Two Essays on International Asset Market and Macroeconomic Dynamics

Author: Tuan Hoang Dao

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Boston College

The Graduate School of Arts and Sciences

Department of Economics

TWO ESSAYS ON INTERNATIONAL ASSET MARKETS AND MACROECONOMIC DYNAMICS

a dissertation

by

TUAN HOANG DAO

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Dissertation Abstract

Two Essays on International Asset Markets and Macroeconomic Dynamics

by

Tuan Hoang Dao

Dissertation Committee:

Fabio Ghironi (chair)
Peter Ireland
Georg Strasser

This dissertation examines the macroeconomic dynamics under different international asset market structures. The dissertation consists of two chapters. The first chapter is my cowork with Taesu Kang, a classmate of mine at Boston College, department of economics. We investigate the dynamics of the U.S and emerging Asian countries during the financial crisis in 2008. We focus on the bank lending channel as the source of shock transmission and explain how the internal default in the U.S can be transmitted to emerging Asian countries. The second chapter of my thesis is my work...
on the international equity home bias and Backus Smith puzzles. I propose a model with a incomplete asset market, endogenous labor supply and non-tradable goods that can generate a high degree of home equity bias, even when the domestic human capital return and equity return are highly correlated. My model also generates a very low correlation between the consumption differential across countries and the real exchange rate. The correlation is more inline with data than the strongly positive correlation predicted by a standard complete asset market framework.
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Chapter 1: International Financial Business Cycles (with Taesu Kang)

1.1 Abstract

Recent international macroeconomics literature on global imbalances explains the U.S.’ persistent current account deficit and emerging countries’ surplus, i.e., the U.S. is the borrower. Little research has been done on the banking-sector level, where U.S. banks are lenders to banks in emerging countries. We build a two-country framework where banks are explicitly modeled to investigate how lending in the banking sector can affect the international macroeconomy during the recent crisis. In the steady state, banks in the developing country borrow from the U.S. banks. When the borrowers in the U.S. pay back less than contractually agreed and damage the balance sheet of the U.S. banks, with the presence of bank capital requirement constraint, U.S. banks raise lending rates and decrease the loans made to U.S. borrowers as well as banks in the developing country. The results are a sharp increase in the lending spread, a reduction in output and a depreciation in the real exchange rate of the developing country. This is the experience of many emerging Asian markets following the U.S. financial crisis starting in late 2007. Another feature of our model captures an empirical fact, documented by Devereux and Yetman (2010), that across different economies, countries with lower financial ratings can suffer more when the lending country deleverages.
1.2 Introduction

International macroeconomics literature on global imbalances explains why the U.S. runs a persistent current account deficit. While the U.S. is the net borrower at the country level, at the banking-sector level, this is not necessarily the case. U.S. banks and banks in other developed economies are net lenders to banks in emerging Asian markets (EAM). In around late 2007 or the beginning of 2008, when losses in the mortgage market began to damage U.S. banks’ balance sheets (Figures 1 and 2), U.S. banks deleveraged and reduced deposits and credits (Figure 3). Not only did they contract loans made to U.S. borrowers, they contracted loans made to foreign borrowing banks as well. Figure 4 documents external (cross-border) assets of banks in developed economies and Figure 5 documents external liabilities of banks in EAM. With the exception of Japan, which was little exposed to U.S. Mortgage Backed Securities (MBS), all major developed economies showed significant contractions in banks’ external assets, which resulted in significant contractions in EAM banks’ external liabilities.\(^1\) The documented contraction in international inter-bank lending was followed by a worldwide drop in GDP growth, both among the developed world (Figure 6) and the developing world (Figure 7). This empirical evidence highlights the importance of the banking system in international transmission of shocks.

The recent financial crisis in the U.S. was characterized by decline in asset prices, disruption in the loan market, sharp increase in interest rate spread and a large drop in GDP. One thing many scholars have agreed is the banking system played a vital role in this crisis. There are a number of recent working papers that include bank in a closed economy dynamic stochastic general equilibrium (DSGE) model to model the recent crisis in the U.S.: Kiyotaki and Gertler (2010), Iacoviello (2010), a series of papers by Ali Dib (2010) and others. Recent development in international macroeconomics literature investigates the effect of the financial linkage that spread the U.S. mortgage crisis worldwide. Devereux and Yetman (2010) and van Wincoop (2011) build international portfolio models where leveraged investors in one country hold the financial asset in the other country. Consequently, any shock that affects the domestic country asset

\(^1\)Kamin and DeMarco 2010 document that the majority of foreign exposure to U.S. MBS are of European Banking Centers.
prices will affect foreign investors’ balance sheets and spread to the foreign economy. Ueda (2010), Kollman (2011) and Kalemi-Ozcan, Papaioannou and Perri (2011) build international business cycle models with banks. In these papers, entrepreneurs in two countries share a common lender(s). Any shock that hits one economy will affect the common lender(s) and thus, its (their) borrowers. While both of these features can be true among the developed world, i.e., U.S. and the Euro Area, they are not the best to describe the recent crisis for the EAM. Contrary to the large portfolio position of European banks in U.S. MBS, banks in EAM have no or very little exposure to the U.S. MBS and firms in these countries have little direct access to foreign bank credits.

We would like to build a two country model with the banking system that plays an important role in international transmission of shock, which has been largely agreed to be the main cause of the recent crisis. Our model is built upon the closed economy version in Iacoviello (2010). In steady state, banks in the developing country (EAM/domestic country) borrow from banks in the developed country (the U.S./foreign country). When some borrowers in the U.S. pay back less than contractually agreed, with the presence of capital requirement constraint, U.S. banks cut back on lending to U.S. borrowers as well as EAM banks and raise the inter-bank lending rate. Domestic banks now face more expensive and less available foreign credit, and will reduce loans made to domestic borrowers. The financial (repayment) shock in the U.S. is transmitted across country via the banking system.

In another exercise, we investigate the behavior of the model under permanent and temporary shocks to the weight of domestic bank loan in the foreign bank’s capital requirement constraint. The permanent shock can be interpreted as a change in bank regulation, such as moving from Basel I to Basel II. A temporary shock can be interpreted as an exogenous drop in domestic banks’ credit ratings. The results for these shocks are reductions in home output, investment and consumption and a depreciation of home real exchange rate.

Our paper is related to a number of empirical papers on global banking. Peek and Rosengren (1997) study the behavior of Japanese banks in the U.S.. During the
financial crisis in the late 1980s and early 1990s in Japan, Japanese banks in the U.S. substantially contracted the amount of loans made to U.S. borrowers. Cetorelli and Goldberg (2008) document that “foreign lending activity of U.S. bank affiliates abroad can rely less on the overall strength of the home office in times of tighter monetary condition in the U.S.”. Popov and Udell (2010) find that financial distress by West European and U.S. parent banks has a significant impact on the availability of business loans for East European firms. Most recently, Imai and Takarabe (2011) use the data from nationwide and local banks in Japan to test whether banking integration plays an important role in transmitting financial shocks across geographical boundaries. They found that nation-wide banks do indeed transmit financial shocks originated from major cities to smaller local economies. The results of our model under different weights of interbank loan in the capital requirement constraint suggests that across countries, lower-rated economies will suffer more when U.S. banks deleverage. This is consistent with empirical evidence for the recent crisis, documented by Devereux and Yetman (2010).

1.3 The Model

There are two countries: the domestic country (EAM) and the foreign country (U.S.).\(^1\) In each country, there are five types of agents: patient households, impatient households, entrepreneurs, firms and banks. There are two sectors in the economy: the tradable and non-tradable good sectors.

Both patient and impatient households (HHs) work for firms in tradable and non-tradable sectors. They earn wage income and consume tradable goods, non-tradable goods and housing. Patient HHs supply deposits for banks and earn a return from the deposits. Impatient HHs, on the other hand, borrow from banks to consume. They can only borrow up to a fraction of the value of their collateral (house).

Domestic bankers take the deposit from domestic depositories and can also borrow in the international inter-bank market. They can only borrow up to a fraction of

\(^1\)The model is built upon Iacoviello (2010) closed economy model
the value of their capital. They pay a return for the fund they borrow and lend it to
domestic borrowers for a higher return. Foreign bankers take the deposit from foreign
depositors. They lend out to foreign borrowers and domestic bankers. Domestic and
foreign bankers face capital requirement constraint.

Entrepreneurs accumulate physical capital used in both tradable and nontrad-
able sectors. They finance their investment with income from capital rental and bank
loan, which is subject to a collateral debt constraint.

Firms in the tradable and nontradable sectors use capital and labor to produce
goods. They pay wages to HHs.

1.3.1 Consumption Basket

Consumers’ consumption aggregate is given by: 
\[ c_t = \left[ (c_t^N)^{\frac{\omega-1}{\omega}} + (c_t^T)^{\frac{\omega-1}{\omega}} \right]^{\frac{1}{\omega-1}}, \]
where \( c_t^T \) and \( c_t^N \) are tradable and non-tradable consumptions. The corresponding price
index is 
\[ P_t = \left[ (P_t^N)^{1-\omega} + (P_t^T)^{1-\omega} \right]^{\frac{1}{1-\omega}}, \]
where \( P_t^T \) and \( P_t^N \) are the tradable and non-
tradable price indices. The consumption aggregate and price indices for the foreign
economy are identical. We denote the price of non-tradable (tradable) relative to the
price of consumption baskets as \( p_t^N \) (\( p_t^T \)).

