Regulation of transmission line capacity and reliability in electric networks

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REGULATION OF TRANSMISSION LINE CAPACITY AND
RELIABILITY IN ELECTRIC NETWORKS

a dissertation

by

METIN CELEBI

submitted in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy

September 2000
The thesis of: Metin Celebi

entitled: REGULATION OF LINE CAPACITY AND

RELIABILITY IN ELECTRIC NETWORKS

submitted to the Department of: Economics

in partial fulfillment of the requirements for the degree of:

Doctor of Philosophy

in the Graduate School of Arts and Sciences has been read and
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ABSTRACT

This thesis is composed of two essays that analyze the incentives and optimal regulation of a monopolist line owner in providing capacity and reliability. Similar analyses in the economic literature resulted in under-investment by an unregulated line owner when line reliability was treated as an exogenous variable. However, reliability should be chosen on the basis of economic principles as well, taking into account not only engineering principles but also the preferences of electricity users.

When reliability is treated as a choice variable, both over- and under-investment by the line owner becomes possible. The result depends on the cross-cost elasticity of line construction and on the interval in which the optimal choices of capacity take place. We present some sufficient conditions that leads to definite results about the incentives of the line owner.

We also characterize the optimal regulation of the line owner under incomplete information. Our analysis shows that the existence of a line is justified for the social planner when the reliability of other lines on the network is not too high, or when the marginal cost of generation at the expensive generating plant is high. The expectation of higher demand in the future makes the regulator less likely to build the line if it will be congested and reliability of other lines is high enough. It is always optimal to have a congested line under complete information, but not necessarily under incomplete information.
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ACKNOWLEDGMENTS

It is a pleasure to thank those who helped through the evolution of this thesis and during my years in the graduate program. I would like to thank the members of my thesis committee, namely Frank Gollop, Richard Arnott, and Marvin Kraus. I am indebted to them for their support during difficult times and for their valuable suggestions and corrections. Of course the responsibility of any possible errors left in the text remains with me.

I would like to thank Christopher Baum for his always generous help in my endless questions about various software and the computer systems. Thanks are also due to Phil Hanser and Thomas-Olivier Nasser for very useful discussions with them. I would like to thank the organizers and the participants of dissertation seminars at Boston College for their valuable questions and comments that kept me improving on the previous drafts of this thesis.

The last but not the least, I am indebted to my family in Turkey for their continuous support in achieving my goals, even though this meant being separated by the Atlantic Ocean for the last five years.
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1. INTRODUCTION

Electric power is a vital input to production and commercial sectors, and plays an important role in our lives with its uses in lighting, air-conditioning, computers and in many other household activities. Electricity sectors around the world account for more than $1 trillion of annual sales revenue, and more than $200 billion in annual investment.¹

Many countries started restructuring their electricity sectors in the last decade to improve competition and to obtain benefits from inter-regional trading. Some countries including the U.S. opened the generation stage to competition, which is no longer believed to be a natural monopoly. But the transmission and the distribution stages still possess the characteristic of a natural monopoly, therefore they are still being regulated.

In the U.S., regulators compensate transmission line owners on a cost-recovery basis, instead of incentive-based or value-based schemes. However, this picture is changing too. First of all, users of transmission networks in some areas (such as New York, New England, or the Pennsylvania-New Jersey-Maryland (PJM)) are being charged on the basis of congestion pricing (a transmission pricing

¹See Joskow [11], p. 25.
mechanism in which users pay for the cost of congestion they create by their transactions). Second, some attempts are being made to change the structure of ownership and control of transmission assets. In February 2000, the Federal Energy Regulatory Commission (FERC) issued an order proposing the formation of Regional Transmission Organizations (RTOs) that will be responsible for operation and planning (investment) of transmission assets. (FERC [9]) Currently, about 35% of the U.S. electricity markets are governed by various versions of RTOs.\(^2\) The basic idea behind the FERC order is to consolidate the operation (and possibly ownership) of transmission assets so that strong externalities in transmission networks can be internalized, and also to stimulate investment in transmission. According to FERC Chairman James Hoecker, the objective is to give new RTOs the necessary incentives to expand transmission to meet a growing market.

Leaving the decisions to invest in transmission networks to RTOs boils down to delegating the investment decisions to a coalition of the owners of transmission assets. The natural alternative to this decentralized mechanism is the regulation of these decisions. This thesis aims to analyze the comparison of these two approaches. The first paper in the thesis considers a setting in which the

transmission pricing of electricity is done by using a locational pricing model, nodal pricing. The transmission prices obtained in this model reflects the true marginal costs of congestion, hence they are efficient. These optimal transmission prices can be used to find the shadow value of transmission capacity for any line on the network. Since the shadow value represents the marginal value to society of transmission capacity, it can be used to reward the owners of transmission lines. We consider such a reward mechanism for providing capacity in order to analyze the incentives of a line owner to invest in line capacity and line reliability in chapter 2.

The third chapter characterizes the optimal regulation of line capacity and reliability for a single transmission line under incomplete information. This paper too uses the nodal pricing model to obtain the shadow value of a line. We also present some results showing the effect of changes in electricity demand, generation cost, and reliability of other lines on the regulator's decision to build a new line.

The conclusion chapter gives a summary of results in the thesis, and provides some policy recommendations.
2. INCENTIVES

AN ANALYSIS OF INCENTIVES TO PROVIDE LINE CAPACITY AND RELIABILITY IN DeregULATED POWER NETWORKS

2.1. Introduction

Electricity markets in many countries are going through a transformation which mostly takes the form of deregulation. The process has started with opening the generation stage to competition so that market forces could operate in this segment which is no longer believed to be a natural monopoly. The next step is the re-configuration of the transmission segment which still possesses the characteristics of a natural monopoly. The current issues are who should operate an existing transmission network and who should decide on the amount of investment in the network. We will concentrate on the latter issue, and examine the incentives to provide transmission capacity and reliability by using a specific mechanism in the operation of those transmission assets.
Reliability in the delivery of electric power used to be seen as an exogenous variable by planners. It was determined using engineering rules, which were viewed as constraints by the decision-making authority. However, reliability should be chosen on the basis of economic principles as well, taking into account not only engineering principles but also the preferences of electricity users. Reliability is truly an endogenous variable. The materials and technology used in the construction of transmission lines, the level of maintenance expenditures, and the transmission capacity of lines constitute the major determinants of the reliability for the whole network.

3 The only exception that I am aware of is an experimental project by Duke Power Company. This project aims to perform planning activities by taking into account the value of reliability to their customers. See Dalton et al [7] for details.

4 See Munasinghe [15], p. 24 for a more detailed treatment of the current approach to reliability. An example of these exogenous constraints is the N-1 rule, or first contingency criterion. It requires that if one component fails, the others must be sufficient to serve the demand. Another example is the practice (in Brazil) of putting a direct constraint on reliability, saying that demand must be served 95% of the time. See Dalton et. al. [7] p. 1400, and Nasser [16] p. 38.

5 I will only mention two examples. First, either rod gaps or surge arresters can be used to protect transformers, reactors, and other components from overvoltages; rod gaps are cheaper, but also less reliable. Second, transmission line poles can be made from wood, steel or concrete; wood is cheaper but can be less reliable. (From the Encyclopedia of Physical Science and Technology [8])

6 For example, a new technology called FACTS allows the operator to automatically redistribute flow when the power flowing on a line reaches its limit. Such a technology enables the system operator to prevent line breakdowns and repair costs due to overflows. Installing FACTS definitely improves reliability, but at an extra cost.

7 The formation of reliability coordinating councils (e.g. North American Electric Reliability Council, NERC) by electric utilities was motivated by the need for operating and design standards for the industry. The existence of these institutions indicates that investors have a multitude of options affecting the reliability of the system. Even under this coordinated structure of the industry, electric utilities do not always conform to those standards. As a senior transmission planner said: "We have a line I would like to monitor, but I do not think we
The model described in this paper investigates the incentives of a monopolistic line owner choosing both reliability and capacity of a transmission line subject to nodal pricing of electricity. I will compare these capacity and reliability levels to the ones chosen by a benevolent social planner. Holding reliability constant, the line owner is likely to choose under-investment to increase congestion rents. Does this result generalize when reliability of the line is also a choice variable? If not, then we could find out conditions under which the line owner chooses the socially optimum level of capacity. That would imply the regulation of line capacity is unnecessary under those conditions, and regulation of reliability would be enough. Moreover, over-investment in line capacity would be a possible outcome.

In this paper I present some conditions each of which imply different results for the comparison of investment by the social planner and the line owner. One of the critical conditions is on cross-cost elasticity of line construction, which basically measures the sensitivity of marginal cost of capacity to an increase in reliability. If marginal cost of capacity is not very sensitive to a marginal increase in reliability (i.e. cross-cost elasticity less than unity) and if optimal

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capacities fall into a specific interval, then the line owner chooses too much capacity and too much reliability compared to the socially optimal levels. But we obtain under-investment in capacity if the elasticity is greater than unity. Another of the results show that the line owner chooses the first-best levels of capacity and reliability if capacity choices fall into a specific interval.

The plan of the paper is as follows. I will describe the institutional setting and the technology of electricity transmission in the next two sections. The third section shows how spot pricing of electricity is done and explains the shadow value of line capacity, which will be used in the analysis of investment. The investment and reliability problems of the social planner and line owner are then presented in section four. The analysis of conditions that yield specific results regarding the choices of the line owner are shown in section five, which is followed by the conclusion.

2.2. Institutional Set-up

Since the focus of the study will be on long-term decisions of investment and reliability, I will ignore the imperfections in the spot pricing of electricity. I aim to highlight the incentives for investment and reliability, rather than analyzing the interactions between the market imperfections in the spot pricing and the
imperfections in investment and reliability choice.

The spot pricing of electricity is assumed to follow nodal pricing as introduced by Schweppe et. al. [18] and used by Hogan [10]. A system operator organizes auctions in which demand and supply bids of users yield the price of electricity at each node of the network. Nodal pricing model in various forms not only is the present paradigm of choice of policy makers (as in PJM (Pennsylvania-New Jersey-Maryland network), NEPOOL (New England) and NYPP (New York)) but its optimality relies on far less restrictive assumptions than its principal rival. The basic assumption for the social efficiency of the nodal pricing model is absence of market power among users of electricity, while the decentralized market model additionally requires the absence of market power among the owners of transmission lines.

For the structure of ownership and control rights of the transmission network, I argue that the line owner should be regulated. Economies of scale in line construction and highly concentrated ownership patterns in most countries make this a more plausible alternative than the decentralized investment systems.

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9 See Schweppe et. al. [18], pp. 151-156 for their nodal pricing model.
10 The alternative is a decentralized pricing system, such as the tradable physical rights approach by Chao and Peck [6].
11 A few utility companies jointly own most of the regional transmission networks in many states of the U.S. In the U.K, there is a single transmission company, called the National Grid Company (NGC).
proposed in the literature. The most prominent model for the latter (by Bushnell and Stoft [4]) uses the instrument of Transmission Congestion Contracts (TCC)\textsuperscript{12} in order to give the right incentives to line owners for investment. Furthermore, Bushnell and Stoft consider only a mechanism for the provision of line capacity, hence investment on reliability is ignored. However, we show in this paper that decisions to provide capacity and reliability do in general affect each other. Therefore, we take the position of regulating both line capacity and reliability.

The regulation of a single line owner was hinted at by Bushnell and Stoft [4]\textsuperscript{13}, and was analyzed by Nasser [16]. The latter considered the dimension of line capacity, but ignored the choice of reliability.

This paper presents the behavior of the line owner in an unregulated environment, to highlight the distortion in incentives. The next step is to examine and formulate the optimal regulation, which I have studied in Chapter 3.

\textsuperscript{12}TCC's were introduced by Hogan [10]. A TCC is defined for a pair of nodes \((i,j)\), and gives its owner a prespecified quantity multiplied by the difference in prices at node \(i\) and \(j\). Nodes specified in the contract are not necessarily connected in physical terms. Hence, a TCC is essentially a financial instrument rather than a certificate to have the right to transfer electric power from node \(i\) to \(j\).

A variant of this approach will be used by New York Power Authority (NYPA). Investors who pay for transmission expansion will receive TCC's as compensation.

\textsuperscript{13}They said, "Transmission assets could be combined into a single entity that would have to be closely regulated due to its substantial market power." (See page 90 in Bushnell and Stoft [4])
The choice of line reliability determines the probability of being in one of two states of the world: either the line works or it fails. I assume that the system operator and users learn the state of the world before the auction for spot pricing is organized. However, investment and reliability decisions are made before the state of the world is known. This is the natural way of timing, because investment decisions are generally made much less frequently than pricing decisions. Hence, the timing of decisions is as follows:

(1) The firm chooses the capacity and reliability of line.

(2) The state of the world becomes known to everyone.

(3) The system operator chooses the dispatch, and calculates the shadow value of line.

I assume perfect competition among users (or bidders) of electricity in auctions that determine spot prices; hence, pricing is done optimally. Optimal prices yield the shadow value of the line, and I assume the line owner gets payment on the basis of shadow value. The shadow value becomes the price of line capacity, or payment per unit of capacity installed on the transmission line. Nasser [16] rightly named this as Natural Transmission Revenue, because shadow value

\[14\text{In practice, auctions are held hourly, but investment decisions take place once in a couple of years.}\]
represents the true value of the marginal unit of capacity to society.

We are aware that rewarding the line owner by shadow value is not the only possible mechanism. For example, a fixed payment or some kind of reimbursement for the cost of line construction are two of many possible reward mechanisms. However, rewarding by shadow value has a nice property that the shadow value represents society's marginal willingness to pay for line capacity. Hence, it constitutes the demand curve for capacity. This is why we chose this mechanism to analyze incentives of the line owner.

I assume that the system operator does not pay anything to the line owner when the line fails, because the reliability of the line is totally controlled by the owner.

Before proceeding to the results, I will present the transmission technology in a simplified network model in the next section.

2.3. Electricity Transmission Technology

The technology of flows on electric power networks is different from the railroad and communications networks. There are three major differences. First, a supplier cannot address the generated power to a specific customer on the network, because power networks are common pools. Second, the path of power flows
cannot be determined by users, the power chooses its own path according to the laws of physics. Finally, storage of electricity is very costly, hence some form of central coordination is needed to equate supply and demand and to make sure that standards related to voltage, frequency and security are satisfied.

Electricity flows according to the laws of physics (or Kirchoff Laws\textsuperscript{15}), not according to pre-specified contract paths. For example, say a utility company in Boston has a contract to sell electricity to a distributor in Michigan. One might think that the power generated in Boston would flow directly on the transmission line(s) connecting Boston and Michigan. But the reality is not that simple: only a portion of the generated power will flow on the direct link, and the remaining part will flow on other paths (possibly from Boston to New York, and then to somewhere else, and finally to Michigan). This is called the "loop-flow" phenomenon. Division of power among different paths depends on relative impedances (a measure of resistance and reactance) of lines. More power flows on lines that have less impedance.

A major consequence of the loop-flow phenomenon emerges when the network is congested, i.e. one or more of transmission lines have reached their capacity to carry power. In this case, generators located at some specific areas

\textsuperscript{15}See the appendix for power-flow technology for a more detailed analysis of Kirchoff Laws.
of the network cannot put power into the network anymore (otherwise transmission assets will be damaged). These generators are located in areas that contribute to the congestion. At the same time, there are some other generators on the network such that their location allows them to alleviate this congestion. Roughly speaking, the first set of generators can be said to create negative externalities because they are limiting the ability of other generators/consumers to use the network. Similarly, the second set of generators can be said to create positive externalities because they are enhancing the ability of some users to transmit electricity by alleviating the transmission congestion.

Now let's illustrate this discussion on the triangular network as shown in Figure 1. Suppose all lines have identical impedances. Then 1/3 of power injected at node 1 will flow on lines (1, 2) and (2, 3), and the remaining 2/3 of it will flow on line (1, 3) (since this path has half the impedance of the first path, which is composed of two serially connected lines). Similarly, 1/3 of power injected at node 2 will flow on lines (1, 2) and (1, 3), and the remaining 2/3 of it will flow on line (2, 3). A derivation of these rules for power flows by using Kirchoff Laws is given in the appendix in Chapter 5.
Figure 1
If one of the lines fail, then the network is no longer triangular, and distribution of power flows will be different. If line (1, 2) fails, then all power generated at node 1 will flow on line (1, 3), and all power generated at node 2 will flow on line (2, 3). If line (1, 3) fails, then all power generated at node 1 will flow on lines (1, 2) and (2, 3), and all power generated at node 2 will flow on line (2, 3).

The last issue that we will illustrate is the congestion management. A system operator solves the congestion problem by switching some of the generation from the congestion-causing generators to congestion-relieving generators. When line (1, 2) is congested and line (1, 3) is up, then we can no longer generate electricity at node 1 (cheap generator) without also generating at node 2 (expensive generator). The optimal way to satisfy an additional unit of demand (for a given level of capacity) is to generate 1/2 more unit at node 1, and 1/2 more unit at node 2. In other words, a unit increase in the capacity of line (1, 2) allows us to generate 3/2 more units at node 1, and 3/2 units less at node 2.\(^\text{16}\)

Similarly, when line (1, 2) is congested and line (1, 3) is down, then a unit increase in the capacity of line (1, 2) allows us to generate one more unit at node 1 and one less unit at node 2.

\(^{16}\)The number 3/2 comes from the fact that generating 3/2 more units at node 1 implies 1/2 more unit of power flowing from node 1 to node 2, and 3/2 less units generated at node 2 implies 1/2 less unit of power flowing from node 2 to node 1. The net flow from node 1 to node 2 becomes one unit, which is the initial increase in the capacity of line (1, 2).
2.3.1. A Simple Model of Electric Networks

The model used in this analysis will be the DC load approximation to the power flow with no line loss. The network will be assumed to have three nodes, which are connected by identical transmission lines in a triangular shape (see Figure 1). Voltage levels throughout the network are assumed to be held constant.

