Essays on Business Cycle Models

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ESSAYS ON BUSINESS CYCLE MODELS

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ABSTRACT

ESSAYS ON BUSINESS CYCLE MODELS

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Empirical studies highlight that countries that trade intermediate goods exhibit more synchronized business cycles. This positive correlation raises the question of causality. Traditional theoretical mechanisms propose the direction where higher bilateral trade in intermediate goods causes increased business cycle correlations. However, the data shows that trade is positively correlated with comovements in GDP as well as total factor productivity (TFP) and the current work in the literature explains only the first relation. I build a small open economy model that makes two contributions — first, it predicts both positive correlations as seen in the data. Second, it explains potential causality in the reverse direction, i.e. countries might choose trade partners based on the properties of their business cycles. Specifically, the model predicts that when the elasticity of substitution between domestic capital and intermediate imports is low, i.e. the country is constrained by domestic technology, there is greater benefit from trading with a positively correlated source and self-insuring through capital accumulation. I provide empirical evidence of this condition in the data by estimating the elasticity of substitution between capital and intermediates by industry using a panel of countries.

We use annual time series data and filtering methods to document the key statistics of the India business cycle. Output, consumption and investment are more volatile than in developed economies. Like in developed countries, consumption is less volatile and investment is more volatile than output in the Indian data. Unlike in the former, investment is not highly correlated with output. We test whether a standard real business cycle model with technology and fiscal shocks, with parameters calibrated for the Indian economy can replicate the features of the business cycle.
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Chapter 1

Comovement of business cycles and trade in intermediate goods

1.1 Introduction

Intermediates in production account for an increasing percentage of world trade as countries take advantage of lower factor costs abroad. Empirical studies have highlighted that countries with such trade linkages exhibit more synchronized business cycles. This positive correlation raises the question of causality. Traditional theoretical mechanisms propose the direction where higher bilateral trade in intermediate goods causes increased business cycle correlations.\(^1\) However, the data shows that trade is positively correlated with comovements in GDP as well as total factor productivity (TFP); and the current work in the literature explains only the first relation.\(^2\) Based on these observations, my model makes two contributions – first, it predicts both

\(^1\)In Burstein, Kurz and Tesar (2008), the low elasticity of substitution between domestic and foreign intermediates in an Armington aggregate production function creates a tight dependence on the foreign good resulting in GDP correlation in response to aggregate shocks. In Arkolakis and Ramanarayanan (2009) and Zlate (2008), endogenous specialization implies that the location of production facilities is responsive to aggregate shocks and this results in the correlated movement of output.

\(^2\)Arkolakis and Ramanarayanan (2009) and Drozd and Nosal (2008) regress the bilateral correlation of TFP on trade intensity for industrialized countries and obtain positive and significant OLS estimates.
positive correlations as seen in the data. Second, it explains potential causality in the reverse direction, i.e. countries might choose trade partners based on the properties of their business cycles.³

In models explaining the traditional direction of causality, a technology shock in the home country leads to an increase in domestic factor productivity and an increase in output. Since Home depends on the foreign country for intermediate goods, there is an increase in demand for the foreign good, and foreign production increases. Thus, the GDP movement is correlated. However, absent a technology shock, the foreign country is producing more output by hiring more factors, so it is not necessary that the factors are more productive. Thus in these models, an increase in trade does not lead to a positive comovement in TFP.

I address this deficiency by introducing the reverse hypothesis, where business cycle comovement affects the trading decisions of a country. In particular, the positive relation suggests that a country imports intermediate goods from countries with whose business cycles its own is synchronized. A theoretical explanation for this is a capital accumulation motive of countries. After a good technology shock, when the marginal cost of production is low, a country has incentive to import intermediate goods from a source country that has also experienced a good technology shock and has a low relative price of the good. In the presence of costs of switching trade partners countries can import during good times, build their capital and run down this capital during bad times. An alternative mechanism in which trade is caused by the properties of the business cycle is based on an insurance motive. Accordingly, countries can smooth consumption or insure against their shocks by importing from countries whose business cycles do not match their own. In this case, there is a negative relation between comovement and trade. An endowment economy will always import from a negatively comoving source to reduce the variance of consumption

³This is not to suggest that there is no causal relationship from trade to business cycles. My claim is that the reverse causality is potentially stronger in replicating the empirical features of the data.
arising from aggregate shocks. Thus it is the presence of capital and the ability to invest that introduces the self insurance channel.\footnote{The role of self insurance in incomplete markets is an established idea in the literature pioneered by Aiyagari (1994). The papers show that in order to decrease fluctuations in consumption in the presence of uninsured shocks, precautionary saving or capital accumulation is generated.}

In order to get better intuition for conditions when the capital accumulation motive is stronger, I model a small open economy with two sources of imports - one that has a positive correlation and the other a negative correlation with the importing country’s business cycle. The country has to choose its trade partner at the beginning of time because the costs of switching trade partners are very high. Assuming that the importing country has full foresight about the distribution of technology shocks, it can make this choice at the beginning by comparing the expected value of its welfare with respect to each trade partner. The idea is to evaluate whether there are conditions under which the model generates higher welfare with a positively comoving trade partner and then to see if the conditions exist in the data.

The key result is that the model predicts a positive correlation between intermediate goods trade and the comovements of GDP and TFP, which is consistent with empirical evidence. This is obtained when the elasticity of substitution between capital and intermediates is low (below one). If the importing country is constrained by domestic technology and cannot easily substitute between the foreign good and domestic capital, then there is no benefit from the insurance provided by trade, and the capital building motive dominates. In the second part of the paper, I provide empirical support for the predicted complementarity by estimating the elasticity of substitution between capital and intermediates using a panel dataset.

The link between trade and cross-country business cycles has been a subject of interest in recent years in the international macroeconomic literature, especially with the proliferation of free trade agreements, currency unions and other integration initiatives. In a seminal paper, Frankel and Rose (1998) argue that joining a currency union is beneficial even if ex ante the business cycles are asynchronized, by showing
empirically that countries with closer trade ties end up with highly correlated business cycles. While the basic positive relation was reiterated in a number of studies, theoretically, the impact of trade on synchronization remained unclear, and empirically, the issue of omitted variables was raised. For example, Imbs (2001, 2004) finds that similarity in sectoral specialization and financial linkages have a significant impact on cycle synchronization, whereas the impact of trade is sensitive to specifications and sub-samples. On the contrary, Baxter and Kouparitsas (2001) find that sectoral similarity does not have a robustly significant effect on output correlations, but intra-industry trade does. Inklaar, Jong-A-Pin and Haan (2007) examine the evidence for OECD countries and find that besides trade, similarity of monetary and fiscal policies as well as specialization has a strong impact on business cycle correlations. In general, the exogeneity of the instruments used in the Frankel and Rose empirical exercise is questioned. Theoretically, production sharing has been examined as a mechanism by which trade affects business cycle synchronization and the traditional direction of causality does not replicate the features of the data. I revisit this issue and point out that looking only at GDP comovement may not be sufficient. In order to assess the welfare that arises from integration initiatives, the models should also replicate the TFP comovement present in the data.

Thus, by suggesting potential causality in the reverse direction, i.e. from business cycles to trade, this paper adds a new dimension to macroeconomic and trade policy. While studying optimum currency areas, regional agreements and trade treaties, current and forthcoming, policy makers should be looking at the TFP correlation as an important determinant of welfare post integration.

The rest of the paper is organized as follows. Section 2 lays out the model for the small open economy with two sources of imports and provides analytical and numerical solutions for the decision problem. Section 3 presents the numerical results and intuition. Section 4 provides supporting empirical evidence of the results obtained. Section 5 concludes and proposes extensions for future work.
1.2 The Model

This section develops an open economy model to obtain conditions under which a country chooses to import intermediate goods from a country with whose business cycle its own comoves positively.

A small open economy (Home) can import intermediate goods from one of two Foreign countries to benefit from the relatively low factor costs there. One foreign country has business cycles that are synchronized with Home (H) and the other country has business cycles that are not. I assume that there are costs of switching partners to capture the significant costs in terms of time and resources that are necessary to develop new supplier relationships or set up new production facilities.\(^5\) In my model, absent switching costs, swings between trading partners would occur since there is always an incentive to import from the lowest cost producer every period. In the simplest case, these switching costs are assumed to be prohibitively high, so that H cannot change its trading partner once it decides to import from one of the Foreign countries. To make this optimal choice, H compares its expected lifetime welfare with each trade partner.

