exactly one-for-one, the average nominal policy rate remains unchanged when volatility increases. Thus, the history-dependent monetary policy rule removes the contractionary bias and allow us to isolate the effects of precautionary saving and working due to uncertainty at the zero lower bound.

### 1.7.3 Impulse Response Analysis

Figure 1.9 plots the impulse responses of a one standard deviation uncertainty shock for our simplified model at the ergodic mean of the model variables. These impulse responses replicate our previous experiments using this alternative model and calibration. Holding the level of the discount factor shock constant, an increase in uncertainty about the future
decreases output by 0.16 percent. In our following analysis of the zero lower bound, we focus on the relative amount that the zero lower bound amplifies the effects of an uncertainty shock compared to this impulse response at the ergodic mean.

To compute the impulse response of an uncertainty shock at the zero lower bound, we generate two time paths for the economy. In the first time path, we simulate a large negative first moment demand shock, which causes the zero lower bound to bind for about two years. In the second time path, we simulate the same large negative first moment demand shock, but also simulate a one-standard-deviation uncertainty shock. We compute the percent difference between the time paths of variables in the two simulations as the impulse response to the uncertainty shock at the zero lower bound.

Figure 1.9 also shows the impulse response to a one-standard-deviation uncertainty shock when the economy hits the zero lower bound constraint for two years. At the zero lower bound, a one standard deviation uncertainty shock produces a 0.35 percent drop in output on impact, and causes a much larger declines in consumption, investment, and hours worked. When compared with the impulse response at the ergodic mean, these results suggest that the zero lower bound more than doubles the decline in output and its components. The desire by households to work and save more translates into a larger drop in equilibrium hours worked and investment when the monetary authority cannot adjust its nominal interest rate. In addition to removing the contractionary bias, simple history-dependent rules like
Equation (1.28) act as a form of commitment by the monetary authority to keep interest rates lower after encountering the zero lower bound. This promise of future lower nominal rates stimulates the economy throughout the zero lower bound episode, but the effect is not strong enough to prevent significant contractions in output and its components. As the monetary authority maintains zero policy rates during the beginning of the recovery, output and its components rise above the ergodic mean impulse responses. As the first moment demand shock subsides and the economy exits the zero lower bound, the time-paths for output and its components rebound sharply and closely follow the impulse response at the ergodic mean.

1.7.4 Revisiting the Role of Uncertainty Shocks in the Great Recession

The impulse responses suggest that adverse effects of uncertainty shocks are amplified at the zero lower bound. The peak drop in output in response to the uncertainty shock is about two times larger when the monetary authority is constrained. As we discuss in Section 1.6.3, the bottom plot of Figure 1.7 shows a three and a half standard deviation VIX-implied uncertainty shock during the end of 2008. Our larger baseline model, without accounting for the zero lower bound, suggests that this large uncertainty shock may explain up to a 0.6 percent drop in output during that period. The results of our zero lower bound experiments, however, suggest that the zero lower bound amplifies uncertainty shocks by at least a factor of two. Thus, our results suggest that the increase in uncertainty when the zero lower bound constraint was binding may have accounted for about a 1.3 percent drop in
output during the Great Recession. The Congressional Budget Office estimates that the gap between actual and potential output for the fourth quarter of 2008 is negative 4.6 percent. Our results suggest that a non-trivial fraction of the decline in output during the Great Recession can be explained by increased uncertainty about the future. Note again that due to our assumption that monetary policy succeeds in fully offsetting the contractionary bias, our results are a lower bound on the effects of uncertainty during the recent crisis. We view our findings as highly complementary to other work on the financial crisis, since our results can be combined with investigations of other channels through which financial crises affect the macroeconomy to obtain a complete picture of the Great Recession.

1.7.5 Complexity of Uncertainty at the Zero Lower Bound

Even after our simplifying assumptions, the problem of modeling uncertainty shocks at the zero lower bound remains computationally intensive in our model. Our alternative model of this section retains the Epstein-Zin preferences, endogenous capital accumulation, and stochastic volatility in the discount factor process from our baseline model of Section 1.3. Many other papers in the zero lower bound literature commonly make one of two simplifying assumptions to reduce the computational burden of the zero lower bound. Some papers, such as Nakov (2008), Nakata (2013), and Eggertsson and Woodford (2003), examine the zero lower bound in a dynamic and stochastic environment using the textbook New-Keynesian model of Woodford (2003). This simple model often features only one exogenous state variable and no endogenous state variables. Other works, such as Erceg
Chapter 1 Uncertainty Shocks in a Model of Effective Demand

and Lindé (2012), use a richer business-cycle model, but rely on a solution technique that imposes perfect foresight. Our paper shows that the transmission of uncertainty to the macroeconomy through precautionary saving and working requires capital accumulation, in a dynamic and stochastic setting where we cannot impose perfect foresight. Therefore, these two simplifications are inappropriate in our framework and we are required to solve a computationally more difficult problem.

1.8 Conclusion

This paper examines the transmission mechanism of uncertainty to the macroeconomy in a standard representative-agent general equilibrium model. Under reasonable assumptions, fluctuations in uncertainty can generate business cycle-like comovements in output, consumption, investment, and hours worked if nominal prices are sticky (or, more generally, if markups are countercyclical). We calibrate our model to be consistent with a well-known and observable index of \textit{ex ante} stock market volatility. We find that the dramatic increase in uncertainty during the fall of 2008, combined with the zero lower bound on nominal interest rates, may be an important factor in explaining the large and persistent decline in output starting at that time.
Chapter 1 Uncertainty Shocks in a Model of Effective Demand

Figure 1.1: Flexible Price Model Intuition

![Flexible Price Model Diagram](image1)

Figure 1.2: Sticky Price Model Intuition

![Sticky Price Model Diagram](image2)
Table 1.1: Baseline Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calibrated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Capital’s Share in Production</td>
<td>0.333</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Household Discount Factor</td>
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<tr>
<td>$\delta$</td>
<td>Depreciation Rate</td>
<td>0.025</td>
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<td>$\phi_K$</td>
<td>Adjustment Cost to Changing Investment</td>
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<tr>
<td>$\phi_P$</td>
<td>Adjustment Cost to Changing Prices</td>
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<tr>
<td>$\Pi$</td>
<td>Steady State Inflation Rate</td>
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</tr>
<tr>
<td>$\rho_r$</td>
<td>Central Bank Interest Rate Smoothing Coefficient</td>
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</tr>
<tr>
<td>$\rho_\pi$</td>
<td>Central Bank Reaction Coefficient on Inflation</td>
<td>1.50</td>
</tr>
<tr>
<td>$\rho_y$</td>
<td>Central Bank Reaction Coefficient on Output Growth</td>
<td>0.50</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Parameter Affecting Household Risk Aversion</td>
<td>60.0</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Intertemporal Elasticity of Substitution</td>
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</tr>
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<td>$\theta_\mu$</td>
<td>Elasticity of Substitution Intermediate Goods</td>
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<td>$\rho_a$</td>
<td>First Moment Preference Shock Persistence</td>
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<tr>
<td>$\rho_{\sigma^a}$</td>
<td>Second Moment Preference Shock Persistence</td>
<td>0.83</td>
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<tr>
<td>$\sigma^a$</td>
<td>Steady-State Volatility of Preference Shock</td>
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<td>$\sigma^{\sigma^a}$</td>
<td>Volatility of Second Moment Preference Shocks</td>
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<tr>
<td>$\rho_z$</td>
<td>First Moment Technology Shock Persistence</td>
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<td>Second Moment Technology Shock Persistence</td>
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<td>$\sigma^z$</td>
<td>Steady-State Volatility of Technology</td>
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<tr>
<td>$\sigma^{\sigma^z}$</td>
<td>Volatility of Second Moment Technology Shocks</td>
<td>0.0037</td>
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</table>
Figure 1.3: Impulse Responses of Quantities to Second Moment Technology Shock

Note: Impulse responses are plotted as percent deviations from their ergodic mean.
Figure 1.4: Impulse Responses of Prices to Second Moment Technology Shock

Note: The impulse responses for inflation and interest rates are plotted in annualized percent deviations from their ergodic mean. All other impulse responses are plotted as percent deviations from their ergodic mean.
Figure 1.5: Impulse Responses of Quantities to Second Moment Preference Shock

Note: Impulse responses are plotted as percent deviations from their ergodic mean.
Chapter 1 Uncertainty Shocks in a Model of Effective Demand

Figure 1.6: Impulse Responses of Prices to Second Moment Preference Shock

Note: The impulse responses for inflation and interest rates are plotted in annualized percent deviations from their ergodic mean. All other impulse responses are plotted as percent deviations from their ergodic mean.
Figure 1.7: VIX and VIX-Implied Uncertainty Shocks
Chapter 1 Uncertainty Shocks in a Model of Effective Demand

Figure 1.8: Model-Implied VIX and Uncertainty Shock Calibration

Note: The impulse response for the VIX is plotted in annualized percent. All other impulse responses are plotted in percent deviations from their ergodic mean.
Chapter 1 Uncertainty Shocks in a Model of Effective Demand

Figure 1.9: Demand Uncertainty Shock Under History-Dependent Taylor Rule

Note: The impulse response for the nominal interest rate is plotted in annualized percent. All other impulse responses are plotted in percent deviations from their ergodic mean.
Chapter 2

Forward Guidance Under Uncertainty

2.1 Introduction

With the federal funds rate currently near zero, the Federal Reserve cannot further stabilize the economy by lowering its short-term nominal policy rate. When constrained by the zero lower bound, research by Eggertsson and Woodford (2003), Wolman (2005), and others advocates using expectations about the future conduct of monetary policy to help support the economy. By committing to expansionary policy in the future, these papers argue that the central bank can mitigate the contractionary effects of the zero lower bound. In practice, central banks often refer to this policy tool as providing forward guidance about the future path of policy. However, much of this previous research relies on models where household decisions can be summarized by the lifetime path of real interest rates. These models fail to analyze how households respond to expectations of future monetary policy when they face
increased uncertainty about the future evolution of the economy. Since the beginning of the Great Recession, many policymakers and economists have expressed significant uncertainty about future economic activity. For example, almost all Federal Open Market Committee (FOMC) participants recently indicated that their uncertainty about future output growth is higher than the norm during the previous two decades. Motivated by the current environment of increased uncertainty, this paper examines the ability of forward guidance to stabilize the economy when the future is more uncertain.

I show that increased uncertainty can reduce the central bank’s ability to stabilize the economy at the zero lower bound. When the economy hits the zero lower bound, the monetary authority can lower the expected path of real interest rates through expectations of future expansionary monetary policy. In making their consumption decisions, however, households care about both the expected path of real interest rates and the conditional distribution of future consumption. When the economy faces significant uncertainty about the future, the inability of the monetary authority to offset shocks endogenously generates higher expected volatility and downside risk for the economy. This increase in risk induces precautionary saving by households, which implies lower consumption for a given path of real interest rates. The decreased demand for consumption goods causes larger contractions in output and inflation when the economy encounters the zero lower bound. In addition, higher uncertainty can result in a dramatically prolonged zero lower bound episode.

\[1\text{See page 53 of the Monetary Policy Report to the Congress on July 17, 2012.}\]
To analyze the quantitative impact of uncertainty, I solve a general-equilibrium model with a zero lower bound constraint on the central bank’s nominal policy rate. I model increased uncertainty about the future as a higher volatility of the exogenous shocks hitting the economy. I examine the effects of increased uncertainty about future discount factors of the representative household, which have the interpretation as uncertainty about future aggregate demand. Using the model, I simulate various zero lower bound scenarios under either a low or high uncertainty calibration. My calibration strategy is motivated by the sub-sample maximum likelihood estimates of Ireland (2011) and Ireland (2003) or implied stock market volatility. I model the occasionally-binding constraint using the global solution method of Coleman (1990).

Increased uncertainty about the future both amplifies and propagates adverse fluctuations at the zero lower bound. Using the constant price-level targeting rule of Eggertsson and Woodford (2003), I simulate a decline in aggregate demand similar to the contraction during the Great Recession. The model predicts that increased uncertainty generates an additional 1.0% decline in the output gap and an additional 0.5% decline in inflation. If the increased uncertainty becomes realized as higher actual shock volatility, the economy experiences significant fluctuations and likely fails to escape the zero lower bound after several years. Without the higher realized shock volatility, price-level targeting can always fully stabilize the economy within a short period after the economy hits the zero lower bound.
Optimal monetary policy under uncertainty responds to the distribution of shocks that agents expect to hit the economy. Optimal monetary policy implies further lowering real rates by committing to a higher price-level target when the economy faces significant uncertainty about the future. To implement the optimal policy, the monetary authority commits to modestly extending its period of zero policy rates after the initial contraction in economic activity. To minimize the downside risk to the output gap and inflation when the economy hits the zero lower bound, the monetary authority must accept higher inflation risk in the future. Thus, the monetary policymaker faces a trade-off between the medium-run distribution of inflation and the short-run distributions of output and inflation. However, optimal monetary policy does not fully eliminate the downside risk in the economy posed by the zero lower bound. Even under optimal policy, the economy may still experience large fluctuations and fail to escape the zero lower bound for an extended period if the volatility of shocks hitting the economy is high. In the face of large shocks, raising the central bank’s inflation target can attenuate much of the downside risk posed by the zero lower bound.

The key parameter in my analysis is the volatility of the demand shocks hitting the economy. To ensure the reasonableness of my calibration, I simulate the model and compare the distribution of possible outcomes with recent macroeconomic data. I use the history-dependent interest-rate rule estimated by Gust, López-Salido and Smith (2013) as a description of recent FOMC behavior. After matching the initial conditions at the end
of 2008, the macroeconomic data since the Great Recession falls within the simulated prediction intervals of the high uncertainty model. Thus, actual data from the U.S. economy is inline with the distribution of possible outcomes that the representative household uses in evaluating their decisions. This exercise provides some evidence that the level of uncertainty in the calibrated model is reasonable. The results suggest the combination of higher volatility and the zero lower bound may play a significant role in explaining the slow recovery of the United States economy. Without higher volatility and the propagation provided by the zero lower bound, the simple model is unable to generate recessions like the most recent macroeconomic data.

2.2 Intuition

This section formalizes the intuition from the introduction using several key equations of the model. For Section 2.2 only, I use Taylor series approximations of these equations to show how increased uncertainty about the future can affect the central bank’s ability to stabilize the economy. These approximations provide analytical tractability which is unavailable when examining the model equations in their original nonlinear form. In Section 2.4, I show that the intuition from these approximations is consistent with the computational results using the full nonlinear model.
2.2.1 Household Consumption Under Uncertainty

The household consumption Euler equation highlights why increased uncertainty about the future may reduce the central bank’s ability to stabilize the economy at the zero lower bound. Under constant relative risk aversion utility from consumption, the following equation links consumption $C_t$ by the representative household to the gross real interest rate $R_t^R$:

$$1 = E_t \left\{ \beta R_t^R \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \right\}, \quad (2.1)$$

where $\beta$ is the household discount factor and $\sigma$ is the risk aversion parameter in the household’s utility function. Using a second-order Taylor series approximation around the steady state, Appendix B.1.1 shows Equation (2.1) can be written as follows:

$$c_t = E_t c_{t+1} - \frac{1}{\sigma} (r_t^r - r^r) - \frac{1}{2} \sigma \text{Var}_t c_{t+1} \quad (2.2)$$

where lowercase variables denote the log of the respective variable, $r^r$ is the steady state net real interest rate, and $\text{Var}_t c_{t+1}$ denotes the conditional variance of future consumption. Iterating Equation (2.2) forward and taking expectations at time $t$ implies the following solution for current consumption:

$$c_t = -\frac{1}{\sigma} \sum_{i=0}^{\infty} \left( E_t r_{t+i}^r - r^r \right) - \frac{1}{2} \sigma \sum_{j=0}^{\infty} \text{Var}_t c_{t+1+j} \quad (2.3)$$

When the economy encounters the zero lower bound, Equation (2.3) shows that the monetary authority can raise household consumption by lowering the expected path of real interest rates. However, Equation (2.3) also shows that households base their consumption decisions on both the expected path of real interest rates and the expected conditional
distribution of future consumption. For any given path of real interest rates, households consume less if they expect a more volatile distribution of future consumption. To achieve a given level of consumption, the monetary authority must choose an even lower path for real rates when households face significant uncertainty about future consumption.