Nodes 1 and 2 are generator nodes, while node 3 is the demand node. Generation (or injection) by generator 1 and generator 2 are labeled $q_1$ and $q_2$, respectively, while the consumption at node 3 is labeled $q_3$. For illustrative purposes, assume that generators have constant marginal cost production technologies with $c_1$ and $c_2$, respectively, and $c_1 < c_2$. Assume the inverse demand function at node 3 is $p_3(q_3) = a - bq_3$, with the implied utility function of $B(q_3) = \left(a - \frac{b}{2}q_3\right)q_3$.\(^{17}\)

Power flowing on line $(i, j)$ from node $i$ to node $j$ is $P_{ij}$. Assume that only line $(1, 2)$, with a line capacity of $K$, has the possibility of congestion.\(^{18}\) Moreover, the reliability\(^{19}\) of line $(1, 2)$ is a decision variable to be chosen at the same time as the line is constructed. The reliability of line $(1, 2)$ will be denoted by

\(^{17}\)The unit of $q_i$'s and $K$ is MW (megawatt), while the unit of $c_i$'s and $p_3$ is \$/MW (dollars per megawatt).

\(^{18}\)A line is congested when the power flowing on the line is just equal to the line capacity

\(^{19}\)Reliability of a line is defined as the probability of the line being 'up'. The line is either 'up' or 'down'. The electricity cannot flow over a 'down' line.
The line connecting nodes 1 and 3 can also fail, but its reliability \( x_{13} \) is exogenously given.\(^{20}\)

### 2.4. The Socially Optimal Dispatch (Spot Pricing)

I start solving the problem backward in time. The problem of the system operator is to choose the dispatch to maximize social surplus in each state of the world, subject to power flow equations and the line constraint. Let’s denote the state of the world by the subscript \( ij = \{0, 1\}, \{0, 1\} \) where \( i \) denotes the state of line (1, 2), and \( j \) denotes the state of line (1, 3), and 0 refers to the state of a line being 'down'. For example, state 10 refers to line (1,2) being 'up', and line (1,3) being 'down'. To simplify the notation, I will use the subscript only for social welfare and the shadow value although all variables in the dispatching problems must be state-dependent.

Then, the optimal dispatch problem for the system operator in state 11 (both lines are 'up') is:

\[
\max_{q_1, q_2, q_3} SW_{11} = \left( a - \frac{b}{2} q_3 \right) q_3 - c_1 q_1 - c_2 q_2
\]

\(^{20}\)If we were to assume that all lines except (1, 2) have perfect reliability (= 1), then we would end up with a trivial problem when we decide on the capacity of line (1, 2). When all other lines are perfect, then a social planner never wants to have the line (1, 2). Demand can be satisfied by using the cheap generator at node 1 when we do not build line (1, 2).
subject to

\[ q_3 = q_1 + q_2 \]  \hspace{1cm} (2.1)

\[ P_{12} = \frac{1}{3}(q_1 - q_2) \leq K \]  \hspace{1cm} (2.2)

\[ q_1, q_2 \geq 0 \]  \hspace{1cm} (2.3)

Equation 2.1 establishes the equality of total power injection and power usage, since there is assumed to be no transmission loss in the system. Equation 2.2 is the line constraint that limits the power flow\(^{21}\) between node 1 and node 2. We need this constraint only when the state of line (1, 2) is 1 (‘up’). We have not listed the flow constraints for other lines, because we have already assumed that their capacities are not binding (or arbitrarily large). Equation 2.3 refers to non-negativity constraints for power injections at node 1 and node 2, respectively. We need the last two constraints because we have defined node 1 and node 2 as power-exporting nodes.

Let \( \lambda_j(K) \) be the Lagrangian coefficient for line constraint. If \( j = 1 \), \( \lambda_j(K) \) refers to shadow value of line (1, 2) when line (1, 3) is 'up'. If \( j = 0 \), \( \lambda_j(K) \) refers to shadow value of line (1, 2) when line (1, 3) is 'down'. We put only one

\[^{21}\text{The relation between power flow and injections comes from the Kirchoff Laws. See the appendix for a review of Kirchoff Laws.}\]
subscript for shadow value, because it is undefined in states where line (1, 2) is down. Assume that the parameters of the problem are such that \( a > c_2 \). The problem yields the following interior solution for the case of a congested line:\(^{22}\)

\[
q_1 = \frac{2a-c_1-c_2}{4b} + \frac{3}{2} K, \quad q_2 = \frac{2a-c_1-c_2}{4b} - \frac{3}{2} K, \quad q_3 = \frac{2a-c_1-c_2}{2b}
\]

Then, the shadow value of the line constraint in state 11 is

\[
\lambda_1(K) = \frac{3}{2}(c_2 - c_1).
\]

The intuition behind the formula for the shadow value of the line is as follows:

If the line is congested (as we have assumed), then increasing the capacity of the line by one unit allows the generation of \( \frac{3}{2} \) more units at node 1, and \( \frac{3}{2} \) less units at node 2. The increase in SW is then equal to the total cost saving.\(^{23}\)

This is \( \frac{3}{2}(c_2 - c_1) = \lambda_1(K) \), as claimed.

The shadow value of the line constraint in state 10 is

\[
\lambda_0(K) = c_2 - c_1.
\]

The formula is similar to the one in state 11, but it is simpler. In this case, our network is reduced to a line (instead of the original triangle), hence there

\(^{22}\)Interior solution refers to \( q_1 > 0 \) and \( q_2 > 0 \). A line is congested when the power flowing on the line is just equal to the line capacity, i.e. \( P_{12} = K \). Note that the dispatching problem is solved for a given level of capacity.

\(^{23}\)Note that the total benefit part of social welfare remains constant in this state. This is because total generation, \( q_3 \), remains the same when line capacity changes. The constant marginal cost technology implies that the price of electricity at the demand node, \( p_3 \), is constant at \( p_3 = \frac{c_1+c_2}{2} \) when both generators are dispatched.
are no loop flows to consider. When the line is congested and line (1, 3) is down, a unit increase in capacity of line (1, 2) allows the dispatcher to generate 1 more unit at node 1, and 1 less at node 2. The increase in SW is again equal to the total cost saving. This is \( c_2 - c_1 = \lambda_0(K) \), as claimed.

2.5. Choice of Capacity and Reliability

2.5.1. Social Planner’s Problem

We will first examine the problem of choosing the socially optimal level of capacity and reliability so that it can be used as a benchmark to be compared to choices by the monopolist line owner. The social planner’s problem in the case of positive capacity is to maximize the expected benefit of having line (1, 2) minus the cost of building it.

\[
\max_{K, x_{12}} x_{12} V^*_1(K) + (1 - x_{12}) V^*_0 - C(K, x_{12})
\]

where

\[
V^*_1(K) = x_{13} SW^*_1(K) + (1 - x_{13}) SW^*_0(K)
\]

is the expected social surplus of consuming and generating electricity evaluated at the optimal dispatch when line (1, 2) is 'up',

\[
V^*_0 = x_{13} SW^*_0 + (1 - x_{13}) SW^*_0
\]

is the expected social surplus when line (1, 2) is 'down',

25
\( SW_{ij}^* \) is the social surplus in state \( ij \), and

\[ C(K, x_{12}) \] is the cost of installing \( K \) MW of capacity with a reliability level of \( x_{12} \). Assume \( C(0, x_{12}) = 0 \), \( C_1 > 0 \), \( C_2 > 0 \), and \( C(K, x_{12}) \) is strictly convex.

It is worth noting that \( V_0^* \) does not depend on the line capacity. This comes from the fact that having a high- or low-capacity line does not make a difference when the line is 'down'. Moreover, I treat the case of being 'down' the same as the case of having no line, i.e. \( K = 0 \).

The problem is intrinsically non-convex, making the solution non-standard. There exists a discontinuity in the objective function at \( K = 0 \). This non-convexity can be dealt with by comparing \( V_0^* \) with the objective function evaluated at the optimum positive level of capacity. If the former turns out to be higher, then we choose \( K = 0 \) as the optimal capacity.

Let the expected shadow value, \( \lambda \), be defined as
\[ \lambda = x_{13} \lambda_1 + (1 - x_{13}) \lambda_0. \]

\[ ^{24} \text{This is another way of saying 'When the line breaks down, it is completely destroyed.'}. \]
\[ ^{25} \text{In order to see that, compare the objective function when there is no line with the one when there is an infinitesimal capacity. The former is just } V_0^*, \text{ while the latter is } x_{12} V_1^* + (1 - x_{12}) V_0^* - C(K, x_{12}). \text{ If we had } \lim_{K \to 0} V_1^*(K) = V_0^*, \text{ there would not be a non-convexity. However, this is not the case: whenever you put an infinitesimal capacity, the Kirchoff Laws imply that you have to start running the expensive generator at node 2 in state 11. Moreover, this change in the amount of power generated at node 2, } q_2, \text{ is a jump from zero to } \frac{2a-c_1-c_2}{4b}. \]
Then, the FOC’s for an interior solution are:

\[ K^* : \quad x_{12}^* \frac{dV^*_1}{dK} - C_1(K^*, x_{12}^*) = 0, \quad \text{or} \]

\[ x_{12}^* \lambda(K^*) - C_1(K^*, x_{12}^*) = 0 \quad (FOC_{K}^S) \]

\[ x_{12}^* : \quad V_1^*(K^*) - V_0^* - C_2(K^*, x_{12}^*) = 0 \quad (FOC_{x_2}^S) \]

The marginal benefit of capacity in equation \( FOC_{K}^S \) is the shadow value multiplied by its reliability, because the benefit of additional capacity is realized only when the line is ‘up’. In equation \( FOC_{x_2}^S \), the marginal benefit of increased reliability is the difference between the expected social surplus when line (1, 2) is ‘up’, and when it is ‘down’.\(^{26}\)

The optimal capacity is \( K^* (> 0) \) if the social welfare evaluated at \( K^* \) is higher than the social welfare evaluated at \( K = 0 \), i.e. if

\[ x_{12}^* \frac{V_1^*(K^*)}{} + (1 - x_{12}^* \frac{V_0^*}{}) - C(K^*, x_{12}^*) \geq V_0^* .\]

\(^{26}\)We assume \( x_{13} \) is not too high to guarantee that marginal benefit of reliability is non-negative. As \( x_{13} \) gets closer to 1 (perfect reliability), then a social planner always chooses not to build a line between nodes 1 and 2. The critical value for \( x_{13} \) is \( x_{13} = \frac{8Kb(c_2 - c_1)}{4(a - c_1)^2 - (2a - c_1 - c_2)^2 - 4bK(c_2 - c_1)} \). Therefore, we assume \( x_{13} < x_{13} \).

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2.5.2. The Line Owner's Problem

As we have mentioned in the section 1, the line owner earns revenue as a function of installed capacity. The system operator pays the line owner $K \lambda(K)$ when the line is 'up' and nothing when the line is 'down'. Hence, the expected shadow value, $\lambda(K)$, is the price of capacity, and a price function that decreases with capacity implies market power for the line owner.

The line owner chooses $K$ and $x_{12}$ to solve

$$\max_{K, x_{12}} x_{12}(K \lambda(K)) - C(K, x_{12})$$

The FOC's for an interior solution are\(^{27}\):

$$K^M : \quad x_{12}^M \left(\lambda(K^M) + K^M \frac{d\lambda}{dK}\right) - C_1(K^M, x_{12}^M) = 0 \quad (FOC_K^M)$$

$$x_{12}^M : \quad K^M \cdot \lambda(K^M) - C_2(K^M, x_{12}^M) = 0 \quad (FOC_x^M)$$

Equation $FOC_K^M$ is the usual MR=MC condition for a monopolist. The marginal revenue of increasing capacity includes both the gain from the marginal unit, and the loss due to decreased revenue for infra-marginal units. Equation $FOC_x^M$ shows that the marginal benefit of reliability is equal to the expected

\(^{27}\)Note that the term $\frac{d\lambda}{dK}$ may have points of discontinuity. We will deal with this complication later.
total revenue when the line is ‘up’.

2.6. Comparison

I will first present the usual under-investment result when reliability is held constant. We only need to compare FOCs for capacity with a given level of reliability. One can easily see that $K^M \leq K^*$ (with reliability fixed) because marginal benefit of the line owner has an extra term, $x_{12}K\lambda'$, which is non-positive. Since the cost function is the same for both the line owner and the social planner, we get $K^M \leq K^*$. This is analogous to the behavior of a monopolist who faces a downward-sloping demand curve for a single product. When $\lambda' < 0$ (downward sloping demand curve), the monopolist under-invests in line capacity. When $\lambda' = 0$ (flat demand curve) the monopolist and the planner chooses the same amount of capacity. Therefore, we obtain the under-investment result when the shadow value of the line is strictly decreasing in capacity.

The next step is to check whether this result can be generalized to the case when reliability is also a decision variable. I will use two methods to compare the two sets of FOC’s that I have presented. The first is an algebraic method that uses a trick to make the set of FOCs comparable. The second method is a graphical analysis of the FOC’s on the capacity-reliability plane by using a
specific functional form for the cost of line construction.

We can combine the problems of social planner and the monopolist into one problem by using a parameter. This parameter, $\theta \in [0, 1]$, measures the relative importance of expected social welfare compared to expected revenue of the monopolist line owner. Now we can write the 'combined' problem as

$$\max_{K, x_{12}} (1-\theta) (x_{12}V_1^*(K) + (1-x_{12})V_0^*) + \theta [x_{12}K\lambda(K) - C(K, x_{12})]$$

where $\theta = 0$ refers to social planner's problem, and $\theta = 1$ refers to the monopolist's problem.

Since the cost function is the same for both, we can re-write the problem as

$$\max_{K, x_{12}} (1-\theta) [x_{12}V_1^*(K) + (1-x_{12})V_0^*] + \theta (x_{12}K\lambda(K)) - C(K, x_{12})$$

The FOCS of the 'combined' problem are:

$$ K : \quad x_{12} \left( \lambda(K) + \theta K \frac{d\lambda}{dK} \right) - C_1(K, x_{12}) = 0 \quad (2.4) $$

$$ x_{12} : \quad (1-\theta) (V_1^*(K) - V_0^*) + \theta K \lambda(K) - C_2(K, x_{12}) = 0 \quad (2.5) $$

Note that we obtain the FOCS of the social planner's problem if we put $\theta = 0$,
and the ones for the monopolist if we put $\theta = 1$. If we take the total derivative of the system of equations 2.4-2.5, then we find how the choices of line capacity and reliability are affected as $\theta$ changes. This will let us compare the problems of the social planner and the line owner. Simple algebra reveals\textsuperscript{28}

\[
\begin{align*}
\frac{dK}{d\theta} &= \frac{(V^*_t-V^*_0-K)\lambda+(\lambda+\theta K)\lambda'-C_{12}}{C_{22}(x_{12}(1+\theta)-C_{11})+(\lambda+\theta K)\lambda'-C_{12}} x_{12} K \lambda' \\
\frac{dx_{12}}{d\theta} &= \frac{-x_{12}(1+\theta)-C_{11})(V^*_t-V^*_0-K)\lambda-(\lambda+\theta K)\lambda'-C_{12}}{C_{22}(x_{12}(1+\theta)-C_{11})+(\lambda+\theta K)\lambda'-C_{12}} x_{12} K \lambda'
\end{align*}
\]
\(2.6\) \(2.7\)

Instead of deriving the complete set of solutions for this combined optimization problem, our approach will be to determine some sufficient conditions in order to get a couple of different results for $\frac{dK}{d\theta}$ and $\frac{dx_{12}}{d\theta}$. The complete solution requires a much more detailed analysis due to several reasons. First, there is a discontinuity in the objective function at $K = 0$ as mentioned in the section where we analyzed the social planner’s problem. Second, $\lambda'(K)$ does not exist at some $K$ values as we will explore later in more detail. Finally, reliability is bounded between $[0, 1]$, which increases the possibility of having corner solutions in a complete solution.

Assume that the second-order conditions\textsuperscript{29} for both the line owner’s and the

\textsuperscript{28}Note that we have used $\lambda'' = 0$ in our derivation.

\textsuperscript{29}The SOCs are $C_{11} -(1+\theta)x_{12} \lambda' > 0$, $C_{22} > 0$, and
social planner's problems hold. One can see that signs of $\frac{dK}{d\theta}$ and $\frac{d\pi_{12}}{d\theta}$ depend on the signs and the magnitudes of terms $V_1^* - V_0^* - K\lambda$ and $\lambda + \theta K\lambda' - C_{12}$. The first term refers to the difference between the marginal benefit of reliability to the social planner and that to the line owner. The second term is related to the shape of the cost function. I will present some sufficient conditions that lead to specific results for the comparison of investment by the social planner and the line owner. But first, we need to obtain an intermediate result to be used in propositions.

The next lemma shows that for some levels of capacity, the marginal benefit of reliability for the social planner can never exceed that for the monopolist.

**Lemma 1:** $V_1^*(K) - V_0^* < K\lambda$ for levels of capacity that imply a congested line in state 11, and $V_1^*(K) - V_0^* > K\lambda$ for levels of capacity that imply an uncongested line only in states 10 and 11. That is, the marginal benefit of reliability to the line owner exceeds that to the social planner for low levels of capacity, and the reverse is true for high levels of capacity.

**Proof:** We will show the result directly by analyzing the behavior of $V_1^*(K) - V_0^* - K\lambda$ for different levels of $K$. The set of values that line capacity can take is

$C_{22}((1 + \theta)x\lambda' - C_{11}) + (\lambda - C_{12} + \theta K\lambda')^2 < 0$ at all values of $K$ and $x_{12}$ except where $\lambda'(K)$ is discontinuous.
composed of five critical intervals each of which corresponds to different functions for $V_1^*(K)$ and $\lambda$. In the first interval, expected shadow value of capacity is constant at a positive value. In the second interval, $\lambda$ is decreasing with $K$, but still positive. In the third interval, line $(1,2)$ becomes uncongested in state 11, but still congested with a constant shadow value in state 10. In the fourth interval, line $(1,2)$ is still uncongested in state 11, but congested in state 10. When $K$ reaches interval five, the line is uncongested in both states, hence expected shadow value becomes zero. We can see the behavior of $\lambda$ in figure 2 for

---

These five intervals arise due to the solution of optimal dispatch problem (for given $K$) mentioned in section 4. The dispatch solution in that section corresponds to capacity levels in interval 1 in state 11. For each $K$ in interval 1, price of electricity is constant due to constant marginal cost of generation. The price becomes $c_1 + c_2$ in state 11, and $c_2$ in state 10. In state 11, an additional unit of demand is optimally satisfied by using both generators. But in state 10, we can only use the expensive generator at node 2 in order to satisfy an additional unit of demand. Since price is constant, quantity demanded also stays constant. All benefit due to a unit increase in $K$ comes from savings in total cost of generation by switching generation from the expensive generator to the cheap one. Since marginal cost of generation is constant for both generators, savings in total cost remain the same whatever the line capacity is. Hence, shadow value of $K$ is constant in this region.