1.2.1 Foreign Countries - Intermediate Good Producers

The time horizon is infinite and time is discrete, \(t = 0, 1, \ldots\). There is an intermediate good \(m\) produced by two foreign countries \(F_i, i = 1, 2\). Country \(i\)'s efficiency in producing good \(m\) at time \(t\) is denoted as \(z_{it}\), which is a random process. If the input cost in country \(i\) is \(x_{it}\), then with constant returns to scale, the cost of producing a

\(^5\)Switching costs can be understood at the firm level, where each firm chooses a supplier ex ante and for relatively small fluctuations doesn’t switch suppliers. The suppliers need not all be located in the same foreign country. Then the import price \(p_m\) for a country should be interpreted as a weighted average of the firm level import prices.
unit of good \( m \) in country \( i \) is \( \frac{x_{it}}{z_{it}} \). Assuming that \( x_{it} \) is the same in \( F_1 \) and \( F_2 \) and normalized to one, the price in the Home country \( H \) of a unit of good produced in country \( i \) is

\[
  p_{it} = \frac{1}{z_{it}}.
\]

Thus, the difference between the two foreign countries lies in the properties of the technology process in relation to \( H \)'s technology as described below.

\subsection*{1.2.2 Home Country}

Facing these prices for intermediate goods, a planner in \( H \) equipped with initial capital stock \( k_0 \) chooses a sequence of future capital stocks \( \{k_t\}_{t=1}^{\infty} \) and a sequence of current and future imports \( \{m_t\}_{t=0}^{\infty} \) to maximize the lifetime utility of a representative household

\[
  U_0 = \sum_{t=0}^{\infty} \beta^t u(c_t),
\]

where \( \beta \in (0,1) \). The economy’s resource constraint is

\[
  f(z_t, k_t, m_t) \geq c_t + k_{t+1} - (1 - \delta)k_t + p_t m_t,
\]

where \( \delta \in (0,1) \) is the rate of depreciation and \( c_t \geq 0 \). The final good output is produced using capital and the imported intermediate good as inputs, and it is allocated to consumption, investment and the purchase of intermediate goods. Labor is assumed to be inelastically supplied The absence of trade in financial assets implies that goods trade is balanced in each period.

Recall that \( z_t \) and \( z_{it} \) \((i = 1, 2)\) are the technology shocks associated with \( H \) and \( F_i \) countries respectively. If \( Cov(z_t, z_{it}) > 0 \) (positive comovement), then \( Cov(z_t, p_{it}) < 0 \), and if \( Cov(z_t, z_{it}) < 0 \) (negative comovement), then \( Cov(z_t, p_{it}) > 0 \). I set up the planning problem to obtain the value function under both cases.
Home’s problem can be written as a recursive formulation of the maximization problem in terms of a Bellman equation:

\[ V(k; z, p) = \max_{k', m} [u(f(k, m) - k' + (1 - \delta)k - pm) + \beta V(k'; z', p')]. \tag{1.2.4} \]

Specifically,

\[ V(k_t; z_t, p_t) = \max_{k_{t+1}, m_t} [u(c_t) + \beta E_t V(k_{t+1}; z_{t+1}, p_{t+1})] \tag{1.2.5} \]

such that

\[ c_t = f(z_t k_t, m_t) - k_{t+1} + (1 - \delta)k_t - p_t m_t, \tag{1.2.6} \]

\[ c_t \geq 0. \tag{1.2.7} \]

The shocks \( z \) and \( p \) follow Markov random processes and are correlated positively or negatively.

A solution to this problem is a value function that satisfies the Bellman equation and the associated policy functions mapping the current state into the optimal choice of \( k \) to carry to the next period.

In the presence of very high switching costs, I solve the problem separately for the cases of positive and negative comovement. Again, since the economy chooses the optimal partner for a lifetime, and not every period, it is important to obtain the unconditional expected value, i.e., the expected value prior to the observation of the first shock. Thus \( V_{pos}(k; z, p) \) (\( V_{neg}(k; z, p) \)) is computed as the unconditional expected lifetime utility of an agent in the home country when intermediate goods are imported from a country with synchronized (asynchronized) business cycles. This lifetime utility takes into account the distribution of positive and negative technology shocks through which the agent is going to live.
1.2.3 Solution

The problem can be solved analytically for the case of logarithmic utility in consumption, Cobb-Douglas production in capital and imports, full depreciation of capital and AR(1) shock processes. These parameters result in a knife-edge case where the covariance of the shocks does not affect the decision of the agent. In other words, the capital building motive and the insurance motive exactly offset each other. The analytical solution is presented in Appendix A.

The case specified above is restrictive and there may be plausible combinations of parameter values for which one motive may dominate the other. Intuition suggests that when the inputs are either complements or substitutes in production, the agent’s expected welfare will be affected by the choice of trade partner depending on the movement of price and the nature of domestic technology. Hence, I introduce a CES production function

\[ f(k_t, m_t) = [(z_t k_t)^\nu + \gamma m_t^\nu]^{\frac{1}{\nu}}, \]

so as to vary the elasticity of substitution \((\frac{1}{1-\gamma})\) between the inputs. The elasticity of substitution provides information about the direction and the degree of difficulty in adjusting the utilization of the inputs. The share of imports in production is determined by the parameter \(\gamma\). I also relax the assumption of full capital depreciation.

The problem is solved numerically by value function iteration for a discrete set of equally spaced points for capital, \(k\). I assume that the shocks \(p\) and \(z\) take on a range of values with some probability. For simplicity, I assume that they take three values each, say \((p_l, p_m, p_h)\) and \((z_l, z_m, z_h)\). Their evolution over time is described by the transition probability matrices. If the shocks are independent, then

\[
P(p'|p) = \begin{bmatrix}
\pi_{lt}^* & \pi_{lm}^* & \pi_{lh}^* \\
\pi_{ml}^* & \pi_{mm}^* & \pi_{mh}^* \\
\pi_{hl}^* & \pi_{hm}^* & \pi_{hh}^*
\end{bmatrix}
\quad \text{and} \quad
P(z'|z) = \begin{bmatrix}
\pi_{lt} & \pi_{lm} & \pi_{lh} \\
\pi_{ml} & \pi_{mm} & \pi_{mh} \\
\pi_{hl} & \pi_{hm} & \pi_{hh}
\end{bmatrix},
\]

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where for example, $\pi_{ll}$ is the probability that $z$ is in the low state at time $t$ and remains in the low state at time $t+1$.

For non-independent shocks, if the transition matrix associated with $p$ is

$$P(p'|p) = \begin{bmatrix} 
\pi_{ll}^* & \pi_{lm}^* & \pi_{lh}^* \\
\pi_{ml}^* & \pi_{mm}^* & \pi_{mh}^* \\
\pi_{hl}^* & \pi_{hm}^* & \pi_{hh}^* 
\end{bmatrix},$$

then, conditional on $p$ being in the low state, the transition matrix associated with $z$ is

$$P(z'|z) = \begin{bmatrix} 
\pi_{ll} + \epsilon & \pi_{lm} - \epsilon/2 & \pi_{lh} - \epsilon/2 \\
\pi_{ml} + \epsilon & \pi_{mm} - \epsilon/2 & \pi_{mh} - \epsilon/2 \\
\pi_{hl} + \epsilon & \pi_{hm} - \epsilon/2 & \pi_{hh} - \epsilon/2 
\end{bmatrix},$$

where $\epsilon > 0$ implies negative comovement and $\epsilon < 0$ implies positive comovement. To understand the pair of matrices, consider the example of negative comovement. Given that $p$ is in the low state, the probability that $z$ is also in the low state or moves to the low state from the medium and high states is higher than the probability that $p$ switches to the medium or high states. Similar matrices are set up conditional on $p$ being in the medium and high states.  

Combining the shocks $z$ and $p$, there are nine possible states of the economy, and $V(k, zp)$ is the optimal value of the objective function, starting from the state $(k, zp)$. In order to evaluate the unconditional value function for each $k$, i.e., the welfare of the agent prior to observing the first shock, I obtain the stationary vector associated with the joint transition probability matrix and the corresponding value function.

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6The initial matrices for $p$ and $z$ are set up so that the relation between $\epsilon$ and $corr(z, p)$ is almost one-to-one for a simulated series of $corr(z, p)$. The entries of the probability matrix are positive and the rows sum to one, hence a nearly symmetric range of values for the $corr(z, p)$ is restricted to $[-0.3, 0.4]$. The details are in Appendix B.
1.3 Results

In this section, I discuss the effects of the correlation properties on the trading decision based on the numerical results of the value function iteration for specified parameters. In addition, I examine how the effects are altered by aspects of the model, namely the elasticity of substitution between domestic capital and the foreign intermediate, the depreciation rate of capital, the share of imports in production and final good imports.