2.2.2 Consumption Uncertainty in General Equilibrium

To illustrate the general-equilibrium effects of the higher-order consumption moments, I embed the approximated household Euler equation into a simple general-equilibrium model. Using a simplified version of the model outlined in Section 2.3, Appendix B.1.2 shows how to derive the following approximate higher-order version of a standard New-Keynesian model:

\[
x_t \approx E_t x_{t+1} - \frac{1}{\sigma} \left( r_t^r - r_t^n \right) - \frac{1}{2} \sigma \text{Var}_t x_{t+1} \tag{2.4}
\]

\[
r_t^r \approx r_t - E_t \pi_{t+1} + \frac{1}{2} \text{Var}_t \pi_{t+1} + \sigma \text{Cov}_t \left( x_{t+1}, \pi_{t+1} \right) \tag{2.5}
\]

\[
\pi_t \approx \beta E_t \pi_{t+1} + \kappa x_t \tag{2.6}
\]

These equations link the output gap \(x_t\) and inflation rate \(\pi_t\) to the nominal interest rate \(r_t\) and real interest rate \(r_t^r\). The output gap \(x_t\) is the percent deviation of equilibrium output from output in an equivalent economy without nominal price rigidities. Shocks in the economy cause changes in the natural real interest rate \(r_t^n\), which is the real interest rate that would prevail in the equivalent flexible-price economy. Changes in the natural rate can cause fluctuations in the output gap and inflation.\(^2\) The monetary authority can minimize

\(^2\)For clarity of exposition, Equations (2.4) and (2.5) omit two additional covariance terms which are related to the exogenous process for the natural rate shocks. The coefficients on these terms are very small and
Chapter 2 Forward Guidance Under Uncertainty

these fluctuations by adjusting the nominal interest rate to offset shocks to the natural real rate. However, the zero lower bound \( r_t \geq 0 \) imposes a limit on the central bank’s ability to offset fluctuations in the natural real rate. When the natural real rate becomes negative, the monetary authority becomes constrained by the zero lower bound and must rely on expectations about future monetary policy to help stabilize the economy.

The expected volatility of the natural real rate governs the amount of uncertainty faced by the economy. Higher expected volatility in \( r^n_t \) makes the future harder to forecast, which increases the uncertainty about the future. Equation (2.5) augments the standard Fisher relation \( r_t^r = r_t - E_t \pi_{t+1} \) to include the impact of uncertainty about future inflation and its expected covariance with future output gaps. Since prices adjust slowly to changing economic conditions, changes in the nominal interest rate affect the economy by altering real interest rates. Solving Equations (2.4) and (2.6) forward:

\[
x_t = -\sum_{i=0}^{\infty} (E_t r^n_{t+i} - E_t r^r_{t+i}) - \frac{1}{2} \sigma \sum_{j=0}^{\infty} \text{Var}_t x_{t+1+j} \tag{2.7}
\]

\[
\pi_t = E_t \left\{ \sum_{i=0}^{\infty} \beta^i \kappa x_{t+i} \right\} \tag{2.8}
\]

Equations (2.7) and (2.8) show that the evolution of the economy is summarized by the expected paths of real interest rates and the expected conditional variance of the output gap. The additional consumption risk in the household Euler equations adds the second-order moments of the output gap to the standard New-Keynesian model.

---

they do not provide any additional intuition. See Appendix B.1.2 for additional details.
The transmission of the household consumption risk to the macroeconomy depends crucially on monetary policy’s ability to stabilize the economy. In absence of the zero lower bound, the monetary authority can always fully stabilize the economy by setting its nominal policy rate equal to the natural real rate. In this scenario, the conditional variances of the output gap and inflation are zero since the monetary authority can stabilize the economy in all future periods. However, suppose the natural real rates becomes negative and the zero lower bound prevents the central bank from fully stabilizing the economy. Households and firms internalize this reduced ability to offset future fluctuations at the zero lower bound. The higher expected volatility affects the economy through two channels. First, Equations (2.4) and (2.6) show that expectations of future output gap fluctuations depress current output and inflation for any given path of real interest rates. In addition, Equation (2.5) shows that a given level of the nominal interest rate and expected inflation are less effective at lowering the real interest rate if agents expect inflation volatility and correlated fluctuations in the output gap and inflation. When the future is more uncertain, the monetary authority must further lower the path of nominal and real interest rates to achieve a given level of the output gap.

2.2.3 Zero Lower Bound and Downside Risk

The intuition discussed thus far suggests that the zero lower bound endogenously generates a more volatile distribution of future consumption for the representative household.
Chapter 2 Forward Guidance Under Uncertainty

In addition to higher expected volatility, however, the asymmetric ability of the central bank to offset shocks generates negative-skewness in the expected distribution of consumption. While the central bank can fully offset expansionary shocks with higher nominal policy rates, the zero lower bound implies a constraint on its ability to offset contractionary shocks. Households internalize this constraint when forming expectations about future consumption. Increased uncertainty amplifies this asymmetry and produces significantly left-skewed distributions for consumption throughout the zero lower bound episode. Thus, the zero lower bound endogenously generates downside tail-risk in household consumption. Returning to Equation (2.1), a third-order approximation of the consumption Euler equation can be written as follows:

\[ c_t = E_t c_{t+1} - \frac{1}{\sigma} \left( r^r_t - r^r \right) - \frac{1}{2} \sigma \text{Var}_t c_{t+1} + \frac{1}{6} \sigma^2 \text{Skew}_t c_{t+1}, \quad (2.9) \]

where \( \text{Skew}_t c_{t+1} \) denotes the conditional skewness of future consumption. Thus, the negative skewness introduced by the zero lower bound provides an additional mechanism that further reduces the responsiveness of consumption to real interest rates.

2.2.4 From Intuition to Model Simulations

The intuition of this section argues that increased uncertainty about the future can amplify adverse fluctuations at the zero lower bound. In the following section, I calibrate and solve a nonlinear model and show that the simulated zero lower bound scenarios are consistent with the intuition developed in this section. In addition, I show that the effects of increased uncertainty in the calibrated model are quantitatively significant. At the zero lower bound,
the precautionary behavior by households amplifies and propagates shocks and dramatically prolongs the zero lower bound episode.

2.3 Model

This section outlines the baseline dynamic stochastic general equilibrium model that I use my analysis. The baseline model shares many features with the models of Ireland (2003) and Ireland (2011). The model features optimizing households and firms and a central bank that systematically adjusts the nominal interest rate to offset adverse shocks in the economy. I allow for sticky prices using the quadratic-adjustment costs specification of Rotemberg (1982). The baseline model considers fluctuations in the discount factor of households, which have the interpretation as demand shocks.

2.3.1 Households

In the model, the representative household maximizes lifetime expected utility over streams of consumption $C_t$ and leisure $1-N_t$. The household receives labor income $W_t$ for each unit of labor $N_t$ supplied in the representative intermediate goods-producing firm. The representative household also owns the intermediate goods firm and receives lump-sum dividends $D_t$. The household also has access to zero net supply nominal bonds $B_t$ and real bonds $B^R_t$. A nominal bond pays the gross one-period nominal interest rate $R_t$ while a real bond pays the gross one-period real interest rate $R^R_t$. The household divides its income from labor and its financial assets between consumption $C_t$ and the amount of the bonds $B_{t+1}$ and $B^R_{t+1}$.
Chapter 2 Forward Guidance Under Uncertainty

to carry into next period. The discount factor of the household $\beta$ is subject to shocks via the stochastic process $a_t$. An increase in $a_t$ induces households to consume more and work less for no technological reason. Thus, I interpret changes in the household discount factor as demand shocks for the economy.

The representative household maximizes lifetime utility by choosing $C_{t+s}, N_{t+s}, B_{t+s+1},$ and $B_{t+s+1}^R$, for all $s = 0, 1, 2, \ldots$ by solving the following problem:

$$\max E_t \sum_{s=0}^{\infty} a_{t+s} \beta^s \frac{(C^n_{t+s}(1 - N_{t+s})^{1-\eta})^{1-\sigma}}{1-\sigma}$$

subject to the intertemporal household budget constraint each period,

$$C_t + \frac{1}{R_t} \frac{B_{t+1}}{P_t} + \frac{1}{R^R_t} B_{t+1}^R \leq \frac{W_t}{P_t} \frac{N_t}{P_t} + \frac{B_t}{P_t} + \frac{D_t}{P_t} + B_t^R.$$  

Using a Lagrangian approach, household optimization implies the following first-order conditions:

$$\eta a_t C_t^{\eta(1-\sigma)-1} (1 - N_t)^{(1-\eta)(1-\sigma)} = \lambda_t$$  

$$ (1 - \eta) a_t C_t^{\eta(1-\sigma)} (1 - N_t)^{(1-\eta)(1-\sigma)-1} = \lambda_t \frac{W_t}{P_t}$$  

$$1 = E_t \left\{ \left( \beta \frac{\lambda_{t+1}}{\lambda_t} \right) \left( \frac{R_t P_t}{P_{t+1}} \right) \right\}$$  

$$1 = E_t \left\{ \left( \beta \frac{\lambda_{t+1}}{\lambda_t} \right) R_t^R \right\}$$

where $\lambda_t$ denotes the Lagrange multiplier on the household budget constraint. Equations (2.13) - (2.14) represent the household intratemporal optimality conditions with respect to consumption and leisure, and Equations (2.15) - (2.16) represent the Euler equations for the one-period nominal and real bonds.
2.3.2 Intermediate Goods Producers

Each intermediate goods-producing firm \( i \) rents labor \( N_t(i) \) from the representative household in order to produce intermediate good \( Y_t(i) \). Intermediate goods are produced in a monopolistically competitive market where producers face a quadratic cost of changing their nominal price \( P_t(i) \) each period. Firm \( i \) chooses \( N_t(i) \), and \( P_t(i) \) to maximize the discounted present-value of cash flows \( D_t(i)/P_t(i) \) given aggregate demand \( Y_t \) and price \( P_t \) of the finished goods sector. The intermediate goods firms all have access to the same constant returns-to-scale Cobb-Douglas production function, subject to a fixed cost of production \( \Phi \).

Each intermediate goods-producing firm maximizes discount cash flows using the household stochastic discount factor:

\[
\max E_t \sum_{s=0}^{\infty} \left( \beta^s \lambda_{t+s} \right) \left[ \frac{D_{t+s}(i)}{P_{t+s}} \right]
\]

subject to the production function:

\[
\left[ \frac{P_t(i)}{P_t} \right]^{-\theta} Y_t \leq N_t(i) - \Phi,
\]

where

\[
\frac{D_t(i)}{P_t} = \left[ \frac{P_t(i)}{P_t} \right]^{1-\theta} Y_t - \frac{W_t}{P_t} N_t(i) - \frac{\phi P}{2} \left[ \frac{P_t(i)}{\Pi P_{t-1}(i)} - 1 \right]^2 Y_t
\]

The first-order conditions for the firm \( i \) are as follows:

\[
\frac{W_t}{P_t} N_t(i) = \Xi_t N_t(i)
\]
\[ \phi_P \left[ \frac{P_t(i)}{PP_{t-1}(i)} - 1 \right] \left[ \frac{P_t}{PP_t(i)} \right] = (1 - \theta) \left[ \frac{P_t(i)}{P_t} \right]^{-\theta} + \theta \Xi_t \left[ \frac{P_t(i)}{P_t} \right]^{-\theta-1} + \phi_P E_t \left\{ \left( \frac{\lambda_{t+1}}{\lambda_t} \right) \frac{Y_{t+1}}{Y_t} \left[ \frac{P_{t+1}(i)}{PP_{t}(i)} - 1 \right] \left[ \frac{P_{t+1}(i)}{PP_{t}(i)} \frac{P_t}{P_t(i)} \right] \right\}, \]

where \( \Xi_t \) is the multiplier on the production function, which denotes the real marginal cost of producing an additional unit of intermediate good \( i \).

### 2.3.3 Final Goods Producers

The representative final goods producer uses \( Y_t(i) \) units of each intermediate good produced by the intermediate goods-producing firm \( i \in [0, 1] \). The intermediate output is transformed into final output \( Y_t \) using the following constant returns to scale technology:

\[ \left[ \int_0^1 Y_t(i) \right]^{\frac{\theta}{\theta-1}} \geq Y_t \]

Each intermediate good \( Y_t(i) \) sells at nominal price \( P_t(i) \) and each final good sells at nominal price \( P_t \). The finished goods producer chooses \( Y_t \) and \( Y_t(i) \) for all \( i \in [0, 1] \) to maximize the following expression of firm profits:

\[ P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \]

subject to the constant returns to scale production function. Finished goods-producer optimization results in the following first-order condition:

\[ Y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\theta} Y_t \]

The market for final goods is perfectly competitive, and thus the final goods-producing firm earns zero profits in equilibrium. Using the zero-profit condition, the first-order condition
for profit maximization, and the firm objective function, the aggregate price index $P_t$ can be written as follows:

$$P_t = \left[ \int_0^1 P_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}$$

### 2.3.4 Monetary Policy

I assume a cashless economy where the monetary authority sets the one-period net nominal interest rate $r_t = \log(R_t)$. Due to the zero lower bound on nominal interest rates, the central bank cannot lower its nominal policy rate below by zero. In my baseline model, I assume that the monetary authority sets its policy rate according to the following constant price-level targeting rule of Eggertsson and Woodford (2003):

$$r_t \left( \theta (p_t - p^*) + x_t \right) = 0, \quad (2.16)$$

where $p_t$ is the log of the price level, $p^*$ is the constant price-level target of the central bank, and $x_t$ is the gap between current output and output in the equivalent flexible-price economy. When the zero lower bound does not bind, the monetary authority uses the nominal interest rate $r_t$ to close the output gap-adjusted price level in parenthesis. When the central bank encounters the zero lower bound, however, the monetary authority cannot perfectly stabilize the economy using its nominal policy rate. By committing to a stable price-level in the long-run, this rule promises to undo any deflation caused by the zero lower bound. By committing to higher inflation and more expansionary policy in the future, Eggertsson and Woodford (2003) shows that this policy rule can help mitigate some of the contractionary effects of the zero lower bound. By committing to a constant price-level target, this rule
implies a zero percent inflation target for the central bank. In Section 2.6.1, I relax this assumption and consider a central bank which chooses a two or four percent inflation target.

2.3.5 Shock Processes

Shocks to the discount rate of households are the only exogenous stochastic process in the baseline model. The stochastic process for these fluctuations is as follows:

\[ a_t = (1 - \rho_a) a + \rho_a a_{t-1} + \sigma^a \epsilon^a_t \]  

(2.17)

Large negative innovations to this process imply a large decline in aggregate demand, which forces the economy to encounter the zero lower bound. The volatility of the preference shock \( \sigma^a \) controls the amount of uncertainty about the future faced by the economy. A higher expected volatility makes forecasting the future time path of the stochastic process more difficult. I specify the stochastic process in levels, rather than in logs, to prevent the volatility \( \sigma^a \) from impacting average value of \( a_t \) through a Jensen’s inequality effect. In the model simulations, \( a_t \) always remains significantly greater than zero.

2.3.6 Equilibrium

In the symmetric equilibrium, all intermediate goods firms choose the same price \( P_t(i) = P_t \) and employ the same amount of labor \( N_t(i) = N_t \). Thus, all firms have the same cash flows and I define gross inflation as \( \Pi_t = P_t/P_{t-1} \) and the markup over marginal cost as \( \mu_t = 1/\Xi_t \). Therefore, I can model our intermediate-goods firms with a single representative intermediate goods-producing firm. Since fluctuations in household discount factors do not
affect the equivalent flexible-price version of my baseline model, I define the output gap as output in deviation from its deterministic steady state $x_t = \ln(Y_t/Y)$. In addition, the gross natural real interest rate that would prevail in the equivalent flexible-price economy can be defined as $R^n_t = \beta^{-1} a_t (E_t a_{t+1})^{-1}$. Thus, shocks to the household discount factor act as fluctuations in the natural real rate for the economy.

### 2.3.7 Solution Method

To formally analyze the impact of the zero lower bound, I solve the model using the policy function iteration method of Coleman (1990) and Davig (2004). This global approximation method, as opposed to local perturbation methods such as linearization, allows me to model the occasionally binding zero lower bound constraint. This method discretizes the state variables on a grid and solves for the policy functions which satisfy all the model equations at each point in the state space. Appendix B.2 contains the details of the policy function iteration algorithm.