1.3.2 Patient HHs

A continuum of domestic patient HHs deposit \( d_t \), consume composite good \( c_{p,t} \) and
housing \( h_{p,t} \), and supply labor to tradable, \( n_{p,t}^T \), and nontradable, \( n_{p,t}^N \), sectors. They earn
wage income and return from their deposits. They maximize the infinite sum of utilities:
\[
\max_{c_{p,t}, h_{p,t}, n_{p,t}^N, n_{p,t}^T} E_0 \sum_{t=0}^{\infty} \beta_p^t \left[ \ln c_{p,t} + \nu \ln h_{p,t} + \tau_p \ln (1 - n_{p,t}^N - n_{p,t}^T) \right],
\]
subject to the budget constraint:
\[
c_{p,t} + d_t + q_t \Delta h_{p,t} = R_{d,t} d_{t-1} + w_{p,t}^N n_{p,t}^N + w_{p,t}^T n_{p,t}^T
\]
where \( R_{d,t} \) is the return from the deposits and \( q_t \) is the price of a house. \( w_{p,t}^T \) and \( w_{p,t}^N \) are wages from the tradable and nontradable sectors respectively. Their first order conditions are:
\[
\frac{1}{c_{p,t}} = \beta_p E_t \left( \frac{R_{d,t+1}}{c_{p,t+1}} \right),
\]
\[
\frac{q_t}{c_{p,t}} = \frac{\nu}{h_{p,t}} + \beta_p E_t \left( \frac{q_{t+1}}{c_{p,t+1}} \right),
\]
\[
\frac{w_{p,t}^N}{c_{p,t}} = \tau_p \frac{1 - n_{p,t}^N - n_{p,t}^N}{c_{p,t}}
\]
\[
\frac{w_{p,t}^T}{c_{p,t}} = \tau_p \frac{1 - n_{p,t}^N - n_{p,t}^T}{c_{p,t}}.
\]

Foreign patient HHs’ optimization problem are identical and indexed with *.

1.3.3 Impatient HHs

Domestic impatient HHs also consume goods and housing, and supply labor. \( c_{i,t}, h_{i,t}, n_{i,t}^N, n_{i,t}^T \) are impatient HHs’ consumptions, houses, labor supply to the tradable and nontradable sectors. Unlike patient HHs, however, they borrow money from banks, \( l_{i,t} \), to finance consumption. They pay interest \( R_{i,t} \) on the loan and can only borrow up to the value of their houses. Their maximization problem is:
\[
\max_{c_{i,t}, h_{i,t}, n_{i,t}^N, n_{i,t}^T, l_{i,t}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta_t^t \left[ \ln c_{i,t} + \nu \ln h_{i,t} + \tau_i \ln (1 - n_{i,t}^N - n_{i,t}^T) \right],
\]

subject to the budget constraint:
\[
c_{i,t} + q_t \Delta h_{i,t} + R_{i,t} l_{i,t} - 1 = l_{i,t} + w_{i,t} n_{i,t}^N + w_{i,t}^T n_{i,t}^T,
\]  
(6)

and the borrowing constraint:
\[
l_{i,t} \leq m_i E_t \left( \frac{q_{t+1} h_{i,t}}{R_{i,t}} \right).
\]  
(7)

Foreign impatient HHs’ problem is equivalent, except that in their budget constraint, there is a repayment shock. Their budget constraint is:
\[
c_{i,t}^* + q_t^* \Delta h_{i,t}^* + R_{i,t}^* l_{i,t}^* - \epsilon_t = l_{i,t}^* + w_{i,t} n_{i,t}^N + w_{i,t}^T n_{i,t}^T.
\]

As in Iacoviello 2010, \( \epsilon_t \) is a mean zero, AR(1) shock that captures the exogenous repayment shock in the U.S.. When \( \epsilon_t \) is greater than 0, U.S. impatient HHs pay back less than their debt obligation.

First order conditions of impatient HHs are:
\[
\frac{1}{c_{i,t}} = \lambda_{i,t} R_{i,t} + \beta_t E_t \left( \frac{R_{i,t+1}}{c_{i,t+1}} \right),
\]
(8)
\[
\frac{q_t}{c_{i,t}} = \frac{\nu}{h_{i,t}} + \lambda_{i,t} m_t E_t(q_{t+1}) + \beta_t E_t \left( \frac{q_{t+1}}{c_{i,t+1}} \right),
\]
(9)
\[
w_{N,i,t} c_{i,t} = \tau_i - n_{N,i,t} - n_{T,i,t},
\]
(10)
\[
w_{T,i,t} c_{i,t} = \tau_i - n_{N,i,t} - n_{T,i,t}.
\]
(11)

\(\lambda_{i,t}\) is the Lagrangian multiplier of impatient HHs' borrowing constraint.

1.3.4 Entrepreneurs

Entrepreneurs’ optimization problem is:

\[
\max_{c_{e,t}, k_{N,i,t}, k_{T,i,t}} E_0 \sum_{t=0}^{\infty} \beta^t_t \ln c_{e,t},
\]
subject to the budget constraint:
\[
c_{e,t} + k_{N,i,t} + k_{T,i,t} + R_{e,t} l_{e,t-1} + \frac{\phi_k}{2} \left( \Delta k_{N,i}^t \right)^2 + \frac{\phi_k}{2} \left( \Delta k_{T,i}^t \right)^2
= l_{e,t} + (r_{k,i,t}^N + 1 - \delta) k_{i-1}^N + (r_{k,i,t}^T + 1 - \delta) k_{i-1}^T,
\]
(12)
and the borrowing constraint:
\[
l_{e,t} \leq m_e(k_{N,i}^t + k_{T,i}^t).
\]
(13)

where \(c_{e,t}\) is entrepreneurs’ consumption. \(k_{N,i}^t, k_{T,i}^t\) are entrepreneurs’ capital in the tradable and nontradable sectors. They finance investment with income from capital rental in the two sectors \(r_{k,i,t}^N + 1, r_{k,i,t}^T + 1\) and bank loan \(l_{e,t}\). The bank loan cannot exceed the value of their capital. Entrepreneurs pay banks a return \(R_{e,t}\) on the loan. Similar to Backus, Kehoe and Kydland (1994), we assume that investment uses the same goods
composite as the consumption basket. $\frac{\phi_k}{2} (\Delta k_t^N)^2$ and $\frac{\phi_k}{2} (\Delta k_t^T)^2$ are convex capital adjustment costs that entrepreneurs face when they change their stock of capital in the tradable and non-tradable sectors. Entrepreneurs’ first order conditions are:

$$\frac{1}{c_{e,t}}(1 + \phi_k \Delta k_t^N) = \frac{\lambda'_{e,t}}{c_{e,t}} m_e + \beta_e E_t \left\{ \frac{1}{c_{e,t+1}^N} [(r_{k,t+1}^N + 1 - \delta) + \phi_k \Delta k_t^N] \right\}, \quad (14)$$

$$\frac{1}{c_{e,t}}(1 + \phi_k \Delta k_t^T) = \frac{\lambda'_{e,t}}{c_{e,t}} m_e + \beta_e E_t \left\{ \frac{1}{c_{e,t+1}^T} [(r_{k,t+1}^T + 1 - \delta) + \phi_k \Delta k_t^T] \right\}, \quad (15)$$

$$\frac{1}{c_{e,t}} = \frac{\lambda'_{e,t}}{c_{e,t}} + \beta_e E_t \left( \frac{R_{e,t+1}}{c_{e,t+1}} \right). \quad (16)$$

where $\lambda'_{e,t}/c_{e,t}$ is the Lagrangian multiplier of entrepreneurs’ borrowing constraint. Foreign entrepreneurs’ problems and first order conditions are similar.

### 1.3.5 Bankers

**Domestic Bankers**: Domestic bankers borrow from domestic depositors and foreign banks and supply loans to impatient HHs and entrepreneurs. The funds they obtain from the foreign bank is in units of tradable goods. They pay returns on the funds they borrow, $R_{d,t}$ and $R_{f,t}$, to depositors and foreign banks respectively. They charge higher interests on the loans they lend out: $R_{i,t}$ and $R_{e,t}$ to impatient HHs and entrepreneurs. They face a capital requirement constraint and a collateral debt constraint. The two constraints together pin down the level of foreign assets in the model. Their optimization problem is:
\[
\max_{c_{b,t},d_t,l_{e,t},l_{i,t},l_{f,t}} E_0 \sum_{t=0}^{\infty} \beta_t^{\prime} \ln c_{b,t},
\]

subject to the budget constraint:
\[
c_{b,t} + R_{d,t}d_{t-1} + l_{e,t} + l_{i,t} + R_{f,t}p_t^T l_{f,t-1} = d_t + R_{e,t}l_{e,t-1} + R_{i,t}l_{i,t-1} + p_t^T l_{f,t} - \left\{ \frac{\phi_e}{2} (\Delta l_{e,t})^2 + \frac{\phi_i}{2} (\Delta l_{i,t})^2 + \frac{\phi_d}{2} (\Delta d_t)^2 + \frac{\phi_f}{2} \Delta (p_t^T l_{f,t})^2 \right\},
\]

(17)

the capital requirement constraint:
\[
d_t + p_t^T l_{f,t} \leq \gamma_e l_{e,t} + \gamma_i l_{i,t},
\]

(18)

and the foreign debt constraint:
\[
p_t^T l_{f,t} \leq m_f \left( \frac{l_{i,t} + l_{e,t} - d_t}{R_{f,t}} \right).
\]

(19)

The international inter-bank loan \( l_{f,t} \) is denominated in tradable good price. In domestic consumption good unit, its value is \( p_t^T l_{f,t} \). Domestic bankers use their capital as collateral, which is equal to total assets \( l_{i,t} + l_{e,t} \) minus liability \( d_t \). Ali Dib (2010) made a similar assumption on the interbank lending constraint. \( m_f \) is the loan to value in the international financial market. \( \frac{\phi_e}{2} (\Delta l_{e,t})^2, \frac{\phi_i}{2} (\Delta l_{i,t})^2, \frac{\phi_d}{2} (\Delta d_t)^2, \frac{\phi_f}{2} (\Delta p_t^T l_{f,t})^2 \) are adjustment costs that banks face when they change their loans and deposits. Their first order conditions are:

\[
\frac{1}{c_{b,t}} \left[ 1 - \phi_d \Delta d_t \right] = \frac{\lambda_{b,t}}{c_{b,t}} + \frac{\lambda_{f,t}}{c_{b,t}} m_f + \beta_b E_t \left\{ \frac{1}{c_{b,t+1}} [R_{d,t+1} - \phi_d \Delta d_{t+1}] \right\},
\]