Figure 1. plots the expected value function for an average value of capital against

Figure 1.1: $\beta = 0.95, \delta = 1, \nu = 0.0001/ - 0.0001, \gamma = 0.15$

a range of values for $corr(z, p)$, for the case of log utility in consumption ($\frac{1}{1-\gamma} \approx 1$), and Cobb-Douglas production function with full depreciation of capital. The parameter for the share of imports in production $\gamma$ is set at 0.15 as in Kose and Yi (2006). This is the case closest to the analytical example discussed. The graph shows that the numerical solution coincides with the analytical result, i.e. the value function is not affected by the correlation of the shocks. A change in $\delta$ to a more realistic value of 0.025 does not alter this result.
1.3.1 Elasticity of substitution

Holding all other parameters constant, I vary the elasticity of substitution ($\sigma$) in a CES production function to deviate from the knife-edge case. Since $\sigma$ is bounded on the lower side by 0 and $\nu$ is bounded on the upper side by 1, the range of possible discrete values for $\nu$ is $[-9, 0.9]$ and for $\sigma$ is $[0.1, 10]$. Figure 2 plots the difference between the expected value functions under the cases of positive and negative comovement, i.e $V_{pos} - V_{neg}$ for average capital against a range of values for elasticity. The graphs indicates that when this parameter is different from one, the comovement of the shocks affects the value function. In particular, when the elasticity is above one, the value function is higher when there is negative comovement and when the elasticity is below one, the result reverses. To gain a clearer understanding, I pick two values for $\nu$ and hence the elasticity and discuss the results for changes in the depreciation rate.

![Figure 1.2: Difference in value function against elasticity of substitution](image)

**Case 1**: $\nu = 0.3, Elasticity = 1.4$

In this case, the value of $\nu$ is 0.3, which implies that the elasticity of substitution is greater than 1 at 1.4.
Figure 1.3: Value function against correlation

Figure 1.4: Difference in value functions against capital

Figure 3 plots the expected value function for an average level of capital against a range of values for \( \text{corr}(z, p) \), for \( \delta = 1 \) and \( \delta = 0.025 \). Recalling that \( \text{corr}(z, p) < 0 \) implies positive comovement, the graphs show that the value function is higher under negative comovement. In Figure 4, I plot the difference between the expected value functions under the cases of positive and negative comovement, i.e. \( V_{\text{pos}} - V_{\text{neg}} \), against capital for two values of \( \delta \). The correlation between \( z \) and \( p \) is set at -0.3 and 0.3 respectively. Again, the graphs show that when the elasticity of substitution is above one, the value function is higher when a country imports from a negatively comoving...
source for all values of capital, so $V_{pos} < V_{neg}$.

When the elasticity of substitution between inputs is high (above 1), depending on price changes, the economy can more easily substitute one input for the other in production. Here, the insurance motive dominates.

**Case 2: $\nu = -1, \text{Elasticity} = 0.5$**

Now I change the value of $\nu$ to equal $-1$, which implies that the elasticity of substitution is below 1 at 0.5. Looking at Figures 5 and 6, it is clear that for a low elasticity of substitution, in general the result is reversed, i.e., $V_{pos} > V_{neg}$ for the range of correlation values and the range of capital. Particularly, when $\delta = 0.025$, the value function is higher when there is positive comovement.

![Figure 1.5: Value function against correlation](image)

This case shows that when the elasticity of substitution between domestic capital and imported intermediates is low (below 1), and capital depreciates slowly, the capital building motive dominates the insurance motive. A low elasticity implies that when the price of the intermediate good falls, imports increase by a smaller proportion. In other words, the economy cannot use the intermediate good instead of domestic capital and is unable to benefit from the fall in price. Conversely, an increase in the price implies that the fall in imports is of a smaller proportion and
hence importing from a country with opposite business cycles reduces welfare. The economy is constrained by the technology and domestic capital and there is not much benefit from insurance. The greater benefit comes from building domestic capital in good times by importing from a country with positively correlated cycle and running down this capital stock during a bad technology shock.

1.3.2 Share of imports

Figure 7 plots $V_{pos} - V_{neg}$ against a range of values for $\gamma$, the share of imports in production. This is for correlations of 0.3 and $-0.3$ and the average value of capital. When the elasticity is above 1, the first plot shows that for a high share of imports in production (above 0.8), $V_{pos} > V_{neg}$. This suggests that when intermediate imports contribute to a significant portion of production, even with a high elasticity of substitution, the self-insurance motive is strong. The second plot shows that when the elasticity of substitution is below 1, for all values of $\gamma$, the value function is higher under positive comovement.
1.3.3 Import of final goods

The above results imply that when an economy imports intermediate goods and cannot easily substitute them with domestic capital in production in response to price changes, it is more beneficial to build capital to smooth consumption over its lifetime. This can be done by trading with a synchronized partner. If on the other hand, the economy imports final goods, then it can insure against consumption variance by trading with an asynchronized partner. To verify this, I modify the model so that the imported good provides utility directly and is not used in production, i.e., the imported good is for final consumption. Thus,

$$ U = \sum_{t=0}^{\infty} \beta^t (b \log c_t + (1 - b) \log m_t) $$  \hspace{1cm} (1.3.1) 

subject to

$$ f(k_t) = c_t + k_{t+1} - (1 - \delta)k_t + p_t m_t, $$  \hspace{1cm} (1.3.2) 

where $f(k_t) = z_t k_t^\alpha$ and $0 < \alpha < 1$. For parameter values $\alpha = 0.3$, $b = 0.5$ and $\delta = 0.025$, the graph shows that the value function is higher under negative comovement for the range of capital, as intuition suggests.
1.4 Empirical tests

In this section, I provide evidence about the empirical validity of the model’s prediction. In particular, the parameter of importance is the elasticity of substitution between capital and intermediates and I explore the data to determine whether these inputs are complements in production in manufacturing industries. A number of empirical papers estimate elasticity parameters between inputs at the industry level. For instance, Saito (2004) estimates the Armington elasticities between intermediate imports and domestic intermediates using bilateral and multilateral data for OECD countries by industry. Gallaway et. al (2003) provide short and long run estimates for the elasticity of substitution between domestic goods and imports for over 300 manufacturing industries in the US. However, since the self insurance role played by domestic capital is key for driving the results of the model described above, it is important to obtain the elasticity parameter specifically between capital and intermediates. Hence I estimate the elasticity of substitution for OECD countries by
industry using a panel dataset. As a proxy, I use data on intermediate goods, which includes both domestic as well as foreign intermediates, due to the lack of data on intermediate imports at the industry level for OECD countries for a long time period.

1.4.1 Estimating the EOS

Assuming the decision maker in the economy is the industry, the inputs of industry specific \((j)\) capital and intermediate goods in country \(i\) enter the production function as follows:

\[
Y_{ijt} = \left[\theta_{ij} k_{ijt}^{\frac{\sigma_{ij}-1}{\sigma_{ij}}} + (1 - \theta_{ij}) m_{ijt}^{\frac{\sigma_{ij}-1}{\sigma_{ij}}} \right]^{\frac{1}{\sigma_{ij}-1}}
\]

(1.4.1)

where \(\sigma_{ij}\) is the time invariant elasticity of substitution between the inputs and \(\theta_{ij}\) is the share of capital in production. Constrained optimization of the equation above yields the log-linear specification

\[
\ln \frac{k_{ijt}}{m_{ijt}} = \sigma_{ij} \ln \frac{\theta_{ij}}{1 - \theta_{ij}} + \sigma_{ij} \ln \frac{P_{ij}^m}{P_{ij}^k}
\]

(1.4.2)

where \(P_{ij}^m\) and \(P_{ij}^k\) are the prices of intermediates and capital respectively. This equation may be stylized to fit the linear regression equation:

\[
\ln y_{ijt} = \beta_{0ij} + \beta_{1ij} \ln x_{ijt} + \epsilon_{ijt}
\]

(1.4.3)

where \(y_{ijt}\) is the capital-intermediate goods ratio, \(x_{ijt}\) is the ratio of intermediate price-capital price and \(\epsilon_{ijt}\) is the independent and identically distributed error term. The elasticity of substitution between capital and intermediate imports, \(\beta_{2ij}\) is the coefficient of interest. \(\beta_{0ij}\) is an unobserved time invariant country-industry specific

\footnote{Note that the same estimation will hold if labor is included in the production function, i.e., \(f(X, L) = \left[\theta k L^{\frac{\sigma_{ij}-1}{\sigma_{ij}}} + (1 - \theta) m L^{\frac{\sigma_{ij}-1}{\sigma_{ij}}} \right]^{\frac{1}{\sigma_{ij}-1}}\).}

\footnote{Johnson and Noguera (2010) construct a global bilateral input-output table by combining input-output tables and bilateral trade data of many countries for the year 2004. This determines foreign import in each industry of the destination country by source.}
Data
The four data series that are required to operationalize equation 4.3 are quantity of intermediate goods inputs, quantity of capital inputs, price of intermediates, and price of capital. I obtain data from the EUKLEMS database at the two-digit industry level for 16 industries (1970 - 2007) and 10 industrial countries. For quantity of capital and intermediate inputs, the series used are Capital services - volume indices, and Intermediate inputs - volume indices. Intermediate inputs - price indices are the series for intermediate goods prices. All the indices use 1995 as the base year. The series for price of capital is constructed by dividing Capital compensation (in millions of local currency) by the capital volume indices. The left-hand side variable in the regression equation is constructed by dividing the volume of capital by the volume of intermediates and the right-hand side variable is obtained by dividing the price of intermediates by the price of capital.