### 2.3.8 Calibration

Table 2.1 lists the calibrated parameters of the model. I calibrate the model at quarterly frequency using standard parameters for one-sector models of fluctuations. Since the model shares features with the estimated models of Ireland (2003) and Ireland (2011), I calibrate many of the parameters to match the estimates reported by those papers. To assist in numerically solving the model, I introduce a multiplicative constant into the production
Chapter 2 Forward Guidance Under Uncertainty

function to normalize output $Y$ to equal one at the deterministic steady state. I choose steady-state hours worked $N$ and the model-implied value for $\eta$ such that the model has a Frisch labor supply elasticity of two. Household risk aversion over the consumption-leisure basket $\sigma$ is 2. The value for $\sigma$ implies an intertemporal elasticity of substitution of 0.5, which is consistent with the empirical estimates of Basu and Kimball (2002). The fixed cost of production for the intermediate-goods firm $\Phi$ is calibrated to $(\mu - 1)Y$, which eliminates pure profits in the deterministic steady state of the model.

The crucial parameter in my calibration is the volatility of the preference shock $\sigma^a$, which controls the amount of uncertainty about the future faced by the economy. I simulate various zero lower bound scenarios using both a low uncertainty calibration with $\sigma^a = 0.015$, and a high uncertainty calibration with $\sigma^a = 0.045$. The low uncertainty calibration is consistent with the maximum likelihood estimates of Ireland (2003) over the post-1979 Great Moderation sample period. When the sample period includes the 2008-2009 Great Recession, however, Ireland (2011) estimates a much larger value for the volatility of the preference shock. My calibrated value for the high uncertainty calibration lies slightly below the estimate of Ireland (2011), but remains inside the standard errors of his estimates. This calibration strategy aims to model the views of the FOMC participants that uncertainty about future economic activity is higher than the norm during the previous two decades. Alternatively, my calibration strategy can also be motivated using implied stock market volatility. Using a similar model, Basu and Bundick (2012) calibrate changes in uncertainty
using fluctuations in the VIX volatility index. The increase in uncertainty from the low to high uncertainty calibration roughly corresponds to a two standard deviation uncertainty shock, which is a conservative estimate for the increase in uncertainty during the Great Recession.\footnote{Basu and Bundick (2012) argue that the increase in uncertainty in late 2008 is consistent with over a three standard deviation uncertainty shock.}

### 2.3.9 Transmission of Precautionary Saving to Macroeconomy

Before examining the computational results, this section shows how precautionary saving by households lowers output and inflation in the macroeconomy. As I discuss in the previous sections, a more volatile and negatively-skewed expected distribution of consumption induces precautionary saving by the representative household. Since consumption and leisure are both normal goods, lower consumption also induces “precautionary labor supply,” or a desire for the household to supply more labor for a given level of the real wage. Figure 2.1 illustrates this effect graphically in real wage and hours worked space. Through the forward-looking marginal utility of wealth denoted by $\lambda_t$, an increase in uncertainty shifts the household labor supply curve outward through a wealth effect. If prices adjust slowly to changing marginal costs, however, firm markups over marginal cost rise when the household increases its desired labor supply. For a given level of the real wage, an increase in markups decreases the demand for labor from firms. As Basu and Bundick (2012) discuss, the increase in markups depends crucially on the behavior of the monetary authority. At
the zero lower bound, the central bank is unable to offset the increase in markups using its nominal interest rate. When the monetary authority aims to stabilize the economy using expectations about future policy, the contractionary higher markups reduce output and inflation throughout the initial recession and recovery. Thus, the higher markups act as contractionary headwinds in the economy during the zero lower bound episode. In a reasonably calibrated New-Keynesian model, the next section shows that these higher markups can significantly amplify and propagate adverse fluctuations at the zero lower bound.

2.4 Quantitative Effects of Uncertainty on Forward Guidance

2.4.1 Single Shock Model Responses

To analyze effects of uncertainty on the central bank’s ability to stabilize the economy, I simulate a large increase in the discount factor of the representative household. This shock acts like a large decline in aggregate demand and causes the zero lower bound to bind for several periods. After the initial shock, I assume the economy experiences no further shocks and the stochastic process for $a_t$ returns to its steady state value using its autoregressive law of motion in Equation (2.17). Figure 2.2 plots the model responses under both low and high levels of uncertainty using the same time path for the discount factor process. The discount factor shock implies a negative natural real rate of about 6%, which Levin et al. (2010) argues is consistent with the initial economic contraction during the Great Recession. The calibrated autoregressive coefficient implies that the natural real interest rate remains

72
Chapter 2 Forward Guidance Under Uncertainty

negative for six quarters following the initial shock.

The constant price-level targeting rule is able to quickly stabilize the economy when uncertainty is low. After the contractionary shock fades and natural real interest rate becomes positive, the central bank maintains a zero nominal policy rate during the economic recovery. This expansionary policy lowers real interest rates, which stimulates household consumption and output by firms. Since the central bank commits to the price-level target, agents in the model fully internalize this future behavior of the monetary authority. This lower path of real interest rates mitigates much of the fall in output and inflation throughout the zero lower bound episode. Despite the severe contraction, the monetary authority is able to quickly stabilize the economy by maintaining a zero policy rate for an additional two quarters during the recovery. Approximately one year after the deflationary forces subside, the central bank is able to fully stabilize the output gap and inflation when uncertainty about the future is low.

Increased uncertainty, however, dramatically affects the ability of the constant price-level targeting rule to stabilize the economy. As the labor supply and labor demand figures illustrate, increased uncertainty raises firm markups and lowers equilibrium hours worked. The model responses show that markups are significantly higher while the natural real interest rate remains negative. These higher markups depress output and inflation through the first several periods of the zero lower bound episode. As the contractionary shock fades
and the monetary authority maintains a zero policy rate, increased uncertainty continues to induce precautionary behavior by households and higher firm markups. These forces generate contractionary headwinds in the economy, which worse macroeconomics outcomes for a given path of real interest rates. As a result, the monetary authority must maintain a zero nominal policy rate for a substantially longer period to implement the same monetary policy rule. Even with the additional periods of zero lower rates, the central bank must implement positive output gaps for an additional three years to generate the necessary inflation to stabilize the price level. Although the negative natural real rates only lasted six quarters, the price-level targeting rule takes several years to fully stabilize the economy.

2.4.2 Model Simulations and Downside Risk in the Economy

In the previous zero lower bound scenarios, a single large shock takes the economy to the zero lower bound and the economy experiences no shocks in the following periods. This analysis allows for easy illustration of the effects of uncertainty at the zero lower bound. However, this method does not show the nonlinear effects of higher uncertainty that becomes realized as higher actual shock volatility. To show the effects of both higher uncertainty and higher realized volatility, Figure 2.3 plots the simulated model predictions after the economy encounters the zero lower bound. To generate these responses, I simulate the same initial shock under both uncertainty calibrations. In the subsequent periods, however, I draw random shocks from either the low or high uncertainty calibration, respectively. I repeat this procedure 50,000 times for both the low uncertainty and high uncertainty calibrations.
Chapter 2 Forward Guidance Under Uncertainty

Figure 2.3 plots the median response across the simulations and 95% prediction intervals for each uncertainty calibration.\footnote{This exercise is similar to the generalized impulse response advocated by Koop, Pesaran and Potter (1996). However, since I am interested in the levels of the output gap, inflation, and the nominal interest rate, I do not difference out the simulated paths using a baseline simulation.}

The model simulations highlight how increased uncertainty about the future can affect the ability of the constant price-level targeting rule to stabilize of the economy at the zero lower bound. Under low uncertainty, the monetary authority can fully stabilize the economy in about four years and likely exits the zero lower bound in about two years. When the economy experiences higher realized volatility, however, the model responses show that the price-level target is much less able to stabilize the economy. Even five years after the initial decline in aggregate demand, the economy may still experience significant fluctuations in output and inflation. In roughly half of the simulated scenarios, the economy fails to escape the zero lower bound after four years. Even if the monetary authority perfectly commits to stabilizing a theoretically-motivated nominal variable, the economy may experience significant fluctuations and fail to escape the zero lower bound if the volatility of shocks in the economy are high.

The simulations at the zero lower bound also highlights a key mechanism in the model. In particular, Figure 2.3 shows that the simulations for the output gap under high uncertainty
Chapter 2 Forward Guidance Under Uncertainty

show a distinct negative skewness. This skewness is a result of the asymmetric ability of the monetary authority to offset shocks at the zero lower bound. While the central bank can fully offset expansionary shocks with higher nominal nominal policy rates, the zero lower bound implies a constraint on their ability to offset contractionary shocks. Increased uncertainty amplifies this asymmetry and produces significantly left-skewed distributions for expected real activity throughout the zero lower bound episode. Throughout the zero lower bound episode, agents internalize this constraint on the monetary authority when forming expectations about future economic activity. Figure 2.4 illustrates this effect by plotting the expected distributions of outcomes after the economy encounters the zero lower bound.\(^5\)

Due to the higher uncertainty and the zero lower bound constraint, households and firms understand that a highly negative macroeconomic outcome is possible even one year in the future. As outlined in the Intuition section, this source of downside tail risk induces the precautionary behavior of households and reduces the ability of the price-level targeting rule to stabilize the economy.

2.4.3 Optimal Monetary Policy Under Commitment

The monetary authority in my baseline model follows the constant price-level targeting rule of Eggertsson and Woodford (2003). I choose this specification for the central bank to examine how uncertainty affects the ability of the same monetary policy rule to stabi-

\(^5\)The expected distributions are computed using a kernel density smoother. A histogram using the raw simulated data produces similar results.
Chapter 2 Forward Guidance Under Uncertainty

lize the economy at the zero lower bound. In this section, I show how an optimal policy maker under commitment responds when the economy faces significant uncertainty about the future. Appendix B.3 outlines the optimal policy problem and its associated solution. Figures 2.5 - 2.7 replicate the previous numerical simulations under the assumption that monetary policy is conducted optimally under commitment.

When uncertainty is low, the zero lower bound hardly constrains the monetary authority’s ability to stabilize the economy. In response to a single adverse shock, Figure 2.5 shows that the central bank is able to quickly stabilize the economy by maintaining a brief period of zero policy rates after the natural rate becomes positive. As Eggertsson and Woodford (2003) discuss, optimal policy commits to a higher price-level target when the economy encounters the zero lower bound. The increase in expected inflation lowers the path of real interest rates and induces higher consumption and output during the initial economic contraction. Even when the economy is continually hit by shocks, Figure 2.6 shows that the economy facing low uncertainty is fully stabilized and almost always exits the zero lower bound in about four years. Despite implying different behavior for the price-level target, Figures 2.3 and 2.6 show that optimal policy and the constant price-level targeting rule generate similar macroeconomic outcomes when uncertainty is low. Thus, optimal monetary policy is closely approximated by the constant price-level target in a low uncertainty environment.
However, the zero lower bound becomes a serious constraint on policy when uncertainty about the future is high. Optimal monetary policy under commitment implies additional increases in the price-level target when the economy faces significant uncertainty about the future. In response to a large contractionary shock, Figure 2.5 shows that the monetary authority raises its price-level target by an additional 0.2% under increased uncertainty. To implement the optimal policy, the monetary authority extends its period of zero policy rates for an additional two quarters after the natural real rate becomes positive. The model shows that optimal monetary policy under uncertainty depends on the nature of shocks that agents expect to hit the economy. Even before the economy is hit by larger realized shocks, optimal monetary policy preemptively responds to the higher expected volatility by raising its price-level target. Even under optimal policy however, increased uncertainty generates an additional 0.7% decline in the output gap and a 0.2% drop in inflation after a single contractionary shock.

Under optimal policy, the monetary authority accepts higher inflation risk in the future to minimize the downside risk to output and inflation when the economy hits the zero lower bound. Despite conducting policy optimally, Figure 2.6 shows that the monetary authority cannot fully stabilize the economy when the volatility of the exogenous shocks is high. In comparison to the constant price-level target, however, the distributions for one-year ahead expected inflation in Figure 2.7 are now positively-skewed. By committing to continually raise its price-level target in response to adverse fluctuations, the monetary authority is
able to minimize some of the downside risk to short-run output and inflation. Thus, the monetary policymaker faces a trade-off between the distribution of medium-run inflation and the distribution of short-run output and inflation when the economy hits the zero lower bound. However, optimal monetary policy does fully eliminate the downside risk posed by the zero lower bound. If the volatility of shocks hitting the economy is high, model simulations show that the economy may still experience large fluctuations and fail to escape the zero lower bound for an extended period.

2.4.4 A Calibration Check Using Recent Macroeconomic Data

In the previous sections, I show that increased uncertainty about the future can reduce the central bank’s ability to stabilize the economy at the zero lower bound. A key parameter in my analysis is the volatility of the demand shocks hitting the economy. Thus, I want to ensure a reasonable calibration for the demand shock volatility. In this section, I simulate a version of my baseline model and compare the distribution of possible outcomes with some recent macroeconomic data since the Great Recession. This exercise allows me to examine whether the distribution of outcomes the representative household uses in evaluating their decisions is in line with the recent experience of the U.S. economy. Figure 2.8 plots the estimated output gap, inflation, and federal funds rate from the end of 2008 to the end of 2012. Since the output gap is difficult to measure precisely, I use the average output gap as computed by Weidner and Williams (2009) using a variety of methods. To match the inflation measure in the model, I use the annualized quarterly percent change in the core
personal consumption expenditures price-index less a two percent inflation target. Since the end of 2008, the United States economy has experienced a significant period of output far below potential, relatively stable but below target inflation, and an extended period of zero nominal policy rates.

To compare the simulated model outcomes with actual macroeconomic data, I need to specify a more reasonable description of monetary policy. Despite the theoretical motivations, no central bank explicitly targets the nominal price level. However, recent empirical evidence suggests that the Federal Reserve’s recent behavior can be described by an interest-rate rule with significant history-dependence. Using Bayesian likelihood methods, Gust, López-Salido and Smith (2013) argues that the following history-dependent interest-rate rule can describe recent Federal Reserve behavior:

\[
\begin{align*}
    r^d_t &= \left(1 - \phi_r\right) r + \phi_r r^d_{t-1} + \phi_\pi \left(\pi_t - \pi\right) + \phi_x x_t, \\
    r_t &= \max\left(0, r^d_t\right),
\end{align*}
\]

where \(r^d_t\) is the desired policy rate of the monetary authority, and \(r_t\) is the actual policy rate subject to the zero lower bound. This history-dependent policy rule is motivated by the work of Reifschneider and Williams (2000). When the monetary authority is unconstrained by the zero lower bound, this policy rule responds exactly as a Taylor (1993)-type policy rule with interest-rate smoothing. However, when the monetary authority encounters the zero lower bound, the history-dependent rule lowers future desired policy rates to offset the
previous higher-than-desired nominal rates caused by the zero lower bound. Similar to a
price-level target, the history-dependent rule commits to a lower path of nominal interest
rates after the economy encounters the zero lower bound. Agents fully internalize this future
conduct of policy, which helps attenuate the contractionary effects of the zero lower bound.
I calibrate $\phi_r = 0.9$, $\phi_\pi = 0.8$, and $\phi_x = 0.25$, which are in line with the estimates of Gust,
López-Salido and Smith (2013).

When volatility in the economy is high, the time-paths for the actual macroeconomic
data fall within the simulated prediction intervals in Figure 2.8. In a similar exercise to
Section 2.4.2, I simulate the model under the low or high uncertainty calibration using the
history-dependent interest-rate rule. However, I calibrate the size of the initial shock such
that both calibrations generates the same estimated output gap as the fourth quarter of
2008. After matching the initial conditions at the end of 2008, the macroeconomic data
since the Great Recession falls within the simulated prediction intervals of the high un-
certainty model. Thus, actual data from the U.S. economy is inline with the distribution
of possible outcomes that the representative household uses in evaluating their decisions.
This exercise provides some evidence that the level of uncertainty in the calibrated model is
reasonable. Through the lens of the model simulations, however, the recent macroeconomic
data is not be the most likely outcome for the economy. Clearly, the model lacks many other
mechanisms which are likely important for fully explaining the dynamics of the economy.
However, the results suggest that volatility and the zero lower bound may be important
Chapter 2 Forward Guidance Under Uncertainty

One potential criticism of the previous exercise is that increasing the volatility of the shocks hitting the economy simply increases the size of the prediction intervals. Thus, the model could generate any arbitrary outcome from the data by simply increasing the width of the prediction intervals. Figure 2.9 attempts to address this concern by simulating a version of the model under high uncertainty but without imposing the zero lower bound. Without the amplification and propagation mechanism provided by the zero lower bound, the high volatility economy cannot generate outcomes for inflation and the output gap similar to the recent data. Without the zero lower bound, I would need to roughly double the volatility of the high uncertainty calibration to make the recent macroeconomic outcomes fall within the simulated prediction intervals. Through the lens of a simple model, the results suggest that both higher volatility and the zero lower bound are jointly necessary to generate simulations that are consistent with the slow recovery after the Great Recession.