But once line capacity enters interval 2, it becomes cheaper to dispatch only the cheap generator in state 11. Therefore, as line capacity increases (in region 2) more power is produced by the cheap generator, and hence the price of electricity falls and quantity demanded increases. But value of each additional capacity decreases as we increase capacity, because utility function of electricity users is concave. Hence, $\lambda' < 0$ in interval 2.

As we increase $K$ further, there comes a point where additional capacity has no value to society in state 11, but still has a constant value in state 10. This occurs when we enter interval 3, where $\lambda_1 = 0$, and $\lambda_0 > 0$.

In interval 4, it becomes cheaper to use only the cheap generator in state 10 (similar to the case in interval 2 for state 11). The value of an additional capacity gets smaller and smaller, hence we end up with $\lambda' < 0$ again.

In interval 5, the line becomes uncongested in both states. In other words, an additional capacity has no longer any value to society. Then, expected shadow value, $\lambda$, takes a value of zero in this interval.
$x_{13} \in (0, 1)$. 
Figure 2

Behavior of expected shadow value over five regions of line capacity
Tables 1, 2, and 3 show the values taken by $V_1^*(K)$, $V_0^*$, and $\lambda$, respectively, over five intervals for capacity.

<table>
<thead>
<tr>
<th>Interval</th>
<th>$V_1^*(K)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$0 &lt; K \leq \frac{2a-c_1-c_2}{6b}$</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{2a-c_1-c_2}{6b} &lt; K &lt; \frac{a-c_1}{3b}$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{a-c_1}{3b} \leq K &lt; \frac{a-c_2}{b}$</td>
</tr>
<tr>
<td>4</td>
<td>$\frac{a-c_2}{b} \leq K &lt; \frac{a-c_1}{b}$</td>
</tr>
<tr>
<td>5</td>
<td>$K \geq \frac{a-c_1}{b}$</td>
</tr>
</tbody>
</table>

*Table 1*

<table>
<thead>
<tr>
<th>Interval</th>
<th>$V_0^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$V_0^*$</td>
</tr>
<tr>
<td>2</td>
<td>$V_0^*$</td>
</tr>
<tr>
<td>3</td>
<td>$x_{13} \left( \frac{(a-c_1)^2}{2b} \right) + (1 - x_{13}) \left( \frac{(a-c_2)^2}{2b} \right)$</td>
</tr>
<tr>
<td>4</td>
<td>$V_0^*$</td>
</tr>
<tr>
<td>5</td>
<td>$V_0^*$</td>
</tr>
</tbody>
</table>

*Table 2*
Then, $\Sigma = V_1^*(K) - V_0^* - K\lambda$ takes the following functional forms as given in the next table.

<table>
<thead>
<tr>
<th>Interval</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$x_{13}\left(\frac{3}{2}(c_2 - c_1)\right) + (1-x_{13})(c_2 - c_1)$</td>
</tr>
<tr>
<td>2</td>
<td>$x_{13}\left(3(a - 3bK - c_1)\right) + (1-x_{13})(c_2 - c_1)$</td>
</tr>
<tr>
<td>3</td>
<td>$(1-x_{13})(c_2 - c_1)$</td>
</tr>
<tr>
<td>4</td>
<td>$(1-x_{13})(a - bK - c_1)$</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

*Table 3*

We also present a graph showing the behavior of $V_1^*(K) - V_0^*$ and $K\lambda$ as $K$ varies in Figure 3.
Line Capacity

Figure 3

$V_1^* - V_0^*$, $\lambda K$
In interval 1, we can re-write $\Sigma$ as $\frac{1}{8b}(c_1 - c_2)(2a - c_1 - c_2 + 2(a - c_1))$. But we know from an earlier assumption in section 3 that $2a - c_1 - c_2 > 0$, hence $a - c_1 > 0$. Therefore, $\Sigma < 0$ in interval 1.

In interval 2, note that $\Sigma$ is an increasing function of $K$. Then, let’s evaluate $\Sigma$ at the lower and upper limits of interval 2. At $K = \frac{2a-c_1-c_2}{6b}$, $\Sigma$ becomes $\frac{1}{2b} \left( \frac{c_1-c_2}{2} \right) \left( 2a - \frac{3c_1+c_2}{2} \right) < 0$. At $K = \frac{a-c_1}{3b}$, $\Sigma$ becomes 0. Since $\Sigma$ is a strictly increasing function in this interval, we conclude that $\Sigma < 0$ in interval 2. Therefore, $V_1^*(K) - V_0^* - K\lambda < 0$ in intervals 1 and 2 (or when the line is congested in state 11).

The sign of $\Sigma$ changes after interval 3. In intervals 4 and 5, $V_1^*(K) - V_0^* - K\lambda > 0$.

This result is surprising, but makes sense. As a very crude intuition, one can see that the social planner attaches a positive value to the state when the line is down (or when the line is not built), but the line owner does not. When the line is not built (or if it fails), the electricity can still go through other lines on the network to serve the demand, hence society gets some positive value even when the line is down (or absent). Moreover, from the point of view of the planner, term $K\lambda$ is analogous to total expenditure, and term $V_1^*(K)$ is analogous to total utility of consumers in a market with a downward-sloping demand curve.
In such a setting, total expenditure is always less than total utility, because some consumers' willingness to pay for the good exceeds the market price. Then, $K\lambda$ is always less than $V_1^*(K)$. But since society gets a surplus of $V_0^*$ when $K = 0$, the sign of $V_1^*(K) - V_0^* - K\lambda$ depends on whether $V_1^*(K) - K\lambda (> 0)$ exceeds $V_0^*$. As $K$ increases, $V_1^*(K) - K\lambda$ gets larger. Therefore, for low levels of capacity, the value of additional reliability for the social planner is less than the one for the monopolist.

One might gain more intuition for the result by considering the functional form of $V_1^*(K)$. In the first interval, shadow value is constant (as shown and explained before). Therefore, $V_1^*(K)$ must be composed of a term linear in $K$, and another one independent of $K$. The first term must be $K\lambda(K)$, which is nothing but the total revenue of the line owner. The second term must be less than $V_0^*$ (social welfare when the line is not built, or when the line is not congested), because as capacity reaches zero we have a positive jump in social welfare. This is the form of non-convexity in the problem as we mentioned before. Therefore, $V_1^*(K) - V_0^*$ must be less than $K\lambda(K)$ in interval 1. In the second interval, shadow value is decreasing in capacity, hence the first term (which depends on $K$) in $V_1^*(K)$ (which is now increasing at a decreasing rate as $K$ changes) must be greater than $K\lambda(K)$. Then, $V_1^*(K) - V_0^*$ must be approaching $K\lambda(K)$.
in this region. It never exceeds $K\lambda(K)$, because these terms are equalized at zero when we reach the third interval. In this interval, $SW_{11} = SW_{01}$ and $(1 - x_{13})(SW_{10} - SW_{00}) = K\lambda$, since $\lambda$ is constant. When we reach interval 4, shadow value of line in state 11 is zero, and shadow value in state 10 is decreasing in $K$. Therefore, $V_1^*(K) - V_0^*$ term must be increasing at a decreasing rate, and hence $V_1^*(K) - V_0^* - K\lambda$ must be greater than zero. In interval 5, value of an additional capacity is zero in both states 11 and 10. Therefore, $V_1^*(K) - V_0^* - K\lambda$ simply reduces to the difference in social welfare of states 10 and 00. But when line (1, 3) fails, having line (1, 2) results in a greater value for social welfare compared to not having it. As a result, $V_1^*(K) - V_0^* - K\lambda$ is greater than zero in interval 5.

Now we can present our results comparing $K^*$ and $K^M$. We assume that there exists an interior solution to the combined optimization problem for all $\theta$. Our first result presents a sufficient condition for the line owner to over-invest in line capacity and line reliability.

**Proposition 1:** If the cost function is such that the cross-cost elasticity$^{31}$ is 

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$^{31}$The cross-cost elasticity measures the responsiveness of the marginal cost of capacity to a change in reliability. When it is greater than unity, a one percent increase in reliability increases the marginal cost of capacity by more than one percent.
less than unity\(^{32}\), i.e. \(\frac{z_{12}}{C_{1}} C_{12} < 1\), and if both \(K^*\) and \(K^M\) fall into interval 1\(^{33}\), then the monopolist chooses a higher level of capacity and reliability compared to the social planner, or \(\frac{dK}{db} > 0\) and \(\frac{dx_{12}}{db} > 0\).

**Proof:** Assume \(\frac{z_{12}}{C_{1}} C_{12} < 1\). Since \(K^*\) and \(K^M\) fall into interval 1 for capacity, \(\lambda' = 0\). Hence, \(\lambda + \theta K \lambda' - C_{12} = \lambda - C_{12} > \lambda - \frac{C_{1}}{x_{12}}\). But we know from equation 2.4 that at an interior optimum in interval 1, \(x_{12} \lambda - C_{1} = 0\), or \(\lambda - \frac{C_{1}}{x_{12}} = 0\). Therefore, \(\lambda - C_{12} > 0\) at an interior optimum in interval 1.

Since \(K^*\) and \(K^M\) fall into interval 1 for capacity, we know from Lemma 1 that \(V_1^* - V_0^* - K \lambda < 0\).

Now we can analyze the signs of \(\frac{dK}{db}\) and \(\frac{dx_{12}}{db}\). The numerator of \(\frac{dK}{db}\) becomes negative, since \(\lambda' = 0\) in this interval. Moreover, the numerator of \(\frac{dx_{12}}{db}\) becomes negative.

The denominators of both \(\frac{dK}{db}\) and \(\frac{dx_{12}}{db}\) are nothing but the negative of the SOC of the combined problem. Since we assume the SOCs hold at the optimum,

\(^{32}\)This assumption is satisfied by some cost functions such as \(C(K, x_{12}) = AK^\alpha + Bx_{12}^\beta\) (for \(\alpha \geq 2; \beta \geq 2; A, B > 0\)), and \(C(K, x_{12}) = AK^\alpha(1 - x_{12})^{-\beta}\) (for \(\alpha \geq 2; 0 < \beta < 1; A > 0\)). However, the assumption is not satisfied for functions of the form \(C(K, x_{12}) = Kx_{12} e^{Kx_{12}}\) (elasticity being greater than one), or of the form \(C(K, x_{12}) = AK^\alpha x_{12}^\beta\) (for \(\alpha \geq 2, \beta > 1, A > 0\)).

\(^{33}\)One can obtain \(K^*\) and \(K^M\) in interval 1 (for capacity) if marginal cost of capacity is high enough. This is a matter of multiplying the cost function with a large parameter (without causing a corner solution at \(K = 0\)), which would not affect any of our assumptions about the cost function. Whether marginal cost of capacity is high enough or not is an empirical question, which is outside the scope of this paper.
or \( C_{22}(x_{12} \lambda (1+\theta)-C_{11})+(\lambda+\theta K \lambda-C_{12})^2 < 0 \), the signs of both denominators are negative.

Therefore, we get \( \frac{dK}{d\theta} > 0 \) and \( \frac{dx_{12}}{d\theta} > 0 \) as claimed. \( \blacksquare \)

This result proves that the under-investment result found in the literature cannot be generalized to cases where line reliability is also an endogenous variable. It is possible to have over-investment in capacity and reliability under the conditions mentioned in the proposition.

Some intuition for the result can be gained by remembering that the marginal benefit of reliability for the line owner exceeds that of the social planner in interval 1. Moreover, the marginal benefit of capacity for both the line owner and the social planner in interval 1 is determined by the same formula, since the shadow value is constant in this interval. Therefore, for a given level of capacity, the line owner chooses a higher level of reliability compared to the social planner. But then, a higher level of reliability implies an upward shift in the marginal benefit of capacity and marginal cost of capacity in the line owner’s problem compared to the social planner’s problem. Since we assumed the cross-cost elasticity being less than one, an increase in reliability does not increase the marginal cost of reliability much. Therefore, the increase in the marginal cost of capacity (due to an increase in reliability) is less than the increase in
the marginal benefit of capacity. This implies that the line owner will choose a higher level of capacity compared to the social planner.

We can see how the proposition works by showing the FOCs on a $K - x_{12}$ plane. The first step is to find the slopes of each FOC. One can easily verify that the slopes are

- $\frac{d\bar{e}_{12}}{dK} = \frac{x_{12} \lambda' - C_{11}}{C_{12} - \lambda}$ (2.8)
- $\frac{d\bar{e}_{12}}{dK} = \frac{\lambda - C_{12}}{C_{22}}$ (2.9)
- $\frac{d\bar{e}_{12}}{dK} = \frac{C_{11} - 2x_{12} \lambda'}{\lambda' + K \lambda' - C_{12}}$ (2.10)
- $\frac{d\bar{e}_{12}}{dK} = \frac{\lambda' + K \lambda' - C_{12}}{C_{22}}$ (2.11)

(Note that we have used the fact that $\lambda'' = 0$ in order to obtain equation 2.10.)

The signs of equations 2.8-2.11 depend on the sign of $\lambda + \theta K \lambda' - C_{12}$ for $\theta = 0, 1$. Moreover, one condition in Proposition 1 is related to the cross-cost elasticity, $\frac{x_{12} C_{12}}{C_{11}}$. Therefore, we will illustrate the proposition by using a specific cost function that yields known values for the terms $\frac{x_{12} C_{12}}{C_{11}}$ and $\lambda + \theta K \lambda' - C_{12}$.

The next example analyzes the problem by using a specific cost function.

**Example 1:** Assume the cost of building a transmission line with capacity
and reliability $x_{12}$ takes the following functional form:

$$ C(K, x_{12}) = A K^3 + B x_{12}^2 \quad \text{(for } A, B >). $$

This separable cost function implies $C_{12} = \frac{x_{12}}{C_1} C_{12} = 0$. Therefore, the signs of the slopes of FOCs depend on the term $\lambda + \theta K \lambda'$. For the social planner's problem, we have $\theta = 0$, hence the sign of $\lambda$ determines the signs of equations 2.8 and 2.9. Since $\lambda > 0$ in intervals 1 through 4, and $\lambda = 0$ in interval 5, $FOC^S_K$ and $FOC^S_x$ are upward sloping in intervals 1 through 4, and $FOC^S_x$ is flat in interval 5. $FOC^S_K$ is not defined in interval 5.\(^{34}\) Moreover, $FOC^S_x$ is not defined whenever $V_t - V_0^* < 0$.\(^{35}\)

The monopolist's problem corresponds to $\theta = 1$, hence the sign of $\lambda + K \lambda'$ determines the signs of equations 2.10 and 2.11. Since $\lambda' \leq 0$, the sign of $\lambda + K \lambda'$ in intervals 1 through 4 depends on the parameters of the spot-pricing problem.

Assume $x_{13} < \frac{c_2 - c_1}{3a - 4c_1 + c_2}$ to make sure that $\lambda + K \lambda' > 0$ in interval 2, and assume\(^{36}\) $\lambda > 2c_2 - c_1$ so that $\lambda + K \lambda' < 0$ in interval 4. In interval 5, $\lambda + K \lambda' = 0$. Under these assumptions, $FOC^M_K$ and $FOC^M_x$ are upward sloping in interval 1 through 3. $FOC^M_x$ is downward sloping in interval 4, but $FOC^M_K$ is not defined in this interval.

\(^{34}\) Note that $V^*_t(K) - V^*_0$ is negative at $K = 0$, and it is an increasing function of $K$. Therefore, $FOC^S_K$ will be undefined for small values of $K$, but will be defined for larger values of $K$.

\(^{35}\) If we had assumed $a < 2c_2 - c_1$, then $\lambda + K \lambda'$ would have become positive for small $K$ in interval 4, and negative for the remaining values in the same interval.

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Neither $FOC^M_K$ nor $FOC^M_x$ are defined in interval 5.

We can see a summary of the behavior of FOCs in Table 5.

<table>
<thead>
<tr>
<th>Interval</th>
<th>$FOC^S_K$</th>
<th>$FOC^S_x$</th>
<th>$FOC^M_K$</th>
<th>$FOC^M_x$</th>
</tr>
</thead>
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<td>1</td>
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<td>Increasing</td>
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<td>2</td>
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The next step is to compare the FOCs in a pairwise manner so that we can determine their relative positions on an $K - x_{12}$ plane. $FOC^S_K$ and $FOC^M_K$ coincides in intervals 1 and 3 (since $\lambda'$ becomes zero), and $FOC^M_x$ makes a positive jump between intervals 1 and 2.

Note that $\lambda + K\lambda' < 0$ in interval 4 means that the marginal benefit part in $FOC^M_K$ becomes negative. Since the marginal cost of capacity is non-negative, $FOC^M_K$ is not defined in interval 4.

One can see this by re-writing $FOC^M_K$ as $\lambda + K\lambda' = C_1(K, x_{12}) / x_{12}$. As $K$ increases infinitesimally at $K = \frac{2a - x_{12} - c_2}{\delta x_{12}}$ (or moving from interval 1 to interval 2), $\lambda + K\lambda'$ decreases discontinuously due to the jump in $\lambda'$. Since $C_1$ is a continuous function of $K$ and $x_{12}$, we must observe a discontinuous increase or decrease in $x_{12}$ so that the FOC is still satisfied. To see the direction of change in $x_{12}$, we need to look at $\frac{\partial}{\partial x_{12}} (C_1(K, x_{12}) / x_{12})$. It turns out that this derivative is equal to $\frac{C_1}{x_{12}} \left( \frac{x_{12}^2 C_{12} - 1}{C_1 C_{12}} \right)$. If $\frac{x_{12}^2 C_{12}}{C_1} < 1$ (or cross-cost elasticity being less than unity), then $x_{12}$ needs to increase discontinuously to compensate for the jump in $\lambda + K\lambda'$. 

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Comparison of $FOC^S_x$ and $FOC^M_x$ reveals that they coincide in interval 3 (since $V^*_1(K) - V^*_0 = K\lambda$ in this interval). For a given $K$ in interval 1, $FOC^S_x$ implies a lower $x_{12}$ compared to $FOC^M_x$, because $V^*_1(K) - V^*_0 < K\lambda$ in this interval.

Finally, $FOC^S_K$ is steeper than $FOC^S_x$ (when they intersect), and $FOC^M_K$ is steeper than $FOC^M_x$ (when they intersect) due to the assumed SOCs.

In the light of this analysis, we can now illustrate Proposition 1 in figure 4. The graph is generated by using the following parameter values:

$a = 110$, $b = 0.2$ : the inverse demand function for electricity implies a maximum price of $110$, and it has a slope of $-0.2$. (Note that $b = 0$ corresponds to a perfectly elastic demand function.)