Estimation Procedure
The goal is to estimate the elasticity of substitution between capital and materials for each industry and country, i.e., the parameter $\sigma_{ij}$ in equation 4.2. This is done by employing panel data techniques to take advantage of the greater variability in the panel data compared to pure time series or pure cross section data, and to be able to estimate the country-industry specific parameters.

Consider the econometric specification

$$\ln y_{ijt} = \beta_{0ij} + \beta_{1ij} \ln x_{ijt} + \epsilon_{ijt},$$  \hspace{1cm} (1.4.4)

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EU KLEMS Growth and Productivity Accounts: November 2009 Release. The countries included are Australia, Austria, Denmark, Finland, France, Italy, Netherlands, Spain, UK and USA. The data for Austria and USA are available from 1977 - 2007.
where \( t = 1, 2, \ldots, T \) and \( \beta_{tij} \) is the elasticity of substitution and \( \beta_{0ij} \) is the unobserved fixed effect. This is a large T panel with fixed effects, endogenous explanatory variables and an error covariance matrix that is not proportional to the identity matrix. Endogeneity could arise because of division bias, i.e., the real quantities are not independent of the prices. They are nominal values deflated by the same \( P \)'s that appear on the RHS and if there is measurement error in \( P \), then there is endogeneity. Given the panel nature of the data, a natural choice for instruments is the lagged values of the right hand side variables. I use two lags of the log of price ratios as instruments. Since the price series is highly persistent, the lags serve as good instruments for the regressor. I report the standard errors that are robust to heteroskedasticity and autocorrelation.\(^10\)

**Results**

The table below reports the estimation results for 10 countries and 16 industries. Of the 160 estimated parameters, 67 percent are positive and significant at the 10 percent level.\(^11\) The average elasticity estimate is 0.35, with a range between 0 and 1.66. 109 estimates are below 0.8 and only two are above one. The results are in line with Bruno (1984) who estimates the elasticity parameter as 0.3 for the manufacturing sector in ten OECD countries.\(^12\) These results show that capital and intermediates are complements in production in most industries in the OECD countries.

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\(^{10}\)The covariance matrix is estimated using a heteroskedasticity and autocorrelation consistent (HAC) estimator (Newey-West).

\(^{11}\)Note that the price (ratio) series in equation is inverted, so the elasticity estimates are positive.

\(^{12}\)Bruno estimates the elasticity of substitution between material inputs and a value added function of capital and labor for the period 1956 - 1978.
<table>
<thead>
<tr>
<th>Sector</th>
<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
<th>Column 4</th>
<th>Column 5</th>
<th>Column 6</th>
<th>Column 7</th>
<th>Column 8</th>
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<td>-0.17*</td>
<td>-0.06</td>
<td>-0.19*</td>
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<td>-0.74*</td>
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<td>0.20*</td>
<td>0.47*</td>
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<td>-0.20</td>
<td>-0.12</td>
<td>0.08</td>
<td>0.08*</td>
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<tr>
<td>Food, Beverages and Tobacco</td>
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<td>-0.09</td>
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<td>0.67*</td>
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<td>0.04</td>
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<td>0.03*</td>
<td>0.03</td>
<td>0.35*</td>
<td>0.23*</td>
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<td>0.43*</td>
<td>0.12*</td>
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<td>0.09*</td>
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<td>Basic Metals and Fabricated Metal</td>
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<td>-0.11</td>
<td>0.47*</td>
<td>-0.00</td>
<td>0.34*</td>
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<td>0.47*</td>
<td>0.02*</td>
<td>0.01</td>
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<td>0.42*</td>
<td>0.16*</td>
<td>0.84*</td>
</tr>
<tr>
<td>Electrical and Optical Equipment</td>
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<td>0.08*</td>
<td>0.14</td>
<td>0.65*</td>
<td>0.16</td>
<td>0.09</td>
<td>0.54*</td>
<td>0.28*</td>
<td>-0.11</td>
<td>0.08*</td>
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<tr>
<td>Transport Equipment</td>
<td>0.10</td>
<td>0.60*</td>
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<td>0.05</td>
<td>1.66*</td>
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<td>-0.06</td>
<td>0.23*</td>
<td>0.22*</td>
<td>-0.21</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses

* p<0.05
1.4.2 Empirical link between TFP comovement and trade

Figure 1.9: Scatter plot for OECD countries

The model predicts a positive link between TFP comovement and intermediate goods trade. To assign a magnitude to the correlation coefficient, I run the following regression:

\[
\log(\text{Trade}_{ij}) = \alpha_0 + \alpha_1(\text{Corr}_{ij}) + \epsilon_{ij}
\]  

(1.4.5)
where $Corr_{ij}$ is the correlation of TFP of pairs of countries. I follow the literature in defining the measure of trade intensity, $Trade_{ij}$, as the ratio of total bilateral intermediate goods trade (measured as the sum of each country’s intermediate goods imports from the other) to total GDP of the two countries.

In order to take the prediction to the data, I construct a comparable measure of GDP ($Y$) from the model’s output. The definition of GDP is the difference between aggregate gross output and aggregate intermediate purchases, or the sum of consumption and investment (assuming balanced trade) as below,

$$f(z_t k_t, m_t) - p_{mt}m_t = c_t + k_{t+1} - (1 - \delta) k_t.$$  

The data for consumption and investment are obtained from the World Bank’s World Development Indicators. TFP is computed as the Solow residual from a Cobb-Douglas production function using the constructed GDP and data from the Penn World Tables.\(^{13}\) The correlation of TFP for pairs of countries is obtained after HP-filtering the annual series.

The data for intermediate goods imports is taken from Comtrade, using the World Bank’s classification of imports into Broad Economic Categories\(^ {14}\). All the data is obtained for OECD countries for the years 1976 - 2003 and divided into five year periods.

The scatter plot shows the positive relation. The OLS coefficient $\alpha_1 = 0.45$ and is significant at the 5 percent level. The interpretation of the value of the coefficient $\alpha_1$ is that a doubling of the correlation between a pair of countries results in an increase of $e^{2\alpha_1}$ of trade intensity between them. For TFP correlation, it would be 2.46.\(^ {15}\)

\(^{13}\)T$FP = \frac{GDP}{K^{\alpha}L^{1-\alpha}}$, $K$ denoting capital and $L$ denoting labor. The data is described in the appendix.

\(^{14}\)Other measures have been used for bilateral intermediate goods trade in the literature. A number of papers, including Burstein, Kurz and Tesar (2008) use data on intermediate trade between owners and US affiliates, reported by the Bureau of Economic Analysis for measuring production sharing. di Giovanni and Levchenko (2009) combine sector level data on production and trade with input-output matrices to measure the extent of vertical linkages at the sectoral level.

\(^{15}\)Arkolakis and Ramanarayanan regress $Corr_{ij}$ on $log(Trade_{ij})$ where the latter refers to bilateral
1.5 Conclusion

Intermediate goods trade accounts for an increasing proportion of world trade. Production sharing creates interdependencies across countries which makes understanding the linkages crucial for trade and macroeconomic policy. This paper takes a step in that direction by shifting attention towards an important feature of the data, namely the positive relation between TFP correlation and trade intensity, while so far the only aspect of the data that has been reproduced is the comovement of GDP correlation and trade intensity. By doing so, I raise the issue of potential causality in the reverse direction, from business cycle comovement to trade. The model predicts that countries import from positively comoving trade partners when the elasticity of substitution between capital and intermediates is low. By estimating the elasticity by industry for a panel of countries, I show that this condition exists in the data. Further, the model explains heterogeneity among countries or industries in the relation between trade and comovement i.e., when the elasticity of substitution is low, an industry imports from a positively comoving source and when the elasticity is high, from a negatively comoving source. Papers based on the original causality do not explain this heterogeneity.