2.5 Discussion and Connections with Existing Literature

2.5.1 Monetary Policy at the Zero Lower Bound

This paper shows that the zero lower bound changes the conditional distribution of consumption when the households face significant uncertainty about the future. This idea is related to work by Adam and Billi (2006), Nakov (2008), and Billi (2008), which also exam-
Chapter 2 Forward Guidance Under Uncertainty

ine the conduct of monetary policy at the zero lower bound. All three papers papers use a linearized New-Keynesian model, but solve the model using a global solution method where agents take account of the future shocks expected to hit the economy. Nakov (2008) shows that higher expected shock volatility causes larger declines in output and inflation even if monetary policy is optimal under commitment. Recent work by Billi (2008) argues that price-level targeting can effectively minimize downside risks in the economy. Returning the intuition from Equation (2.2), household consumption in the first-order linearized model is based on the expected value of consumption and the current real interest rate. Thus, the models in these papers are able to capture changes in the conditional mean caused by the presence of uncertainty at the zero lower bound. However, the models are unable to other changes in the consumption distribution caused by the zero lower bound. Figures 2.4 and 2.6 show that higher uncertainty about the future affects the conditional mean, variance, and skewness of the expected distributions at the zero lower bound. Thus, the linearized model likely underestimates the true effects of uncertainty and downside risks by restricting the analysis to changes in the conditional mean.

This paper is related to work by Wolman (2005), Nakata (2013), Fernández-Villaverde et al. (2012), Braun, Körber and Waki (2012), Christiano and Eichenbaum (2012), Gust, López-Salido and Smith (2013), and Judd, Maliar and Maliar (2012), which also solve a nonlinear New-Keynesian model with a zero lower bound constraint. These papers are complementary to this study as they examine a different set of economic questions. This

83
study is closest to Nakata (2013), which studies optimal government spending and monetary policy at the zero lower bound. Nakata compares a deterministic economy ($\sigma^{a} = 0$) to an economy with uncertainty ($\sigma^{a} > 0$) and shows that optimal government spending under discretion increases when the economy faces uncertainty about the future. While Nakata (2013) and I use a similar model of households and firms, his work primarily focuses on the role of fiscal policy at the zero lower bound. In this paper, I focus on how uncertainty about the future can affect the monetary authority’s ability to stabilize the economy using expectations about future policy.

2.5.2 Contractionary Bias in the Nominal Interest-Rate Distribution

In addition to the precautionary working mechanism, increases in uncertainty at the zero lower bound can produce an additional source of fluctuations. This additional amplification mechanism, which Basu and Bundick (2012) defines as the contractionary bias in the nominal interest rate distribution, can dramatically affect the economy when uncertainty increases at the zero lower bound. The contractionary bias emerges when the zero lower bound prevents the monetary authority from attaining its inflation goal on average. For this discussion, assume monetary policy sets its desired policy rate using the following simple rule:

$$r^{d}_{t} = r + \phi_{\pi}(\pi_{t} - \pi)$$  \hspace{1cm} (2.20)

$$r_{t} = \max\left(0, r^{d}_{t}\right)$$  \hspace{1cm} (2.21)
Chapter 2 Forward Guidance Under Uncertainty

For a given monetary policy rule, the volatility of the exogenous shocks determines the volatility of inflation. Through the monetary policy rule in Equation (2.20), the volatility of inflation dictates the volatility of the desired nominal policy rate. However, since the zero lower bound left-truncates the actual policy rate distribution, more volatile desired policy rates lead to higher average actual policy rates. Figure 2.10 illustrates this effect by plotting hypothetical distributions of the nominal interest rate under low and high levels of exogenous shock volatility. The plot shows that the average actual policy rate is an increasing function of the volatility of the exogenous shocks when monetary policy follows a simple Taylor (1993)-type rule. Reifschneider and Williams (2000) first discuss this phenomenon and Mendes (2011) analytically proves this result using a simple New-Keynesian model.

Changes in the contractionary bias caused by higher uncertainty have dramatic general-equilibrium effects on the economy. Figure 2.11 plots the average Fisher relation \( r = \pi + r^r \) and the average policy rule under both high and low levels of volatility. The upper-right intersection of the monetary policy rule and the Fisher relation dictates the normal general-equilibrium average levels of inflation and the nominal interest rate. An increase in volatility shifts the policy rule inward and increases the average nominal interest rate for a given level of inflation. Higher volatility thus raises average real interest rates, since it implies a higher level of the nominal interest rate for a given level of inflation. All else equal, higher real interest rates discourages output and puts downward pressure on the average level of inflation in the economy. Appendix A.2 provides numerical evidence that small changes in the
contractionary bias caused by higher expected volatility can have dramatic effects on the model economy. For example, Basu and Bundick (2012) shows that a small 0.25 percentage point increase in uncertainty about future demand produces a 0.35 percent decrease in aggregate output when monetary policy follows a simple Taylor (1993)-type rule at the zero lower bound.

Throughout this paper, I follow Reifschneider and Williams (2000) and focus on specifications for monetary policy that remove this alternative mechanism. The constant price-level target automatically removes the contractionary bias by offsetting any deflation with equivalent inflation in the future. When monetary policy is conducted optimally under commitment or the history-dependent interest rate rule, I add or subtract a constant bias correction to the steady state inflation rate $\Pi$ under high uncertainty to ensure the simulated model delivers zero inflation on average. This modeling strategy allows me to isolate the effects of the precautionary behavior by households and show how it makes the economy less responsive to the expected path of interest rates. Without these corrections, an increase in the contractionary bias caused by higher uncertainty implies that the monetary authority misses its unconditional inflation target simply because the zero lower bound binds in a few more states of the world.

This discussion of the contractionary bias helps clarify the economic mechanisms at work in some recent papers in the literature. Recent work by Nakata (2012) and Johannsen (2013)
show that higher demand and fiscal uncertainty at the zero lower bound greatly depresses the economy. Both papers use nonlinear New-Keynesian models and assume that monetary policy follows a Taylor (1993)-type rule subject to the zero lower bound. However, neither of these papers make any adjustments for the contractionary bias. Therefore, their results contain the effects of both the contractionary bias and precautionary working mechanisms. While quantitatively large, the effects of the contractionary bias channel emerge as a technical consequence of examining changes in uncertainty under a particular simple monetary policy rule. In addition, the uncorrected Taylor (1993)-type rule probably does not represent the actual conduct of Federal Reserve policy at the zero lower bound. Therefore, while these papers also examine the effects of uncertainty at the zero lower bound, they primarily rely on a very different economic mechanism to generate their results.

2.5.3 Uncertainty and the Effectiveness of Monetary Policy

Recent papers by Vavra (2012) and Aastveit, Natvik and Sola (2013) argue that monetary policy is less effective at altering economic activity when uncertainty is high. As the micro level, Vavra (2012) shows that the output response to a nominal shock in an $S_s$ pricing-model is substantially reduced when the volatility of firm-level productivity increases. Aastveit, Natvik and Sola (2013) shows that the responses of output and investment to an identified monetary policy shock are reduced under higher macro-level uncertainty. Both of these papers share a common message with this paper: Higher expected volatility can affect the transmission of monetary policy to the macroeconomy. However, these papers
focus on the effects of higher uncertainty on different agents in the economy. Vavra (2012) examines how uncertainty affects the forward-looking decisions of price-setting firms, while Aastveit, Natvik and Sola (2013) follows the intuition of Bloom (2009) and emphasizes non-convex adjustment costs for investment. In this paper, I focus on precautionary saving and working behavior by households. I view these works as highly complementary to this paper as all three papers illustrate the various effects of uncertainty on the effectiveness of monetary policy. However, both of these papers are silent on the effects of the zero lower bound, which currently remains a real constraint on many central banks.

2.6 Extensions

2.6.1 Raising the Central Bank’s Inflation Target

In response to the recent macroeconomic outcomes around the world, some economists argue that central banks should raise their inflation targets above the conventionally accepted two percent. Ball (2013) states, “A four percent target would ease the constraints on monetary policy arising from the zero lower bound on interest rates, with the result that economic downturns would be less severe.” In the previous sections, I assume that the central bank targets a zero average inflation rate with a four percent average real interest rate. To examine the effects of a higher inflation target, I re-calibrate the steady state inflation rate \( \Pi \) such that the average inflation rate is either two or four percent with a two percent real interest rate. Figure 2.12 shows the simulation results under two or four percent inflation...
Chapter 2 Forward Guidance Under Uncertainty

targets under optimal monetary policy where the economy faces high uncertainty about the future.

In choosing an inflation target, the central bank faces a trade-off between the average level of inflation and the amount of fluctuations caused by the zero lower bound. Under the two percent inflation target, the results are similar to the previous sections since the average nominal interest rate in the economy remains unchanged. In the face of large shocks, Figure 2.12 shows that a higher inflation target helps attenuate much of the fluctuations and downside risk associated with the zero lower bound. In addition, the economy exits the zero lower bound over two years earlier in the median simulation. While the model clearly does not properly model the costs of higher average inflation, the results are consistent with the ideas of Ball (2013) and Blanchard, Dell’Ariccia and Mauro (2010) that higher average inflation can reduce fluctuations associated with the zero lower bound. The results also supplement the work of Williams (2009) by showing that a two-percent inflation target may provide insufficient buffer even under optimal monetary policy.

2.7 Conclusions

The aim of this paper is to show how uncertainty about the future can affect the ability of the monetary authority to stabilize the economy. In the absence of the zero lower bound, monetary policy could simply alleviate the contractionary effects of uncertainty by lowering its nominal policy rate. When the monetary authority encounters the zero lower bound,
Chapter 2 Forward Guidance Under Uncertainty

the central bank must rely on expectations about the future path of policy. This paper shows that the monetary authority must commit to a more expansive policy when the future is more uncertain. This study emphasizes that policymakers must consider the entire distribution of possible outcomes when evaluating trade-offs at the zero lower bound.
Table 2.1: Calibration of Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calibrated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Household Discount Factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\phi_P$</td>
<td>Adjustment Cost to Changing Prices</td>
<td>160.0</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>Steady State Inflation Rate</td>
<td>1.000</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Parameter Affecting Household Risk Aversion</td>
<td>2.0</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Consumption Share in Period Utility Function</td>
<td>0.24</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Elasticity of Substitution Intermediate Goods</td>
<td>6.0</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Preference Shock Persistence</td>
<td>0.85</td>
</tr>
<tr>
<td>$\sigma^a$</td>
<td>Low Uncertainty Preference Shock Volatility</td>
<td>0.015</td>
</tr>
<tr>
<td>$\sigma^a$</td>
<td>High Uncertainty Preference Shock Volatility</td>
<td>0.045</td>
</tr>
</tbody>
</table>
Figure 2.1: Precautionary Labor Supply Intuition
Figure 2.2: Model Responses to Zero Lower Bound Episode under Price-Level Targeting

Note: The output gap and markup impulse responses are plotted as percent deviations. The nominal and real interest rates and inflation are plotted in annualized percent. The price level plotted in percent deviation from its pre-shock level.
Figure 2.3: Model Simulations After Hitting Zero Lower Bound under Price-Level Targeting

Note: The output gap is plotted in percent deviations. The nominal interest rate and inflation are plotted in annualized percent.
Chapter 2 Forward Guidance Under Uncertainty

Figure 2.4: Expected Distributions at Zero Lower Bound under Price-Level Targeting

Note: The output gap is plotted in percent deviation and inflation is plotted in annualized percent.
Chapter 2  Forward Guidance Under Uncertainty

Figure 2.5: Model Responses to Zero Lower Bound Episode under Optimal Policy

Note: The output gap and markup impulse responses are plotted as percent deviations. The nominal and real interest rates and inflation are plotted in annualized percent. The price level plotted in percent deviation from its pre-shock level.

96
Chapter 2 Forward Guidance Under Uncertainty

Figure 2.6: Model Simulations After Hitting Zero Lower Bound under Optimal Policy

Note: The output gap is plotted in percent deviations. The nominal interest rate and inflation are plotted in annualized percent.
Figure 2.7: Expected Distributions at Zero Lower Bound under Optimal Policy

Note: The output gap is plotted in percent deviations. The nominal interest rate and inflation are plotted in annualized percent.
Figure 2.8: Model Simulations and Recent Macroeconomic Data

Note: The output gap is plotted in percent deviations. The nominal interest rate and inflation are plotted in annualized percent. See main text for data sources.
Figure 2.9: Model Simulations and Recent Macroeconomic Data Without Zero Lower Bound

High Uncertainty Without Zero Lower Bound

Output Gap

Inflation

Nominal Interest Rate

Note: The output gap is plotted in percent deviations. The nominal interest rate and inflation are plotted in annualized percent. See main text for data sources.
Chapter 2 Forward Guidance Under Uncertainty

Figure 2.10: Nominal Interest Rate Distribution with Zero Lower Bound Constraint

Figure 2.11: Simple Monetary Policy Rules & Fisher Relation with Zero Lower Bound Constraint
Figure 2.12: Model Simulations Under Two and Four Percent Inflation Targets

High Uncertainty With 2% Inflation Target

High Uncertainty With 4% Inflation Target

Note: The output gap is plotted in percent deviations. The nominal interest rate and inflation are plotted in annualized percent. Monetary policy is conducted optimally under commitment.
Chapter 3

Real Fluctuations at the Zero Lower Bound

3.1 Introduction

Since the end of 2008, the United States economy has remained stuck at the zero lower bound. With the federal funds rate currently near zero, the Federal Reserve cannot further help stabilize the economy by lowering its short-term nominal policy rate. Since the monetary authority can no longer use its standard policy tool, many economists have argued that the economy is fundamentally different at the zero lower bound. For example, Krugman (2011) argues that improvements in technology that would normally cause output to expand can cause economic contractions at the zero lower bound. Work by Christiano, Eichenbaum and Rebelo (2011) and many others argue that the benefits of increased gov-
government spending are much larger when the economy is in a liquidity trap. Finally, recent work by Eggertsson, Ferrero and Raffo (2013) argues that structural reforms in Europe that would decrease firm market power do not support economic activity in the short-run since the European Central Bank is also near the zero lower bound.

In this paper, I argue that these results about the differential nature of the economy at the zero lower bound crucially hinge on particular assumptions about monetary policy. Previous work on the effects of real shocks at the zero lower bound commonly uses dynamic general equilibrium models with nominal rigidites. In almost all studies, the previous literature assumes that monetary policy conducts policy according to some form of the following Taylor (1993)-type policy rule subject to the zero lower bound:

\[
\begin{align*}
    r_t^d &= r + \phi_\pi \pi_t + \phi_x x_t \\
    r_t &= \max \left( 0, r_t^d \right)
\end{align*}
\]  (3.1)  (3.2)

where \( r_t^d \) is the desired policy rate of the monetary authority, \( \pi_t \) is the inflation rate, \( x_t \) is some measure of real economic activity, \( r_t \) is the actual policy rate subject to the zero lower bound, and \( \phi_\pi \geq 1 \) and \( \phi_x \geq 0 \) are parameters. During normal times, this policy rule prescribes that the central bank responds to contemporaneous fluctuations in inflation and economic activity using its short-term nominal policy rate. When inflation and economic activity fall too low, however, the central bank becomes constrained by the zero lower bound. Under this standard policy rule, government spending multipliers can be very large,
and increases in technology and firm competitiveness can cause large declines in output at the zero lower bound.