(20)

\[
\frac{1}{c_{b,t}} \left[ 1 + \phi_i \Delta l_{i,t} \right] = \frac{\lambda_{b,t}}{c_{b,t}} \gamma_i + \frac{\lambda_{f,t}}{c_{b,t}} m_f + \beta_b E_t \left\{ \frac{1}{c_{b,t+1}} [R_{i,t+1} + \phi_i \Delta l_{i,t+1}] \right\},
\]

(21)

\[
\frac{1}{c_{b,t}} \left[ 1 + \phi_e \Delta l_{e,t} \right] = \frac{\lambda_{b,t}}{c_{b,t}} \gamma_e + \frac{\lambda_{f,t}}{c_{b,t}} m_f + \beta_b E_t \left\{ \frac{1}{c_{b,t+1}} [R_{e,t+1} + \phi_e \Delta l_{e,t+1}] \right\},
\]

(22)

\[
\frac{1}{c_{b,t}} \left[ 1 - \phi_f \Delta (p_t^T l_{f,t}) \right] = \frac{\lambda_{b,t}}{c_{b,t}} + \frac{\lambda_{f,t}}{c_{b,t}} R_{f,t} + \beta_b E_t \left\{ \frac{1}{c_{b,t+1}} \left[ R_{f,t+1} \frac{p_{t+1}^T}{p_t^T} l_{f,t+1} - \phi_f \Delta (p_{t+1}^T l_{f,t+1}) \right] \right\}.
\]

(23)
where $\lambda'_{b,t}$ and $\lambda'_{f,t}$ are multipliers on the capital requirement and foreign debt constraints, multiplied by banker consumptions. The intuition here is similar to that of Iacoviello (2010), with one exception, the presence of $\lambda'_{f,t}$. To increase one unit of consumption today, bankers can either increase one unit of today’s deposit or today’s inter-bank loan (today’s liabilities), or reduce one unit of today’s consumers’ loan or business loan (today’s assets). If he, for example, chooses to increase $d_t$, re-arranging the equations gives:

$$1 - \lambda'_{b,t} - \lambda'_{f,t}m_f - \phi_d \Delta d_t = E_t \left\{ \beta_b \frac{c_{b,t}}{c_{b,t+1}} [R_{d,t+1} - \phi_d \Delta d_{t+1}] \right\}.$$ 

The right hand side of the equation is the cost of increasing one unit of deposit this period, which is equal to the additional return tomorrow that bankers have to pay on the deposit, less the lower cost that bankers pay on adjustment cost tomorrow, discounted into today value by bankers’s stochastic discount factor $\left\{ \beta_b \frac{c_{b,t}}{c_{b,t+1}} \right\}$. The left hand side is the marginal benefit of consuming one more unit today, minus the cost of tightening capital requirement constraint, $\lambda'_{b,t}$, minus the cost of tightening foreign debt constraint, $\lambda'_{f,t}m_f$, minus the adjustment cost in changing deposit that bankers face today. A similar argument holds if bankers choose, instead, to increase foreign loans or decrease loans made to domestic borrowers.

**Foreign Bankers:** Foreign bankers borrow the fund from foreign depositors and supply loans to foreign impatient HHs and entrepreneurs. Foreign banks also lend to domestic banks in the form of tradable goods. They only face budget constraint and capital requirement constraint. They are subject to the endowment shock $\epsilon_t$. Their maximization problem is:
\[
\max_{c_{b,t}, d_{t}, l_{e,t}, l_{i,t}, l_{f,t}} E_0 \sum_{t=0}^{\infty} (\beta^t) t \ln c_{b,t},
\]
subject to the budget constraint:
\[
c_{b,t} + R_{e,t} d_{t-1} + l_{e,t} + l_{i,t} + l_{f,t} + p_{t}^T l_{f,t}
= d_{t} + R_{e,t} l_{e,t-1} + R_{f,t} l_{f,t-1} - 1 + R_{e,t} l_{e,t} - 1 + R_{f,t} l_{f,t} - 1 - \epsilon_t - \left\{ \frac{\phi_e}{2} (\Delta l_{e,t})^2 + \frac{\phi_i}{2} (\Delta l_{i,t})^2 + \frac{\phi_{f}}{2} (\Delta l_{f,t})^2 + \frac{\phi_{f}}{2} \Delta (p_{t}^T l_{f,t})^2 \right\},
\]
and the capital requirement constraint:
\[
d_{t} \leq \gamma_e l_{e,t} + \gamma_i l_{i,t} + \gamma_f l_{f,t}.
\]
Their first order conditions are similar to those of domestic banks without the multiplier on the foreign debt constraint \(\lambda_{f,t}^t\). When foreign banks increase their consumption by increasing deposits or reducing loans, only their capital requirement constraint is tightened.

1.3.6 Firms

Firms in the tradable and nontradable sectors use labor from HHs and capital from entrepreneurs to produce tradable and nontradable goods. They pay wages to HHs and capital rental fees to entrepreneurs. Their maximization problem is

\[
\max_{k_{t-1}, n_{p,t}^j, n_{l,t}^j} \pi_t^j = p_t^j y_t^j - r_{k,t} k_{t-1}^j - w_{p,t}^j n_{p,t}^j - w_{l,t}^j n_{l,t}^j
\]
subject to \(y_t^j = z_t^{\alpha_j} (k_{t-1}^j)^{\alpha} \left( (n_{p,t}^j)^{1-\sigma} (n_{l,t}^j)^{\sigma} \right)^{1-\alpha_j}\),

where \(j = T, N\). The Cobb-Douglas aggregate of labor is to control for the economic size of patient and impatient HHs in the economy, as in Iacoviello (2005 and 2010). The
higher $\sigma$ is, the larger the size of impatient HHs vs. patient HHs. The Cobb-Douglas aggregate is used, instead of a simple linear combination, to pin down the steady state labor supply to each sector. In the model with two sectors and two agents, even though total labor demand in each sector and total labor supply of each type of agents are determined, a linear aggregate cannot determine what fraction of labor effort of each agent is allocated to each sector.

1.3.7 Market Clearing Conditions

The housing market clearing conditions are:

$$h_{p,t} + h_{i,t} = 1,$$

$$h_{p,t}^* + h_{i,t}^* = 1.$$

The good market clearing conditions for tradable goods are:

$$y_t^T + l_{f,t} = (p_t^T)^{-\omega} \left[ c_{p,t} + c_{i,t} + c_{e,t} + c_{b,t} + k_t^N + k_t^T - (1 - \delta) (k_t^{N-1} + k_t^{T-1}) + \text{adj}_t \right] + R_{f,t} l_{f,t-1},$$

$$y_t^{T,*} + R_{f,t} l_{f,t-1} = (p_t^{T,*})^{-\omega} \left[ c_{p,t}^* + c_{i,t}^* + c_{e,t}^* + c_{b,t}^* + k_t^{N,*} + k_t^{T,*} - (1 - \delta) (k_t^{N,*-1} + k_t^{T,*-1}) + \text{adj}_t^* \right] + l_{f,t},$$

where $\text{adj}_t$ ($\text{adj}_t^*$) is the sum of all adjustment costs the domestic (foreign) bankers and entrepreneurs face. The market clearing conditions for non-tradables are implied from the budget constraints of all agents and the above four market clearing conditions.

1.4 Key Assumptions and Calibration

1.4.1 Key Assumptions

The steady state deposit and lending rates are as followed:

$$\delta_1 = \frac{\lambda_{f}(R_f - m_f)}{(1 - \beta_b R_d - \lambda_{f} m_f)(1 - \gamma_i)}$$

$$\delta_2 = \frac{\lambda_{f}(R_f - m_f)}{(1 - \beta_b R_d - \lambda_{f} m_f)(1 - \gamma_i)}.$$ Detailed solutions can be found in the Appendix.

In steady state, foreign banks takes the deposit from foreign savers (patient HHs) and lend out to foreign impatient HHs, foreign entrepreneurs and domestic banks. In
<table>
<thead>
<tr>
<th></th>
<th>Domestic Country</th>
<th>Foreign Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deposit Rates</td>
<td>$R_d = \frac{1}{\beta_0}$</td>
<td>$R_d^* = \frac{1}{\beta_0^*}$</td>
</tr>
<tr>
<td>Interbank Rate</td>
<td>$R_f = (1 - \gamma_f) \frac{1}{\beta_0} + \gamma_f R_d$</td>
<td>$R_f^* = (1 - \gamma_f) \frac{1}{\beta_0^*} + \gamma_f R_d$</td>
</tr>
<tr>
<td>Loan to IHS</td>
<td>$R_i = R_d + \frac{1}{\beta_1} (R_d - R_f)$</td>
<td>$R_i^* = (1 - \gamma_i) \frac{1}{\beta_0} + \gamma_i R_d$</td>
</tr>
<tr>
<td>Loan to Entrepreneur</td>
<td>$R_e = R_d + \frac{1}{\beta_2} (R_d - R_f)$</td>
<td>$R_e^* = (1 - \gamma_e) \frac{1}{\beta_0} + \gamma_e R_d$</td>
</tr>
</tbody>
</table>

order for foreign banks to accept the deposit, the return on deposits that foreign banks must pay should be "low enough" for foreign banks. Specifically, $\frac{1}{\beta_0} > R_d^* = \frac{1}{\beta_0^*}$, or foreign bankers are more impatient than foreign depositors. In order for foreign impatient HHs and entrepreneurs to borrow from foreign banks, the interest rates the foreign banks charge must be "low enough" for them, or $\frac{1}{\beta_0} > R_e^* = (1 - \gamma_e) \frac{1}{\beta_0} + \gamma_e \frac{1}{\beta_0^*}$ and $\frac{1}{\beta_i} > R_i^* = (1 - \gamma_i) \frac{1}{\beta_0} + \gamma_i \frac{1}{\beta_0^*}$. Foreign entrepreneurs and impatient HHs are more impatient than the weighted average of foreign bankers and foreign depositors. The intuition here is similar to that of Iacoviello (2010).