An interesting implication of the model’s predictions is that policy makers should be looking at the TFP correlation as an important determinant of welfare post integration.

Extensions of the model could be devoted to a fully specified characterization of the decision of the optimal trade partner in the presence of switching costs. This would allow calibration of the model to match the bilateral trade and business cycle correlation facts. The current model is sufficient however in paving the path for in depth empirical investigations regarding firstly, the direction of causality that explains trade intensity of final goods and obtain a positive correlation.
the features of the data more accurately and secondly, the role of TFP correlation in trade integration initiatives.

1.6 Appendix

1.6.1 Analytical solution

The value function $V(k_t; z_t)$ must satisfy

$$V(k_t; z_t) = \max_{k_{t+1}, m_t} [\ln c_t + \beta E_t V(k_{t+1}; z_{t+1})]$$

(1.6.1)

such that

$$c_t = z_t k_t^\alpha m_t^{1-\alpha} - k_{t+1} - p_m m_t, \quad (1.6.2)$$

and

$$\ln z_{t+1} = \rho \ln z_t + \epsilon_{t+1}, \quad (1.6.3)$$

where $\epsilon_t$ is an i.i.d shock.

Assume that the relation between Foreign and Home technology is

$$z_{Ft} = z_t^a$$

which implies that

$$p_{mt} = \frac{1}{z_t^a},$$

If $a > 0$, then there is positive comovement and if $a < 0$, then there is negative comovement between the two countries’ technologies.

Guess the value function,

$$V(k_t; z_t) = E + F \ln k_t + G \ln z_t.$$  

(1.6.4)
Substituting the guess in the Bellman equation gives,

\[ E + F \ln k_t + G \ln z_t = \max \left[ \ln c_t + \beta E + \beta F \ln (z_t k_t^{\alpha} m_t^{1-\alpha} - k_{t+1} - \frac{m_t}{z_t}) + G (\rho \ln z_t + \epsilon_{t+1}) \right]. \]  

The policy functions are:

\[ m_t = \left[ (1 - \alpha) z_t^{1+\alpha} k_t^{\alpha} \right]^{\frac{1}{\alpha}} \]  

\[ c_t = (1 - \beta) \alpha (1 - \alpha) \frac{1-\alpha}{\alpha} k_t z_t^{\frac{1+a(1-\alpha)}{\alpha}} \]  

\[ k_{t+1} = \alpha \beta (1 - \alpha) \frac{1-\alpha}{\alpha} k_t z_t^{\frac{1+a(1-\alpha)}{\alpha}}. \]

Substituting the policy functions and equating the coefficients:

\[ E = \frac{1}{(1 - \beta)^2} \left[ \beta \ln \beta + (1 - \beta) \ln (1 - \beta) + \ln (\alpha (1 - \alpha) \frac{1-\alpha}{\alpha}) \right] \]  

\[ F = \frac{1}{1 - \beta} \]  

\[ G = \frac{1 + a(1 - \alpha)}{\alpha (1 - \rho) (1 - \beta)} \]

The unconditional expected lifetime utility of the agent, i.e. the agent’s expected welfare prior to the realization of the first shock is:

\[ E_t V(k_t) = \text{constant} + \frac{1}{1 - \beta} E_t \ln k_t + \frac{1 + a(1 - \alpha)}{(1 - \rho) (1 - \beta) \alpha} E_t \ln z_t. \]  

\( E(\ln z_t) = 0 \) which implies that the covariance of the shocks \( a \) does not affect the decision of the agent.

**1.6.2 Simulation**

I plot the simulated series of \( corr(z, p) \) against \( \epsilon \) for specific values in the transition probability matrices. For example, starting with
Figure 1.10: Relation between $\epsilon$ and simulated series for $corr(z, p)$

\[
P(p'|p) = \begin{bmatrix}
0.9668 & 0.0332 & 0.0000 \\
0.0109 & 0.9782 & 0.0109 \\
0.0000 & 0.0332 & 0.9668 \\
\end{bmatrix}
\quad \text{and} \quad
P(z'|z) = \begin{bmatrix}
0.4 & 0.3 & 0.3 \\
0.3 & 0.4 & 0.3 \\
0.3 & 0.3 & 0.4 \\
\end{bmatrix},
\]

the relation between $\epsilon$ and $corr(z, p)$ is almost one-for-one as shown in the graph.

1.6.3 Data

1. Construction of TFP

List of countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, UK, USA

Years: 1976 - 2003

Data source: Penn World Tables

Method: The procedure follows Caselli(2005). The series for capital is constructed using the perpetual inventory method, where $g$ is the geometric growth rate between the first available year and 1980, $\alpha$ is 0.3 and $\delta$ is 0.06 as in the literature.
2. Descriptive statistics for 4.2

<table>
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<th>TFP correlation</th>
<th>Bilateral intermediate trade intensity</th>
</tr>
</thead>
<tbody>
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<td>Mean</td>
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<td>0.0021</td>
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<tr>
<td>Standard deviation</td>
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<td>0.0034</td>
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<tr>
<td>Minimum</td>
<td>−0.99</td>
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</tr>
<tr>
<td>Maximum</td>
<td>0.99</td>
<td>0.0469</td>
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Table 1.1: Data for OECD countries

3. Results of the OLS regression using TFP correlation as the independent variable.

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<tr>
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<th>TFP</th>
</tr>
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<td>0.45  (0.04)</td>
</tr>
<tr>
<td>constant</td>
<td>−7.15  (0.03)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.05</td>
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</table>

Table 1.2: Regression of trade intensity on TFP correlation
Bibliography


Chapter 2

India’s Business Cycle: Evidence and Theory

2.1 Introduction

India is one of the fastest growing emerging economies in the world today with an average quarterly GDP growth of 8.1 percent between 2004-2010. The growth is accompanied by volatility in investment, output and consumption. The Real Business Cycle (RBC) model is regarded as the workhorse in dynamic macroeconomic research. While RBC models are successfully applied to developed economies, their ability to replicate the data in developing countries is still being explored. In this paper, we make a two-fold contribution. On the empirical side, we use annual data and appropriate filtering techniques to document business cycle statistics for the Indian economy. The main findings in the data are that output, consumption and investment are more volatile than in developed economies. Like in developed countries,  

\footnote{\begin{itemize}
  \item Seasonally adjusted quarterly data is not produced by the statistical system in India. For business cycle measurement, the NIPFP-DEA program has computed seasonally adjusted quarterly GDP data from 1999.
  \item With a labor force of about half a billion, the labor market and employment fluctuations are also an important facet of India’s business cycle. However due to severe lack of data regarding the labor market, we exclude labor from the current analysis.
\end{itemize}}
consumption is less volatile and investment more volatile than output in the Indian data. Unlike in the former, investment is not highly correlated with output.

On the theory side, we test whether a standard RBC model driven by a transitory technology shock and a shock to capital income tax, and parameters calibrated for the Indian economy can replicate the stylized features of the data. Using Generalized Method of Moments (GMM), we estimate the parameters (persistence and volatility) of the technology process. In the baseline model, we assume that the government levies a tax on capital income. With only productivity shocks driving fluctuations, the model cannot capture the low correlation between investment and output movement. When there is a positive tax shock and especially if it is likely that there is a high tax rate in subsequent periods, capital becomes more expensive as the return to capital is reduced and there is less incentive to invest in future capital. But on impact of the tax shock, the movement of output is not affected. The results suggest that a high volatility of the tax shock is required to match the low investment-output correlation.

Due to the lack of data on labor force and employment, we abstract from labor movements in the model. To put our results in perspective, we calibrate the RBC model with labor and without labor movements for the US, and comment on the bias that results from excluding labor in the analysis of the model for India. We find that by fixing labor supply at unity, the results of the moments are underestimated.

The focus on business cycle models for emerging economies is relatively recent. In a seminal paper, Aguiar and Gopinath (2007) find that if the technology shock is appropriately modeled to include transitory and trend components, a standard RBC model can explain the high volatility of developing country business cycles. Neumeyer and Perri (2005) evaluate the role played by interest rates in the business cycles of emerging economies and contrast it with the role played by productivity shocks. Some authors argue that a standard RBC model might not be the most relevant theoretical benchmark to study and replicate key business cycle properties.
of all developing countries. Bergoeing and Soto (2005) incorporate characteristics of
the Chilen economy in the RBC framework and find that in addition to technology
shocks, fiscal policies and labor market rigidities constitute the main sources of the
business cycle. A major part of business cycle analysis in India relates to developing
leading and coincident indicators to facilitate forecasting of booms and recessions as
well as dating cycles. Chitre, in a series of studies, presents evidence of synchronous
movements in non-agricultural output, industrial production, capital formation and
other monetary variables. He identifies indicators of growth cycles and characterizes
the Indian economy as passing through five growth cycles of economic activity during
the period 1951−1975. Dua and Banerji (1999) adopt the NBER approach and report
six business cycle recessions from 1964 − 1997, averaging less than a year. Though
there is a history of business cycle research pertaining to India, it has been mainly
confined to descriptive investigations. In this paper, we document the empirical fea-
tures of the main macroeconomic variables and test whether the basic RBC theory
explains the stylized facts for India.