However, standard Taylor (1993)-type policy rules imply that the central bank stops responding to the state of the economy at the zero lower bound. The contemporaneous policy rule implies that the monetary authority will only begin to respond to the economy once inflation and real economic activity increase enough to allow the economy to liftoff from the zero lower bound. Even if the economy is continually hit by bad shocks at the zero lower bound, the central bank will not respond to the economy until conditions improve. This assumption is inconsistent with many recent statements and actions by policymakers. For example, the December 2011 statement from the Federal Open Market Committee states, “The Committee will continue to assess the economic outlook in light of incoming information and is prepared to employ its tools to promote a stronger economic recovery in a context of price stability.” Since encountering the zero lower bound, many central banks have relied on “unconventional” policy tools of forward guidance about the future conduct of policy and quantitative easing to help improve macroeconomic outcomes. While the exact effectiveness of these policies remains unknown, central banks continue to endogenously respond to macroeconomic conditions.¹

In this paper, I examine the effects of real shocks at the zero lower bound when the central  

¹See Bernanke (2012) for a review of the effectiveness of these policies.
bank continues to respond to the state of the economy. I use a plausible deviation from a standard policy rule that allows the central bank respond to the economy using expectations about future policy. By adjusting the future path of policy in response to current economic conditions, previous work by Eggertsson and Woodford (2003), Wolman (2005), and others advocates that the central bank can mitigate some of the contractionary effects of the zero lower bound. As originally discussed by Reifschneider and Williams (2000), these policies can be easily modeled through the following history-dependent monetary policy rule:

\[
\begin{align*}
    r_d^t &= r + \phi_\pi \pi_t + \phi_x x_t + \phi_d (r_d^{t-1} - r_{t-1}) \\
    r_t &= \max(0, r_d^t)
\end{align*}
\]

where \(\phi_d > 0\) is a parameter. When the monetary authority is unconstrained by the zero lower bound, the policy rule in Equation (3.3) responds exactly as a simple Taylor (1993)-type policy rule. However, when the monetary authority encounters the zero lower bound, the history-dependent monetary policy rule lowers future desired policy rates to offset the previous higher-than-desired nominal rates that occurred due to the zero lower bound. Agents in the economy internalize the future expansionary monetary policy, which helps stabilize the economy at the zero lower bound. Thus, the central bank continues to respond to the state of the economy at the zero lower bound by adjusting its future conduct of policy.

I show that these assumptions about future monetary policy are crucial in determining the effects of real shocks at the zero lower bound. If monetary policy continues to respond to
the economy, government spending multipliers can be smaller than one and improvements in technology and firm competitiveness can increase output. Thus, the responses of the economy to real shocks do not look as fundamentally different than an economy away from the zero lower bound. This dependence of the model responses to the specification of policy echoes the previous work of Basu and Kimball (2005). They show that shocks that cause output to rise under “good” monetary policy rules generally tend to have contractionary effects under more sub-optimal policy rules. However, this paper argues that differences in policy specifications still matter greatly even when current policy rates are zero. The results suggest that the model-implied benefits of policy inventions such as increased government spending, quantitative easing, and central bank liquidity facilities all depend crucially on the assumptions about the future conduct of monetary policy.

3.2 Model

This section outlines the baseline dynamic stochastic general equilibrium model that I use my analysis. The baseline model shares many features with the models of Ireland (2003) and Ireland (2011). The model features optimizing households and firms and a central bank that systematically adjusts the nominal interest rate to offset adverse shocks in the economy. I allow for sticky prices using the quadratic-adjustment costs specification of Rotemberg (1982). The baseline model considers fluctuations in the discount factor of households, technology, government spending, and firms’ desired markups.
3.2.1 Households

In the model, the representative household maximizes lifetime expected utility over streams of consumption $C_t$ and leisure $1 - N_t$. The household receives labor income $W_t$ for each unit of labor $N_t$ supplied in the representative intermediate goods-producing firm. The representative household also owns the intermediate goods firm and receives lump-sum dividends $D_t$. The household also has access to zero net supply nominal bonds $B_t$ and real bonds $B^R_t$. A nominal bond pays the gross one-period nominal interest rate $R_t$ while a real bond pays the gross one-period real interest rate $R^R_t$. The household divides its income from labor and its financial assets between consumption $C_t$ and the amount of the bonds $B_{t+1}$ and $B^R_{t+1}$ to carry into next period. The government levies lump-sum taxes $T_t$ on households each period. The discount factor of the household $\beta$ is subject to shocks via the stochastic process $a_t$. An increase in $a_t$ induces households to consume more and work less for no technological reason. Thus, I interpret changes in the household discount factor as demand shocks for the economy.

The representative household maximizes lifetime utility by choosing $C_{t+s}, N_{t+s}, B_{t+s+1},$ and $B^R_{t+s+1}$, for all $s = 0, 1, 2, \ldots$ by solving the following problem:

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} a_{t+s} \beta^s \left( \log (C_{t+s}) - \chi \frac{N_{t+s}^{1+\eta}}{1 + \eta} \right)$$

subject to the intertemporal household budget constraint each period,

$$C_t + \frac{1}{R_t} \frac{B_{t+1}}{P_t} + \frac{1}{R^R_t} B^R_{t+1} \leq \frac{W_t}{P_t} N_t + \frac{B_t}{P_t} + \frac{D_t}{P_t} + B^R_t - T_t.$$
Chapter 3 Real Fluctuations at the Zero Lower Bound

Using a Lagrangian approach, household optimization implies the following first-order conditions:

\[ a_t c_t^{-1} = \lambda_t \]  
\[ n_t^\eta = \lambda_t \frac{w_t}{p_t} \]  
\[ 1 = E_t \left\{ \left( \beta \frac{\lambda_{t+1}}{\lambda_t} \right) \left( \frac{r_t p_t}{p_{t+1}} \right) \right\} \]  
\[ 1 = E_t \left\{ \left( \beta \frac{\lambda_{t+1}}{\lambda_t} \right) r_t^R \right\} \]

where \( \lambda_t \) denotes the Lagrange multiplier on the household budget constraint. Equations (3.5) - (3.6) represent the household intratemporal optimality conditions with respect to consumption and leisure, and Equations (3.7) - (3.8) represent the Euler equations for the one-period nominal and real bonds.

3.2.2 Final Goods Producers

The representative final goods producer uses \( Y_t(i) \) units of each intermediate good produced by the intermediate goods-producing firm \( i \in [0, 1] \). The intermediate output is transformed into final output \( Y_t \) using the following constant returns to scale technology:

\[ \left[ \int_0^1 Y_t(i) \frac{\theta_{t+1}}{\eta} \, di \right] \frac{\theta_1}{\eta+1} \geq Y_t \]

\( \theta_t \) is the elasticity of substitution across different intermediate goods, which is subject to exogenous fluctuations. Each intermediate good \( Y_t(i) \) sells at nominal price \( p_t(i) \) and each final good sells at nominal price \( p_t \). The finished goods producer chooses \( Y_t \) and \( Y_t(i) \) for
all $i \in [0, 1]$ to maximize the following expression of firm profits:

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$$

subject to the constant returns to scale production function. Finished goods-producer optimization results in the following first-order condition:

$$Y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\theta_t} Y_t$$

The market for final goods is perfectly competitive, and thus the final goods-producing firm earns zero profits in equilibrium. Using the zero-profit condition, the first-order condition for profit maximization, and the firm objective function, the aggregate price index $P_t$ can be written as follows:

$$P_t = \left[ \int_0^1 P_t(i)^{1-\theta_t} di \right]^{\frac{1}{1-\theta_t}}$$

### 3.2.3 Intermediate Goods Producers

Each intermediate goods-producing firm $i$ rents labor $N_t(i)$ from the representative household in order to produce intermediate good $Y_t(i)$. Intermediate goods are produced in a monopolistically competitive market where producers face a quadratic cost of changing their nominal price $P_t(i)$ each period. Firm $i$ chooses $N_t(i)$, and $P_t(i)$ to maximize the discounted present-value of cash flows $D_t(i)/P_t(i)$ given aggregate demand $Y_t$ and price $P_t$ of the finished goods sector. The intermediate goods firms all have access to the same constant returns-to-scale Cobb-Douglas production function, subject to a fixed cost of production $\Phi$. Changes in $\theta_t$ cause fluctuations in the desired markups of intermediate-goods producing
Chapter 3 Real Fluctuations at the Zero Lower Bound

firms over marginal cost.

Each intermediate goods-producing firm maximizes discount cash flows using the household stochastic discount factor:

$$\max E_t \sum_{s=0}^{\infty} \left( \beta^s \frac{\lambda_{t+s}}{\lambda_t} \right) \left[ \frac{D_{t+s}(i)}{P_{t+s}} \right]$$

subject to the production function:

$$\left[ \frac{P_i(i)}{P_t} \right]^{-\theta_t} Y_t \leq Z_t N_t(i) - \Phi,$$

where

$$\frac{D_t(i)}{P_t} = \left[ \frac{P_t(i)}{P_t} \right]^{1-\theta_t} Y_t - \frac{W_t}{P_t} N_t(i) - \frac{\phi_P}{2} \left[ \frac{P_t(i)}{\Pi P_{t-1}(i)} - 1 \right]^2 Y_t$$

The first-order conditions for the firm $i$ are as follows:

$$\frac{W_t}{P_t} N_t(i) = \Xi_t Z_t N_t(i) \quad (3.9)$$

$$\phi_P \left[ \frac{P_t(i)}{\Pi P_{t-1}(i)} - 1 \right] \left[ \frac{P_t}{\Pi P_{t-1}(i)} \right] = (1 - \theta_t) \left[ \frac{P_t(i)}{P_t} \right]^{-\theta_t} + \theta_t \Xi_t \left[ \frac{P_t(i)}{P_t} \right]^{-\theta_t-1}$$

$$+ \phi_P \mathbb{E}_t \left\{ \left( \frac{\lambda_{t+1}}{\lambda_t} \right) Y_{t+1} \left[ \frac{P_{t+1}(i) \Pi P_t(i)}{P_t(i)} - 1 \right] \left[ \frac{P_{t+1}(i)}{\Pi P_t(i)} \frac{P_t}{P_t(i)} \right] \right\}, \quad (3.10)$$

where $Z_t$ is the level of technology and $\Xi_t$ is the multiplier on the production function, which denotes the real marginal cost of producing an additional unit of intermediate good $i$.

3.2.4 Monetary and Fiscal Policy

I assume a cashless economy where the monetary authority sets the one-period net nominal interest rate $r_t = \log(R_t)$. Due to the zero lower bound on nominal interest rates, the central
Chapter 3 Real Fluctuations at the Zero Lower Bound

bank cannot lower its nominal policy rate below by zero. I assume that monetary policy follows one of following specifications, which are both subject to the zero lower bound:

\[
r_{d}^{t} = r + \phi_{\pi} \log \left( \Pi_{t}/\Pi \right) + \phi_{y} \log \left( Y_{t}/Y \right) + \phi_{d} \left( r_{d}^{t-1} - r_{t-1} \right) \tag{3.12}
\]

\[
r_{t} = \max \left( 0, r_{d}^{t} \right) \tag{3.13}
\]

Equation (3.11) is a standard Taylor (1993)-type policy rule, where the central bank responds only to contemporaneous deviations from output and inflation from their steady state values. However, when the economy encounters the zero lower bound, the central bank is unable to respond to the economy. Equation (3.12) endogenously adjusts the future path of monetary policy when the economy encounters the zero lower bound. The additional parameter \( \phi_{d} \) controls the amount of response of the future path of policy when the economy hits the zero lower bound. When the economy is unconstrained by the zero lower bound, the two previous rules will imply the same monetary policy rule. Fiscal policy finances government spending \( G_{t} \) using lump-sum taxes on households.

3.2.5 Shock Processes

The model features fluctuations in household discount factors, technology, the desired markup of firms, and government spending. The stochastic process for these fluctuations is
Chapter 3 Real Fluctuations at the Zero Lower Bound

as follows:

\[ a_t = (1 - \rho_a)a + \rho_a a_{t-1} + \sigma^a \varepsilon^a_t \]  

\[ Z_t = (1 - \rho_Z)Z + \rho_Z Z_{t-1} + \sigma^Z \varepsilon^Z_t \]  

\[ \theta_t = (1 - \rho_\theta)\theta + \rho_\theta \theta_{t-1} + \sigma^\theta \varepsilon^\theta_t \]  

\[ G_t = (1 - \rho_G)G + \rho_G G_{t-1} + \sigma^G \varepsilon^G_t \]  

The steady state levels of the discount factor and technology shock are normalized to one.

In this paper, I am interested in comparing the responses of the model economy both at and away from the zero lower bound. To simulate a large recession that takes the economy to the zero lower bound, I simulate a large increase in the household discount factor. This shock acts like a large decline in aggregate demand, which forces the economy to encounter the zero lower bound. After encountering the zero lower bound, I examine the impact of the three other real shocks conditional on either specification of monetary policy.

3.2.6 Equilibrium and Solution Method

In the symmetric equilibrium, all intermediate goods firms choose the same price \( P_t(i) = P_t \) and employ the same amount of labor \( N_t(i) = N_t \). Thus, all firms have the same cash flows and I define gross inflation as \( \Pi_t = P_t/P_{t-1} \) and the markup over marginal cost as \( \mu_t = 1/\Xi_t \). Therefore, I can model the intermediate-goods firms with a single representative intermediate goods-producing firm.
To formally analyze the impact of the zero lower bound, I solve the model using the policy function iteration method of Coleman (1990) and Davig (2004). This global approximation method, as opposed to local perturbation methods such as linearization, allows me to model the occasionally binding zero lower bound constraint. This method discretizes the state variables on a grid and solves for the policy functions which satisfy all the model equations at each point in the state space. Appendix C.1 contains the details of the policy function iteration algorithm. In the following sections, I use a first-order linearized version of the model to convey the main intuition of my results. In the quantitative numerical exercises, however, I use the full non-linear model outlined in Section 3.2. Recent work by Braun, Körber and Waki (2012) shows that the linearized model can produce qualitatively government spending multipliers at the zero lower bound. Therefore, I use the linearized model to build intuition, but use the nonlinear model to assure an accurate solution at the zero lower bound.

3.2.7 Calibration

Table 3.1 lists the calibrated parameters of the model. I calibrate the model at quarterly frequency using standard parameters for one-sector models of fluctuations. Since the model shares features with the estimated models of Ireland (2003) and Ireland (2011), I calibrate many of the parameters to match the estimates reported by those papers. I choose $\chi$ to normalize output $Y$ to equal one at the deterministic steady state. I calibrate $\eta$ such that the model has a Frisch labor supply elasticity of one half. The fixed cost of production for
the intermediate-goods firm $\Phi$ is calibrated to $(\mu - 1)Y$, which eliminates pure profits in the deterministic steady state of the model.

The crucial parameter in my calibration is the policy parameter $\phi_d$, which controls amount of history-dependence in the policy rule. In my baseline model, I calibrate set $\phi_d = 0.9$, which implies a fair amount of history-dependence when the economy hits the zero lower bound. In addition to the anecdotal evidence from the Introduction, work by Gust, López-Salido and Smith (2013) suggests that the Federal Reserve’s recent behavior can be described by an interest-rate rule with significant history-dependence. In the following simulations, I compare the effects of real shocks under a standard Taylor (1993)-type rule to the history-dependent policy rule. When the economy is away from the zero lower bound, these rules imply the exact same behavior of the monetary authority.

3.3 Effects of Real Shocks

3.3.1 Aggregate Demand and Aggregate Supply

Before turning to my quantitative findings, I present the key intuition for my results heuristically using simple aggregate demand and aggregate supply diagrams as in ?. These relations can be derived using a first-order approximation around the deterministic steady state. Figure 3.1 plots the aggregate demand and supply curves for the economy both at and away from the zero lower bound. The key difference at the zero lower bound is that the aggregate
Chapter 3 Real Fluctuations at the Zero Lower Bound

demand curve becomes upward sloping in output and inflation space. This result emerges
from the interactions of the Fisher relation \( r = \pi + r^r \) and the zero lower bound. During
normal times, the central bank follows the Taylor principle and adjusts its nominal interest
rate \( r \) more than one for one with movements in inflation. If inflation rises by one percent,
nominal interest rates rise by more than one percent through the Taylor principle. Thus,
real interest rates \( r^r \) real interest rates rise after an increase in inflation. Higher real interest
rates discourage consumption, which causes a decline in output. The endogenous response
of the monetary authority to movements in inflation is the key component in generating the
downward sloping aggregate demand curve.

At the zero lower bound, however, the slope of the aggregate demand curve becomes
positive. Higher inflation cannot be immediately offset by higher nominal policy rates to-
day. An increase in inflation lowers real interest rates, which induces higher consumption
and output and the positive slope of the aggregate demand curve. This differential slope
of the aggregate demand curve is why the previous work argues that the economy is fun-
damentally different at the zero lower bound. For example, Figure 3.2 analyzes a shock
which causes a downward shift in the aggregate supply curve holding the aggregate de-
mand curve fixed. During normal times, this shock would normally cause a modest decline
in inflation and an increase in output. All else equal, the same shock at the zero lower
bound would imply a larger disinflation and a fall in economic activity. Thus, the inability
of the monetary authority to adjust its current nominal policy rate generates the upward
sloping demand curve and the potential for changing the response of the economy to shocks.