$c_1 = 10$, $c_2 = 40$ : the marginal cost of generation is $10$ at node 1, and $40$ at node 2.

$x_{13} = 0.8636$ : the probability of failure in line $(1, 3)$ is $86.36\%$.

$A = 0.0002$, $B = 6500$ : cost of line capacity is multiplied by $0.0002$, and cost of line reliability is multiplied by $9000$. 

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Figure 4
The thick lines on figure 4 represent the FOCs of the social planner, and the thin lines represent the FOCs of the monopolist. The dashed lines refer to the FOCs for reliability, and the solid lines refer to the FOCs for capacity.

The figure shows that the optimal capacity choices of both the planner and the line owner falls into interval 1 under the assumptions made earlier. As we can see on the figure, the monopolist chooses a higher capacity ($K^M > K^*$) and higher reliability ($x_{12}^M > x_{12}^*$) compared to the social planner. (End of Example 1).

The next proposition sets a sufficient condition for the line owner to choose first-best levels of capacity and reliability.

**Proposition 2:** If both $K^*$ and $K^M$ fall into interval 3, then the line owner chooses the first-best levels of capacity and reliability, or $\frac{dK}{d\theta} = \frac{dx_{12}}{d\theta} = 0$.

**Proof:** Since $K^*$ and $K^M$ fall into interval 3, $\lambda'(K) = 0$. Moreover, $V_1^* - V_0^* - K\lambda = 0$ since we are in interval 3. This implies that the numerators of both equations 2.6 and 2.7 become zero. Therefore, $\frac{dK}{d\theta} = \frac{dx_{12}}{d\theta} = 0$ as claimed.  

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39 The reader can see that $FOC^R_K$ and $FOC^S_x$ intersects twice (both in interval 1). However, only one of them corresponds to the global optimum. To see that, we check the value of the SOC for the social planner's problem. The SOC takes a value of $C_{22}((x\lambda' - C_{11}) + (\lambda - C_{12})^2 \approx +500 > 0$ at the first intersection, and it takes a value of about $-400 < 0$ at the second intersection. Therefore, the SOC is only satisfied at the second intersection, which gives us the unique global optimum.

Similarly, $FOC^M_K$ and $FOC^M_x$ coincide twice (both in interval 1). The first intersection occurs at point $(0,0)$ on the $K - x_{12}$ plane, but the SOC is not satisfied at this point. The second intersection satisfies the SOC, hence it is the unique global optimum.
Note that interval 3 is where the marginal benefit of reliability for the line owner is equalized to that for the social planner for a given level of capacity. Moreover, the marginal benefit of capacity is the same for both the social planner and the line owner, since the shadow value is constant. This implies the result, because the set of FOCs are the same for both of them.

This proposition deserves attention due to two reasons. First, it gives another set of conditions under which the under-investment result does not hold. Second, it shows that an unregulated line owner might choose the first-best levels of capacity and reliability. If we know that capacity choices are going to be in interval 3, then we do not need to regulate either capacity nor reliability.

We will illustrate this result by an example on a $K - x_{12}$ plane as we did for Proposition 1. The next example sets the assumptions on functional forms and parameters in order to illustrate Proposition 2.

**Example 2:** Since we have not employed any assumption for cross-cost elasticity in Proposition 2, we can pick any cost function for line construction that satisfies the SOCs. We pick w.l.o.g. that the cost function is the same as in Example 1. Therefore, $C(K, x_{12}) = A K^3 + B x_{12}^2$.

Assume again that $x_{13} < \frac{\alpha_2 - c_1}{3a - 4c_1 + c_2}$ to make sure that $\lambda + K \lambda' > 0$ in interval 2, and assume $a > 2c_2 - c_1$ so that $\lambda + K \lambda' < 0$ in interval 4. Then, the shapes and
slopes of FOCs will be the same as in Figure 4. We will only change parameters of the cost function for line construction to illustrate Proposition 2. Hence, the parameters are assumed to take the following values:

\[ a = 110, \quad b = 0.2, \quad c_1 = 10, \quad c_2 = 40, \quad x_{13} = 0.8636, \text{ and} \]

\[ A = 0.00006, \quad B = 9000. \]

Figure 5 shows the FOCs implied by these parameters.
As in Figure 4, the thick lines refer to the FOCs for the social planner’s problem, and the dashed lines refer to the FOCs for reliability. The FOCs of the social planner’s problem and the FOCs of the monopolist’s problem intersect in interval 3. Since the set of FOCs are the same for both the social planner and the line owner in this interval, they choose the same amount of capacity and reliability. (End of Example 2).

The next proposition presents a sufficient condition for obtaining the under-investment in capacity and over-investment in reliability.

**Proposition 3:** If the cost function is such that the cross-cost elasticity is greater than unity, and if both \( K^* \) and \( K^M \) fall into the same interval in one of intervals 1 or 2, then the line owner chooses under-investment in capacity and over-investment in reliability, or \( \frac{dK}{d\theta} < 0 \) and \( \frac{dx_{12}}{d\theta} > 0 \).

**Proof:** Assume \( \frac{x_{12}}{C_1} C_{12} > 1 \), then \( \lambda + \theta K \lambda' - C_{12} < 0 \) at the optimum. Since \( K^* \) and \( K^M \) fall into one of intervals 1 or 2, \( V_1^* - V_0^* - K\lambda < 0 \). These observations imply that the numerator of equation 2.6 will be positive, and the numerator of equation 2.7 will be negative. Since both denominators are negative, then we obtain \( \frac{dK}{d\theta} < 0 \) and \( \frac{dx_{12}}{d\theta} > 0 \) as claimed. ■

**Example 3:** Assume that the cost function takes the form \( C = A (K + K_0)^2 x_{12}^{1.5} \). Hence, the cross-cost elasticity becomes \( \frac{x_{12}}{C_1} C_{12} = \frac{3}{2} > 1 \). In order to
get both $K^*$ and $K^M$ in interval 1, we assume the parameters take the following values:

\[ a = 110, \quad b = \frac{1}{3}, \quad c_1 = 10, \quad c_2 = 30, \quad x_{13} = 0.1, \text{ and} \]

\[ A = \frac{2}{3}, \quad K_0 = 10. \]

Figure 6 shows the FOCs implied by these parameters.
Figure 6
The thick lines refer to the FOCs for the social planner’s problem, and the dashed lines refer to the FOCs for reliability. We dropped the labels for FOCs to improve the visibility of intersections. As claimed in Proposition 3, when the cross-cost elasticity is greater than unity and both $K^*$ and $K^M$ fall into interval 1, the line owner over-invests in line reliability ($x_{12}^M > x_{12}^*$), and under-invests in line capacity ($K^M < K^*$). (End of Example 3)

2.7. Conclusion

We have analyzed the incentives of a monopolist line owner in choosing the line capacity and line reliability. The common idea that the line owner invests too little in order to maximize the congestion rent does not always hold true when reliability is also a choice variable.

We present a couple of sufficient conditions each of which leads to a different result regarding the investment choices of the line owner and the social planner. The conditions are based on the cross-cost elasticity of line construction and on the interval in which the optimal choices of capacity take place. The first result finds that the line owner over-invests in both capacity and reliability if the elasticity is less than unity and if capacity choices of the social planner and the line owner are not too high. Another result shows that we get the first-best
levels of capacity and reliability from the line owner if the optimal choices of capacity fall into an interval where the set of FOEs are the same for the social planner and the line owner. Finally, we present a sufficient condition that yields under-investment in capacity.

Therefore, under the reward mechanism analyzed in this paper, regulation of both capacity and reliability is in general better than leaving those decisions to market forces. 'Natural Transmission Revenue' mechanism (which sets shadow value of line as its price) does not necessarily result in under-investment in line capacity. We are aware of the fact that regulation by itself has a cost to society due to administrative expenses and due to cost arising from asymmetric information. Hence, empirical studies are needed to compare regulatory mechanisms with decentralized mechanisms.

Our results indicate the need for empirical studies to determine the shape of the function that relates the cost of providing capacity and reliability in transmission networks. An estimate of the cost function together with a simulation analysis for modeling the spot pricing of electricity (and hence the shadow value of capacity) will determine which of our conditions hold true for different networks.
3. REGULATION

REGULATION OF TRANSMISSION LINE INVESTMENT AND RELIABILITY

3.1. Introduction

Successful deregulation of electric power industry requires open and fair access to the transmission network. Attention has focused on transmission pricing issues but an equally important long-term issue is the design of a market and regulatory framework which induces owners to make optimal investments in network capacity and reliability. This is the topic of this paper.

Reliability of electric power used to be seen as an exogenous variable by planners. It was determined using engineering rules, which were viewed as constraints by the decision-making authority. However, reliability should be

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40 The only exception that I aware of is an experimental project by Duke Power Company. This project aims to perform planning activities by taking into account the value of reliability to their customers. See Dalton et al [7] for details.

41 See Munasinghe [15], p. 24 for a more detailed treatment of the current approach to reliability. An example of these exogenous constraints is the N-1 rule, or first contingency criterion. It requires that if one component fails, the others must be sufficient to serve the demand. Another example is the practice (in Brazil) of putting a direct constraint on reliability, saying that demand must be served 95% of the time. See Dalton et. al. [7] p. 1400, and Nasser [16] p. 38..
chosen on the basis of economic principles as well, taking into account not only engineering principles but also the preferences of electricity users and the cost of reliability. The materials\textsuperscript{42} and technology\textsuperscript{43} used in the construction of transmission lines, the level of maintenance expenditures, and the transmission capacity of lines constitute the major determinants of the reliability for the whole network\textsuperscript{44}. In this paper, reliability of a line is considered as a choice variable for the investor who constructs the line. The optimal regulation of a monopolist firm choosing reliability and capacity of the line is formulated.

For the structure of ownership and control rights of the transmission network, there are two basic alternatives in the literature. The first one involves the regulation of the line owner(s). The second alternative is to leave the in-

\textsuperscript{42}I will only mention two examples. First, either rod gaps or surge arresters can be used to protect transformers, reactors, and other components from overvoltages; rod gaps are cheaper, but also less reliable. Second, transmission line poles can be made from wood, steel or concrete; wood is cheaper but can be less reliable. (From the Encyclopedia of Physical Science and Technology [8])

\textsuperscript{43}For example, a new technology called FACTS allows the operator to automatically redistribute flow when the power flowing on a line reaches its limit. Such a technology enables the system operator to prevent line breakdowns and repair costs due to overflows. Installing FACTS definitely improves reliability, but at an extra cost.

\textsuperscript{44}The formation of reliability coordinating councils (e.g. North American Electric Reliability Council, NERC) by electric utilities was motivated by the need for operating and design standards for the industry. The existence of these institutions indicates that investors have a multitude of options affecting the reliability of the system. Even under this coordinated structure of the industry, electric utilities do not always conform to those standards. As a senior transmission planner said: “We have a line I would like to monitor, but I do not think we would like the results.” Another quote: “A district engineer, after being informed that a copper conductor was annealed and had lost most of its strength, said, “I do not think we can afford to replace it this year.” (From Electrical World [20], p.18).
vestment decision to a decentralized market mechanism. The most prominent model for the latter (by Bushnell and Stoft [4]) uses the instrument of Transmission Congestion Contracts (TCC)* in order to give the right incentives to line owners for investment. The study assumes there are many potential investors who are free to make expansions to any line on the network subject to some rules that determine the reward for investments. Their basic idea is to let the investor choose any set of TCC’s over the network under one condition: the new aggregate set of TCC’s must imply a feasible^46^ dispatch. The system achieves the socially optimum investment under quite restrictive assumptions, such as the exact match of contracts to the actual dispatch at all times. Moreover, the lumpiness of line investment is not captured in their analysis.

Holding reliability fixed, the line owner is likely to choose under-investment to increase congestion rents even when the spot-pricing of electricity is done optimally^47^.

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45 TCC’s were introduced by Hogan [10]. A TCC is defined for a pair of nodes (i, j), and gives its owner a prespecified quantity multiplied by the difference in prices at node i and j. Nodes specified in the contract are not necessarily connected in physical terms. Hence, a TCC is essentially a financial instrument rather than a certificate to have the right to transfer electric power from node i to j.

A variant of this approach will be used by New York Power Authority (NYPA). Investors who pay for transmission expansion will receive TCC’s as compensation.

46 A dispatch is feasible if it does not violate the constraints of the dispatching problem, such as line flow constraints, the constraint for the conservation of power, etc.

47 See Nasser [16] p. 175, and Bushnell and Stoft [3].
mission Revenue\textsuperscript{48}, for the line owner in order to show the under-investment result with exogenous reliability. I have shown in Celebi [5] that (when reliability as well is a choice variable) the owner might invest too little or too much capacity depending on the cross-cost elasticity of line construction with respect to capacity and reliability. If the marginal cost of capacity is very sensitive to a marginal increase in reliability, then the monopolist chooses too much capacity and too little reliability compared to the socially optimal levels. The common result of both studies is the need for regulation.

The regulation alternative is studied in Nasser [16], which formulates the optimal regulation of a monopolist firm owning all the lines in the network. The study considers only capacity as the decision variable, ignoring the firm's ability to affect the reliability of a line. Moreover, Nasser [16] does not question the existence of lines, and implicitly assumes that it is to the benefit of society to have all the lines on the network. However, in the absence of line failures, some lines on the network may be either detrimental to social welfare, or just redundant. My analysis shows that the existence of a line is justified when the reliability of other lines on the network is not too high. Another result relates

\textsuperscript{48}Natural Transmission Revenue is the shadow value of line capacity multiplied by the capacity. It is "natural" in the sense that shadow value represents the true value of the marginal capacity to society.
the decision to build a line to the marginal cost of the expensive generator. As the marginal cost gets higher, the regulator is more likely to build the line under some conditions. I further argue that the expectation of higher demand in the future makes the regulator less likely to build the line if it will be congested and reliability of other lines is high enough. Finally, I analyze whether it is socially optimal to have a non-congested line under complete and incomplete information structures. It is always optimal to have a congested line under complete information, but that result does not always hold under incomplete information.

The plan of the paper is as follows. I will describe the institutional setting, and the technology of electricity transmission in the next two sections. The fourth section shows how spot pricing is done and explains the shadow price of line capacity, which will be used in the analysis of investment. Regulation is analyzed in section five, followed by the conclusion.

3.2. Institutional Set-up

The spot pricing of electricity is assumed to follow nodal pricing as introduced by Schwegge et. al. [18][49], and used by Hogan [10]. A system operator organizes

[49]See Schwegge et. al. [18], pp. 151-156 for their nodal pricing model.

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auctions in which demand and supply bids of users yield the price of electricity at each node of the network. The nodal pricing model not only is the present paradigm of choice of policy makers (as in New England and U.K.) but its optimality relies on far less restrictive assumptions than its principal rival\textsuperscript{50}. The basic assumption in the nodal pricing model is the absence of market power among users and suppliers of electricity, while the decentralized market model also requires the absence of market power among the owners of transmission lines. Since the focus of the study will be on long-term decisions of investment and reliability, I will ignore the imperfections in the spot pricing of electricity. I aim to formulate the optimal regulation of investment and reliability, rather than analyzing the interactions between the market imperfections in the spot pricing and the imperfections in investment and reliability choice.

For the structure of ownership and control rights of the transmission network, the line owner is assumed to be regulated. Economies of scale\textsuperscript{51} in line construction and highly concentrated ownership patterns in most countries\textsuperscript{52} make this a more plausible alternative than the decentralized investment systems proposed

\textsuperscript{50}The alternative is a decentralized pricing system, such as the tradable physical rights approach by Chao and Peck [6].

\textsuperscript{51}See Baldick-Kahn [2] for more details on the technology of line construction.

\textsuperscript{52}A few utility companies jointly own most of the regional transmission networks in many states of the U.S. In the U.K, there is a single transmission company, called the National Grid Company (NGC).
in the literature. Moreover, the regulation of a single line owner was hinted at by Bushnell and Stoft [4]53, and was analyzed by Nasser [16].

The choice of line reliability determines the probability of being in one of the two states of the world: either the line works or it fails. I assume that the system operator and users learn the state of the world before the auction for spot pricing is organized. However, investment and reliability decisions are made before the state of the world is known. This is the natural way of timing, because investment decisions are generally made much less frequently than pricing decisions 54. The regulator offers a contract to the firm (before the decision on investment and reliability) on the basis of observable variables. At the time of writing the contract, the regulator does not know the firm's type and the level of cost-reducing activities of the firm. We can think of the type of firm as a measure of its productivity, which is exogenous. 55 Cost-reducing activities, or effort, represent any managerial effort or maintenance activity that is not observable to anyone other than the firm itself.

53 They said, "Transmission assets could be combined into a single entity that would have to be closely regulated due to its substantial market power." (Bushnell and Stoft [4], p. 90)

54 In practice, auctions are held hourly, but investment decisions take place once in a couple of years.

55 An alternative interpretation of the firm's type is its ability to access state-of-the-art technology of line construction and materials. This case may arise when construction firms are also involved in developing the technology of line construction. A firm can find itself in a relatively advantageous position compared to others once it completes the development of a new technology.
I assume perfect competition among users (or bidders) of electricity in auctions that determine spot prices, hence pricing is done optimally. Optimal prices yield the shadow price of the line, which is the value to the society of a marginal unit of capacity evaluated at optimal spot prices. Instead of rewarding the firm by the shadow price per unit of capacity, I consider a more general transfer which is a function of observable variables when the line is built, and zero when the line is not built. The only observable variables to the regulator are capacity, reliability and total cost. One might be surprised that the firm is paid a transfer even when the line is 'down'. But it should also be kept in mind that the regulator has the option of paying less for low reliability levels, hence there is no need to devise a different transfer function for the case of a 'down' line.

Before proceeding to the economic analysis, I will present the power technology in a simplified network model in the next section.

3.3. A Simple Model of Electric Networks

The model used in this analysis will be the DC load approximation to the power flow with no line loss. The network will be assumed to have three nodes, which are connected by identical transmission lines in a triangular shape. Voltage levels
throughout the network are assumed to be held constant. Figure 1 illustrates the triangular network.