In the interbank market, domestic banks borrow from foreign banks because the funds supplied from foreign banks are cheaper than the funds supplied from domestic depositors. From the Appendix solution for the multiplier on the interbank borrowing constraint, one can easily verify that the condition $R_f < R_d$ ensures the binding of the constraint in steady state. It is equivalent to: $(1 - \gamma_f) \frac{1}{\beta_0} + \gamma_f \frac{1}{\beta_0^*} < \frac{1}{\beta_0}$, or savers in domestic country are more impatient than the weighted average of savers and bankers in the foreign country. For domestic borrowers to accept the rates that the domestic bank charges, they have to be "impatient enough" or $\frac{1}{\beta_e} > R_e$ and $\frac{1}{\beta_i} > R_i$.

Within the large literature on the global imbalance, to generate the observed current account in the U.S. and other developing nations, especially China, the common assumption is the representative agent in the U.S. is more impatient than a representative in the developing country. To generate the flow of funds at the banking sector level from the U.S. to EAM, we only assume that the savers in EAM are more impatient than the weighted average of savers and bankers in the U.S. Other agents in the EAM can be more patient than the U.S. Thus, our assumption does not contradict the assumption in the global imbalance literature.
1.4.2 Calibration

Table 1: Agents Discount Factor

<table>
<thead>
<tr>
<th>Domestic Agent Discount Factor</th>
<th>Value</th>
<th>Foreign Agent Discount Factor</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_p$</td>
<td>0.9875</td>
<td>$\beta^*_p$</td>
<td>0.9925</td>
</tr>
<tr>
<td>$\beta_i$</td>
<td>0.95</td>
<td>$\beta^*_i$</td>
<td>0.94</td>
</tr>
<tr>
<td>$\beta_e$</td>
<td>0.95</td>
<td>$\beta^*_e$</td>
<td>0.94</td>
</tr>
<tr>
<td>$\beta_b$</td>
<td>0.96</td>
<td>$\beta^*_b$</td>
<td>0.975</td>
</tr>
</tbody>
</table>

The discount factors for each agent are given by table 1. All these values are within the range of two standard deviation bands interval (0.91, 0.99) estimated by Carroll and Samwick (1997). They are chosen according to the key assumptions. The fraction of impatient HHs $\sigma$ is 0.5. Campbell and Mankiw (1990) estimated the fraction of liquidity constrained HHs to be 0.5. Iacoviello (2005 and 2010) set the fraction of impatient HHs to be 0.36 and 0.3 respectively. Setting $\sigma$ to be 0.5 is at the upper bound of the values used in the literature. It gives the convenience of algebraically solving the model in closed form without changing its fundamentals. Elasticity of substitution between tradable and non-tradable goods $\omega$ is 0.44 as estimated by Stockman and Tesar (1995). $\gamma_i, \gamma_e$ are 0.9 as in Iacoviello (2010). We choose $\gamma_f$ to be 0.9. Parameters controlling bankers’ adjustment cost $\phi_d, \phi_i, \phi_e, \phi_f$ are 0.25. Loan to values $m_i, m_e, m_f$ are 0.9, 0.9 and 0.7 respectively. Capital depreciation rate $\delta$ is 0.025. The rest of the model’s parameters are chosen from the closed economy model by Iacoviello (2010)

Table 2: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.35</td>
<td>$\nu$</td>
<td>0.08</td>
</tr>
<tr>
<td>$\tau_p$</td>
<td>2</td>
<td>$\tau_i$</td>
<td>2</td>
</tr>
<tr>
<td>$\phi_k$</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1.5 Results

1.5.1 Repayment shock

The repayment shock is exogenous. Alternatively, one can endogenize the default shock as function of the underlying state of the economy. For example, in Forlati and Lambertini (2011), borrowers default endogenously, when they find that the value of their collateral is lower than the value of the loan they borrow. Within the context of
this paper, we treat the repayment shock as exogenous for simplicity and tractability. A further step, to describe how default can happen endogenously and depend on the fundamentals of the lending country, and through the banking sector, spread to the borrowing country, is worthwhile for future investigation.

Figure 10 plots the impulse response results of foreign macroeconomic variables for the foreign repayment shocks. Default coming from foreign impatient HHs forces the foreign banks to contract both loans and deposits to maintain their required capital-asset ratio. The results are a fall in output, asset price, investment, employment and loan and an increase in lending interest rates. Similar results have been obtained in Iacoviello’s (2010) closed economy version.

Figure 11 plots the impulse response of international interbank loan and interest rate. When the lending banks from the developed country contract the loan for all of their borrowers, they do so for the borrowing banks as well.

Figure 12 plots the impulse response results of domestic macroeconomic variables. When foreign banks contract assets by raising lending rates to maintain their capital requirement ratio, domestic banks now face more expensive (as $R_f$ increases) and less available credits (international borrowing constraint tightens when $R_f$ increases), they have to raise domestic lending rates and reduce the loans made to domestic borrowers. A domestic credit crunch, characterized by a decrease in loan and an increase in borrowing interest rates has occurred following the default from abroad.

Domestic output, investment and asset prices fall, which are the typical results following a credit crunch. What is interesting here is the movement of resources across sectors and the dynamics of the real exchange rate. The international loan is denominated in tradable goods. When the loans that foreign banks made to domestic banks suddenly decrease, in the foreign country, the demand for tradable goods decreases and the price of tradables relative to non-tradables decreases. In the domestic country, the supply of tradable goods suddenly decreases, which increases the price of tradables relative to non-tradables. As a result, the real exchange rate decreases on impact. Over
time, in the domestic country, labor and investment move from the non-tradable to the tradable sector to equalize the prices in two sectors, exchange rate appreciates toward its steady state value. Figure 13 plots the impulse responses of real exchange rate and price of tradable and non-tradable goods in the foreign and domestic countries. Figure 8 documents the real exchange rate movements of Chinese Taipei, India and Korea. The sharp reduction in the real exchange rate of these countries against the U.S. happened around the time when U.S. banks substantially deleveraged their balance sheet with respect to Asia.

Figure 14 plots the impulse responses of repayment shock under different values of $\gamma_f$. A lower value of $\gamma_f$ can be interpreted as banks’ strategy to contract foreign loans and give priority to long-term domestic borrowers. Peek and Rosengren (1997) documented this behavior among Japanese banks. It can also be interpreted as a lower credit rating of the domestic economy. With a smaller $\gamma_f$, the repayment shock generates much larger volatilities of domestic variables while decreasing the volatilities of foreign variables. In other words, a lower $\gamma_f$ helps mitigate the effects of the financial shock in the developed country where it originates, while amplifying the effects on the developing country. The intuition for this comes from foreign banks’ capital requirement constraint:

$$d^*_t \leq \gamma_e l^*_e + \gamma_i l^*_i + \gamma_f p^*_T l^*_{f,t}. \tag{26}$$

Since deposit equals assets minus equity:

$$l^*_t + l^*_e + p^*_T l^*_{f,t} - E^*_t \leq \gamma_e l^*_e + \gamma_i l^*_i + \gamma_f p^*_T l^*_{f,t},$$

$$(1 - \gamma_e)l^*_e + (1 - \gamma_i)l^*_i + (1 - \gamma_f)p^*_T l^*_{f,t} \leq E^*_t. \tag{26}$$

When default happens and decreases foreign banks’ equity, $E^*_t$, these banks will have to
decrease the left hand side of the above equation. When $\gamma_f$ is smaller than $\gamma_i$, $\gamma_e$, it is more beneficial for the foreign banks to contract international loans. One unit decrease in $l_{f,t}$ will loosen the capital requirement constraint by $1 - \gamma_f$, which is larger than $1 - \gamma_e$, or $1 - \gamma_i$, if banks contract business loans, or consumer loans. The adjustment costs banks face are convex and together with $\gamma$ will determine how banks contract their portfolios. Without the convexity in costs, when $\gamma_f$ is lower relative to $\gamma_i$ and $\gamma_e$, banks will find it most beneficial to contract foreign loans only.

Devereux and Yetman (2010), using the data for the recent crisis, found that the magnitude of capital flow from one country to the U.S. depends on the country's foreign currency credit rating. A lower rating resulted in a larger capital outflow of the country to the U.S., following the recent U.S. crisis. A lower rating asset will have a higher weight in banks' risk weighted asset (RWA) portfolio in equation 26, or a lower $\gamma_f$ in our model. Thus, the empirical evidence is in line with our model prediction, that countries perceived as more risky will suffer more from the U.S. crisis than less risky countries.

1.5.2 $\gamma_f$ shock

Permanent Shock

A permanent shock to $\gamma_f$ can be interpreted as a change in regulation. A real world example of this is the change from the Basel I Accord to the Basel II Accord. Under the Basel I Accord, banks' assets were classified into categories such as sovereign, banks, collateral, etc. All debts under the same category carried the same weight in banks' RWA and banks were required to hold capital equal to 8% of banks' total RWA. For example, all corporate debts had the weight of 100% and all government debts had the weight of 0%. The Basel II Accord no longer gives the same weight to all assets in one category if they have a different level of risks. Borrowing banks in developing countries, if considered risky by Basel II's new assessment of risk, will have a higher weight in the lending bank's RWA.

Figures 15 to 18 have impulse response for a 10% permanent negative shock to $\gamma_f$. As the international inter-bank loan has a higher weight in the lending banks' RWA, lending banks permanently increase the lending rate, $R_f$, and decrease the amounts of
loans made to borrowing banks in the developing country, \( l_f \). The steady state interbank lending rate is: 
\[
R_f = \frac{1}{\beta^*} - \left[ \frac{1}{\beta^*} - \frac{1}{\beta_H} \right] \gamma_f.
\]
When \( \gamma_f \) decreases, \( R_f \) converges to a higher steady state. The steady state lending rates to domestic borrowers are the weighted average of interbank lending rate and domestic deposit rate. Thus, they converge to a new higher steady state. As a result, domestic consumption, output and investment converge to a lower steady state.