The rest of the paper is organized as follows. In Section 2, we discuss the data
and compute the business cycle statistics for the Indian economy. Section 3 presents
the RBC model, a description of the parameters used for calibration and the es-
timation of the parameters of the technology process. In Section 4, we present the
key results of the paper. Section 5 concludes and proposes extensions for future work.

2.2 Business Cycle Statistics for India

2.2.1 Methodology

A serious constraint for business cycle research in India, both empirical and theoreti-
cal, is the unavailability of long time-series data of monthly or quarterly frequency for
macroeconomic variables. We compute the statistics using the longest annual times

32
series data. The data on investment (the sum of household investment and private corporate investment) and consumption (private consumption expenditure) are extracted from the Reserve Bank of India for the period 1951 - 2008.\(^3\) Assuming a closed economy framework, total output is computed as the sum of private consumption and investment.\(^4\) We log the data and employ the technique of Hodrick-Prescott (HP) filtering to remove the long-run trend and isolate the cyclical component of the time series. Given the annual frequency of the data, the value for the smoothing parameter \(\lambda\) has to be adjusted appropriately. We follow Ravn and Uhlig (2002) and set \(\lambda = 6.25\).\(^5\)

While the value of \(\lambda = 1600\) has been shown to be a good approximation for quarterly data, for annual data, there is less agreement in the literature.\(^6\) Hence we also present results using the Band-Pass Filter (BP) based on the algorithm proposed by Baxter-King (1999). In particular, we employ the specification \(BP - K(p, q)\) where \(K = 3\) (lag length for the moving average), \(p = 2\) (shortest cycle length) and \(q = 8\) (longest cycle length) with reference to annual data.\(^7\) The two filters provide very similar results for the statistics computed.\(^8\)
Figure 2.1: Cyclical GDP, Private consumption and Private investment

2.2.2 Findings

Figure 1 shows the cyclical evolution of GDP, consumption and investment in India between 1950 and 2007. We observe that there exist several cycles with distinct mag-

The data for household and corporate investment is available at current prices. Hence we convert to real values by using the GDP deflator.

We explore alternate definitions of private investment and hence output. We use (a) corporate investment (b) household + corporate investment and (c) household + corporate investment + net exports. In this last case, the correlation between investment as measured and output is 0.61. The results are reported in the appendix.

Ravn and Uhlig (2001) provide empirical evidence for the US that the smoothing parameter should be adjusted using the fourth power of frequency change when moving from quarterly to annual data. The standard value for $\lambda$ used at quarterly frequency is 1600, so this adjustment yields a value of 6.25 for annual data. It is also close to the value of 10 recommended by Baxter and King(1999) for this frequency of data.

Backus and Kehoe (1992) use $\lambda = 100$, while Correia, Neves and Rebelo (1992) and Cooley and Ohanian (1991) suggest a value of 400 for annual data.

Dua and Banerji (2001) find that the average length of the business cycle is six years.

Baxter and King (1999) show that a value of $\lambda$ close to 10 results in a strong correspondence between the Hodrick-Prescott and the Bandpass filters.
nitude and length. Consumption moves closely with output as in developed countries. As a fraction of GDP, private consumption decreased from 85 percent in the beginning of the sample to 58 percent at the end. The share of private investment increased from 7 percent to 25 per cent from 1951 to 2009. The expansion in investment is accompanied by high volatility.

The following table provides a summary of the business cycle statistics for the Indian economy. We report three key statistics namely; amplitude of fluctuations or volatility (the percent standard deviation of the cyclical component of each series); volatility relative to that of output (the ratio of standard deviation of the given series to that of output) and comovement (the contemporaneous correlation of the cyclical component of each series with that of the other).

Using the table, we make some observations regarding the features of India’s business cycle. Private consumption displays a relative volatility of 0.8 to that of output. It is procyclical with a correlation coefficient of 0.8. Investment is 6 times as volatile as output and displays a correlation with output of 0.5.

A relevant exercise at this stage would be to compare India’s business cycle statistics to the corresponding values for the US cycle. We calculate the same statistics.
using annual data for the US for the similar time period (1950-2009). The data is extracted from the US National Income and Product Accounts. We use seasonally adjusted annual time series for personal consumption expenditure and gross private domestic investment. Output is the sum of private consumption and investment. We compute the statistics using the HP filter.\(^9\)

Consumption and investment display higher volatility in India compared to the US.\(^10\) Like in the US, private consumption in India is less volatile than output. Private consumption in India is procyclical as in the US with a correlation coefficient of 0.8. An important difference is that the correlation between consumption and investment is negative in India while it is 0.9 in the US. Moreover, the correlation of investment with output is 0.96 in the US, while it is 0.5 in India. The low correlation is investigated further.

2.2.3 Investment dynamics

Since the Indian economy went through deregulation and liberalization reforms during the latter half of this period (1950-2009), it is instructive to analyze investment data across the decades and in different sectors. Total investment (gross capital formation, constant prices) as a share of GDP (at constant prices) increased from 12 percent in the beginning of the sample to 35 percent towards the end (see Figure 2).\(^12\) Splitting up total investment into household sector, private corporate sector and public sector investment shows that prior to the reforms, the bulk of investment

\(^9\)Cooley-Hansen (1981) document the main business cycle statistics in the US using quarterly data for the time period 1955-1984. Some of the key values reported are as follows: output volatility (1.74), consumption volatility (0.81), investment volatility (8.45), correlation of consumption with output (0.65) and correlation of investment with output (0.91).

\(^10\)Others have documented this fact for developing countries. Aguiar and Gopinath (2007) find that on average emerging market economies have a business cycle twice as volatile as their developed counterparts.

\(^11\)Aguiar and Gopinath (2007) find that in emerging markets, consumption is generally more volatile than output. Only two of the thirteen countries studied by them show a ratio of consumption to output volatility of less than one. However, note that the data they use is total GDP, whereas
took place in the public sector (see Figure 3). The share of household investment in total investment has been steady at around 35 percent across the years. In contrast, the share of corporate sector investment in total investment which was roughly 14 percent till 1980, jumped to 30 percent after 1980.

A look at the relative volatility of investment to output shows that government investment was relatively stable, except in the early decades. Investment was mainly undertaken by the public sector based on five year plans and not by forward looking private firms. However, post the reforms, entry barriers were eliminated and firms had flexibility in making investment decisions. As Shah (2008) points out, in an environment of greater competition from domestic and foreign firms where profit expectations drive investment decisions, as well as exposure to financial markets, the investment by firms was highly variable. The relative volatility of corporate private investment to output was around 10 after 1970. While the correlation of household investment to output increased over the decades, the correlation of corporate investment to output remained low.

---

12The data for investment in the various sectors is obtained from the Handbook of Statistics, Reserve Bank of India. This data is available at current prices, hence it is converted to real value by using the GDP deflator.
Comparing this to the US shows that non-residential investment in the US moves very closely with output and the correlation has been increasing over the decades (in
the period 1990-2010, the correlation is 0.94).

2.3 Model

In this section we present the standard real business cycle model that incorporates a productivity shock as well as a capital tax shock. The economy consists of a representative household that has preferences over the sequence of consumption $C_t$ described by

$$U_t = E_t \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma}$$

(2.3.1)

where $\beta \in (0, 1)$ is the time discount factor and $1/\gamma$ is the intertemporal elasticity of substitution.

The budget constraint is given by

$$C_t + K_t - (1 - \delta)K_{t-1} = r_t K_t + w_t + T_t - \tau_t (r_t - \delta) K_t$$

(2.3.2)

where $\delta \in (0, 1)$ is the rate of depreciation and $C_t \geq 0$. $r_t$ and $w_t$ denote the rental rate of capital and wages respectively. We assume that in every period, agents supply one unit of labor inelastically. The government levies a tax on capital income $(\tau_t)$ and redistributes the tax revenue as a lump sum transfer $(T_t)$ to the households. The tax allows a depreciation deduction.