Nodes 1 and 2 are generator nodes, while node 3 is the demand node. Generation (or injection) by generator 1 and generator 2 are labeled $q_1$ and $q_2$, respectively, while the consumption at node 3 is labeled $q_3$. Power flowing on line $(i,j)$ from node $i$ to node $j$ is $P_{ij}$. Assume that only line $(1,2)$, with a line capacity of $K$, has the possibility of congestion. Moreover, the reliability of line $(1,2)$ is a decision variable to be chosen at the same time as the line is constructed. Reliability of a line is defined as the probability of the line being 'up'. The line is either 'up' or 'down', and the electricity cannot flow over a 'down' line. The reliability of line $(1,2)$ is denoted by $x_{12}$. The line connecting nodes 1 and 3 has a possibility of being 'down' too. However, its reliability, $x_{13}$, is exogenous\textsuperscript{56} for decision makers, and independent of the reliability of line $(1,2)$.

\textsuperscript{56}This assumption is made to keep the model simple, and to be able to see the role of line $(1,2)$ in satisfying the demand at node 3 when line is $(1,3)$ 'down'. If it were not exogenous, our set-up would imply that building a line between nodes 1 and 2 may be inferior to just increasing the reliability of existing line $(1,3)$. Both alternatives improve the reliability of system, but the first one brings the disadvantage of having to use the expensive generator in the case of congestion. Hence, the second alternative (just increasing the reliability of line $(1,3)$) is better than having an additional line between nodes 1 and 2 (assuming that an improvement in reliability costs less than building an additional line). However, the first alternative is viable if the construction cost between nodes 1 and 2 is less than the cost between nodes 1 and 3 (This might be due to a difference in distance, or difference in environmental costs, etc.). The first alternative is again viable if line $(1,2)$ is needed in order to diversify risk in case of climate shocks to transmission lines. As a result, I will concentrate on choosing the reliability of one line to be able to isolate the trade off between congestion and reliability.
Assume that generators have increasing marginal cost production technologies with $c_1'(q_1) = q_1$ and $c_2'(q_2) = mq_2$, respectively, where $m > 1$. That is, we assume that the marginal cost of generator 1 is less than the marginal cost of generator 2 for each generation level except zero. Assume the inverse demand function at node 3 is $p_3(q_3) = a - bq_3$, with the implied utility function of $B(q_3) = (a - \frac{b}{2}q_3)q_3$.\(^{57}\)

3.4. The Socially Optimal Dispatch (Spot Pricing)

I start solving the problem backward in time. The problem of the system operator is to choose the dispatch to maximize social surplus in each state of the world, subject to power flow equations and the line constraint with a given capacity. Denote the state of the world by $rs$, where $r$ and $s$ take two values, 0 and 1. That is, 00 refers to the state when both line (1, 2) and line (1, 3) are 'down', 01 refers to the state when line (1, 2) is down, and line (1, 3) is 'up', so on. To simplify the notation, I will use a state subscript only for social welfare and the shadow price, although all variables in the dispatching problems must be state-dependent.

\(^{57}\)The unit of $q$'s and $K$ is MW (megawatt), while the unit of $c$'s and $p_3$ is $$/MW (dollars per megawatt).
Then, the optimal dispatch problem for the system operator in state 11 is:

$$Max \quad SW_{11} = (a - \frac{b}{2} q_3) q_3 - \frac{q_1^2}{2} - \frac{mq_2^2}{2}$$

subject to

$$q_3 = q_1 + q_2 \quad (3.1)$$

$$P_{12} = \frac{1}{3} (q_1 - q_2) \leq K \quad (3.2)$$

$$q_1 \geq 0, \quad q_2 \geq 0$$

Equation 3.1 establishes the equality of total power injection and power usage, since there is assumed to be no transmission loss in the system. The constraint 3.2 is the line constraint that limits the power flow between node 1 and node 2. When the state is 01, then constraint 3.2 is not needed, and the optimal value of the objective function becomes $SW_{01}$. In state 10, the constraint 3.2 reduces to $q_1 \leq K$, because all power injected at node 1 goes through line (1,2). The optimal value for this state is $SW_{10}$. In state 00, the demand node is essentially disconnected from generator 1, hence only the more expensive

---

58The formula for the power flow is a result of Kirchoff Laws, which states that power coming to an intersection divides itself in inverse proportion to the relative resistance of paths. That is, more power flows on a path which has less resistance than others.

In our simple network (with identical lines), a unit power injection from node 1 causes a 1/3 unit power flow from node 1 to node 2 on line (1,2). Similarly, a unit injection from node 2 causes a 1/3 unit flow from node 2 to node 1 on line (1,2). The net flow is their difference.
generator can be used. The solution of the optimization problem at \( q_1 = 0 \), and no line constraint yields \( SW_{00} \).\(^{59}\)

Let \( \lambda_i \) be the Lagrangian coefficient for the constraint 3.2 in state 1\(i\) (\(i = 0, 1\)). The problem yields the following set of FOC's in state 11:

\[
q_1 : \quad p_3 = c'_1(q_1) + \frac{\lambda_1}{3} \\
q_2 : \quad p_3 = c'_2(q_2) - \frac{\lambda_1}{3}
\]

The first equation demonstrates the necessary condition for the optimal injection from node 1. The marginal social cost of a unit injection is the sum of the marginal cost of generation and the cost of congestion created due to this injection. Each unit of injection at node 1 requires \( 1/3 \) units of capacity on line (1, 2), which has a marginal benefit of \( \lambda_1 \) to the society. Marginal benefit of injection on the LHS is the price of electricity that consumers at node 3 pay. The second equation is similar to the first one, except this one is for the injection at node 2. The injection at node 2 relieves the congestion of line (1, 2), hence the social marginal cost is reduced by one-third of \( \lambda_1 \). The optimal solution for a congested line (where \( K < \frac{a(m-1)}{3(m+b+bm)} \)) is:

\[
q_1 = \frac{2a+3K(2b+m)}{4b+1+m}, \quad q_2 = \frac{2a-3K(2b+1)}{4b+1+m}, \quad q_3 = \frac{4a+3K(m-1)}{4b+1+m}, \quad p_3 = \frac{a(1+m)-3bK(m-1)}{4b+1+m}
\]

\(^{59}\)Results for each state of the world are in Appendix.

\(^{60}\)Note that we are not introducing any notation for the state when line (1, 2) is down, because the shadow value is not defined when the line is not there.
Then, the shadow price of the line constraint becomes

\[ \lambda_1 = 3(p_3 - c_1'(q_1)) = 3 \left( \frac{a(m-1) - 3K(m+b+bm)}{4b+1+m} \right) \]

The intuition behind the formula for the shadow price of the line is as follows: The generator at node 1 has to pay \$\frac{\lambda_1}{3}$ for each unit of injection, because only \(1/3\) of that injection flows on line \((1,2)\). On top of that, the generator incurs a marginal generation cost of \(c_1'(q_1)\). The benefit of that injection is the marginal benefit to consumers, \(p_3\). Optimality requires marginal benefit being equal to marginal cost at the margin, which gives us the formula for the shadow price.

When the line constraint 3.2 is not binding, the optimal dispatch is such that marginal cost of generators are equalized, that is \(q_1 = mq_2\). Note that \(q_1 < mq_2\) when the constraint 3.2 is binding, that is when the line is congested.

Now we know the optimal dispatch and shadow price for each state of the world and for each level of capacity. The complete solution and a numerical example is given in the Appendix. The next step is to choose the capacity and the reliability of the line, and we will use a model based on the analysis in Laffont and Tirole\(^{61}\) [13].

\(^{61}\)See Chapter 3, page 168 for their analysis.
3.5. Regulation of Capacity and Reliability

3.5.1. The Model

The monopolist firm chooses the capacity, $K$, and reliability level, $x_{12}$, of the transmission line connecting nodes 1 and 2 on the triangular network.

The firm incurs a cost which depends on the type of firm, on the cost-reducing effort level, on the transmission capacity of the line, and on the reliability of the line. The regulator observes the cost, the capacity, and reliability, but does not observe the firm's type, $\beta$, and its effort level, $e$. ($\beta \in [\underline{\beta}, \overline{\beta}]$, and is distributed with density $f(\beta)$, and cdf $F(\beta)$).

The timing is as follows:

1. The regulator chooses the contract.
2. The firm chooses $K$, $x_{12}$, and $e$.
3. The state of the world is revealed.
4. The system operator chooses the dispatch.

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$^{62}$We realize that reliability is not completely observable in the real world. The choice of materials and technology in the construction of a transmission line can yield an estimate of the actual reliability of the line, but other factors such as maintenance activities may not be observable to the regulator. A rigorous way to model this fact is to consider reliability of a line as a function of a decision variable which represents all observable variables, and an effort level specific to line reliability. However, this would make the model more difficult to solve, and hence will be omitted.
The importance of the timing comes only from its effect on the information set of the regulator and the firm when they make their decisions. The regulator designs the contract without knowing the type of firm or the state of the world. The contract is binding for both parties, and is verifiable in court. This assumption eliminates the possibility of the regulator learning the type of firm, and re-writing the contract.\textsuperscript{63}

The firm makes its decision without knowing the state of the world, and maximizes the expectation of profit over the states of the world at stage 2. The probability of having line (1, 2) 'up' is equal to the reliability of the line, which is chosen by the firm. This is an interesting problem in the sense that the decision maker chooses both the probability distribution and payoffs in each state of the world in order to maximize expected profit.

The cost is determined by the type of the firm, the effort level, line capacity, and reliability:

\[
C = C_0 + C_1 (\beta, e, K, x_{12})
\]

where there exists a cardinalization of \( e \) and \( \beta \) such that \( C_\beta > 0, C_e < 0 \); and \( C_K > 0, C_{x_{12}} > 0 \). \( C_0 \) refers to fixed cost in line construction\textsuperscript{64} for \( K > 0 \).

\textsuperscript{63}It seems that this setup is not renegotiation proof, but this is beyond the scope of this paper.

\textsuperscript{64}The fixed cost results from the cost of land (or rights-of-way) on which the lines are built, and from the cost of transmission towers, support structures, etc. which we can safely assume
Following the convention in Laffont and Tirole [13], assume that the cost is reimbursed to the firm, plus a transfer is given. The line built by the firm is not explicitly rented by the regulator to the users of the network because the price of using the line (transmission price) is implicitly included in the price of electricity\textsuperscript{65}. In other words, we view the line as a good with no revenue at the time of the contract. Moreover, we formulate the regulation problem as a direct revelation mechanism, in which the firm reports its type to the regulator, who then calculates the transfer and other observable variables according to a pre-specified optimal formula.

Therefore, the risk-neutral firm gets a utility of $U \equiv t - \psi(e)$, where $t$ is the transfer to the firm, and $\psi(e)$ is the disutility of effort to the firm ($\psi' > 0$, $\psi'' > 0$).

Suppose that transferring money to the firm is distortionary, i.e. each dollar transferred to the firm costs $(1 + \mu)$ to the society\textsuperscript{66}. Then, social welfare is the to be independent of the line capacity.

\textsuperscript{65}Transmission price of sending power from node $i$ to node $j$ is defined as $p_j - p_i$, where $p_i$ is the optimal spot price of electricity at node $i$.

\textsuperscript{66}One might argue that the system operator could use the profit at the stage of spot-pricing to help the regulator pay transfers to the line owner. This is a reasonable argument considering the fact that raising funds to pay transfers is costly to the regulator. Therefore, such use of profits from the spot-pricing would reduce the social cost of each dollar of transfer to the firm. However, we assume that the system operator and the regulator do not have an agreement to share profits, and that the system operator returns the profit back to electricity consumers and producers as a lump-sum transfer.
sum of the expected social price of the line, minus the cost of total transfers, and the utility of firm:

$$\Phi(K > 0) = x_{12} V_1(K) + (1 - x_{12}) V_0 - (1 + \mu) [C + t] + U$$

where

$$V_1(K) = x_{13} SW_{11}(K) + (1 - x_{13}) SW_{10}(K)$$: the expected social value when line (1,2) is 'up'.

$$V_0 = x_{13} SW_{01} + (1 - x_{13}) SW_{00}$$: the expected social value when line (1,2) is 'down'.

The following lemma presents the non-convexity in the objective function.

**Lemma 1.** The social value function, $V_1(K)$, has a discontinuity at $K = 0$.

Proof: In Appendix.

The intuition of this result can be stated as follows. When the line (1,2) does not exist, generators at nodes 1 and 2 run according to the unconstrained economic dispatch rule. Our assumption on the generation cost implies that this corresponds to generating $m$ times as much electricity at the cheap generator as at the expensive generator. Moreover, this dispatch is exactly the same as the dispatch when the line (1,2) exists but is not congested. Hence, the social welfare\(^{67}\) when the line does not exist is the same as that when the line

---

\(^{67}\)To avoid any confusion, we should note that the social welfare here corresponds to the
exists but is not congested. We also know that social welfare increases with line capacity, and does not make a negative jump when the line becomes non-congested. Therefore, there must be a discontinuity in social welfare function at $K = 0$.

We can re-write the social welfare function by using the definition of the utility of the firm (when the line capacity is strictly positive) as

$$\Phi(K > 0) = x_{12}V_1(K) + (1 - x_{12})V_0 - (1 + \mu) [C + \psi(e)] - \mu U$$

Social welfare in the absence of line $(1, 2)$ is

$$\Phi(K = 0) = V_0$$

The next section will establish the benchmark case, in which the regulator observes everything. Then, we will be able to see the consequences of asymmetric information on regulated variables.

---

68 More precisely, an increase in line capacity increases social welfare when the line is congested, and does not have any impact when the line is not congested.
3.5.2. Complete Information

The regulator observes both $e$ and $\beta$.\(^{69}\) It is possible to choose not to have the line, i.e. $K = 0$, which creates the non-convexity. The solution strategy is first to solve for the optimum when $K > 0$, and then to compare the implied social welfare to the one when $K = 0$.

Therefore, the regulator chooses $U(\beta), e(\beta), K(\beta), x_{12}(\beta)$ given\(^{70}\) $\beta$ to solve

$$\text{Max } x_{12} V_1(K) + (1 - x_{12}) V_0 - (1 + \mu) [C + \psi(e)] - \mu U$$

s.to $U(\beta) \geq 0 \quad \forall \beta$

Let $\lambda = x_{13} \lambda_1(K) + (1 - x_{13}) \lambda_0(K)$ be the expected shadow price of line (1, 2) over the states of line (1, 3). Then, the optimum must satisfy the following FOC’s\(^{71}\) for an interior solution (when the line exists)\(^{72}\):

$$U(\beta) = 0 \quad \forall \beta$$ (3.3)

\(^{69}\)This is only a sufficient condition for obtaining complete information. The necessary and sufficient condition is to observe either $e$ or $\beta$, because the regulator can infer the unobserved one by observing only one of them.

\(^{70}\)Note that $\beta$ is observed under complete information.

\(^{71}\)These FOC’s do not necessarily imply the uniqueness of the solution. The required conditions for uniqueness involve restrictions on the third derivatives of the cost function, hence they are too restrictive, and will be omitted. Interested readers are referred to Kamien and Schwarz [12] (pp. 204-211), Arrow and Kurz [1] (pp. 43-51), and Seierstad and Sydsaeter [19] (pp. 367-391) for some sufficiency results in optimal control problems.

\(^{72}\)In order to assure $K > 0$, assume that the benefit of having infinitesimal capacity exceeds the fixed cost, i.e. $\lim_{K \to 0} (x_{12} \cdot V_1(K) - (1 + \mu) \cdot C_0) > 0$.
\[
\psi'(e) = -C_e\tag{3.4}
\]
\[
x_{12} \lambda(K) = (1 + \mu) C_K\tag{3.5}
\]
\[
V_1(K) - V_0 = (1 + \mu) C_{x_{12}}\tag{3.6}
\]

The first equation says that the firm must get its reservation payoff (which is assumed to be zero without any loss of generality), i.e. the transfer must be equal to the disutility from expending the optimal effort level. Since the type of firm is known to the regulator, the firm cannot earn any information rent. Equation 3.4 shows that the effort level must be chosen such that the marginal disutility is equal to the decrease in cost due to increased effort. Equation 3.5 says the line capacity must be at the level where the marginal social value, \(x_{12} (x_{13} \lambda_1(K) + (1 - x_{13})\lambda_0(K))\), is equal to the social marginal cost of capacity, which takes into account the fact that transfers to the firm are costly to the society. Finally, equation 3.6 is the FOC for line reliability. The marginal benefit of reliability is the difference in value to the society of having the line 'up' and 'down', i.e. \(V_1(K) - V_0\).

Social welfare when \(K > 0\) must be compared to that when \(K = 0\). Hence, the regulator chooses to have the line with type \(\beta\) if
\[ x_{12}^* V_1(K^*) + (1 - x_{12}^*) V_0 - (1 + \mu) [C^* + \psi(e^*)] \geq V_0, \text{ or} \]
\[ \Phi(K^* > 0) \geq \Phi(K = 0) \]

where stars denote optimal values under complete information.

Before going any further, we can discuss the contribution of this analysis with endogenous (compared to exogenous) reliability using FOC's 3.5 through 3.6. As opposed to having an arbitrary level of reliability, equation 3.6 tells us that reliability should be chosen by considering the additional benefit and cost at the margin. But more importantly, equation 3.5 implies that the optimal choice of line capacity potentially depends on reliability. Although a higher reliability may or may not imply a higher capacity, there is a relation between them in general. Under some conditions, the capacity may be independent of reliability. The following lemma gives a sufficient condition for this independence.

**Lemma 2.** The first-best capacity level is independent of the line reliability if

i-) reliability is non-zero, and ii-) the cost function is such that

\[ C = I(\beta, e) + J(K, x_{12}), \text{ where } \frac{\partial}{\partial K} J(K, x_{12}) = x_{12} \ G(K) \text{ for any functions } I \text{ and } G. \]

**Proof:** Under the second condition, the equation 3.5 can be written as

\[ \text{More precisely, for functions } I \text{ and } G \text{ that satisfy Second-Order Conditions.} \]

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\[ x_{12} \lambda(K) = (1 + \mu) x_{12} G(K). \]

Since \( x_{12} \neq 0 \), then the equation reduces to

\[ \lambda(K) = (1 + \mu) G(K), \]

which is independent of \( x_{12} \). Moreover, the separability of the cost function implies that the FOC for effort (equation 3.4) is only determined by effort, \( e \). Therefore, the optimal capacity is independent of reliability. QED.

This result is quite intuitive. If reliability affects the marginal benefit and marginal cost of capacity in the same way, then its level will not be decisive in the choice of capacity.

3.5.3. Incomplete Information

After finding the complete information optimum, we now analyze the optimal regulation when the regulator does not observe \( e \) and \( \beta \).