As \( \gamma_f \) permanently decreases, from equation 25, we see that foreign banks’ capital requirement constraint tightens. Foreign banks can ”loosen” the constraint by either deleveraging (reducing the total size of its RWA and deposit) or restructuring its portfolio (holding less assets with high weight and more assets with low weight). The foreign banks’ adjustment cost helps pin the optimal path for their deposit demand and loan supply. Contrary to the repayment shock, when the only option is to deleverage, foreign banks in this case also restructure their portfolios and hold more assets with lower weight in their RWA. As a result, foreign deposit goes down (deleverage effect) and loans to foreign IHs and entrepreneurs go up (portfolio restructuring effect). The foreign investment, consumption and output go up. New steady state foreign domestic lending rates, which only depend on foreign banks and patient HHs time preference, stay the same.

Temporary Shock

Figures 19 to 22 have the impulse responses for the temporary negative shock to \( \gamma_f \). The temporary shock can be interpreted as an exogenous temporary drop in domestic banks’ credit rating. A real world example for this is the drop in domestic bank credit rating of South Korean banks during the Asian financial crisis in 1997. Figure 9 has the graph of credit ratings of nationwide South Korean banks and the South Korean Won - US Dollar exchange rate. Credit Ratings of major banks in South Korea dropped significantly before and right at the beginning of the crisis. The results of the impulse response show a drop in domestic gdp, consumption and investment. The foreign loan given to domestic banks contracts and interest rate increases. The real exchange rate also depreciates as a result of tightening foreign credit. This was also the experience of South Korea during the financial crisis.
1.6 Relation to empirical facts and existing literature

For the foreign repayment shock, our model generates a drop in output, consumption, investment, loans and housing prices and an increase in bank lending rates in both home and foreign countries. The borrowing country’s real exchange rate also depreciates. Qualitatively, our model matches the empirical facts. The lowest drop in the foreign and domestic consumption are \(2 \times 10^{-3}\) and \(2 \times 10^{-4}\), respectively. The drops of foreign and domestic investment are \(5 \times 10^{-3}\) and \(5 \times 10^{-4}\). The transmission of shock to the foreign country is just 10%. Quantitatively, our model does not match the magnitude of international transmission observed in data.

Devereux and Yetman (2010) build an international portfolio model to describe the recent crisis. Leveraged investors in each country holds foreign equity in their portfolios. The total value of their portfolios has to be greater than a constant times their equities. When a shock hits the home country and decreases home asset prices, the value of portfolios of home and foreign investors decreases, forcing them to deleverage. Eric van Wincoop (2011) build a model with leveraged financial institutions, who invest in both home and foreign assets. The default shock in his model is similar to the repayment shock in ours. Since foreign financial institutions hold domestic assets, the domestic default shock damages the foreign bank balance sheet and spreads the crisis to the foreign country. The main difference between our model and theirs is in our model, the leveraged domestic bank does not hold foreign asset. In our model, shock is transmitted through a credit crunch in the interbank loan market. Their models fit well for the comovement between the U.S. and Europe since European Banking Centers were the majority foreign holders of U.S. MBS. Our model fits the story between the U.S. and EAM. EAM were not directly exposed to the U.S. MBS as only 3% of U.S. ABS are held outside of the U.S., Europe and the Carribean.

Ueda (2010), Kollmann et.al. (2011) and Kalemi-Ozcan et.al. (2011) also build international business cycle models with leveraged bank(s). In their models, borrowers in both countries share a common lender(s). When shock hits one country and damages the balance sheet of the common lender(s), the common lender(s) contract loans in both countries. In their model, borrowers in one country have direct access to the credit of the foreign lenders. The story works in the developed world. For EAM, this is not the case as few borrowers in EAM have direct access to U.S. bank credit. In our model,
borrowers in EAM only borrow funds from the U.S. through domestic banks. Thus, in steady state, banks in EAM are net borrowers in our model, which is an empirical fact and cannot be generated with a model of two symmetric countries.

Another main difference between our model and previous models with leveraged financial investors (banks) is in our capital requirement (leverage) constraint, we separate the weights of different assets in the lending bank RWA. Thus, we are able to investigate the behavior of international transmission of shock when borrowing banks have different credit rating. We found that when the borrowing economy has a lower rating, the magnitude of capital that flows back to the U.S. in the crisis is higher. Our result is consistent with the empirical findings in Devereux and Yetman (2010).

1.7 Conclusion

The recent financial crisis in the U.S. highlights the role that the banking sector plays in the global macroeconomy. There has been substantial empirical evidence that suggests financial crisis can be transmitted across borders through the contraction in cross-border loans in the banking system. The very first empirical papers were written by Peek and Rosengren (1997 and 2000), who studied the Japanese financial crisis and its effects on the U.S.. More recent empirical papers study the U.S. financial crisis and its effects on lending in other countries. Such papers are Cetorelli and Goldberg (2008 and 2009) and Popov and Udell (2010). Our model provides a theoretical framework to support the hypothesis. When financial shock hits one country, the cross border inter-bank loan contracts and transmits the shock to another country.

Our paper is also related to a number of papers that study the effects of shocks to international lending rates on a small open economy. These include Buyukkarabacak (2008), Christensen et al. (2009) and Faia and Iliopulos (2010). These papers treat the source of shock as exogenous. Our paper goes one step further and points out that the lending country’s financial shock could be what is behind the increase in the international lending rate. Our paper also differs from other recent papers with leveraged banks (investors) in three dimensions. First, the shock from the source country is not directly transmitted by damaging the foreign banks’ balance sheet, but rather, from contracting the loan in the interbank market. This helps apply our model for EAM, which were
not directly exposed to the U.S. MBS. Second, borrowers in one country do not borrow
directly from foreign banks, but through domestic banks. Thus, in the steady state, at
the banking level, EAM are net borrowers from the U.S. Third, we separate the weight
of international loans from weights for consumer and business loans in the capital re-
quirement constraint. This helps us investigate the dymanics of the borrowing country
when its banks have different credit ratings and when there is a bank regulation change
in the lending country.
1.8 Appendix

The steady state equations of domestic and foreign patient HHs’ FOCs for deposits give the deposit rates: \( R_d = \frac{1}{\beta_p} \) and \( R_d^* = \frac{1}{\beta_p^*} \). Foreign banks’ steady state FOCs for deposits and loans can be written as:

\[
1 - \beta_b^* R_d^* = \lambda_b^* \gamma_d^*, \\
1 - \beta_b^* R_i^* = \lambda_b^* \gamma_i^*, \\
1 - \beta_b^* R_e^* = \lambda_b^* \gamma_e^*, \\
1 - \beta_b^* R_f = \lambda_b^* \gamma_f^*.
\]

where \( \lambda_b^* \) is the multiplier on foreign banks’ capital requirement constraint, multiplied by foreign bankers’ consumption, i.e., \( \lambda_b^* = \lambda_b^* c_{b,t}^* \). Simple algebra, replacing \( \lambda_b^* \) with \( (1 - \beta_b^* R_d^*) \), then yields the foreign banks’ lending rates. Domestic banks’ steady state FOCs for deposits and loans are:

\[
1 - \beta_b R_d = \lambda_b^* + \lambda_f^* m_f, \\
1 - \beta_b R_i = \lambda_b^* \gamma_i^* + \lambda_f^* m_f, \\
1 - \beta_b R_e = \lambda_b^* \gamma_e^* + \lambda_f^* m_f, \\
1 - \beta_b R_f = \lambda_b^* + \lambda_f^* R_f.
\]

From the first and the last equation of the above system of four equations, one can solve for the value of \( \lambda_f^* \):

\[
\lambda_f^* = \frac{\beta_b (R_d - R_f)}{R_f - m_f}.
\]

The bottom of the equation is greater than 0, since \( R_f > 1 > m_f \). Thus, \( \lambda_f^* > 0 \) when \( R_d > R_f \). Combining the above system of equations to solve for domestic lending rates,
we have

\[
\frac{R_f - R_d}{R_i - R_d} = \frac{X_f'(m_f - R_f)}{X_b'(1 - \gamma_i)},
\]

\[
\frac{R_f - R_d}{R_i - R_d} = \frac{X_f'(m_f - R_f)}{(1 - \beta_b R_d - X'_f m_f)(1 - \gamma_i)} = -\delta_1,
\]

\[
R_i = R_d + \frac{1}{\delta_1}(R_d - R_f).
\]

\[
\frac{R_f - R_d}{R_e - R_d} = \frac{X_f'(m_f - R_f)}{X_b'(1 - \gamma_e)},
\]

\[
\frac{R_f - R_d}{R_e - R_d} = \frac{X_f'(m_f - R_f)}{(1 - \beta_b R_d - X'_f m_f)(1 - \gamma_e)} = -\delta_2,
\]

\[
R_e = R_d + \frac{1}{\delta_2}(R_d - R_f).
\]

All deposit and lending rates are now determined.
1.9 Figures

Figure 1:
New Delinquent Balances by Loan Type

Source: Federal Reserve Bank of New York

Figure 2:
Percent of Mortgage Debt 90+ Days Late by State

Source: Federal Reserve Bank of New York
Figure 3:

U.S Bank Assets & Liabilities

Source: Board Governors of Federal Reserve System

Figure 4:

External Assets of Banks in Developed Economies

Source: Bank for International Settlement
Figure 5:
External Liabilities Banks in Emerging Asian Markets

Source: Bank for International Settlement

Figure 6:
GDP Growth of Developed Economies

Source: World Bank
Figure 7:

GDP Growth of U.S and Developing Economies

Source: World Bank

Figure 8:

RER movements of Emerging Markets

Source: Bank for International Settlement
Figure 9:

Source: Moody’s

Figure 10:

Impulse Response: Foreign repayment shock
Figure 11:

Impulse Response: Foreign repayment shock

Figure 12:

Impulse Response: Foreign repayment shock
Figure 13:

Impulse Response: Foreign repayment shock

Figure 14:

Impulse Response: Foreign repayment shock
Figure 15:

Impulse Response: Permanent Shock to $\gamma_f$

Figure 16:

Impulse Response: Permanent Shock to $\gamma_f$
Impulse Response: Permanent Shock to $\gamma_f$

Figure 17:

Figure 18:

Impulse Response: Permanent Shock to $\gamma_f$
Figure 19:

Impulse Response: Temporary Shock to $\gamma_f$

Figure 20:

Impulse Response: Temporary Shock to $\gamma_f$
Figure 21:

Impulse Response: Temporary Shock to $\gamma_f$

Figure 22:

Impulse Response: Temporary Shock to $\gamma_f$
2.10 Abstract

In a standard dynamic stochastic general equilibrium model with a complete asset market, home agents should hold a foreign equity biased portfolio to hedge the non-traded labor income risk, which contradicts home equity biased portfolios observed worldwide. As the labor income share increases, the degree of home bias should decrease because there is more incentive to hold foreign equity. In the data, there is not any evidence that the labor income share and the degree of home bias are negatively correlated. The standard model also predicts that the consumption differential-real exchange rate correlation is positive, while it is negative in the data. I show that a combination of market incompleteness, non-tradable goods and labor supply can explain the three features above. My model can generate a large equity home bias, despite the strong positive correlation of non-traded human capital return with domestic equity return. The home bias is not sensitive to the labor income share. The consumption differential-real exchange rate unconditional correlation generated by my model simulation is zero.
2.11 Introduction

To diversify risks, investors in country $n$, who consume a fraction $\mu^n$ of the world’s output, should buy the same fraction $\mu^n$ of global financial assets, (Obstfeld and Rogoff (1996)). However, investors all over the world hold mostly home equities in their portfolios. Table 1 illustrates the degrees of home equity bias for selected countries. Following Ahearne et al. (2004) and Coeurdacier and Rey (2011), home equity bias is defined as:

$$EHB_i = 1 - \frac{\text{Share of Foreign Equities in Country i Equity Holdings}}{\text{Share of Foreign Equities in the World Market Portfolio}}$$

Table 1: Home Equity Bias for Selected Countries in 2011

<table>
<thead>
<tr>
<th>Country</th>
<th>Domestic Market in % of World Market Capitalization</th>
<th>Share of Portfolio in Domestic Equity in %</th>
<th>Degree of Equity Home Bias $EHB_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>33.0</td>
<td>74.6</td>
<td>0.62</td>
</tr>
<tr>
<td>Canada</td>
<td>4.0</td>
<td>71.7</td>
<td>0.70</td>
</tr>
<tr>
<td>Germany</td>
<td>2.5</td>
<td>47.5</td>
<td>0.46</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>6.6</td>
<td>62.8</td>
<td>0.60</td>
</tr>
<tr>
<td>Australia</td>
<td>2.6</td>
<td>76.8</td>
<td>0.76</td>
</tr>
<tr>
<td>Japan</td>
<td>7.0</td>
<td>79.5</td>
<td>0.78</td>
</tr>
</tbody>
</table>

Baxter and Jermann (1997) point out that since non-traded human capital return can be highly correlated with domestic equity return, the optimal portfolio should be foreign biased, which makes the puzzle “worse than you think.” A standard dynamic stochastic general equilibrium (DSGE) model predicts that home investors should hold mostly foreign equity. In addition, the fraction of domestic equity in home portfolios should depend on the fraction of labor income share in home GDP. The larger the labor income share is, the less domestic equity home investors should hold, or the smaller the degree of home equity bias. This is not the case in the data. Figure 1 plots the degree of home equity bias against labor income shares across OECD countries in 2005. Figures 2 and 3 graph labor income shares and the degree of home equity bias over time for selected countries. The data suggests that there is not a negative relation between labor income share and home equity bias.

1 Data source: World Federation of Exchanges and CPIS.
income shares and the degree of home equity bias.

The early literature on international portfolios that tried to explain the observed level of home bias is based on endowment economy models without labor income. Such models are in Tesar (1993), Baxter, Jermann and King (1998), Pesenti and van Wincoop (2002), Collards et al. (2009) and etc. With an endowment economy, one can avoid the tendency of labor income to generate a foreign biased portfolio. Although endowment economy models help build our initial foundation for the understanding of optimal international portfolios, they ignore half of the puzzle.

The world wide increase of asset trade in the last two decades, together with its importance in the global transmission of shocks has generated renewed interest in understanding international portfolios in a DSGE context. Tille and van Wincoop (2010), Devereux and Sutherland (2011) and Evans and Hnatkovska (2012) developed methods to solve for optimal international portfolios in DSGE models. Matsumoto (2007), Heathcote and Perri (2008), Engel and Matsumoto (2009), Coeurdacier et al. (2009 and 2010) are among those who applied these methods in a complete market framework. Matsumoto (2007) builds an international portfolio model with tradable and non-tradable sectors. He assumes complete markets, and the stocks in both sectors are traded internationally. He finds that the optimal portfolio depends on parameters’ values. A very foreign biased portfolio of stocks in the non-tradable sector is needed to generate equity home bias in the portfolio of stocks in the tradable sector. In the data, however, the degree of equity home bias in the non-tradable sector is much higher relative to that of the tradable sector (see Kang and Stulz (1997), Denis and Huizinga (2004), Hnatkovska (2010)). Heathcote and Perri (2008) generate home bias with capital accumulation. This results in a pro-cyclical investment expenditure and counter-cyclical dividends. Thus, home equity is perfectly negatively correlated with home labor income, which makes it useful to hedge labor income risk. However, labor income and dividend payment are positively correlated in the data for G7 countries, casting doubt on Heathcote and Perri’s key mechanism for generating equity home bias (Coeurdacier et al. (2010)). Engel and Matsumoto (2009) show that a forward position in the foreign exchange market can ensure perfect risk sharing with nominal rigidity. Thus, a com-
plete market equilibrium can be achieved even with an equity home biased portfolio. However, the implied long position of home investors on domestic bond contradicts the fact that the U.S appears short on the dollar and long on foreign exchange (Obstfeld (2007)). Tille and van Wincoop (2010) use cost in asset trade to generate equity home bias. Fitzgerald’s (2012) empirical tests find that the null hypothesis of frictionless asset markets within developed countries cannot be rejected. Coeurdacier et al. (2009) use redistributive shocks and “iPod” shocks. Such shocks are of debatable and need more micro-foundation (Ghironi (2007)). Coeurdacier et al. (2010) explain home bias with investment efficiency shocks.

International portfolio models that assume market incompleteness include Pesenti and van Wincoop (2002), Hnatkovska (2010) and Feng (2012). Pesenti and van Wincoop (2002) build a portfolio balance, endowment economy model where stocks of the non-tradable endowment are not traded. They obtain moments of stock returns and tradable and non-tradable consumptions, and they conclude that the optimal portfolio should be slightly home biased. Hnatkovska (2010) builds a DSGE model with similar assumptions. In her model, bias in consumption of tradable goods generates home bias. When home non-tradable consumption increases above foreign, home demand for tradable goods increases. Since home consumption of tradable goods consists of largely home goods, home agents should hold home equity in the tradable sector to hedge non-tradable sector technology shocks. The findings of these papers suggest that market incompleteness could be an answer to the international portfolio puzzle. However, it is uncertain whether their results still hold when the labor income is present. In addition, it is complicated, if not yet possible, for one to extend their models to include labor income in a standard DSGE framework. The numerical method used to solve for dynamic portfolio choice in Hnatkovska (2010) relies critically on the closed form solution for dynamic portfolio holdings given conditional means and variance of returns, which was developed by Campbell, Chan and Viceira (2003). With labor income, this method does not yield a closed form solution for portfolio holdings (Viceira (2001) and Campbell and Viceira (2003)). Feng (2012) builds a model that can generate home equity bias with incomplete market, endogeneous labor supply and taste shock. She solves for the optimal portfolio that depends on the covariance of labor income and taste shock.
with foreign equity excess return. With the correlation measured from data, a home equity bias portfolio is implied. However, it is unclear whether the model generate a high positive correlation between home equity return and home labor income. To see why this is the case, log linearize the consumption, leisure first order conditions to get: 
\[ \hat{\tau}_t + \hat{w}_t - \rho \hat{C}_t = \kappa \hat{L}_t, \]
where \( \tau_t, w_t, C_t \) and \( L_t \) are taste shock, wage, consumption and labor supply. \( \kappa \) is the inverse of the Frisch elasticity of labor supply. For simplicity, assume further that labor is inelastically supplied and therefore \( \kappa = 0 \), the equation becomes: 
\[ \hat{\tau}_t + \hat{w}_t - \rho \hat{C}_t = 0. \]
Thus, when a positive taste shock hits, wage tends to be negatively correlated with consumption, and consequently, negatively correlated with domestic equity return. Since it is unclear whether the model generates a strong positive correlation between domestic equity return and human capital return in a DSGE setting, it is unclear whether the model has solved the puzzle identified by Baxter and Jermann (1997).

In this paper, I extend the work by Pesenti and van Wincoop (2002) and Hnatkovska (2010) and include a production economy. The percentage of home equity held in the home portfolio of stocks in the tradable sector generated by my model is 94%, despite a 64% labor income share in the GDP and the strongly positive unconditional correlation of human capital return and equity return. The optimal portfolio is insensitive to the change in the labor income share. In addition, the unconditional correlation of consumption differential and real exchange rate is zero.