The government balances its budget every period which means that it transfers the tax revenue lump sum to the household. Thus,

$$T_t = \tau_t (r_t - \delta) K_t.$$  

(2.3.3)

The representative firm faces the standard profit maximization problem and produces output $Y_t$ using capital $K_t$ according to the Cobb-Douglas production function,

$$Y_t = A_t K_t^{1-\alpha} (\mu)^\alpha$$

(2.3.4)
where $1 - \alpha$ is the share of capital in production and $\mu_t$ represents the deterministic trend. Thus,

$$r_t = (1 - \alpha) \frac{Y_t}{K_t} \quad (2.3.5)$$

and

$$w_t = \alpha Y_t \quad (2.3.6)$$

The transitory productivity shock and the tax shock are assumed to follow AR(1) processes in logs:

$$\log A_{t+1} = \rho_a \log A_t + \epsilon_{a_{t+1}}. \quad (2.3.7)$$

$$\log \tau_{t+1} = \rho_\tau \log \tau_t + \epsilon_{\tau_{t+1}}. \quad (2.3.8)$$

A competitive equilibrium is defined as a set of stochastic processes for $a_t$ and $\tau_t$ as well as quantities $C_t$ and $K_t$ that solves the consumer’s and firm’s decision problems.

Note that in order to explain the dynamics of investment and output as well as their low correlation in the data, it is essential that the two shocks in the model occur simultaneously but are uncorrelated with each other. A positive productivity shock will, on impact, cause output, consumption and investment to jump up. A positive tax shock will lead to lower expected return on future capital, thereby dampening the incentive to invest. If both the shocks are highly correlated, then the favourable effect on investment as a result of the productivity shock will be offset by the adverse effect due to the tax shock, leading to a relatively stable path for investment across time. However, this is in contrast with what we observe in the data, where investment displays considerable volatility.

2.3.1 Choice of Parameter Values

We use the model equations and the relevant data series to calibrate the key parameters of the model which are:
\( \beta \): The Euler condition for consumption yields \( \beta = \frac{1}{1+r} \). The annualized average of the real interest rate in India from 1978 - 2000 is roughly 2 percent, resulting in a value for \( \beta = 0.98 \).

\( \delta \): The capital stock series for India is obtained using the standard perpetual inventory method. The depreciation rate is computed by regressing the depreciation series on the capital stock resulting in \( \delta = 4.5 \) percent. This is close to the value for depreciation rate assumed in the literature using annual data for India (Virmani, 2004), which is about 5 percent.

\( \alpha \): For the share of capital, we follow Virmani (2004) and set \( 1 - \alpha = 0.3 \).

\( \gamma \): We follow the literature and set the inter-temporal elasticity of substitution, \( \gamma = 2 \).

\( \rho_a \) and \( \sigma_a \): We follow NIPFP that uses the accounting method of Verma (2010) and extends the dataset to 2008 to compute the TFP series for India. They estimate the parameters of an AR(1) process as 0.92 and 0.005 respectively for persistence and standard deviation respectively.

In addition, we estimate the parameters governing the amplitude and persistence of the technology shock process in a basic model. In particular, we estimate \( \rho_a \) and \( \sigma_a \) by applying the generalized method of moments (GMM) using annual data on output, consumption and investment from India. We follow Cicco-Garcia, Pancrazi and Uribe (2009) and include 11 moment conditions: the standard deviations of detrended output, consumption and investment, the correlation of output with consumption and investment and the first and second order autocorrelations of output, consumption and investment. The sample period is 1951 – 2009. The estimated parameters are 0.9 and 0.05.
The parameters are listed in the table below.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 - \alpha$</td>
<td>0.3</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.98</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.05</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>0.92</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Table 2.2: Parameters

$\rho_\tau$ and $\sigma_\tau$: We vary the parameters for the persistence and standard deviation of the shock process and report the investment output correlation statistic.

<table>
<thead>
<tr>
<th>Std dev</th>
<th>$\rho_\tau$</th>
<th>0.5</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.88</td>
<td>0.87</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.87</td>
<td>0.58</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.3: Correlation between output and investment

2.4 Results

The table below reports the moments from the model and compares it to the data moments reported earlier. The values of the parameters for the tax shock process are chosen so as to match the investment-output correlation in the data (i.e. around 0.5). We then check whether the tax series exhibits these properties.

The results show that business cycle models can replicate observed fluctuations of the economy. A model with a capital income tax shock is able to match the low investment-output correlation as seen in the data. However, it produces higher consumption volatility, lower investment volatility and a more negative correlation between investment and output than in the data. While the model with a tax shock
Table 2.4: Business Cycle Statistics - Data and Model

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Volatility</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>1.72</td>
<td>1.8</td>
</tr>
<tr>
<td>PC</td>
<td>1.39</td>
<td>2.8</td>
</tr>
<tr>
<td>PI</td>
<td>9.61</td>
<td>4.3</td>
</tr>
<tr>
<td><strong>Relative Volatility</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>1.00</td>
<td>1.0</td>
</tr>
<tr>
<td>PC</td>
<td>0.81</td>
<td>1.5</td>
</tr>
<tr>
<td>PI</td>
<td>5.59</td>
<td>2.4</td>
</tr>
<tr>
<td><strong>Correlation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>PC, Output</td>
<td>0.76</td>
<td>0.50</td>
</tr>
<tr>
<td>PI, Output</td>
<td>0.55</td>
<td>0.58</td>
</tr>
<tr>
<td>PC, PI</td>
<td>-0.05</td>
<td>-0.46</td>
</tr>
</tbody>
</table>

does not replicate all the features of the data, it performs better than a basic model with only a TFP shock, especially by replicating the negative correlation between consumption and investment.

**2.4.1 Tax rate - what does the data show?**

The data on capital income tax rate is not available. We take the corporate tax reported by about 6500 companies listed on the Stock Exchanges in India for the years 1995 to 2010. This is divided by the profits before tax to obtain the effective tax rate.\(^{13}\) The estimates for the AR(1) process are based on a specification of the tax rates in levels. The choice seems consistent with the outcome of the DFGLS test applied to the series. The DFGLS test has the best performance in a small sample. The test applied to the series rejects the null of a unit root at the 5 per cent significant level. The modified AIC is a powerful lag selection criterion and picks the lag of 1, suggesting an AR(1) process. This is complemented by the KPSS test, where we cannot reject the null of stationarity at the 5 per cent significance level. By fitting

\(^{13}\)The data is obtained from the Centre for Monitoring the Indian Economy
the data to an AR(1) model, we obtain the persistence and standard deviation as 0.6 and 0.31 respectively.

Further, we check whether the data and the model are similar by computing the correlations of the tax series with output, consumption and investment and comparing them to results obtained from the model.\textsuperscript{14} Two out of three model correlations have the same sign as in the data. In general, the tax series shows high volatility and persistence as obtained in the model. With regard to correlation, we obtain mixed results.

\begin{table}[h]
\centering
\begin{tabular}{lcc}
\hline
Correlation & Data & Model \\
\hline
\text{Y, tax rate} & $-0.44$ & 0.25 \\
\text{C, tax rate} & $-0.50$ & $-0.59$ \\
\text{I, tax rate} & 0.17 & 0.79 \\
\hline
\end{tabular}
\caption{Statistics of the tax rate}
\end{table}

\textsuperscript{14}Since DSGE models have been tested mainly for their ability to match the second moments of the data, we compute the correlations between the tax series and the other macroeconomic variables, rather than estimate the nature of the shocks.
2.5 A comment on excluding labor supply movements from the model

Due to the severe lack of data on employment and wages, we assume that labor is inelastically supplied in the model. This means that we are abstracting from the choice between consumption and leisure that agents make when they face aggregate shocks. This could bias the results of the moments computed based on the current model. We try to understand the direction and extent of the bias by calibrating a model with variable labor and with fixed labor (one unit inelastically supplied) for US parameters and comparing the two sets of moments.

In the first model, the economy consists of a representative household that has preferences over the sequence of consumption $C_t$ described by

$$U_t = E_t \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma}$$

(2.5.1)

where $\beta \in (0, 1)$ is the time discount factor and $\gamma$ is the intertemporal elasticity of substitution.

The budget constraint is given by

$$C_t + K_t - (1 - \delta)K_{t-1} = Y_t$$

(2.5.2)

where $\delta \in (0, 1)$ is the rate of depreciation. Output is produced using capital according to a Cobb-Douglas production function,

$$Y_t = (z_t)^{\alpha} K_t^{1-\alpha}$$

(2.5.3)

where $z_t$ is the technology shock which follows an AR(1) process.