Let \( E(\beta, C, K, x_{12}) \) be the effort level required for a firm of type \( \beta \) to supply capacity \( K \) with reliability \( x_{12} \) at cost \( C \). Our assumptions for the cost function imply

\[ E_\beta > 0, \quad E_C < 0, \quad E_K > 0, \quad \text{and} \quad E_{x_{12}} > 0. \]

Since the regulator does not observe the type of the firm, optimal regulation
is chosen by maximizing expected social welfare over possible types of the firm.

A truth-telling mechanism requires two types of constraints: incentive compatibility (IC), and individual rationality (IR). The first one requires the firm to reveal its type truthfully, and the second guarantees participation of all types of firms. IC constraints can be represented by the FOC of the firm's problem under some sufficiency conditions as stated in the following Lemma.

Lemma 3. IC constraint can be replaced by $\dot{U}(\beta) = -\psi'(e) E_\beta$, if

i) $C_{ee}, C_{e\beta}, C_{eK}, C_{ex12}, C_{\beta K}, C_{\beta x12}$ are all non-negative, and

ii) $C(\beta) \geq 0$, $K(\beta) \leq 0$, $x_{12}(\beta) \leq 0$ at the optimum.

Proof: (In Appendix)

The original IR constraint is to give at least zero utility to any type, i.e. $U(\beta) \geq 0$. But we can replace it with $U(\beta) \geq 0$, because $\dot{U}(\beta)$ is negative.\footnote{A formal representation of IC constraints can be stated as follows: let $\varphi(\beta, \beta')$ denote the expected utility of a type $\beta$ firm when it reports $\beta'$ to the regulator. Then, a truth-telling mechanism is such that for each type $\beta$, $\varphi(\beta, \beta') \geq \varphi(\beta, \beta') \forall \beta$.}

\footnote{The firm reports its type to the regulator in order to maximize its payoff. If that problem implies a unique solution, then the firm's behaviour can be represented by the FOC.}

\footnote{$C_{ee} \geq 0$ means that the cost reducing effect of effort does not increase as the line capacity gets larger. $C_{\beta K} \geq 0$ says the marginal cost of capacity is not less for a less productive firm.}

\footnote{These conditions are not imposed, but rather are to be checked at the optimum to guarantee the uniqueness of the solution for the firm's problem. In fact, these are sufficient but not necessary conditions. We need only $\varphi_{21}(\beta, \beta') \geq 0$, which can be satisfied by weaker conditions on $C, K$, and $x_{12}$. See Mirrlees [14] (pp. 3-20) and Rogerson [17] (pp. 1361-1367) for further analyses on the validity of the First-Order Approach to solve Principal-Agent problems.}

\footnote{This can be seen from the FOC of the firm's problem, $\dot{U}(\beta) = -\psi'(e) \cdot E_\beta$. Since $E_\beta > 0$}
The regulator chooses $U(\beta), e(\beta), K(\beta), x_{12}(\beta)$ to maximize the expected value (over types) of the social welfare function $\Phi$ subject to the IC and IR constraints:

$$\max_{\beta} \int_{\beta} \{x_{12}V_1(K) + (1 - x_{12})V_0 - (1 + \mu) [C + \psi(e)] - \mu U \} f(\beta) d\beta$$

subject to:

1. $U(\bar{\beta}) \geq 0$ (3.7)
2. $\dot{U}(\beta) = -\psi'(e) \cdot E_\beta$ (3.8)

Again, the problem is solved for $K > 0$, hence this should be checked after finding the solution. The implied social welfare must again be compared to the social welfare when $K = 0$.

Let $U$ be the state variable, $e, K, x_{12}$ be the control variables, and $\eta$ be the co-state variable. Then, we have an optimal control problem with a bounded state variable. At the optimum, $U(\bar{\beta}) = 0$ due to the transversality condition\textsuperscript{79}.

\textsuperscript{79}Either the co-state variable or the state variable must be zero at the terminal point, $\bar{\beta}$. The co-state variable takes a positive value, hence $U(\bar{\beta})$ must be zero. To see that, take the FOC for the state variable, which is

$$\dot{\eta}(\beta) = \mu \cdot f(\beta),$$

and integrate it over $\beta$, and get $\eta(\beta) = \mu \cdot F(\beta)$ for any $\beta$. Then, $\eta(\bar{\beta}) > 0$. 

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Hence, we can ignore constraint 3.7. The FOC's for the interior solution\textsuperscript{80} become:

\[
U(\beta) = 0
\]  
(3.9)

\[
\psi'(e) + C_e = -\frac{\mu}{1 - \mu} \frac{F(\beta)}{f(\beta)} [\psi'' E_\beta + \psi' E_{\beta C} C_e]
\]  
(3.10)

\[
(x_{12} \lambda(K) - (1 + \mu) C_K) = \frac{\mu F(\beta)}{f(\beta)} \psi' \frac{d}{dK} E_\beta
\]  
(3.11)

\[
(V_1(K) - V_0 - (1 + \mu) C_{x_{12}}) = \frac{\mu F(\beta)}{f(\beta)} \psi' \frac{d}{dx_{12}} E_\beta
\]  
(3.12)

Let’s denote the variables satisfying these FOC’s by $\tilde{U}, \tilde{e}, \tilde{K},$ and $\tilde{x}_{12}$. Equation 3.9 shows the information rent for the least efficient firm $(\beta = \beta)$ under the optimal regulation. That type of firm does not receive any information rent because it is unable to act like a less efficient firm by reducing its effort level. However, all other types $(\beta < \beta)$ have that imitation ability, hence they get a positive rent. We can see this by looking at equation 3.8, which implies that $U(\beta) < 0$.

For all types of firms except the most efficient one, the effort level under

\textsuperscript{80}The Second-Order conditions will be omitted, since they are too restrictive. The interested reader can be referred to Kamien and Schwarz [12] and Arrow and Kurz [1] for some sufficiency results.
incomplete information is less than the level under complete information when all other variables have the same values. This can be seen by noting that equation 3.10 differs from equation 3.4 only by the RHS of the equation. For $\beta > \bar{\beta}$, $-[\psi^\prime \cdot E_{\beta} + \psi^\prime \cdot E_{\beta C} \cdot C_e] < 0$ represents the change in the total rent received by types within $[\beta, \bar{\beta}]$ due to an increase in the effort level of type $\beta$. This has a probability $F(\beta)$, and each unit of increase in rent costs $\mu$ to society. This must be weighed against the distortion created by moving away from the first-best effort level, which is determined by solving $\psi^\prime(e) = -C_e$. The distortion creates a social cost of $(\psi^\prime(e) + C_e)(1 + \mu)$ for type $\beta$, which happens with probability $f(\beta)$. Therefore, the second-best effort level is determined by the balance of these two costs, as shown in equation 3.10.

The capacity level is determined by equation 3.11. Similar to the analysis of effort level, an increase in capacity for type $\beta$ changes the total rent of types within $[\beta, \bar{\beta}]$ by $\psi^\prime \cdot \frac{d}{dK} E_{\beta}$, which occurs with probability $F(\beta)$. Moreover, moving away from the first-best level of capacity for type $\beta$ creates distortion, which costs $(x_{12}\lambda(K) - (1 + \mu) \cdot C_K)$ with probability $f(\beta)$.

Equation 3.12 shows the rule for selecting the level of reliability. The RHS of

\[81\text{In the special case of } C_{eK} = C_{ex12} = 0, \text{ we do not need the last reservation.}\]

\[82\text{This term becomes negative due to our previous assumptions: } E_{\beta} > 0, \ C_{ee} \geq 0, \text{ and } C_{\beta e} \geq 0.\]
the equation includes the marginal cost to society of increased rents, and LHS includes the marginal cost of distortion due to incomplete information.

The last comment about FOC's concerns the comparison of the second-best outcome among different types of firms. The first-best levels of effort, capacity, and reliability are obtained when the firm is the most efficient one ($\beta = \bar{\beta}$), but at the expense of rewarding the highest information rent. For less efficient types, the optimal regulation implies a lower information rent and more distortion. In the extreme case of the least efficient type with $\beta = \underline{\beta}$, the information rent is zero and the distortion is the highest.

The following result establishes the condition under which the first-best capacity and reliability levels can be obtained for all types of firms:

**Proposition 1.** (From Laffont and Tirole [13]) The optimal regulation under incomplete information yields the first-best capacity and reliability levels ($\tilde{K} = K^*$, and $\bar{x}_{12} = x_{12}^*$) if and only if there is a function $\xi$ such that

$$C = C(\xi(\beta, e), K, x_{12})$$

**Proof:**

We can write the cost function as $C = C(\beta, E(\beta, C, K, x_{12}), K, x_{12})$. Differentiating with respect to $\beta$ yields

$$0 = C_\beta + C_e E_\beta, \text{ or } E_\beta = -\frac{C_\beta}{C_e}.$$
Hence, \( \frac{d}{dK} E_\beta = \frac{\partial}{\partial K} \left( -\frac{C_\theta}{C_e} \right) \), and \( \frac{d}{dx_{12}} E_\beta = \frac{\partial}{\partial x_{12}} \left( -\frac{C_\theta}{C_e} \right) \).

But the separability assumption in the proposition implies that \( \frac{C_\theta}{C_e} = \frac{\xi_\theta}{\xi_e} \), which is independent of \( K \) and \( x_{12} \). Therefore, \( \frac{d}{dK} E_\beta = \frac{d}{dx_{12}} E_\beta = 0 \), meaning that the first-best capacity and reliability levels are obtained in equations 3.11 and 3.12. The “only if” part of the proposition can be proved similarly.

(For the original proof, see Proposition 3.4 in Laffont and Tirole [13] ■)

If the cost function takes this special form, then capacity and reliability do not affect the firm’s ability to obtain information rent\(^{83}\). In general, the regulator uses observable variables \((C, K, x_{12})\) as instruments in order to limit this information rent. But the special functional form of the cost function in the proposition makes it impossible for the regulator to use \( K \) and \( x_{12} \) for that purpose because the relation between \( \beta \) and \( e \) is fixed by the known function \( \xi \). Hence, we obtain a dichotomy between the selection of \( K \) and \( x_{12} \) on one hand, and the regulation of effort level on the other hand.

Algebraically, this independence is satisfied if we have \( \frac{d}{dK} E_\beta = \frac{d}{dx_{12}} E_\beta = 0 \). Then, equations 3.11 and 3.12 reduce to the FOC’s for \( K \) and \( x_{12} \) under the complete information case. Therefore, separability of the cost function in

\(^{83}\)Note that a firm can act like a less efficient type by reducing its effort such that the observed cost remains the same.
the form given in the proposition enables the regulator to obtain the first-best capacity and reliability levels even under incomplete information.

Another implication of the special functional form of cost function is related to the implementation of the optimal capacity and reliability. In general, transfer to the firm depends on all observable variables. However, the dichotomy that we mentioned above implies that the firm can be rewarded only as a function of the realization of $\xi(\beta, e)^{84}$. Hence, the optimal transfer function takes the form of $t = T(\xi)$ for the appropriate $T(\cdot)$ function to be determined in the implementation phase.

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84 Even though the regulator cannot separately observe $\beta$ and $e$, their aggregate contribution $\xi$ is inferred from the observable variables $C$, $K$ and $x_{12}$. 

86
Do We Need the Line at All?

The analysis so far has concentrated on finding the optimal capacity and reliability levels if the regulator has already decided to build the line. The more fundamental question is whether building the line is more beneficial to society than not building it. Due to the non-convexity of the problem at \( K = 0 \), we were unable to answer this question when we solved the optimal regulation problem under incomplete information. However, the solution that we obtained for the optimal regulation will be used in deciding whether to build the line or not. In particular, building the line is optimal if the expected value (over types) of the social welfare function evaluated at the optimum (\( \widetilde{K} > 0 \)) is greater than the social welfare of not having the line, i.e. if

\[
\int_0^\beta \Phi(\widetilde{K}) f(\beta) \, d\beta > \Phi(K = 0) = V_0.
\]

Intuitively speaking, a line should be built between nodes 1 and 2 if sending electricity to node 3 is not reliable enough with the existing network. The next result formalizes this intuition.

**Proposition 2.** The regulator is more likely to have a line between nodes 1 and 2 as the reliability of line (1, 3) decreases. More precisely, let \( \mathcal{A}(x_{13}) \) be the set of technologies and parameters for which line (1, 2) is desirable for each level of \( x_{13} \). Then for \( \overline{x}_{13} < \overline{x}_{13} \), \( \mathcal{A}(\overline{x}_{13}) \subset \mathcal{A}(\overline{x}_{13}) \).
Proof: In Appendix.

One reason for not building line (1, 2) seems to be the high generation cost that has to be incurred in case of congestion. If you build that line, then the expensive generator at node 2 has to be used more than optimally when the line is congested. This increases the cost of supplying electricity. However, one must also consider the fact that the demand has to be met by only the expensive generator when line (1, 3) fails and line (1, 2) is not there. The following result presents that trade off.

Proposition 3. The regulator is more likely to have a line between nodes 1 and 2 as the marginal cost of generator 2 increases (due to an increase in parameter $m$) if i) line (1, 2) is not congested, or ii) the reliability of line (1, 3) is not too high. More precisely, let $B(c_2)$ be the set of technologies and parameters for which line (1, 2) is desirable for each level of $c_2$. Then for $\bar{c}_2 < \bar{c}_2$, $B(\bar{c}_2) \subset B(\bar{c}_2)$ under the two given conditions.

Proof: In Appendix.

If we know\footnote{We 'know' whether the line will be congested or not because the optimal choice of capacity (given the line is there) determines the existence of congestion.} that the line will not be congested when we build the optimal capacity, then we use generators up to the point where their marginal costs are
An increase in the marginal cost of generator 2 will affect the social welfare the same whether line (1, 2) is there or not. On the other hand, if we know that the line will be congested, then we need to compare the benefit of not being stuck with the expensive generator to the increased electricity cost due to congestion. In this case, the benefit outweighs cost if the probability of being in the state when generator 2 is the only available option is high enough, i.e. \((1 - x_{13})\) is high enough.

One might think that the need for extra lines on the network increases as the demand for electricity grows. The following result establishes the relation between the magnitude of demand at node 3 and the decision to build the line (1, 2).

**Proposition 4.** An increase in demand at node 3 (due to an increase in demand parameter \(a\)) makes it more likely to choose building the line if i-) the optimal capacity (given the line is built) does not imply congestion, or ii-) reliability of line (1, 3) is not too high. More precisely, let \(D(a)\) be the set of technologies and parameters for which line (1, 2) is desirable for each level of \(a\). Then for \(\bar{a} < \tilde{a}\), \(D(\bar{a}) \subset D(\tilde{a})\) when the line is congested.

**Proof:** In Appendix.

The following result compares the decision to build a line under complete
and incomplete information.

**Proposition 5.** If it is optimal not to build line $(1,2)$ under complete information, then the same is true under incomplete information.

*Proof:* In Appendix.

If building a line is not optimal for society under incomplete information, then the maximum value of social welfare with the line has to be less than social welfare without the line. The addition of incomplete information to this picture adds incentive compatibility and individual rationality constraints. Therefore, maximum value of social welfare in this constrained problem has to be less than the maximum value in the unconstrained problem. This is why the non-optimality of building a line under complete information implies non-optimality of building it under incomplete information.

An interesting question is whether it is of benefit to society to have a line which is not congested, i.e., the power flowing on the line is less than the line capacity. In a static analysis of investment like ours, building an extra unit of capacity which will not be used to transmit power does not benefit society, yet it has a cost. This straightforward reasoning implies that the line capacity should not be more than the amount of electricity flowing on it. This intuition is correct under complete information, but not necessarily under incomplete information.
When the regulator cannot observe type and effort of the firm, the optimal capacity might imply a non-congested line for a large class of cost functions. Specifically, when the cost of building a line is not separable in the sense of the condition given in Proposition 1, then the regulator uses capacity and reliability as instruments in order to affect the information rent of the firm. If an increase in capacity can be used to reduce the firm's information rent, then the optimal capacity can be more than the power flowing on the line. The next proposition formalizes that result.

**Proposition 6.**

i-) Under complete information, line (1, 2) must be congested in some states of the world. More precisely, the shadow price of line (1, 2) must be strictly greater than zero in state 10 when $x_{13} < 1$.

ii-) Under incomplete information, the line can be non-congested in all states of the world if an increase in capacity reduces the firm's information rent. More precisely, the shadow price in state 10 can be equal to zero if $\frac{d}{dK} E_\beta < 0$ and $\beta > \underline{\beta}$.

**Proof:** In Appendix.
3.5.4. An Illustration

In this section, we present a numerical example to illustrate some of the results derived in the paper. We assume the line construction cost takes the following functional form:

\[ C = 100 - e + \frac{1+\beta}{2} K^2 x^2_{12} \]

The disutility of effort to the firm is assumed to be

\[ \psi(e) = \frac{e^2}{2} \]

For ease of exposition, suppose that there are only two types of firms (efficient type and inefficient type) instead of a continuum of types as assumed in the paper. The firm is of type \( \beta \) (efficient) with probability \( p \), and \( \bar{\beta} \) (inefficient) with probability \( 1 - p \), where \( \beta < \bar{\beta} \). Then, we can obtain the FOC's similar to equations 3.9-3.12 under incomplete information as follows\(^{86}\):

\[
U(\beta) = \frac{e^2(\bar{\beta})}{2} - \frac{1}{2} \bar{\beta}^2 > 0 \\
U(\bar{\beta}) = U(\beta) = 0 \\
e(\beta) = 1 - e(\beta) = 0 \\
e(\bar{\beta}) = 1 - e(\beta) = \frac{p}{1-p} \frac{\bar{\beta} - \beta}{1 + \mu} K^2(\bar{\beta}) x_{12}(\bar{\beta})
\]

\(^{86}\)A sketch for the derivation of FOC's for this discrete-type example is given in appendix.
\[ K(\bar{\beta}) : x_{12}(\bar{\beta}) V'_1(K(\bar{\beta})) - (1 + \mu)(1 + \bar{\beta})K(\bar{\beta}) x_{12}^2(\bar{\beta}) = 0 \]

\[ K(\bar{\beta}) : x_{12}(\bar{\beta}) V'_1(K(\bar{\beta})) - (1 + \mu)(1 + \bar{\beta})K(\bar{\beta}) x_{12}^2(\bar{\beta}) = \mu \frac{p}{1 - p} \frac{\bar{\beta} - \beta}{2} K(\bar{\beta}) x_{12}^2(\bar{\beta}) \Gamma \]

\[ x_{12}(\bar{\beta}) : V_1(K(\bar{\beta})) - V_0 - (1 + \mu)(1 + \bar{\beta})K^2(\bar{\beta}) x_{12}(\bar{\beta}) = 0 \]

\[ x_{12}(\bar{\beta}) : V_1(K(\bar{\beta})) - V_0 - (1 + \mu)(1 + \bar{\beta})K^2(\bar{\beta}) x_{12}(\bar{\beta}) = \mu \frac{p}{1 - p} \frac{\bar{\beta} - \beta}{2} K^2(\bar{\beta}) x_{12}(\bar{\beta}) \Gamma \]

where \( \Gamma = e(\bar{\beta}) - \frac{\bar{\beta} - \beta}{2} K^2(\bar{\beta}) x_{12}^2(\bar{\beta}) \) is the marginal disutility of potential reduction in effort level for the efficient type (type \( \beta \)) by mimicking the inefficient type (or by reporting \( \bar{\beta} \) as its type).