In my model, market incompleteness and non-tradable goods tilt home portfolio toward home equity. When the market is incomplete, home agents cannot fully insure against non-tradable sector relative technology shock. When favorable non-tradable sector relative technology shock hits, home non-tradable consumption is high and therefore, home marginal utility of tradables is high, due to the complementary relationship between the two goods. At the same time, labor mobility across sectors increases home tradable output, making home tradable sector equity a good asset to hedge non-tradable sector relative technology shocks. When labor income is a negligible part of GDP, it is intuitive that home agents will hold a home biased portfolio of tradable sector equity, as seen in Pesenti and van Wincoop (2002) and Hnatkovska (2010). As labor income share
increases, on the one hand home agents would like to hold more foreign equity to hedge the positive correlation of home equity return and labor income. On the other hand, as capital share decreases, home agents need more home equity to hedge non-tradable sector relative technology shocks. The change in the degree of home bias when the labor income share increases is small, which is what we observe in the data.

This paper is also related to the literature on the consumption differential-real exchange rate correlation puzzle. In a standard complete market framework, consumption differential and real exchange rates between two countries should be perfectly positively correlated (Backus and Smith (1993)). This is not the case in the data since the correlation is low and often negative. Figures 5 and 6 graph the consumption differential and real exchange rate for the last 37 years between the U.S. and U.K., and the U.S. and Japan. Table 2 reports the correlation between the two series.\(^2\)

<table>
<thead>
<tr>
<th></th>
<th>(Cor(C^H_t, RER_t))</th>
<th>(Cor(\Delta C^H_t, \Delta RER_t))</th>
</tr>
</thead>
<tbody>
<tr>
<td>US-UK</td>
<td>-0.33</td>
<td>-0.17</td>
</tr>
<tr>
<td>US-JPN</td>
<td>0.42</td>
<td>0.39</td>
</tr>
</tbody>
</table>

Benigno and Thoenissen (2008) show that with an incomplete market, tradable sector technology shocks generate a strong negative consumption differential-real exchange rate correlation while non-tradable sector technology shocks generate a strong positive correlation. The model is convincing if one believes that tradable sector technology shocks are the prevailing source of fluctuation. The model can generate a low consumption-real exchange rate correlation when the tradable sector productivity shocks are seven times more volatile and three times more persistent than the non-tradable sector. Corsetti, Dedola and Leduc (2008) generate a negative consumption differential-real exchange rate in line with data using highly persistent shocks perfectly correlated across sectors.

In my model, I investigate the consumption differential-real exchange rate puz-

\(^2\)Data is from the World Bank. The HP filter parameter is 6.25 as in Ravn and Uhlig (2002) for annual data
zle jointly with the home equity bias puzzle. Doing so helps me identify one channel that can generate low correlation that has not been identified before. Following tradable sector relative technology shocks, the correlation between consumption and real exchange rate is negative, which is similar to the results in the previous literature. Following non-tradable sector relative technology shocks, home non-tradable consumption, and therefore tradable consumption, rise above foreign. Since the home portfolio does not contain enough home equity to support such an increase in consumption, home agents have to spend a fraction of their permanent wealth. Thus, in subsequent periods when home wealth deteriorate, home consumption decays at much faster rate than other variables in the model, including the real exchange rate. The results with non-tradable sector productivity shocks are different to those found in previous literature, which usually find that non-tradable technology shocks generate perfect correlation between consumption differential and real exchange rate. The unconditional consumption differential-real exchange rate in my model is close to zero with more convincing shock processes.

2.12 Model

The model framework is built upon Ghironi, Lee and Rebucci (2009) and Devereux and Sutherland (2010). I use Devereux and Sutherland’s (2011) solution method to solve for the optimal portfolio. There are two symmetric countries, each has size 1/2, with tradable and non-tradable sectors. Following Pesenti and van Wincoop (2002) and Hnatkovska (2010), I assume market incompleteness, and equities of firms in the non-tradable sector are not traded internationally (see Kang and Stulz (1997), Denis and Huizinga (2004), and Hnatkovska (2010) for empirical evidence). Prices are flexible and labor is endogenous and mobile across sectors. Shocks are log AR(1), sectoral technology shocks and uncorrelated across countries and sectors.

The basket of tradable goods consumed at home is given by:

\[
C^T_t = \left[ \left( \frac{1}{2} \right)^{\frac{1}{\omega}} (C^H_t)^{\frac{1}{\omega} - 1} + \left( \frac{1}{2} \right)^{\frac{1}{2}} (C^F_t)^{\frac{1}{2} - 1} \right]^{\frac{1}{\omega} - 1}, \omega > 0,
\]

where \(C^H_t\) and \(C^F_t\) denote consumption sub-baskets consumed at home of both home
and foreign tradable goods, given by Dixit-Stiglitz aggregates:

\[ C^H_t = \left[ 2^{\frac{1}{\epsilon}} \int_0^1 c^H_t(z)^{\frac{1}{\epsilon}} \, dz \right]^{\frac{1}{1-\epsilon}}, \quad C^F_t = \left[ 2^{\frac{1}{\epsilon}} \int_0^1 c^F_t(z^*)^{\frac{1}{\epsilon}} \, dz^* \right]^{\frac{1}{1-\epsilon}} \]

with \( \epsilon > 1 \).

The corresponding price indexes are:

\[ P^T_t = \left[ \frac{1}{2} (P^H_t)^{1-\omega} + \frac{1}{2} (P^F_t)^{1-\omega} \right]^{\frac{1}{1-\omega}}, \]

\[ P^H_t = \left[ 2^{\frac{1}{\epsilon}} \int_0^1 (P^H_t(z))^{1-\epsilon} \, dz \right]^{\frac{1}{1-\epsilon}}, \quad P^F_t = \left[ 2^{\frac{1}{\epsilon}} \int_0^1 (P^F_t(z^*))^{1-\epsilon} \, dz^* \right]^{\frac{1}{1-\epsilon}}. \]

The non-tradable consumption aggregate and price index are:

\[ C^N_t = \left[ \int_0^1 c^N_t(z)^{\frac{1}{\epsilon}} \, dz \right]^{\frac{1}{1-\epsilon}}, \quad P^N_t = \left[ \int_0^1 P^N_t(z)^{1-\epsilon} \, dz \right]^{\frac{1}{1-\epsilon}}. \]

Home agents’ maximization problem is:

\[ \max E_0 \sum \gamma_t \left\{ \frac{C_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \chi L_t^{1+\frac{1}{\varphi}} \right\}, \sigma > 0, \varphi > 0. \]

I follow Schmitt-Grohe and Uribe (2003) and assume endogenous discount factors that follow the following process:

\[ \gamma_{t+1} = \gamma_t \beta (C^T_{A_t})^{-\eta} / (C^T_A)^{-\eta}, \eta > 0, \]

where \( C^T_A \) and \( C^T_{A_t} \) are country aggregate tradable good consumption at time \( t \) and its initial symmetric steady state. Agents take \( \gamma_t \) as exogeneous and do not internalize the impact of their consumption on the discount factor. Consumption is an aggregate of tradable and non-tradable consumption: \( C_t = \left[ a^{\frac{1}{\theta}} (C^T_t)^{\frac{\theta-1}{\theta}} + (1-a)^{\frac{1}{\theta}} (C^N_t)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \theta > 0 \). The parameter \( a \) controls for the relative size of tradable and non-tradable sectors. The budget constraints in units of tradable consumption baskets is given by:

\[ C^T_t + p^N_t C^N_t + \alpha_{1t} + \alpha_{2t} = r_{1t} \alpha_{1t-1} + r_{2t} \alpha_{2t-1} + d^T_t + d^N_t + w_t L_t, \]

where \( p^N_t \) is the price of the basket of non-tradables in terms of tradables (i.e. \( p^N_t = P^N_t / P^T_t \)). \( \alpha_{1t} \) and \( \alpha_{2t} \) are home real holdings of domestic and foreign tradable equities.
$r_{1t}$ and $r_{2t}$ are returns on home and foreign equities in the tradable sector. $d^T_t$ and $d^N_t$ are dividends of home tradable and non-tradable sectors. $w_t$ is the wage and $L_t$ is the total labor supply. The problem for foreign agents is similar. Foreign variables are denoted with asterisks.

The first-order conditions for home agents are:

\[
\frac{C_t^{\frac{1}{\theta} - \frac{1}{\sigma}} a^{\frac{1}{\sigma}}}{(C^T_t)^{\frac{1}{\theta}}} = \lambda_t, \tag{1}
\]

\[
\frac{C_t^{\frac{1}{\theta} - \frac{1}{\sigma}} (1 - a)^{\frac{1}{\sigma}}}{(C^N_t)^{\frac{1}{\theta}}} = p^N_t \lambda_t, \tag{2}
\]

\[
\chi L_t^\theta = w_t \lambda_t, \tag{3}
\]

\[
\lambda_t = \frac{\gamma_{t+1}}{\gamma_t} E_t [\beta \lambda_{t+1} r_{1t+1}], \tag{4}
\]

\[
\lambda_t = \frac{\gamma_{t+1}}{\gamma_t} E_t [\beta \lambda_{t+1} r_{2t+1}], \tag{5}
\]

where $\lambda_t$ is the Lagrangian multiplier of the budget constraint. The discount factor at $t + 1$ is known at time $t$ and appears outside the expectation operator.