The point of this paper is to highlight that the central bank can still have a stabilizing role even in the face of the upward sloping demand curve. Although it cannot adjust its current policy rate, the central bank can endogenously adjust its future path of policy in order to offset adverse fluctuations. By committing to a lower path of nominal policy rates for any given level of expected inflation, the central bank can affect expectations of future inflation and output at the zero lower bound. Although the central bank cannot affect the slope of the aggregate demand curve in Figure 3.2, it can still affect the equilibrium outcomes of inflation and output through shift variables of expected inflation and output. Expectations of higher inflation in the future cause an additional upward shift in the aggregate supply curve as firms choose higher prices for any given level of current output and inflation. Higher inflation and output in the future cause the aggregate demand curve to shift outward by lowering real interest rates and inducing higher consumption.

The simple demand and supply diagrams show that the equilibrium outcomes for inflation and output clearly depend on the magnitude of the shifts. The amount of endogenous response of the monetary authority at the zero lower bound is controlled by the amount of history-dependence in the policy rule. In the next section, I show that a reasonably calibrated model with a fair amount of history dependence is enough to dramatically alter the responses of several real shocks. Thus, the differential nature of the economy at the
zero lower bound depends crucially on the assumptions about future policy.

### 3.3.2 Technology Shocks

Figure 3.3 plots the impulse response of a one percent increase in technology both at and away from the zero lower bound. Figure 3.4 shows the impact of the shock heuristically using the aggregate supply and demand diagrams from the previous section. Away from the zero lower bound, the increase in productivity decreases firm marginal costs. Lower current and future marginal costs cause a shift down in the aggregate supply curve as firms choose lower prices for any given level of current output. Under a standard Taylor (1993)-rule, the central bank aggressively lowers its nominal policy rate. By following the Taylor principle, the central bank lowers its policy rate more than one for one with inflation, which helps lower real interest rates. Since the technology shock is persistent, agents expect higher output throughout the duration of the shock. Higher output expectations shift out the aggregate demand curve for any given level of real interest rates. However, the central bank does not response aggressively enough and the economy experiences some disinflation as a result of the technological improvement. At impact of the shock, output rises by one half percent and inflation falls by three-quarters of one percent when the central bank follows a standard policy rule away from the zero lower bound.

Under a standard Taylor (1993) rule at the zero lower bound, the technology shock becomes highly contractionary. Under a positively slope aggregate demand curve, a shift
downward in the aggregate supply curve reduces output and generates a large disinflation. Expectations of lower output in the future also cause a leftward shift in the aggregate demand curve, as households reduce consumption for a given level of real interest rates. At impact, inflation and output fall by one percent and remain negative throughout the zero lower bound episode. Under its contemporaneous policy rule, the monetary authority begins responding to the economy only after the zero lower bound episode and doesn’t alter its future course of policy after the large contractionary shock.

If monetary policy follows the history-dependent rule, then the central bank responds to the technology shock using forward guidance about future policy. In response to the large contractions in output and inflation, the central bank commits to a lower path of policy in the future. Expectations of future expansionary policy raises inflation expectations and causes an upward shift in the aggregate supply curve. Higher expected output after the zero lower bound episode helps attenuate much of the leftward shift in the aggregate demand curve. For the calibrated model from Section 3.2, the additional shifts implied by the history-dependent rule are enough to offset much of the contraction in output. After a smaller initial contraction of one half percent, output rises and remains positive throughout the duration of the zero lower bound episode. The time path for inflation starts below and then rises above the the impulse responses away from the zero lower bound. Overall, the results indicate that the equilibrium response of a technology shock depends crucially on future assumptions about monetary policy. If the monetary authority uses forward guidance
Chapter 3 Real Fluctuations at the Zero Lower Bound

to stabilize the economy, the response to a technology shock may not look too qualitatively different at the zero lower bound.

3.3.3 Markup Shocks

Figure 3.5 plots the impulse response of a one percent decrease in firm desired markups both at and away from the zero lower bound. This shock is also acts as a positive supply shock which causes a downward shift in the aggregate supply curve. Thus, Figure 3.6 shows the results through the shifts in aggregate supply and demand, which looks qualitatively similar to the technology shock responses. Recent work by Eggertsson, Ferrero and Raffo (2013) argues that a decrease in desired markups can model the proposed structural reforms in Europe designed to increase overall competitiveness. The calibrated model implies that a one percent decrease in desired markups causes a 0.2 percent increase in output and 0.4 percent decline in inflation at impact away from the zero lower bound. If the central bank fails to respond to the economy at the zero lower bound, the same shock causes declines in output and inflation of over one percent. However, if the central bank tries to stabilize the economy using expectations of future policy, the time path for inflation looks similar to the responses away from the zero lower bound. After an initial small decline, output remains positive throughout the zero lower bound episode. Thus, the equilibrium responses to a markup shock may not be fundamentally different if the central bank continues to respond to the state of the economy.
3.3.4 Government Spending Shocks

Finally, I examine the effects of increased government spending. Recent work by Christiano, Eichenbaum and Rebelo (2011) and many others argues that government spending multipliers are substantially larger when the economy is constrained by the zero lower bound. Figure 3.8 plots the impulse responses of the model to a one percent increase in government spending and Figure 3.9 illustrates the results in aggregate supply and demand space. Higher government spending induces a negative wealth effect on households, which causes them to work more for any given level of the real wage. Higher labor input, with the increased government spending, creates expectations of higher output in the future. Higher expected output shifts out the aggregate demand curve. However, the household’s desire to work more increases firm marginal costs. Higher future marginal costs raise inflation expectations and shifts out the aggregate supply curve as well. When the economy is away from the zero lower bound, the negative wealth effect on households causes them to reduce overall consumption. Since the model does not have capital, $Y_t = C_t + G_t$, so a fall in consumption implies a government spending multiplier of less than one.

Under the standard Taylor (1993) rule at the zero lower bound, the outward shift in aggregate demand raises both inflation and output. The increased inflation expectations help further lower real interest rates. These lower real interest rates induce higher consumption and cause consumption to increase after the shock. Thus, the government spending multiplier becomes larger than one when the central bank follows a standard contemporaneous
policy rule. While the exact size of the multiplier is subject to specific conditions, the upward sloping aggregate demand curve and the non-responsive monetary policy are crucial elements in getting consumption to rise after a spending shock.

Under the history-dependent policy, the central bank endogenously adjusts its future path in an effort to curb future inflation. Thus, the monetary policy rule implies a tighter policy in the future because of the large increase in government spending. Expectations of higher nominal rates in the future induce lower output and inflation expectations. Expectations of lower output and inflation shift the aggregate demand curve backward and induce a slight downward shift in the aggregate supply curve. By continuing to respond to the economy, the central bank curtails much of the expansion caused by fiscal policy in order to maintain stable inflation. Consumption now falls under the history-dependent policy, which reduces the spending multiplier to less than one. The results highlight that the model-implied benefits of higher government spending, even when current nominal interest rates are zero, depend crucially on the assumptions about monetary policy.

3.3.5 Amount of History-Dependence

The previous result highlight that history-dependence in the monetary policy rule can significantly alter the effects of real shocks. In my baseline model, I calibrate the history-dependence parameter $\phi_d$ to be 0.9. This value is consistent with the empirical evidence of Gust, López-Salido and Smith (2013), which I discuss in the next section. In this section,
I show the degree of history-dependence in the policy rule needed to alter the effects of shocks at the zero lower bound. Figure 3.9 plots the model responses of a positive technology shock at the zero lower bound under alternative level of the history-dependence parameter $\phi_d$. The results show that the policy rule needs a history-dependence coefficient of larger than 0.8 in order to eliminate much of the contractionary nature of the technology shock at the zero lower bound. The reason for this result is largely mechanical: The degree of history-dependence controls the length of the memory for the policy rule. Suppose that the economy has been stuck at the zero lower bound for $n$ periods. Then, solving backward using the history-dependent rule implies the following value for the desired nominal policy rate at the zero lower bound:

$$r_d^t = r + \phi_d \pi_t + \phi_d x_t + \phi_d r_{t-1}$$

$$r_d^t = r + \phi_d \pi_t + \phi_d (r + \phi_d \pi_{t-1} + \phi_d x_{t-1} + \phi_d r_{t-2})$$

$$r_d^t = r + \phi_d \pi_t + \phi_d x_t + \sum_{i=1}^{n} \phi_d^i (r + \phi_d \pi_{t-i} + \phi_d x_{t-i})$$

Thus, the value of $\phi_d$ controls the weight of current versus past economic conditions in the monetary policy rule. Alternatively, $\phi_d$ controls how the monetary authority alters the future path of policy when they become constrained by the zero lower bound. Without sufficiently high values for the history-dependence parameter, the amount of memory in the policy rule decays very quickly. Also, the backward solution shows that the product
Chapter 3 Real Fluctuations at the Zero Lower Bound

of $\phi_d^i$ and the policy rule coefficients $\phi_\pi$ and $\phi_x$ determines the overall degree of history-
dependence. Policy rules that are closer to optimal policy (Higher values of $\phi_\pi$ and $\phi_x$) during normal times require less history-dependence in order to achieve a given level of memory in the policy rule.

3.3.6 Forms of History-Dependence

The previous result highlight that history-dependence in the monetary policy rule can significantly alter the effects of real shocks. In my baseline model, I model this history-dependence as the Reifschneider and Williams (2000) rule. This policy rule acts as a standard contemporaneous policy rule, but adds an additional term $r_d^{t-1} - r_{t-1}$ that only affects the economy during and after a zero lower bound episode. The advantage of this rule is that it nests the standard policy rule, and thus produces the same responses away from the zero lower bound. However, the form of the history-dependence can take many alternative forms. For example, Gust, López-Salido and Smith (2013) suggests that the Federal Reserve’s recent behavior is described by a the following interest-rate rule with significant history-dependence:

$$r_t^d = (1 - \phi_r) r + \phi_r r_{t-1}^d + \phi_\pi \pi_t + \phi_x x_t$$

(3.18)

$$r_t = \max \left( 0, r_t^d \right).$$

(3.19)

This policy rule implies history-dependence by smoothing through the desired nominal policy rate of the central bank. At the zero lower bound, the policy rule is operationally similar to the Reifschneider and Williams (2000) rule in my baseline model. However, the smoothing in the desired policy rate will imply a more moderately sloped exit from the zero lower
In addition to smoothing in the desired nominal policy rate, the history-dependence policy could also target nominal income, the nominal price-level, or any other backward-looking non-jump variable. Even small weights on these backward-looking variables in the policy rule would imply significant history-dependence. For example, suppose the central bank places weight on the nominal price level in its policy rule, which Gorodnichenko and Shapiro (2007) is consistent with the Federal Reserve behavior in the late 1990s. Agents in the economy understand that any deflation caused by the zero lower bound will eventually be offset with higher inflation in the future to stabilize the price level. Thus, the central bank continues to adjust its future path of policy in response to current economic conditions at the zero lower bound. Regardless of the form of the history-dependence, the key insight is that the central bank continues to respond to the economy at the zero lower bound. In the next section, I explore the implications of a specific-type of history-dependence, which is of particular interest to policymakers.

3.3.7 Unemployment and History-Dependence

The previous section shows that the history-dependence in the policy rule can take a number of forms. In this section, I show that a response of monetary policy to unemployment automatically implies history-dependence in a model of labor search frictions. Recent work by Blanchard and Galí (2010) embeds a Diamond-Mortensen-Pissarides search and matching
model of unemployment into a standard model of nominal price rigidity. In their framework, the number of employed workers $N_t$ evolves according to the following law of motion:

$$N_t = (1 - \psi) N_{t-1} + m_t$$  \hspace{1cm} (3.20)$$

where $m_t$ denotes a matching function that depends on the amount of unemployed workers and the number of vacancies, and $\psi$ denotes the job destruction rate. Under the assumption of full participation, the unemployment rate $u_t$ can be written as follows:

$$u_t = 1 - N_t$$  \hspace{1cm} (3.21)$$

Solving backward, we can write the unemployment rate as a backward looking state variable with $\psi > 0$

$$u_t = 1 - \sum_{i=0}^{\infty} (1 - \psi)^i m_{t-i}$$  \hspace{1cm} (3.22)$$

If monetary policy reacts to fluctuations in unemployment in its policy rule, the backward-looking nature of the unemployment rate automatically implies history-dependence in the monetary policy rule. At the zero lower bound, agents internalize the central bank’s commitment to stabilizing the labor market. This link between current and past labor market conditions implies that the central bank continues to respond to the state of the economy even when constrained by the zero lower bound. This response to unemployment is consistent with recent Federal Open Market Committee statements and actions. Since the beginning of the Great Recession, the Federal Reserve consistently cites the elevated unemployment rate as justification for using its policy tools to improve labor market conditions.
3.3.8 Discussion and Connections with Existing Literature

This paper aims to extend results from the literature on monetary policy in models of nominal price rigidities. Prior work such as Basu and Kimball (2005), Woodford (2003), and many others shows that the effects of real shocks crucially depend on the assumptions about monetary policy. This paper argues that the assumptions about monetary policy continue to matter greatly even the economy is stuck at the zero lower bound. Zero nominal policy rates alone are not a sufficient definition to pin down the conduct of monetary policy.

This work is related to independent work by Erceg and Lindé (2012), which also examines the effects of government spending multipliers at the zero lower bound. Under a standard Taylor (1993)-type policy rule, Erceg and Linde show that the government spending multiplier can be very large under a variety of assumptions about the structure of fiscal policy and the frictions and agents in the real economy. In an Appendix, however, they show numerically that government spending multipliers are considerably smaller at the zero lower bound if monetary policy responds to the nominal price-level. The point of my paper is to highlight why government spending multipliers differ greatly under the history-dependent policy. By continuing to respond to the state of the economy at the zero lower bound, the monetary authority endogenously adjusts future policy in response to current economic conditions. This response offsets much of the inflation generated by the fiscal expansion and limits the “free lunch” for the fiscal authority. In addition, I aim to highlight the inconsistency of the standard contemporaneous policy rule assumption and the recent actions
and statements by policymakers.

This paper is also related to work by Eggertsson and Woodford (2004). Under optimal monetary and fiscal policy, they show that the gains from optimal monetary policy are smaller when fiscal policy is conducted optimally in a liquidity trap. This paper aims to show the converse of this result in a positive rather than normative setting. The benefits from fiscal expansions in a liquidity trap are smaller if monetary policy acts in a history-dependent manner.