The parameters of the problem are assumed to take the following values throughout the example unless otherwise stated:

\[ a = 50, \quad b = 0.2, \quad m = 1.5, \quad x_{13} = 0.8, \quad \mu = 0.5, \quad \beta = 0.2, \quad \bar{\beta} = 0.21, \quad p = 0.1 \]

As a reminder, \( a \) and \( b \) are parameters of electricity demand, and \( m \) measures how expensive the marginal cost of generation at node 2 relative to the one at node 1. The parameter \( \mu \) is the cost to society of raising funds in order to make transfers to the line owner, and \( \beta \) measures the type of the line owner (the higher the \( \beta \), the less productive the firm). Finally, \( p \) is the proportion of efficient types (type \( \beta \)) of firms.
First of all, we will illustrate Proposition 1 by showing that the optimal capacity and reliability under complete information are not the same as the ones under incomplete information.

<table>
<thead>
<tr>
<th>Complete Information</th>
<th>Incomplete Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K(\beta)$</td>
<td>$x_{12}(\beta)$</td>
</tr>
<tr>
<td>$a = 20$</td>
<td>3.000</td>
</tr>
<tr>
<td>$a = 150$</td>
<td>13.006</td>
</tr>
</tbody>
</table>

(The table continued from page 93)

The first thing to note is that the efficient firm supplies the same amount of capacity and reliability under both complete and incomplete information. This first-best outcome for the efficient type is obtained at the expense of giving information rent ($U(\bar{\beta}) > 0$) to that type. On the other hand, the regulator does not leave any rent to the inefficient type ($U(\bar{\beta}) = 0$) at the expense of not getting the first-best levels of capacity and reliability.

Moreover, we observe that the inefficient type does not necessarily supply lower reliability compared to the efficient type under incomplete information. When the demand for electricity is such that $a = 20$, the inefficient type builds a line with a perfect reliability of 1.000, as opposed to an inferior reliability of 0.588 by the efficient type. The values given in Table 1 are optimal assuming
that we have already decided to build a line. However, it is optimal not to build
the line when \( a = 20 \), and optimal to build when \( a = 150 \).

We will now illustrate Proposition 2, which is about the sensitivity of the
decision to build a line to the reliability of another line on the network. As we
can see in the second column of Table 2, the advantage\(^{87}\) of having a line between
nodes 1 and 2 diminishes as the reliability of line \((1, 3)\), \( x_{13} \), increases. In fact,
it is optimal to build the line when \( x_{13} \leq 0.374 \), but not when \( x_{13} \geq 0.374 \) (not
shown on the table). Moreover, the optimal reliability (assuming we want to
build it) of line \((1, 2)\) decreases as the reliability of line \((1, 3)\) increases. Finally,
a look at the results for \( x_{13} = 1.0 \) reveals that the optimal reliability (given the
line is built) for line \((1, 2)\) is zero. It means that we cannot improve welfare
by building a *reliable*\(^{88}\) line between nodes 1 and 2 when the line connecting 1
and 3 never fails. In this case, the new line will only create congestion, but no
improvement in network reliability.

\(^{87}\)The advantage is defined as the difference in welfare when we have the line with optimal
capacity versus the welfare when we do not build the line.

\(^{88}\)We use the word *reliable* for any reliability level greater than zero.
The next illustration is for Proposition 3, which analyzes the relation between the decision to build a line and the marginal generation cost of the expensive generator on the network. The result depends partly on whether the existing lines on the network are reliable enough. Therefore, we present two cases, one with $x_{13} = 0.7$ (low reliability of existing lines), and another with $x_{13} = 0.99$ (high reliability). In the former case, an increase in the marginal generation cost (an increase in $m$ from 2.3 to 2.4) makes the regulator more likely to build the line (1,2). This can be seen from the second column of Table 3, where the difference in welfare of having and not having the line decreases. But in the latter case, an increase in $m$ makes it less likely to have the line. This is because of the trade-off between being stuck with the expensive generator when line (1,3) fails versus having to use the expensive generator when line (1,2) is congested. As $x_{13}$ increases, the probability of being stuck with the expensive generator
falls, hence the regulator is concerned more with congestion implications.

When \( x_{13} = 0.7 \), we have

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
m & \Phi(\bar{K} > 0) - \Phi(K = 0) & K(\beta) & K(\bar{\beta}) & x_{12}(\beta) & x_{12}(\bar{\beta}) & U(\beta) \\
\hline
2.3 & -106.012 & 7.050 & 7.050 & 0.987 & 0.979 & 0.208 \\
2.4 & -106.008 & 7.300 & 7.300 & 0.953 & 0.945 & 0.208 \\
\hline
\end{array}
\]

(Table 8)

When \( x_{13} = 0.99 \), we have

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
m & \Phi(\bar{K} > 0) - \Phi(K = 0) & K(\beta) & K(\bar{\beta}) & x_{12}(\beta) & x_{12}(\bar{\beta}) & U(\beta) \\
\hline
2.3 & -149.202 & 8.337 & 7.313 & 0.027 & 0.031 & 0.001 \\
2.4 & -149.250 & 6.166 & 7.357 & 0 & 0 & 0 \\
\hline
\end{array}
\]

(Table 9)
3.6. Conclusion

This paper formulates the optimal regulation of investment and reliability of transmission lines in a deregulated electricity market set-up. The first contribution of this paper is to analyze the influence of line reliability as an endogenous variable in a regulatory framework under incomplete information. The second is to consider the structure of the network as endogenous by investigating whether to build a line between two nodes of the network.

Under certain assumptions, I show that the regulator can design a contract for the line owner so that the first-best capacity and reliability levels are obtained. Moreover, I find that the existence of a line is justified when reliability of other lines on the network is not too high. In an extreme case when all other lines on the network are perfectly reliable with enough capacities, there is no need to connect some nodes by building additional lines. Such extra lines will only increase the possibility of network congestion without improving the reliability of network. However, when some of the other lines do not have enough capacity or when they are not perfectly reliable, society may get benefit from additional lines.

Another result argues that under some conditions building a line between the cheap and the expensive generator is more advantageous as the marginal
cost of the expensive generator increases. I further argue that the optimality of building a line with the expectation of a growing demand in the future depends on whether we expect the new line to be congested, and also on the reliability of other lines. Finally, I analyze whether it is socially optimal to have a non-congested line under complete and incomplete information structures. It is always optimal to have a congested line under complete information, but that result does not always hold under incomplete information.

I plan to check the generality of these results under more complex network structures in my future studies. Moreover, I will study the implementation of the optimal regulation contract, because my analysis so far has only provided a general characterization of the outcome of regulation, rather than specific reward formulas to be used in real world applications.
3.7. Appendix

Results for the optimal dispatch

The following table gives social welfare evaluated at the optimal dispatch for given line capacity in three states of the world.

<table>
<thead>
<tr>
<th>State</th>
<th>(0 &lt; K &lt; \frac{a(m-1)}{3(m+b+bm)})</th>
<th>(\frac{a(m-1)}{3(m+b+bm)} &lt; K &lt; \frac{ma}{m+b+bm})</th>
<th>(K &gt; \frac{ma}{m+b+bm})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(SW_{11})</td>
<td>(\frac{4a^2+3K(2a(m-1)-3K(m+b+bm))}{2(1+4b+m)})</td>
<td>(\frac{a^2(1+m)}{2(m+b+bm)})</td>
<td>(\frac{a^2(1+m)}{2(m+b+bm)})</td>
</tr>
<tr>
<td>(SW_{10})</td>
<td>(\frac{a^2+K(2ma-K(b+m+bm))}{2(b+m)})</td>
<td>(\frac{a^2+K(2ma-K(b+m+bm))}{2(b+m)})</td>
<td>(\frac{a^2(1+m)}{2(m+b+bm)})</td>
</tr>
<tr>
<td>(SW_{01})</td>
<td>(\frac{a^2(1+m)}{2(m+b+bm)})</td>
<td>(\frac{a^2(1+m)}{2(m+b+bm)})</td>
<td>(\frac{a^2(1+m)}{2(m+b+bm)})</td>
</tr>
<tr>
<td>(SW_{00})</td>
<td>(\frac{a^2}{2(b+m)})</td>
<td>(\frac{a^2}{2(b+m)})</td>
<td>(\frac{a^2}{2(b+m)})</td>
</tr>
</tbody>
</table>

- \(SW_{00} < SW_{10} < SW_{11} < SW_{01}\)
- \(SW_{11} = SW_{01}\) if \(\lambda_1 > 0\)
- \(SW_{10} = SW_{00}\) if \(\lambda_0 > 0\)
- \(V_1 > V_0\) if \(x_{13} < \frac{SW_{10} - SW_{00}}{SW_{10} - SW_{00} + SW_{01} - SW_{11}}\)

Table A1

The first interval for the line capacity corresponds to congestion in state 11 and 10. The second one refers to no congestion in state 11, and congestion in state 10. The congestion does not happen in either state when the capacity is in the last interval. Note that the expected benefit of having line (1, 2) does not
always exceed the expected benefit of not having it ($V_1$ vs. $V_0$). For capacity levels less than $\frac{a(m-1)}{3(m+b+bm)}$, the comparison depends on the reliability of line (1, 3). This is the interval of capacity levels for which the price at the demand node exceeds the marginal cost of the cheapest generator when both lines are 'up'.

**Example 4:** When $m = 1.5$, $a = 600$, $b = 0.2$, we have the following results:

<table>
<thead>
<tr>
<th></th>
<th>$0 &lt; K &lt; 50$</th>
<th>$50 &lt; K &lt; 450$</th>
<th>$K &gt; 450$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SW_{11}$</td>
<td>$218182 + \frac{600K-6K^2}{2.2}$</td>
<td>$225000$</td>
<td>$225000$</td>
</tr>
<tr>
<td>$SW_{10}$</td>
<td>$105882 + \frac{1800K-2K^2}{3.4}$</td>
<td>$105882 + \frac{1800K-2K^2}{3.4}$</td>
<td>$225000$</td>
</tr>
<tr>
<td>$SW_{01}$</td>
<td>$225000$</td>
<td>$225000$</td>
<td>$225000$</td>
</tr>
<tr>
<td>$SW_{00}$</td>
<td>$105882$</td>
<td>$105882$</td>
<td>$105882$</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>$\frac{300-6K}{1.1}$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>$\frac{900-2K}{1.7}$</td>
<td>$\frac{900-2K}{1.7}$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

*(Table A2)*

A graphical representation of the comparison of social welfare in each state of the world is given in Figure 7 in the following page.
Proof of Lemma 1:

When line (1, 2) is not built, expected social value is

$$x_{13} \, SW_{01} + (1 - x_{13}) \, SW_{00}.$$

Once infinitesimal capacity is there, the expected social value becomes

$$\lim_{K \to 0} x_{13} \, SW_{11}(K) + (1 - x_{13}) \, SW_{10}(K).$$

From the first table in Appendix, we can see that

$$\lim_{K \to 0} SW_{10}(K) = SW_{00}, \text{ but } \lim_{K \to 0} SW_{11}(K) < SW_{01}.$$

Therefore, the social value function has a discontinuity at \( K = 0 \). ■

Proof of Lemma 3:

A firm with type \( \beta \) announces a type, \( \hat{\beta} \), (which may not be the true type) to maximize its expected utility:

$$\varphi(\beta, \hat{\beta}) = t(\hat{\beta}) - \psi \left( E \left[ \beta, C(\hat{\beta}), K(\hat{\beta}), x_{12}(\hat{\beta}) \right] \right) \quad (3.13)$$

Then, the FOC is \( \varphi_2(\beta, \hat{\beta}) = 0 \). Hence, a truth telling mechanism must satisfy

$$\varphi_2(\beta, \beta) = 0.$$

Now suppose that \( \hat{\beta} = \beta \) is not the global optimum, or \( \hat{\beta} \neq \beta \). Then,

$$\varphi(\beta, \hat{\beta}) > \varphi(\beta, \beta), \text{ or}$$

$$\int_{\beta}^{\hat{\beta}} \varphi_2(\beta, x) \, dx > 0.$$
But we can re-write this by subtracting $\varphi_2(\beta, \beta) = 0$ as

$$
\int_{\beta}^{\beta} \int \varphi_{21}(y, x) dy dx > 0
$$

(3.14)

We can use equation 3.13 to get

$$
\varphi_{21}(\beta, \tilde{\beta}) = \frac{\partial}{\partial \beta} [-\psi'(e) E_\beta]
$$

$$
= -\psi'' E_\beta \left[ E_C \dot{C} + E_K \dot{K} + E_{x_12} \dot{x}_{12} \right] - \psi' \left[ E_{\beta C} \dot{C} + E_{\beta K} \dot{K} + E_{\beta x_{12}} \dot{x}_{12} \right]
$$

If $E_{\beta C} \leq 0$, $E_{\beta K} \geq 0$, $E_{\beta x_{12}} \geq 0$ (which are equivalent to the conditions in part (i) of the lemma) and if conditions in part (ii) holds, then $\varphi_{21}(\beta, \tilde{\beta}) \geq 0$.

Now consider two cases: $\tilde{\beta} < \beta$, and $\tilde{\beta} > \beta$. In the first case $x \in [\tilde{\beta}, \beta]$, which contradicts equation 3.14. In the second case $x \in [\beta, \tilde{\beta}]$, which again contradicts equation 3.14. Therefore, $\tilde{\beta} = \beta$ is the global optimum.

Proof of Proposition 2:

As $x_{13}$ increases, the marginal increase in $\int_\beta^{\tilde{\beta}} \Phi(\tilde{K}) f(\beta) d\beta$ is less than the marginal increase in $V_0$. To see that, differentiate w.r.t. $x_{13}$, and get

$$
\frac{\partial}{\partial x_{13}} \left[ \int_\beta^{\tilde{\beta}} \Phi(\tilde{K}) f(\beta) d\beta - V_0 \right]
$$

$$
= \int_\beta^{\tilde{\beta}} [x_{12}(SW_{11}(K) - SW_{10}(K)) + (1 - x_{12})(SW_{01} - SW_{00})] f(\beta)d\beta - (SW_{01} - SW_{00})
$$
\[ \int_{\beta} x_{12}(SW_{11}(K) - SW_{01}(K)) f(\beta) d\beta + \int_{\beta} x_{12}(SW_{00} - SW_{10}(K)) f(\beta) d\beta \leq 0 \]

The reason for the last inequality comes from the fact that \( SW_{11}(K) - SW_{01}(K) \leq 0 \), and \( SW_{00} - SW_{10}(K) < 0 \) for all \( K \). Therefore, an increase in \( x_{13} \) reduces the difference between the social welfare of having line (1,2) and the social welfare of not having it. \( \blacksquare \)

Proof of Proposition 3:

Again, we start from the expression used to determine whether to build the line: \( \int_{\beta} \Phi(\widetilde{K}) f(\beta) d\beta - V_0 \). Differentiating w.r.t. \( m \) yields:

\[ \int_{\beta} x_{12} \left[ x_{13} \left( \frac{\partial SW_{11}}{\partial m} - \frac{\partial SW_{01}}{\partial m} \right) + (1 - x_{13}) \left( \frac{\partial SW_{10}}{\partial m} - \frac{\partial SW_{00}}{\partial m} \right) \right] f(\beta) d\beta = \beta \text{(3.15)} \]

The terms in brackets take different values for different levels of capacity.

For \( 0 < K < \frac{a(m-1)}{3(m+b+bm)} \), the term equals \( x_{13} \left( \frac{3K}{2} - \frac{a-c}{2b} \right) + K(1 - x_{13}) \).

Hence, the term is positive if \( x_{13} < \frac{bK(2a-bK)}{(b+m)^2} \left( \frac{bK(2a-bK)}{(b+m)^2} + \frac{a^2(2-a)(b+m)}{(1+4b+m)^2} \right) \).

For \( \frac{a(m-1)}{3(m+b+bm)} < K < \frac{ma}{m+b+bm} \), the term becomes \( (1 - x_{13}) \left( \frac{bK(2a-bK)}{2(m+b)^2} \right) \), which is positive if \( x_{13} \neq 1 \).

For \( K > \frac{a-c}{b} \), the terms in brackets become \( (1 - x_{13}) \left( \frac{a^2(2-a)(b+m)}{(m+b)^2} \right) \),
which is greater than zero if \( x_{13} \neq 1 \).

In sum, expression 3.15 is positive for \( 0 < K < \frac{a(m-1)}{3(m+b+bm)} \) if \( x_{13} \) is not too high. This capacity interval corresponds to a congested line. It is greater than zero for \( K > \frac{a(m-1)}{3(m+b+bm)} \) if \( x_{13} \neq 1 \). This capacity interval corresponds to a non-congested line in state 11. Therefore, as the marginal cost of the expensive generator gets higher, the regulator is more likely to build the line i) when the line is congested, or ii) when the reliability is not too high.

**Proof of Proposition 4:**

Under incomplete information, building line (1,2) is optimal if

\[
\bar{\beta} \int_{\beta} \Phi(K) f(\beta) \, d\beta - V_0 > 0.
\]

Differentiating the LHS of the inequality w.r.t. \( a \) yields:

\[
\bar{\beta} \int_{\beta} x_{12} \left[ x_{13} \left( \frac{\partial SW_{11}}{\partial a} - \frac{\partial SW_{01}}{\partial a} \right) + (1 - x_{13}) \left( \frac{\partial SW_{10}}{\partial a} - \frac{\partial SW_{00}}{\partial a} \right) \right] f(\beta) \, d\beta.
\]

But \( \frac{\partial SW_{11}}{\partial a} - \frac{\partial SW_{01}}{\partial a} < 0 \) for \( K < \frac{a(m-1)}{3(m+b+bm)} \), and equals zero for \( K \geq \frac{a(m-1)}{3(m+b+bm)} \), and \( \frac{\partial SW_{10}}{\partial a} - \frac{\partial SW_{00}}{\partial a} > 0 \) for all \( K \). Then the integral becomes greater than zero when the line is not congested \( (K \geq \frac{a(m-1)}{3(m+b+bm)}) \) or \( x_{13} \) is not too high.