In the second model, the agent has preferences over consumption and labor given
by

$$U_t = E_t \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma} - \eta \frac{l_t^{1+\phi}}{1+\phi}$$ (2.5.4)

where $\frac{1}{\phi}$ is the Frisch elasticity of labor supply and $\eta$ is the share of labor in utility. The production function is

$$Y_t = (z_tL_t)^{\alpha}K_t^{1-\alpha}$$ (2.5.5)

The parameters used in the calibration are given in the table. The model moments reported for output, consumption and investment are the standard deviation, the standard deviation relative to that of output and the correlation with output. The results show that with inelastic labor, the volatility of output, consumption and investment are reduced. For example, in terms of moments, the standard deviation of output falls from 5.9 percent to 4.7 percent when the labor supply elasticity goes from 0.3 to zero. In a model with elastic labor, both the substitution and income effects of a wage change play a role in moving consumption, since the agent can choose between consumption and leisure. Flexible labor supply provides an extra margin of adjustment when analyzing the dynamic response of key macroeconomic variables to a productivity shock. In addition, the variation in output due to a transitory technology shock can be decomposed into variation in labor supply and changes in capital stock and the latter are of relatively smaller magnitude in the short run. So when the labor supply is fixed, we shut down the first channel thereby dampening
Table 2.7: Moments from the model

<table>
<thead>
<tr>
<th>Volatility</th>
<th>Model with $L$</th>
<th>Model with $L = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$ 0.059</td>
<td>0.047</td>
<td></td>
</tr>
<tr>
<td>$C$ 0.051</td>
<td>0.039</td>
<td></td>
</tr>
<tr>
<td>$I$ 0.097</td>
<td>0.077</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Relative volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$ 1.00</td>
</tr>
<tr>
<td>$C$ 0.86</td>
</tr>
<tr>
<td>$I$ 1.64</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$ 1.00</td>
</tr>
<tr>
<td>$C$ 0.99</td>
</tr>
<tr>
<td>$I$ 0.96</td>
</tr>
</tbody>
</table>

the overall effect and reducing output volatility. King and Rebelo(1999) also show a reduction in the standard deviation of output and labor when the elasticity is reduced. Thus we can conclude that the results obtained in the model described in the previous section, when labor supply is assumed to be inelastic, underestimate the volatilities.

2.6 Conclusion

In this paper, we document the features of the business cycle in India using annual data and appropriate filtering techniques. A key feature is that investment and output are not highly correlated over the cycle, and the correlation of consumption and investment is negative, unlike in developed countries. In the model we include a tax on capital income which acts as a disincentive for future investment, and the results show that high volatility of the tax rate shock is required to produce the low investment output correlation. While the model does not replicate all the features of the data, it does better than a model with only productivity shocks.

The data on tax series is not very reliable, but we use the available data and find that the volatility of corporate taxes is high. However, the correlations between tax
rate and the macroeconomic variables provide mixed results. A better series is required to improve our understanding of the corporate tax in India.

We also find that by excluding labor movements from the model, our results underestimate the true volatilities.

Extensions to the model include introducing a government that consumes and invests, and also opening the economy to reflect the features of the Indian economy.

2.7 Appendix

2.7.1 Alternate measures of private investment

<table>
<thead>
<tr>
<th>Volatility</th>
<th>Corporate inv</th>
<th>Household + corporate</th>
<th>Household + corporate + Net exports</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>1.62</td>
<td>1.72</td>
<td>1.97</td>
</tr>
<tr>
<td>C</td>
<td>1.38</td>
<td>1.39</td>
<td>1.39</td>
</tr>
<tr>
<td>I</td>
<td>32.78</td>
<td>9.6</td>
<td>15.72</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Relative volatility</th>
<th>Corporate inv</th>
<th>Household + corporate</th>
<th>Household + corporate + Net exports</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>C</td>
<td>0.86</td>
<td>0.81</td>
<td>0.44</td>
</tr>
<tr>
<td>I</td>
<td>20.28</td>
<td>5.59</td>
<td>8.14</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Corporate inv</th>
<th>Household + corporate</th>
<th>Household + corporate + Net exports</th>
</tr>
</thead>
<tbody>
<tr>
<td>C,Y</td>
<td>0.80</td>
<td>0.76</td>
<td>0.74</td>
</tr>
<tr>
<td>I,Y</td>
<td>0.27</td>
<td>0.55</td>
<td>0.60</td>
</tr>
<tr>
<td>C,I</td>
<td>−0.14</td>
<td>−0.05</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Table 2.8:
2.7.2 The model’s solution

Optimality conditions of the Household’s problem

Assuming that the variables grow at the constant rate \( \mu \), we make the variables stationary. Define \( y_t = Y_t / X_{t-1} \), \( c_t = C_t / X_{t-1} \), \( k_t = K_t / X_{t-1} \), \( i_t = I_t / X_{t-1} \), \( tr_t = T_t / X_{t-1} \) and \( g = G_t / X_{t-1} \). Thus the constraints and the first order conditions are:

\[
y_t = A_t k_t^{1-\alpha} \mu^\alpha \quad (2.7.1)
\]

\[
\mu k_{t+1} - (1 - \delta) k_t = r_t k_t + w_t + tr_t - \tau_t (r_t - \delta) k_t - c_t \quad (2.7.2)
\]

\[
tr_t = \tau_t (r_t - \delta) k_t - g \quad (2.7.3)
\]

\[
(\frac{\mu}{c_t})^\gamma = \beta E_t \frac{1}{c_{t+1}} \left[ r_{t+1} - \tau_{t+1} (r_{t+1} - \delta) + 1 - \delta \right] \quad (2.7.4)
\]

**Steady State**

\[
\frac{y}{k} = \frac{1}{(1-\alpha)(1-\tau)} \left( \frac{\mu}{\beta} - \delta (\tau - 1) - 1 \right) \quad (2.7.5)
\]

\[
k = \mu \left( \frac{y}{k} \right)^{-1/\alpha} \quad (2.7.6)
\]

\[
\frac{c}{k} = (1 - \tau (1 - \alpha)) \frac{y}{k} + \frac{tr}{k} + \delta (\tau - 1) - \mu + 1 \quad (2.7.7)
\]

\[
\frac{tr}{k} = \tau (1-\alpha) \frac{y}{k} - \delta \tau \quad (2.7.8)
\]

\[
\frac{i}{k} = \mu - 1 + \delta \quad (2.7.9)
\]
2.7.3 GMM Estimation Procedure

Let $\theta = [\rho_g, \rho_z, \sigma_g, \sigma_z]'$ be the $4 \times 1$ vector of structural parameters to be estimated. The moment conditions are written as:

$u_t(\theta) = \begin{bmatrix}
\sigma_y(\theta) - (y_t - \overline{y})^2 \\
\sigma_c(\theta) - (c_t - \overline{c})^2 \\
\sigma_i(\theta) - (i_t - \overline{i})^2 \\
\rho_{y,c}(\theta) - \frac{(y_t - \overline{y})(c_t - \overline{c})}{\sigma_y(\theta)\sigma_c(\theta)} \\
\rho_{y,i}(\theta) - \frac{(y_t - \overline{y})(i_t - \overline{i})}{\sigma_y(\theta)\sigma_i(\theta)} \\
\rho_{y_1}(\theta) - \frac{(y_t - \overline{y})(y_{t-1} - \overline{y})}{\sigma_y^2(\theta)} \\
\rho_{y_2}(\theta) - \frac{(y_t - \overline{y})(y_{t-2} - \overline{y})}{\sigma_y^2(\theta)} \\
\rho_{c_1}(\theta) - \frac{(c_t - \overline{c})(c_{t-1} - \overline{c})}{\sigma_c^2(\theta)} \\
\rho_{c_2}(\theta) - \frac{(c_t - \overline{c})(c_{t-2} - \overline{c})}{\sigma_c^2(\theta)} \\
\rho_{i_1}(\theta) - \frac{(i_t - \overline{i})(i_{t-1} - \overline{i})}{\sigma_i^2(\theta)} \\
\rho_{i_2}(\theta) - \frac{(i_t - \overline{i})(i_{t-2} - \overline{i})}{\sigma_i^2(\theta)}
\end{bmatrix}$

where $\sigma_x(\theta)$, $\rho_{xy}(\theta)$ and $\rho_{xz}(\theta)$ denote the standard deviation of $x_t$, the correlation between $x_t$ and $y_t$ and the autocorrelation of order $j$ of $x_t$ respectively, implied by the theoretical model. These are functions of the vector $\theta$ of structural parameters. We compute the moments implied by the theoretical model by solving a log-linearized system of equilibrium conditions. Define $Q = u'Wu$, where $u(\theta)$ denotes moment conditions and $W$ is a symmetric positive definite matrix. The GMM estimate of $\theta$ denoted by $\hat{\theta}$ is given by $\hat{\theta} = argmin_\theta Q(\theta, W)$. Since the number of moment conditions exceed the number of estimated parameters, the weighting matrix $W$ is updated optimally.
Bibliography