### 3.4 Conclusions

Even when current nominal policy rates are zero, this paper argues that assumptions about monetary policy are crucial in determining the equilibrium effects of real shocks. Alternative assumptions about policy can generate government spending multipliers that are greater or less than one and cause increases in technology and firm competitiveness to become expansionary or highly contractionary. Given the importance of the assumed policy rule, future versions of this work will aim to identify the amount of history-dependence in the recent Federal Reserve actions. I plan to use both direct and indirect methods to argue that this recent behavior is characterized by at least a moderate amount of history-dependence.
Table 3.1: Calibration of Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calibrated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Household Discount Factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\phi_P$</td>
<td>Adjustment Cost to Changing Prices</td>
<td>160.0</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>Steady State Inflation Rate</td>
<td>1.000</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Inverse of the Frisch Labor Supply Elasticity</td>
<td>2.0</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Scalar Affecting Steady State Hours</td>
<td>0.72</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Steady State Elasticity of Substitution Intermediate Goods</td>
<td>6.0</td>
</tr>
<tr>
<td>$G/Y$</td>
<td>Steady State Government Spending Over Output</td>
<td>0.2</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Preference Shock Persistence</td>
<td>0.90</td>
</tr>
<tr>
<td>$\rho_Z$</td>
<td>Technology Shock Persistence</td>
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</tr>
<tr>
<td>$\rho_G$</td>
<td>Government Spending Shock Persistence</td>
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</tr>
<tr>
<td>$\rho_\theta$</td>
<td>Markup Shock Persistence</td>
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</tr>
<tr>
<td>$\sigma^a$</td>
<td>Preference Shock Volatility</td>
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</tr>
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<td>$\sigma^A$</td>
<td>Government Spending Shock Volatility</td>
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</tr>
<tr>
<td>$\sigma^G$</td>
<td>Government Spending Shock Volatility</td>
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</tr>
<tr>
<td>$\sigma^\theta$</td>
<td>Markup Shock Volatility</td>
<td>0.0025</td>
</tr>
</tbody>
</table>
Figure 3.1: Aggregate Supply and Demand and the Zero Lower Bound

\[ r = \pi + r' \]

\[ -\pi = r' \]
Figure 3.2: Downward Shift in Aggregate Supply
Chapter 3 Real Fluctuations at the Zero Lower Bound

Figure 3.3: Model Responses to Positive Technology Shock

Note: Output, consumption, and technology are plotted as percent deviations. The nominal interest rate is plotted in annualized percent, while the real interest rate and inflation are plotted in annualized percent deviations.
Figure 3.4: Effects of Positive Technology Shock
Chapter 3 Real Fluctuations at the Zero Lower Bound

Figure 3.5: Model Responses to Decrease in Firm Desired Markups

Note: Output, consumption, and desired markups are plotted as percent deviations. The nominal interest rate is plotted in annualized percent, while the real interest rate and inflation are plotted in annualized percent deviations.
Chapter 3 Real Fluctuations at the Zero Lower Bound

Figure 3.6: Effects of Decrease in Firm Desired Markups
Chapter 3 Real Fluctuations at the Zero Lower Bound

Figure 3.7: Model Responses to Increase in Government Spending

Note: Output, consumption, and government spending are plotted as percent deviations. The nominal interest rate is plotted in annualized percent, while the real interest rate and inflation are plotted in annualized percent deviations.
Figure 3.8: Effects of Increase in Government Spending
Figure 3.9: Model Responses Under Alternative Calibrations of History-Dependence

Note: Output, consumption, and government spending are plotted as percent deviations. The nominal interest rate is plotted in annualized percent, while the real interest rate and inflation are plotted in annualized percent deviations.
Appendix A

Uncertainty Shocks in a Model of Effective Demand

A.1 Solving the Model with a Zero Lower Bound Constraint

A.1.1 Numerical Solution Method

To analyze the impact of uncertainty shocks at the zero lower bound, we solve our model using the policy function iteration method of Coleman (1990). This global approximation method allows us to model the occasionally binding zero lower bound constraint. This section provides the details of the algorithm when monetary policy follows a simple Taylor (1993)-type interest-rate rule. The algorithm is implemented using the following steps:

1. Discretize the state variables of the model: \( \{K_t \times a_t \times \sigma_t^a\} \)
Appendix A  Uncertainty Shocks in a Model of Effective Demand

2. Conjecture initial guesses for the policy functions of the model
   \[ N_t = N(K_t, a_t, \sigma_t^a), \]
   \[ I_t = I(K_t, a_t, \sigma_t^a), \]
   \[ \Pi_t = \Pi(K_t, a_t, \sigma_t^a), \]
   and
   \[ E_t V_{t+1}^{1-\sigma} = EV(K_t, a_t, \sigma_t^a). \]

3. For each point in the discretized state space, substitute the current policy functions
   into the equilibrium conditions of the model. Use interpolation and numerical inte-
   gration over the exogenous state variables \( a_t \) and \( \sigma_t^a \) to compute expectations for
   each Euler equation. This operation generates a nonlinear system of equations. The
   solution to this system of equations provides an updated value for the policy functions
   at that point in the state space.

4. Repeat Step (3) for each point in the state space until the policy functions converge
   and cease to be updated.

We implement the policy function iteration method in FORTRAN using the nonlinear equation
solver DNEQNF from the IMSL numerical library. When monetary policy follows the history-
dependent policy rule in Equation (1.28), we include the lagged difference between the
actual and desired policy rates \( (r_{t-1} - r_{t-1}^d) \) in the discretized state space.

A.2 Uncertainty, the Zero Lower Bound, and the

Contractionary Bias

As we discuss in the main text, the interaction between uncertainty and the zero lower
bound can produce an additional source of fluctuations beyond the precautionary working
Appendix A Uncertainty Shocks in a Model of Effective Demand

and saving channel. We refer to this additional amplification mechanism as the contractionary bias in the nominal interest rate distribution. In this Appendix, we show that the contractionary bias can dramatically affect the economy when uncertainty increases at the zero lower bound. In addition, we show that the assumptions regarding this new mechanism are crucial in assessing the general-equilibrium effects of changes in uncertainty at the zero lower bound. For Sections A.2.1-A.2.3 only, we reduce the unconditional volatility of demand shocks $\sigma^a$ to 0.5 percent from our baseline calibration of 2.0 percent and decrease the standard deviation of uncertainty shocks $\sigma^\sigma_a$ to 0.50. In Section A.2.4, we explain the rationale for temporarily reducing the volatility of the exogenous shocks hitting the economy.


We begin our analysis by assuming the monetary authority sets the nominal interest rate according to the following simple rule:

$$r^d_t = r + \rho_\pi(\pi_t - \bar{\pi}) \quad (A.1)$$

$$r_t = \max (0, r^d_t), \quad (A.2)$$

where $r^d_t$ is the desired policy rate of the monetary authority, and $r_t$ is the actual policy rate subject to the zero lower bound. Figure A.1 plots the impulse responses of a one standard deviation uncertainty shock at the ergodic mean of the model variables. These impulse responses replicate our previous experiments using this simplified model and alternative
Appendix A Uncertainty Shocks in a Model of Effective Demand

calibration. Due to the considerably smaller calibration of the exogenous shocks, this alternative calibration produces an extremely small drop in output: Holding the level of the discount factor shock constant, a 50 percent increase in the volatility of the shock process decreases output by less than one basis point. Figure A.1 also plots the impulse responses of a one standard deviation uncertainty shock under a zero lower bound scenario similar to the simulation in Section 1.7.3. At the zero lower bound, a one standard deviation uncertainty shock produces a 0.35 percent drop in output. Compared to the impulse responses at the ergodic mean, the decline in output due is magnified by over an order of magnitude when the monetary authority is unable to change its nominal policy rate. This result explains our claim in the text that we could explain all of the output drop in the Great Recession as being due to uncertainty alone if we did not remove the contractionary bias.

A.2.2 Contractionary Bias in the Average Nominal Interest Rate

The previous results suggest that the zero lower bound massively amplifies uncertainty shocks. However, our assumed monetary policy rule may be overstating the effects of the zero lower bound. In the model, the volatility of the exogenous shocks determines the volatility of inflation. Through the monetary policy rule in Equation (A.1), the volatility of inflation dictates the volatility of the desired nominal policy rate. However, since the zero lower bound left-truncates the actual policy rate distribution, more volatile desired policy rates lead to higher average actual policy rates. Figure A.2 illustrates this effect by plotting the distribution of the nominal interest rate under both low and high levels of exogenous
Appendix A Uncertainty Shocks in a Model of Effective Demand

shock volatility. Figure A.2 shows that the average actual policy rate is an increasing function of the volatility of the exogenous shocks when monetary policy follows a simple Taylor (1993)-type rule.\(^1\) We refer to this link between the volatility of the exogenous shocks and the level of the nominal interest rate as the contractionary bias in the actual policy rate distribution.\(^2\)

We argue that accounting for the contractionary bias is crucial in assessing the general-equilibrium effects of changes in uncertainty at the zero lower bound. Figure A.3 plots the average Fisher relation \(\ln(R) = \ln(\Pi) + \ln(R^R)\) and the average policy rule under both high and low levels of volatility. The upper-right intersection of the monetary policy rule and the Fisher relation dictates the normal general-equilibrium average levels of inflation and the nominal interest rate. An increase in volatility shifts the policy rule inward and increases the average nominal interest rate for a given level of inflation. Higher volatility thus raises average real interest rates, since it implies a higher level of the nominal interest rate for a given level of inflation. All else equal, higher real interest rates discourage consumption

\(^1\)Using a simple New-Keynesian model without capital, Mendes (2011) analytically proves that the average nominal interest rate is increasing in the volatility of the exogenous shocks when monetary policy follows a simple Taylor (1993)-type rule.

\(^2\)Nakata (2013) and Nakov (2008) also use a New-Keynesian model to examine the zero lower bound in a dynamic and stochastic setting. Both papers also discuss this link between the volatility of the exogenous shocks and the average level of the nominal interest rate under a simple policy rule or optimal monetary policy under discretion.
and investment and depress output in the economy.

Using this intuition regarding the contractionary bias, we can identify two distinct sources of fluctuations in the impulse responses in Figure A.1. An increase in uncertainty induces precautionary saving and working, which we discuss in detail in the main text of the paper. In addition, the uncertainty shock temporarily increases the contractionary bias in the expected average nominal interest rate. The transitory increase in the contractionary bias implies higher expected nominal interest rates for any given level of inflation. Even though current nominal rates remain at the zero lower bound, an increase in expected nominal rates after the zero lower bound episode raises expected real interest rates. Higher future real interest rates reduce expected future output and inflation, which lowers current output and inflation through forward-looking consumption and investment decisions. Like the precautionary saving and working channel, the transitory increase in the contractionary bias produces declines in output and its components. Our previous impulse responses in Figure A.1 show the effects of both mechanisms. However, the previous results obscure the relative contribution of each mechanism in explaining the amplification of the uncertainty shock.

A.2.3 Impulse Response Analysis Under History-Dependent Policy Rule

To quantify the contribution of each mechanism, we also examine the impact of an uncertainty shock at the zero lower bound under the history-dependent policy rule in Equation
Appendix A Uncertainty Shocks in a Model of Effective Demand

(1.28). As we discuss in the main text, this alternative specification for monetary policy removes the contractionary bias by promising to offset deviations from the desired policy rule caused by the zero lower bound. Figure A.1 also plots the impulse responses to an uncertainty shock for the history-dependent policy rule under the alternative shock calibration. A demand uncertainty shock at the zero lower bound produces a two basis point drop in output when the monetary authority follows the history-dependent policy rule. The differences in the impulse responses under each monetary policy rule allows us to quantify the relative contributions of the contractionary bias and the precautionary saving and working channels. Under the simple Taylor (1993) rule, Figure A.1 shows that the increase in the contractionary bias and the precautionary behavior channel combine to produce a decline in output of 35 basis points. This decline is much larger than the 2 basis point decline under the history-dependent policy rule, which only features the precautionary saving and working channel. These results suggest that the increase in the contractionary bias explains much of the decline in output after an uncertainty shock when monetary follows a simple interest-rate rule.

A.2.4 Uncertainty, Contractionary Bias, and Equilibrium Existence

In addition to greatly amplifying fluctuations due to changes in uncertainty, this section provides evidence that the contractionary bias can even interfere with equilibrium existence under some calibrations. When the monetary authority follows the simple policy rule in Equation (A.1), Figure A.3 shows an increase in volatility shifts the policy rule to the left
Appendix A  Uncertainty Shocks in a Model of Effective Demand

and increases the average nominal interest. For high levels of volatility, however, the policy rule shifts far enough to the left such that the policy rule no longer intersects the Fisher relation. In this situation, Mendes (2011) shows that a rational expectations equilibrium fails to exist because the contractionary bias is too large. Mendes (2011) also conjectures that a simple history-dependent rule like Equation (1.28) should remove the contractionary bias since the average nominal interest rate is no longer increasing in the volatility of the exogenous shocks.

Our computational experiments provide numerical support to the analytical results and conjectures of Mendes (2011). When monetary policy follows the simple Taylor (1993) rule in Equation (A.1), we are unable to solve our model numerically for our baseline calibration of $\sigma^a = 0.02$ and $\sigma^{\sigma^a} = 0.019$. This numerical failure suggests that the contractionary bias is large enough that a rational expectations equilibrium fails to exist for this calibration.\(^3\) However, we are able to solve our model when we decrease the size of the exogenous shocks to $\sigma^a = 0.005$ and $\sigma^{\sigma^a} = 0.0025$. This result suggests that the smaller exogenous shock volatility decreases the size of the contractionary bias to a level consistent with a rational expectations equilibrium. However, when monetary policy follows the history-dependent rule in Equation (A.1), we are able to solve our model using our baseline calibration of

\(^3\)The contractionary bias only affects equilibrium existence when the monetary authority follows a simple Taylor (1993)-type rule subject to the zero lower bound. Without the zero lower bound, an increase in volatility increases the volatility of the nominal interest rate, but leaves the average level of the nominal interest rate unchanged.
Appendix A Uncertainty Shocks in a Model of Effective Demand

$\sigma^a = 0.02$ and $\sigma^c = 0.019$. This numerical result suggests that the conjecture by Mendes (2011) is correct and the history-dependent rule removes the contractionary bias in the decision rules. Maintaining the considerably lower volatility calibration of $\sigma^a = 0.005$ and $\sigma^c = 0.0025$ in Sections A.2.1-A.2.3 allows us to solve the model under both monetary policy specifications and decompose the relative contributions of the precautionary working and contractionary bias channels.

Even for small increases in uncertainty, the temporary increase in the contractionary bias produces large declines in output and its components. However, we choose to eliminate the contractionary bias channel and assume that the monetary authority follows the history-dependent rule in the main text of the paper. Mechanically, the history-dependent rule allows us to solve our model using our baseline volatility of Table 1.1. In addition, we believe the increase in the contractionary bias at the zero lower bound produces implausibly large declines in output and its components. We view the contractionary bias channel as a technical consequence of examining changes in uncertainty at the zero lower bound under a particular simple monetary policy rule. Therefore, we focus our main analysis of uncertainty at the zero lower bound on the more economically interesting precautionary working and savings channel.
Appendix A Uncertainty Shocks in a Model of Effective Demand

Figure A.1: Demand Uncertainty Shock Under Alternative Policy Rules

Note: The impulse response for the nominal interest rate is plotted in annualized percent. All other impulse responses are plotted in percent deviations from their ergodic mean.
Figure A.2: Nominal Interest Rate Distribution with Zero Lower Bound Constraint

Figure A.3: Simple Monetary Policy Rules & Fisher Relation with Zero Lower Bound
Appendix B

Forward Guidance Under Uncertainty

B.1 Derivations of Approximated Model

B.1.1 Approximation of Consumption Euler Equation

This section provides a detailed derivation of the equations from Section 2.2. Using the consumption Euler equation in Equation (2.1), complete the following steps to derive Equations (2.2) or (2.9):

1. Multiply and divide the right side of the Euler equation by the steady state values of the real interest rate $R^R$ and consumption $C$ raised to the power $-\sigma$. Apply the natural logarithm and exponential functions inside the conditional expectations. Denote $\hat{X}_t = \log(X_t/X)$ to write the variables in log-deviations from steady state.

$$1 = E_t \left\{ \beta R_t^R \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \right\} = E_t \left\{ \left( \frac{R_t^R}{R_t^R} \right) \left( \frac{C_t}{C} \right)^{\sigma} \left( \frac{C_{t+1}}{C} \right)^{-\sigma} \right\}$$
Appendix B  Forward Guidance Under Uncertainty

\[ 1 = E_t \left\{ \exp \left( \log \left( \frac{R_t^R}{R^R} \right) - \sigma \log \left( \frac{C_{t+1}}{C} \right) + \sigma \log \left( \frac{C_t}{C} \right) \right) \right\} \]

\[ 1 = E_t \left\{ \exp \left( \hat{R}_t^R + \sigma \hat{C}_t - \sigma \hat{C}_{t+1} \right) \right\} \]

2. Factor out the time \( t \) variables outside of the conditional expectations, reorganize, and take the logarithm of both sides.

\[ 1 = E_t \left\{ \exp \left( \hat{R}_t^R + \sigma \hat{C}_t \right) \exp \left( -\sigma \hat{C}_{t+1} \right) \right\} \]

\[ \left( \exp \left( \hat{R}_t^R + \sigma \hat{C}_t \right) \right)^{-1} = E_t \left\{ \exp \left( -\sigma \hat{C}_{t+1} \right) \right\} \]

\[ -\hat{R}_t^R - \sigma \hat{C}_t = \log \left( E_t \left\{ \exp \left( -\sigma \hat{C}_{t+1} \right) \right\} \right) \]

3. Replace \( \exp \left( -\sigma \hat{C}_{t+1} \right) \) with its Taylor series expansion around \( \hat{C}_{t+1} = 0 \) and take conditional expectations at time \( t \).

\[ -\hat{R}_t^R - \sigma \hat{C}_t = \log \left( E_t \left\{ 1 - \sigma \hat{C}_{t+1} + \frac{1}{2} \sigma^2 \hat{C}^2_{t+1} - \sigma^3 \hat{C}^3_{t+1} + \ldots \right\} \right) \]

\[ -\hat{R}_t^R - \sigma \hat{C}_t = \log \left( 1 - \sigma E_t \hat{C}_{t+1} + \frac{1}{2} \sigma^2 E_t \hat{C}^2_{t+1} - \sigma^3 E_t \hat{C}^3_{t+1} + \ldots \right) \]

4. Define \( Z = \sigma E_t \hat{C}_{t+1} - \frac{1}{2} \sigma^2 E_t \hat{C}^2_{t+1} + \sigma^3 E_t \hat{C}^3_{t+1} - O \left( C^4_{t+1} \right) \) and use the Taylor series expansion of \( \log(1 - Z) = -Z - (1/2)Z^2 - (1/3)Z^3 - O \left( Z^4 \right) \) to expand the previous equation. To compute a second-order approximation, drop all terms that are third-order or above. Reorganize the remaining terms to form the conditional variance \( \text{Var}_t \hat{C}_{t+1} = E_t \hat{C}^2_{t+1} - \left( E_t \hat{C}_{t+1} \right)^2 \).

\[ -\hat{R}_t^R - \sigma \hat{C}_t = -\sigma E_t \hat{C}_{t+1} + \frac{1}{2} \sigma^2 \text{Var}_t \hat{C}_{t+1} \]

\[ \hat{C}_t = E_t \hat{C}_{t+1} - \frac{1}{\sigma} \hat{R}_t^R - \frac{1}{2} \sigma \text{Var}_t \hat{C}_{t+1} \]
Appendix B Forward Guidance Under Uncertainty

5. Denote variables in logs using lowercase letters and normalize steady state consumption $C$ to equal one to derive Equation (2.2):

$$c_t = E_t c_{t+1} - \frac{1}{\sigma} \left( r^r_t - r^r \right) - \frac{1}{2} \sigma \text{Var}_t c_{t+1}$$

To derive the third-order approximation in Equation (2.9), retain the third-order terms in Step 4 and reorganize the remaining terms to form the conditional skewness$
\text{Skew}_t \hat{C}_{t+1} = E_t \hat{C}_{t+1}^3 - 3E_t \hat{C}_{t+1}^2 E_t \hat{C}_{t+1} + \left( E_t \hat{C}_{t+1} \right)^3$.