The upper limit for \( x_{13} \) is 1 when the line is not congested, and it is equal to \( \frac{\frac{\partial SW_{10}}{\partial a} - \frac{\partial SW_{00}}{\partial a}}{\frac{\partial SW_{11}}{\partial a} - \frac{\partial SW_{01}}{\partial a}} \) when the line is congested. Hence, an increase in demand makes it more likely to choose building the line when the line is not congested, and makes it less likely when the line is congested.
Proof of Proposition 5:

Since it is optimal not to build the line under complete information, we have \( \Phi(\beta, K^*) < V_0 \). Taking the expectation over types gives

\[
\mathbb{E}_\beta \Phi(\beta, K^*) f(\beta) d\beta < V_0.
\]

But \( \tilde{K} \) was obtained by maximizing \( \mathbb{E}_\beta \Phi(\beta, K) f(\beta) d\beta \) subject to the IC constraint. The solution to an unconstrained problem yields an optimal value of the objective function which is greater than or equal to the one obtained from a constrained problem. Therefore,

\[
\mathbb{E}_\beta \Phi(\beta, \tilde{K}) f(\beta) d\beta \leq \mathbb{E}_\beta \Phi(\beta, K^*) f(\beta) d\beta < V_0.
\]

Therefore, the regulator does not build the line under incomplete information either. ■

Proof of Proposition 6:

First of all, recall that the expected shadow price was defined as \( \lambda(K) = x_{13} \lambda_1(K) + (1 - x_{13}) \lambda_0(K) \), where \( \lambda_1(K) \) refers to the shadow price of line (1, 2) when line (1, 3) is 'up', and \( \lambda_0(K) \) refers to the shadow price when line (1, 3) is 'down'. Moreover, we can see from the spot-pricing results in Appendix that if \( \lambda_1(K) > 0 \), then \( \lambda_0(K) > 0 \); and if \( \{ \lambda_0(K) = 0 \text{ and } x_{13} < 1 \} \), then \( \lambda_1(K) = 0 \). Therefore, we can conclude that \( \lambda(K) = 0 \leftrightarrow \lambda_0(K) = 0 \text{ and } x_{13} < 1 \). Now we can prove the results:
i-) Under complete information, the FOC for a non-zero\(^{90}\) capacity is

\[ x_1^* \cdot \lambda(K^*) = (1 + \mu) \ C_K \]

Suppose \( \lambda(K^*) = 0 \), or \( \lambda_0(K^*) = 0 \). Then the FOC implies that \( C_K = 0 \), which contradicts with the assumption that \( C_K > 0 \), i.e. cost of an additional capacity is always positive. Therefore, we must have \( \lambda_0(K^*) > 0 \) when \( x_{13} < 1 \).

ii-) Under incomplete information, the FOC for a non-zero capacity is

\[ \bar{x}_{12} \lambda(\bar{K}) - (1 + \mu) \ C_K = \frac{\mu F(\beta)}{f(\beta)} \ \psi' \ \frac{d}{dK} E_\beta \]

Suppose \( \lambda(\bar{K}) = 0 \). Then the LHS of the equation will be strictly less than zero. If \( \frac{d}{dK} E_\beta < 0 \) and \( \beta > \beta_0 \), then the RHS will also be strictly less than zero since \( F(\beta) > 0 \) for \( \beta > \beta_0 \). Therefore, it is possible to find a set of cost parameters and firm types such that the equation is satisfied. I will skip finding the specific conditions for parameters. \( \blacksquare \)

Derivation of FOC’s for the discrete-case illustration

We will only provide a sketch of the derivation. The main program of the social planner is to maximize the expected social welfare (over firm's type) subject to Incentive Compatibility (IC) and Individual Rationality (IR) constraints for each type. We will first state all these constraints, then eliminate some of

\( ^{90} \)The possibility of a corner solution for capacity level is only possible at \( K = 0 \), and FOC's are also sufficient for any \( K > 0 \). Since a non-congested line can occur only with \( K > 0 \), the analysis of the FOC is enough for our result.
them.

IC constraints are

\[
\begin{align*}
\text{Efficient} & : U(\beta) \geq U(\bar{\beta}) + \psi(e(\bar{\beta})) - \psi \left( e(\bar{\beta}) - K^2(\bar{\beta}) x_{12}(\bar{\beta}) \right) \\
\text{Inefficient} & : U(\bar{\beta}) \geq U(\beta) + \psi(e(\beta)) - \psi \left( e(\beta) - K^2(\beta) x_{12}(\beta) \right)
\end{align*}
\]

IR constraints are

\[
\begin{align*}
\text{Efficient} & : U(\beta) \geq 0 \\
\text{Inefficient} & : U(\bar{\beta}) \geq 0
\end{align*}
\]

Note that the IC constraint for the efficient type and the IR constraint for the inefficient type implies the IR constraint for the efficient type. Hence, we can eliminate the IR constraint for the efficient type. Moreover, the IR constraint for the inefficient type must be binding, i.e. \( U(\beta) = 0 \). Otherwise, we can decrease both \( U(\bar{\beta}) \) and \( U(\beta) \) by the same small amount without violating any constraints. Furthermore, the IC constraint for the efficient type must be binding, i.e. the efficient type is indifferent between telling the truth and mis-reporting. Otherwise, we can decrease \( U(\beta) \) without violating any remaining
constraint, which is the IC constraint for the inefficient type. Finally, we assume that the remaining constraint is not binding at the optimal solution. This needs to be checked at the end.

In summary, the social planner chooses $e(\beta), e(\bar{\beta}), U(\beta), K(\beta), x_{12}(\beta), x_{12}(\bar{\beta})$ to solve the maximization problem subject to one equality constraint, which is the IC constraint for the efficient type. This equality constraint can be used to eliminate one variable, $U(\beta)$, after which the problem reduces to an unconstrained maximization problem. Since $U(\beta)$ will be expressed only in terms of variables related to the inefficient type, the FOC's for the inefficient type will be distorted by these additional terms. We skip writing the objective function, since it is too long to fit here and it is very similar to the function for the continuous-type case.
4. CONCLUSION

Our analysis gives us two main insights. First, the result of under-investment in line capacity by a line owner cannot be generalized to cases where line reliability is also a choice variable. Therefore, empirical studies are needed to test the specific conditions that lead to various results about the comparison of investment by the line owner with the first-best level of investment for society. Second, an increase in electricity demand, or a higher cost of generation at expensive generating units does not necessarily increase the expected benefit of building transmission lines on the network. This somewhat surprising result follows from the trade-off between reliability benefits and congestion costs of having a new transmission line.

Throughout the analysis in this thesis, we treat line reliability as a choice variable instead of being fixed by industry standards, or only by engineering rules. We argue that line reliability needs to be determined according to economic principles, which takes into account the benefits and costs of reliability. But these benefits and costs depend partly on line capacity, hence our treatment of both capacity and reliability as decision variables captures possible interdependencies between them.
In the context of regulation, if a policy-maker focuses only on the regulation of line capacity, the regulated company will still have opportunity to exercise its market power by varying the level of reliability. Depending on the structures of costs and of demand, the profit-maximizing monopolist may under- or over-supply reliability relative to the optimum, whether or not line capacity is regulated. Therefore, both capacity and reliability have to be regulated instead of only capacity.

We find that a benevolent social planner may assign less value to a marginal increase in line reliability compared to the owner of that transmission line. Specifically, in a setting where the line owner is paid nothing when the line is down, and rewarded by the shadow value of capacity when the line is up, expected marginal benefit of reliability for society is less than expected marginal benefit for the monopolist for small levels of capacity. Therefore, an unregulated line owner can pick a higher level of reliability compared to the social planner.

It is even possible for the line owner to over-invest in both capacity and reliability. The result depends on the cross-cost elasticity of the cost of building a line, and on whether the first-best level of capacity is too low. Such a contradiction with the established under-investment result in the literature can be regarded as a warning sign to regulators who may be designing regulatory
policies to alleviate the under-investment by line owners. A regulation policy which does not take line reliability into account may lead to sub-optimal results.

We characterized the optimal regulation contract for capacity and reliability under incomplete information. We used a well-known property in regulation theory to show that the first-best levels of capacity and reliability can be obtained if the cost function takes a special separable form. This means that the existence of asymmetric information does not necessarily cause the regulator to distort incentives to provide capacity and reliability.

We also examined how changes in some exogenous parameters affect the regulator’s decision to build a new transmission line. One result shows that an exogenous increase in electricity demand does not necessarily improve the attractiveness of building a line. If the new line will be congested once built, then reliability benefits of having the line can be offset by the costs of newly created congestion. Although this result seems counter-intuitive, it follows from the special characteristics of transmission technology. One can reduce capacity of a network to transmit electricity by building new transmission capacity.

Our analysis suggests the need for empirical research in order to obtain more specific results about the incentives and regulation of a line owner to provide line capacity and reliability. We plan to extend the analysis in this thesis to the
context of recently proposed RTOs in the U.S. Since an RTO can be thought to
be a coalition of line owners, such a study needs to employ tools of cooperative
game theory.

Another future research project is the formulation of a mechanism to im-
plement the optimal regulation that we characterized in this thesis. One can
analyze the conditions under which the optimal contract can be implemented
by linear payment contracts.
5. APPENDIX: TRANSMISSION TECHNOLOGY

Electricity divides itself in a network according to Kirchoff Laws, and the amount of power flow between any two connected nodes (or junctions) is determined by a specific function of voltage angles at those nodes. The first Kirchoff Law states that total power flowing into a node must be equal to total power flowing out of that node. The second Kirchoff Law is related to voltage: total change in voltage within a closed circuit loop must be zero.

We will first present the AC (Alternating Current) flow technology. Consider a network with \( n \) nodes and \( m \) lines. Each line is characterized by its impedance, which is roughly a measure of friction on the line. The impedance of line \( k \) is expressed in complex numbers as \( r_k + i x_k \), where \( r_k \) represents the resistance and \( x_k \) represents the reactance of line \( k \). Each node is associated with a voltage level, a voltage angle, and a net power injection (which can be negative). Let’s denote the voltage level at node \( i \) by \( V_i \), the voltage angle by \( \theta_i \), and the net power injection by \( q_i \). If line \( k \) connects nodes \( i \) and \( j \), then the amount of real power flowing from node \( i \) to node \( j \) is determined by the

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91 The other power flow technology is the DC (or Direct Current) flow. In DC circuits, power flow is constant over time, but it alternates in sinusoidal waves in AC circuits. Both technologies are used in modern networks.

92 Electricity power is composed of two parts: real power and reactive power. Real power can be transformed into useful work, but reactive power cannot produce useful work. Instead,
following formula:

\[ P_k = G_k V_i^2 - G_k V_i V_j \cos(\theta_i - \theta_j) + Y_k V_i V_j \sin(\theta_i - \theta_j) \]

where \( P_k \) is the power flowing on line \( k \), \( G_k = \frac{r_k}{r_k^2 + x_k^2} \), and \( Y_k = \frac{x_k}{r_k^2 + x_k^2} \).

The first Kirchoff Law can then be written as

\[ q_i = \sum_{k \in C_i} P_k \quad i = 1, \ldots, n. \]

where \( C_i \) is the set of lines that are connected to node \( i \).

Although AC power technology is more widely used than DC technology, the sinusoidal power-flow function is difficult to analyze both analytically and numerically. Therefore, we will employ a DC-flow approximation\(^{93}\) to simplify the analysis. Assume that voltage levels at each node is stabilized at unity by supplying necessary reactive power to the system. Then, \( V_i = 1 \ \forall i \). Moreover, assume that voltage angle differences are small, i.e. \( \theta_i - \theta_j \approx 0 \). Then, \( \sin(\theta_i - \theta_j) \approx 0 \), and \( \cos(\theta_i - \theta_j) \approx 1 \). To simplify the analysis, we further assume that there is no transmission loss on the network, i.e. \( r_k = G_k = 0 \). Then, power flow equation becomes

\[ P_k = \frac{1}{x_k} (\theta_i - \theta_j) \quad k = 1, \ldots, m. \quad (5.1) \]

\(^{93}\)The DC-flow approximation is employed quite extensively in both Electrical Engineering literature and Economics literature. See Hogan [10], Nasser [16], and Chao and Peck [6].
The next step is to solve for power flows as functions of injections at nodes. Once we obtain the solution, we will be able to tell the amount of power flowing on each line in response to power injections (generation of power, or demand for power). Let $P = [P_1 \ P_2 \ \ldots \ P_m]'$ be the vector of power flows, $\bar{\theta} = [\theta_1 \ \theta_2 \ \ldots \ \theta_n]'$ be the vector of voltage angles, and $\bar{Q} = [q_1 \ q_2 \ \ldots \ q_n]'$ be the vector of net power injections at nodes.

We will introduce an $m \times n$ matrix, $\tilde{T}$, that shows topology of the network, that is the relation between nodes and lines. The matrix is composed of only $+1$, $-1$, and $0$ as possible entries, that gives us information on which lines are connected to each node, and also on the direction of power flows. In order to form the matrix, one first chooses an arbitrary \footnote{The specific direction chosen at this stage is not important, because the solution of power flow as a function of net injections at nodes will give us the same amount of power flow (in absolute value) regardless of the direction that we have chosen for power flows.} direction of power flow for each line. If line $k$ connects nodes $i$ and $j$, and if it flows from node $i$ to node $j$, then the $ki$'th entry of the matrix $\tilde{T}$ will take a value of $+1$, the $kj$'th entry will be $-1$, and all the remaining entries on the $k$'th row will be $0$.

Let $a_k = \frac{1}{z_k}$ be the admittance of line $k$, and $A$ be the $m \times m$ diagonal matrix whose entries on the diagonal are $a_k$'s. Then we can write the power-flow equation 5.1 in vector form as
The first Kirchoff Law in vector form can be written as
\[
P_{m\times1} = A_{m\times m} \tilde{T}_{m\times n} \tilde{\theta}_{n\times1}.
\]

Substituting \( P \) into \( Q \) reveals
\[
\tilde{Q}_{n\times1} = (\tilde{T}_{m\times n})' P_{m\times1}.
\]

Note that the matrix \( \tilde{T}'A \tilde{T} \) is not of full rank, because \( \sum_{i=1}^{n} q_i = [1 \ldots 1]_{1\times n} \tilde{Q} = 0 \). One can see this by looking at the columns of the matrix \( \tilde{T}' \): each column consists of one +1, one -1, and zeros since each line connects only two nodes. This is another way of saying that the sum of positive injections (that we put into the network) must be equal to the sum of negative injections (that we consume) in a lossless \(^{95}\) network. Therefore, we will choose an arbitrary node as the reference node, and express the voltage angle at all other networks in terms of the angle at the reference node. Assume w.l.o.g. that \( \theta_n = 0 \). Then, we can drop the last column of \( \tilde{T} \) (since the last column will only have zeros after multiplying it with \( \tilde{\theta} \)), and the last elements of \( \tilde{Q} \) and \( \tilde{\theta} \). Now we can redefine our reduced matrices as \( T_{m\times(n-1)} \), \( Q_{(n-1)\times1} \), and \( \theta_{(n-1)\times1} \) in order to re-write

\[
\tilde{Q} = \tilde{T}'A \tilde{T}\tilde{\theta}
\]

\(^{95}\)In a network with losses, \( \sum_{i=1}^{n} q_i \) = Total Losses.
equation 5.2 as follows

\[ Q = T' A T \theta, \text{ or} \]

\[ \theta = (T' A T)^{-1} Q. \]

But \( \theta_n = 0 \) also implies that \( P_{m \times 1} = A_{m \times m} \theta 
\tilde{T}_{m \times n} \tilde{\theta}_{n \times 1} = A T \theta \). Substituting for \( \theta \) from the previous line gives us the relation between flows and injections as we aimed:

\[ P = A T (T' A T)^{-1} Q \quad (5.3) \]

We will now illustrate equation 5.3 by using the triangular network in Figure 1, which has 3 nodes and 3 lines \((n = m = 3)\). Assume that all three lines have identical reactance values. Then, \( A = a I_{m \times m} \) where \( a \) is a positive scalar, and \( I \) is the identity matrix. Let \( P_1 \) denote the power flowing from node 1 to node 3, \( P_2 \) denote the power flowing from node 1 to node 2, and \( P_3 \) denote the power flowing from node 2 to node 3. Then the matrix \( \tilde{T} \) becomes

\[
\tilde{T} = \begin{bmatrix}
+1 & 0 & -1 \\
+1 & -1 & 0 \\
0 & +1 & -1
\end{bmatrix}.
\]

Hence the reduced matrix \( T \) (which was obtained by trimming the last column of \( \tilde{T} \)) becomes
Simple matrix algebra reveals that equation 5.3 will take the following form:

\[ T = \begin{bmatrix}
  +1 & 0 \\
  +1 & -1 \\
  0 & +1
\end{bmatrix} \]

\[ P = \begin{bmatrix}
  +2/3 & +1/3 \\
  +1/3 & -1/3 \\
  +1/3 & +2/3
\end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}. \]

Now we can see how injections to the network creates power flows on each line. For example, amount of power flowing on line 2 (connecting nodes 1 and 2) is given by the formula \( P_2 = \frac{1}{3} (q_1 - q_2) \). Moreover, the power flow equation also gives us information on how much power will flow on each line as power is injected at a particular node\(^{96}\). For example, 2/3 of power injected at node 1 flows on line 1 (between nodes 1 and 3), and the remaining 1/3 of it will flow on lines 2 and 3.

\(^{96}\)One might wonder what happens to power flows as \( q_3 \) (or the injection at the reference node) changes. We cannot see it from the power flow equation that we obtained. The reason for that comes from the relation among power injections. Since \( \sum_{i=1}^{n} q_i = 0 \), it means that \( q_3 = -(q_1 + q_2) \). That is, one cannot change the injection at the reference node without changing at least one of other injections. Since we know the relation between the injections at nodes 1 or 2 and power flows, we can find how an additional injection at the reference node changes the power flows.
References