B.1.2 Derivation of Higher-Order New-Keynesian Model

In this section, I outline the derivation of the approximate higher-order New-Keynesian model from Section 2.2. To simplify the derivations, I assume that the exogenous process for $a_t$ follows an autoregressive process in logs and the household utility function is additively separable in consumption and leisure. To derive Equations (2.4) - (2.6), I combine a second-order approximation of the household Euler equations with a first-order approximation of the remaining model equations. Clearly, this approach neglects some higher-order terms that are present in a complete second-order approximation of the underlying model. However, the approximations in this section provide analytical tractability which is unavailable when examining the model equations in their original nonlinear form. In Section 2.4, I show that the intuition from these approximations is consistent with the computational results using the full nonlinear model. To derive Equations (2.4) - (2.6), complete the following steps:
Appendix B Forward Guidance Under Uncertainty

1. Apply Steps 1 - 5 from Appendix B.1.1 to the consumption Euler equations for real and nominal bonds and the Euler equation for the natural real interest rate. Use the law of motion \( \log (a_{t+1}) = (1 - \rho_a) \log (a_t) + \sigma_a \varepsilon_{t+1} \) to write \( a_{t+1} \) as function of \( a_t \) and \( \varepsilon_{t+1} \). Impose \( E_t \varepsilon_{t+1} = 0 \) and \( E_t \varepsilon_{t+1}^2 = 1 \).

\[
1 = E_t \left\{ \left( \frac{a_{t+1}}{a_t} \right) \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} R_t^R \right\}
\]

\[
1 = E_t \left\{ \left( \frac{a_{t+1}}{a_t} \right) \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{R_t}{\Pi_{t+1}} \right\}
\]

\[
1 = E_t \left\{ \left( \frac{a_{t+1}}{a_t} \right) R_t^N \right\}
\]

\[
- \hat{R}_t^R - \sigma \hat{C}_t + (1 - \rho_a) \hat{a}_t = \log \left( E_t \left\{ \exp \left( \sigma_a \varepsilon_{t+1} - \sigma \hat{C}_{t+1} \right) \right\} \right)
\]

\[
- \hat{R}_t - \sigma \hat{C}_t + (1 - \rho_a) \hat{a}_t = \log \left( E_t \left\{ \exp \left( \sigma_a \varepsilon_{t+1} - \sigma \hat{C}_{t+1} - \hat{\Pi}_{t+1} \right) \right\} \right)
\]

\[
- \hat{R}_t^N + (1 - \rho_a) \hat{a}_t = \log \left( E_t \left\{ \exp \left( \sigma_a \varepsilon_{t+1} \right) \right\} \right)
\]

\[
- \hat{R}_t^R - \sigma \hat{C}_t + (1 - \rho_a) \hat{a}_t = -\sigma E_t \hat{C}_{t+1} + \frac{1}{2} \sigma^2 \text{Var}_t \hat{C}_{t+1} - \sigma (\sigma^a) \text{Cov}_t \left( \hat{C}_{t+1}, \varepsilon_{t+1} \right) + \frac{1}{2} (\sigma^a)^2
\]  

(B.1)

\[
- \hat{R}_t - \sigma \hat{C}_t + (1 - \rho_a) \hat{a}_t = -\sigma E_t \hat{C}_{t+1} - E_t \hat{\Pi}_{t+1} + \frac{1}{2} \sigma^2 \text{Var}_t \hat{C}_{t+1} + \frac{1}{2} \text{Var}_t \hat{\Pi}_{t+1} - \sigma (\sigma^a) \text{Cov}_t \left( \hat{C}_{t+1}, \hat{\Pi}_{t+1} \right) + \frac{1}{2} (\sigma^a)^2
\]  

(B.2)

\[
- \hat{R}_t^N + (1 - \rho_a) \hat{a}_t = \frac{1}{2} (\sigma^a)^2
\]  

(B.3)
2. Subtract Equation (B.1) from Equation (B.2). Also, subtract Equation (B.3) from Equation (B.1).

\[
-\hat{R}_t + \hat{R}_t^N - \sigma \hat{C}_t = -\sigma E_t \hat{C}_{t+1} + \frac{1}{2} \sigma^2 Var_t \hat{C}_{t+1} - \sigma (\sigma^a) Cov_t \left( \hat{C}_{t+1}, \varepsilon_{t+1} \right)
\]

\[
-\hat{R}_t + \hat{R}_t^R = -E_t \hat{\Pi}_{t+1} + \frac{1}{2} Var_t \hat{\Pi}_{t+1} + \sigma Cov_t \left( \hat{C}_{t+1}, \hat{\Pi}_{t+1} \right) - \sigma^a Cov_t \left( \hat{\Pi}_{t+1}, \varepsilon_{t+1} \right)
\]

3. After reorganizing and dividing by \( \sigma \), the resulting equations can be written as follows:

\[
\hat{C}_t = E_t \hat{C}_{t+1} - \frac{1}{\sigma} \left( \hat{R}_t^R - \hat{R}_t^N \right) - \frac{1}{2} \sigma Var_t \hat{C}_{t+1} + (\sigma^a) Cov_t \left( \hat{C}_{t+1}, \varepsilon_{t+1} \right) \quad (B.4)
\]

\[
\hat{R}_t^R = \hat{R}_t + E_t \hat{\Pi}_{t+1} + \frac{1}{2} Var_t \hat{\Pi}_{t+1} + \sigma Cov_t \left( \hat{C}_{t+1}, \hat{\Pi}_{t+1} \right) - \sigma^a Cov_t \left( \hat{\Pi}_{t+1}, \varepsilon_{t+1} \right) \quad (B.5)
\]

4. Under the assumption of separable utility from consumption and leisure, Equations (2.10) and (2.11) are now modified as follows:

\[
\eta a_t C_t^{-\sigma} = \lambda_t \quad (B.6)
\]

\[
\chi a_t N_t^{\eta} = \lambda_t \frac{W_t}{P_t} \quad (B.7)
\]

\( \eta \) is now the inverse Frisch labor supply elasticity and \( \chi \) is a constant which pins down steady state hours worked.

5. Using a first-order linear approximation of Equations (B.6), (B.7), (2.14), (2.15), the aggregate production function, and the national income accounting identity, the following equations can be derived:

\[
x_t = \hat{Y}_t = \hat{C}_t = \mu \hat{N}_t \quad (B.8)
\]
Appendix B Forward Guidance Under Uncertainty

\[
\begin{align*}
\hat{W}_t^R &= \hat{\zeta}_t = -\hat{\mu}_t = (\eta + \sigma\mu) \hat{N}_t = \frac{\eta + \sigma\mu}{\mu} x_t \\
\hat{\Pi}_t &= \beta E_t \hat{\Pi}_{t+1} + \frac{1}{\phi_p} \hat{\zeta}_t = \beta E_t \hat{\Pi}_{t+1} + \frac{1}{\phi_p} \frac{1}{\mu} x_t
\end{align*}
\]

(B.9)  (B.10)

Define \( \kappa \) as the slope of the Phillips curve in Equation (B.10).

6. To derive Equations (2.4) - (2.6) in the main text, combine the results from Equations (B.8) - (B.9) and Equations (B.4) - (B.5). In addition, use lowercase variables to denote log variables and normalize steady state output \( Y \) and consumption \( C \) to equal one. For clarity of exposition, I omit the two additional covariance terms in Equations (B.4) and (B.5) which are related to the specific exogenous process for the preference shocks in the main text. The coefficients on these terms are very small and they do not provide any additional intuition. Hence, I replace the equality sign with the approximately equal sign in the main text to reflect these omissions.

B.2 Numerical Solution Method

To analyze the impact of uncertainty on the effectiveness of forward guidance, I solve the model using the policy function iteration method of Coleman (1990). This global approximation method allows me to model the occasionally-binding zero lower bound constraint. This section provides the details of the algorithm when monetary policy follows a price-level targeting rule of Eggertsson and Woodford (2003). The algorithm is implemented using the following steps:

1. Discretize the state variables of the model: \( \{a_t \times P_{t-1}\} \)
Appendix B Forward Guidance Under Uncertainty

2. Conjecture initial guesses for the policy functions of the model \( N_t = N(a_t, P_{t-1}) \),
\( \Pi_t = \Pi(a_t, P_{t-1}) \), \( R_t = R(a_t, P_{t-1}) \), and \( R_t^R = R^R(a_t, P_{t-1}) \).

3. For each point in the discretized state space, substitute the current policy functions into the equilibrium conditions of the model. Use interpolation and numerical integration over the exogenous state variable \( a_t \) to compute expectations for each Euler equation. This operation generates a nonlinear system of equations. The solution to this system of equations provides an updated value for the policy functions at that point in the state space. The solution method enforces the zero lower bound for each point in the state space and in expectation.

4. Repeat Step (3) for each point in the state space until the policy functions converge and cease to be updated.

I implement the policy function iteration method in FORTRAN using the nonlinear equation solver DNEQNF from the IMSL numerical library. When monetary policy follows an alternative specification, the state variables and Euler equations are adjusted appropriately.

B.3 Optimal Monetary Policy Under Commitment

The optimal monetary policy maker under commitment aims to maximize the representative household’s utility subject to the constraints of the economy. Some of the constraints include expectations of future variables. Following Khan, King and Wolman (2003), I introduce lagged Lagrange multipliers to make the solutions time-invariant. The augmented
Lagrangian for the optimal policy problem under commitment can be written as follows:

\[
L = \min_{\{\omega_{t+s}\}_{s=0}^{\infty}} \max_{\{d_{t+s}\}_{s=0}^{\infty}} \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \left( \frac{a_{t+s}}{a_t} \right) \left( \frac{C_{t+s}^\eta (1 - N_{t+s})^{1-\eta}}{1 - \sigma} \right) \right. \\
+ \omega_{1t+s} \left( Y_{t+s} - C_{t+s} - \frac{\phi_P}{2} \left( \frac{\Pi_{t+s}}{\Pi} - 1 \right) \right)^2 Y_{t+s} \\
+ \omega_{2t+s} \left( N_{t+s} - \Phi - Y_{t+s} \right) \\
+ \omega_{3t+s} \left( W_R^{t+s} - \frac{1 - \eta}{\eta} C_{t+s} (1 - N_{t+s})^{-1} \right) \\
+ \omega_{4t+s} \left( \phi \right) - \theta W_R^{t+s} + \phi_P \left( \frac{\Pi_{t+s}}{\Pi} - 1 \right) \left( \frac{\Pi_{t+s}}{\Pi} \right) \\
\times \left( a_{t+s} C_{t+s}^{\eta(1-\sigma)-1} (1 - N_{t+s})^{(1-\eta)(1-\sigma)} Y_{t+s} \right) \\
- \omega_{4t+s-1} \phi_P \left( \frac{\Pi_{t+s}}{\Pi} - 1 \right) \left( \frac{\Pi_{t+s}}{\Pi} \right) \left( a_{t+s} C_{t+s}^{\eta(1-\sigma)-1} (1 - N_{t+s})^{(1-\eta)(1-\sigma)} Y_{t+s} \right) \\
+ \omega_{5t+s} \left( a_{t+s} C_{t+s}^{\eta(1-\sigma)-1} (1 - N_{t+s})^{(1-\eta)(1-\sigma)} R_{t+s}^{-1} \right) \\
- \omega_{5t+s-1} \left( a_{t+s} C_{t+s}^{\eta(1-\sigma)-1} (1 - N_{t+s})^{(1-\eta)(1-\sigma)} \Pi_{t+s}^{-1} \right) \\
+ \omega_{6t+s} \left( R_{t+s} - 1 \right) \right\},
\]

where \( d_t = \{ Y_t, C_t, N_t, W_R^t, \Pi_t, R_t \} \) is the set of decision variables and \( \omega_t = \{ \omega_{1t}, \omega_{2t}, \omega_{3t}, \omega_{4t}, \omega_{5t}, \omega_{6t} \} \) is the vector of Lagrange multipliers. The final constraint imposes the zero lower bound constraint since the gross nominal policy rate \( R_t \) must be greater than or equal to one. After solving for the first-order conditions, the optimal policy problem is solved using the algorithm outlined in Appendix B.2. To determine the
Appendix B  Forward Guidance Under Uncertainty

equilibrium real interest rate $R_t^R$, I also include the Euler equation for a zero net supply real bond as well. The algorithm solves for the policy functions for $N_t = N(a_t, \omega_{4t-1}, \omega_{5t-1})$, \( \Pi_t = \Pi(a_t, \omega_{4t-1}, \omega_{5t-1}) \), $R_t = R(a_t, \omega_{4t-1}, \omega_{5t-1})$, $R_t^R = R^R(a_t, \omega_{4t-1}, \omega_{5t-1})$, \( \omega_{4t} = \omega_4(a_t, \omega_{4t-1}, \omega_{5t-1}) \), and \( \omega_{5t} = \omega_5(a_t, \omega_{4t-1}, \omega_{5t-1}) \) on a discretized state space for \( \{a_t \times \omega_{4t-1} \times \omega_{5t-1}\} \).
Appendix C

Real Fluctuations at the Zero Lower Bound

C.1 Numerical Solution Method

To analyze the impact of real shocks at the zero lower bound, I solve the model using the policy function iteration method of Coleman (1990). This global approximation method allows me to model the occasionally-binding zero lower bound constraint. This section provides the details of the algorithm when monetary policy follows a standard Taylor (1993)-type policy rule. The algorithm is implemented using the following steps:

1. Discretize the state variables of the model: \( \{a_t \times Z_t \times \theta_t \times G_t\} \)

2. Conjecture initial guesses for the policy functions of the model \( N_t = N(a_t, Z_t, \theta_t, G_t) \),
Appendix C Real Fluctuations at the Zero Lower Bound

\[ \Pi_t = \Pi(a_t, Z_t, \theta_t, G_t), \quad \text{and} \quad R_t^R = R^R(a_t, Z_t, \theta_t, G_t). \]

3. For each point in the discretized state space, substitute the current policy functions into the equilibrium conditions of the model. Use interpolation and numerical integration over the exogenous state variables \( a_t, Z_t, \theta_t, G_t \) to compute expectations for each Euler equation. This operation generates a nonlinear system of equations. The solution to this system of equations provides an updated value for the policy functions at that point in the state space. The solution method enforces the zero lower bound for each point in the state space and in expectation.

4. Repeat Step (3) for each point in the state space until the policy functions converge and cease to be updated.

I implement the policy function iteration method in FORTRAN using the nonlinear equation solver DNEQNF from the IMSL numerical library. When monetary policy follows an alternative specification, the state variables and Euler equations are adjusted appropriately.
Bibliography


Bibliography


Bibliography


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164
Bibliography


Bibliography


Bibliography