Firm $z$’s production is linear in labor and is given by:

\[
y_t^j(z) = Z_t^j L_t^j(z), \quad j = T, N,
\]

where $y_t^j(z)^T$, $y_t^N(z)^T$, $L_t(z)^T$ and $L_t^N(z)$ are the outputs and labor demands of individual firms in the tradable and non-tradable sectors. $Z_t^T$ and $Z_t^N$ are technologies in the tradable and non-tradable sectors and their log deviations from steady state, $\tilde{z}_{t+1}^T$ and $\tilde{z}_{t+1}^N$, follow AR(1) processes as follows:

\[
\begin{bmatrix}
\tilde{z}_{t+1}^T - \tilde{z}_{t+1}^* \\
\tilde{z}_{t+1}^N - \tilde{z}_{t+1}^*
\end{bmatrix}
= \begin{bmatrix}
\rho^T & 0 \\
0 & \rho^N
\end{bmatrix}
\begin{bmatrix}
\tilde{z}_t^T - \tilde{z}_t^* \\
\tilde{z}_t^N - \tilde{z}_t^*
\end{bmatrix}
+ \begin{bmatrix}
\tilde{e}_{t+1}^T \\
\tilde{e}_{t+1}^N
\end{bmatrix}, \tag{6}
\]
where $e_T^T$ and $e_T^N$ are jointly normally distributed with mean zero and covariance matrix:

$$
E_t \begin{bmatrix}
  e_{t+1}^T \\
  e_{t+1}^N \\
  e_{t+1}^T \\
  e_{t+1}^N
\end{bmatrix}
= \begin{bmatrix}
  (\sigma_T^T)^2 & COV(e_{t+1}^T, e_{t+1}^N) \\
  COV(e_{t+1}^T, e_{t+1}^N) & (\sigma_N^N)^2
\end{bmatrix}
$$

$$
= (\sigma_N^N)^2 \begin{bmatrix}
  \nu & \rho^{TN} \sqrt{\nu} \\
  \rho^{TN} \sqrt{\nu} & 1
\end{bmatrix}.
$$

$\sigma_T$, $\sigma_N$, $\rho^{TN}$ are standard deviations and correlation of $e_T^T$ and $e_T^N$. $\nu$ is the variance ratio of the two shocks. Firm revenues are distributed as labor income and dividends. Firms’ profit maximizing behaviors yield the following conditions for dividends, prices, and labor incomes:

$$
d_t(z)^j = \frac{p_t(z)^j y_t(z)^j}{\epsilon}, \quad p_t(z)^j = \frac{\epsilon w_t^j}{\epsilon - 1 Z_t^j},
$$

$$
w_t L_t(z)^j = p_t(z)^j y_t(z)^j \epsilon - 1 \epsilon, \quad j = T, N,
$$

where prices are in units of the tradable consumption basket. Aggregate over tradable and non-tradable sectors to get the total dividends, prices and labor income payments in each sector:

$$
d_t^j = \frac{p_t^j y_t^j}{\epsilon}, \quad p_t^j = \frac{\epsilon w_t}{\epsilon - 1 Z_t^j},
$$

$$
w_t L_t^j = p_t^j y_t^j \epsilon - 1 \epsilon, \quad j = T, N
$$

2.13 Solving for the optimal portfolio

Combining equations (4) and (5) from the consumers’ first-order conditions, we have:

$$
E_t \begin{bmatrix}
  \frac{1}{2} \frac{\hat{x}_t^T}{(C_t^T)^{\frac{3}{2}}} r_{1t+1} \\
  \frac{1}{2} \frac{\hat{x}_t^T}{(C_t^T)^{\frac{3}{2}}} r_{2t+1}
\end{bmatrix}
= E_t \begin{bmatrix}
  \frac{1}{2} \frac{\hat{x}_t^T}{(C_t^T)^{\frac{3}{2}}} r_{1t+1} \\
  \frac{1}{2} \frac{\hat{x}_t^T}{(C_t^T)^{\frac{3}{2}}} r_{2t+1}
\end{bmatrix}.
$$

Denote $\hat{x}_t$ the log deviation of variable $x_t$ from its steady state. To solve for the optimal portfolio, I follow Devereux and Sutherland (2011) and take a second-order Taylor expansion of the above equation around the steady state, which yields the
following equation:

\[
E_t \left[ \hat{r}_{xt+1} + \frac{1}{2} (\hat{r}_{1t+1}^2 - \hat{r}_{2t+1}^2) - \frac{1}{\theta} \hat{C}_{t+1}^T \hat{r}_{xt+1} + \left( \frac{1}{\theta} - \frac{1}{\sigma} \right) \hat{C}_{t+1} \hat{r}_{xt+1} \right] = 0 + 0(\epsilon^3)
\]

\(0(\epsilon^3)\) is a residual which contains all terms of order higher than two, which can be ignored in a second-order approximation. \(r_{xt}\) is the return differential between home and foreign stocks: \(r_{xt} \equiv r_{1t} - r_{2t}\) and \(\hat{r}_{xt} \equiv \hat{r}_{1t+1} - \hat{r}_{2t+1}\). Applying a similar procedure to the foreign first-order conditions gives us:

\[
E_t \left[ \hat{r}_{xt+1} + \frac{1}{2} (\hat{r}_{1t+1}^2 - \hat{r}_{2t+1}^2) - \frac{1}{\theta} \hat{C}_{t+1}^T \hat{r}_{xt+1} + \left( \frac{1}{\theta} - \frac{1}{\sigma} \right) \hat{C}_{t+1} \hat{r}_{xt+1} \right] = 0 + 0(\epsilon^3)
\]

One can rearrange the above two equations to get the following equations:

\[
E_t \left[ \left\{ -\frac{1}{\theta} (\hat{C}_{t+1}^T - \hat{C}_{t+1}^T) + \left( \frac{1}{\theta} - \frac{1}{\sigma} \right) (\hat{C}_{t+1} - \hat{C}_{t+1}^T) \right\} \hat{r}_{xt+1} \right] = 0 + 0(\epsilon^3) \tag{9}
\]

\[
E_t[\hat{r}_{xt+1}] = E_t \left[ -\frac{1}{2} (\hat{r}_{1t+1}^2 - \hat{r}_{2t+1}^2) + \frac{1}{2\theta} (\hat{C}_{t+1}^T + \hat{C}_{t+1}^T) \hat{r}_{xt+1} - \frac{1}{2} \left( \frac{1}{\theta} - \frac{1}{\sigma} \right) (\hat{C}_{t+1} + \hat{C}_{t+1}^T) \hat{r}_{xt+1} \right] + 0(\epsilon^3) \tag{10}
\]

Equation (6) is the portfolio optimality condition. Note that when the size of the non-tradable sector is zero, and \(C_{t+1} = C_{t+1}^T\), we get the equation in Devereux and Sutherland (2011): \(E_t[(\hat{C}_{t+1} - \hat{C}_{t+1}^T) \hat{r}_{xt+1}] = 0\). Equation (7) indicates that up to first-order approximation, \(E_t[\hat{r}_{xt+1}] = 0\). This is the same result as in Devereux and Sutherland (2011).

Define \(W_t = \alpha_{1t} + \alpha_{2t}\) to be total net claims of home agents on the foreign country at the end of period \(t\) (i.e. the net foreign assets of home agents). The log deviation of \(W_t\) is defined as: \(\hat{W}_t = (W_t - \bar{W})/p^Hy^T\), where \(\bar{W}, p^H\) and \(y^T\) are initial steady state values of home net foreign assets, tradable price and tradable output respectively. Let \(\bar{\alpha} = \alpha_1/\beta p^Ty^T\). Combining home and foreign budget constraints, first-order conditions for asset holdings and shock processes, one can derive the dynamics of tradable consumption.
differentials and net foreign assets:

\[
\dot{\mathbf{W}}_t = \frac{1}{\beta} \dot{\mathbf{W}}_{t-1} + \bar{\alpha} \hat{r}_{x,t} + \frac{1}{2} \left[ (AB - E)(\hat{z}^T_t - \hat{z}^T_{t-1}) - (G - AC)(\hat{C}^T_t - \hat{C}^T_{t-1}) - (AD - F)(\hat{z}^N_t - \hat{z}^N_{t-1}) \right],
\]

(8)

\[
(1 - \xi)^i (\hat{C}^T_t - \hat{C}^T_{t-1}) + \frac{1 - \rho^T}{1 - \xi - \rho^T} [(1 - \xi)^i - (\rho^T)^i] I (\hat{z}^T_t - \hat{z}^T_{t-1})
\]

\[- \frac{1 - \rho^N}{1 - \xi - \rho^N} [(1 - \xi)^i - (\rho^N)^i] K (\hat{z}^N_t - \hat{z}^N_{t-1}) = E_t \left[ \hat{C}^T_{t+1} - \hat{C}^T_{t+1} \right], \forall i \geq 0 ,
\]

(9)

where

\[
A = \left( \frac{1 - \omega}{\epsilon} + \frac{1 - 1}{a} \right) + \frac{\varphi}{\sigma} \left( \frac{\sigma - (\sigma - \theta)(1 - a)}{\sigma} \right) - \frac{(\epsilon - 1)}{\sigma} \left( 1 - a \right)
\]

\[
B = \frac{\varphi}{\sigma} \left[ (\sigma - (\sigma - \theta)(1 - a)) + (\sigma(\sigma + \theta(1 - a)) \right]
\]

\[
D = \frac{\varphi}{\sigma} \left[ (1 - a) + \varphi(\sigma - \theta)(1 - a) \right]
\]

\[
F = \left[ \frac{\epsilon - 1}{a} \frac{\varphi}{\sigma}, (\sigma - (\sigma - \theta)(1 - a)) + \frac{\epsilon - 1}{1 - a} \right] \left( 1 - a \right)
\]

\[
t = \frac{\varphi}{\sigma} \left( \frac{(\sigma - (\sigma - \theta)(1 - a))}{\sigma} \right)
\]

\[
\xi = \frac{\varphi}{1 + C(1 - a)(\sigma - \theta)}
\]

Without the non-tradable sector, \( a = 1, I = 0 \) and \( K = 0 \). When the stationary inducing device is removed, \( \eta = 0 \) and equation 8 becomes: \( (\hat{C}^T_t - \hat{C}^T_{t-1}) = E_t \left[ \hat{C}^T_{t+1} - \hat{C}^T_{t+1} \right] \), \( \forall i \geq 0 \). The consumption differential is a random walk that jumps immediately to its long-run level on the impact of shocks, which is the same result as in Ghironi, Lee and Rebucci (2009) and Devereux and Sutherland (2010). We can combine equation (8), equation (9) and the no-Ponzi condition to solve for the on-impact tradable consumption differential, \( (\hat{C}^T_t - \hat{C}^T_{t-1}) \), as a function of technology shocks:

\[
\hat{C}^T_t - \hat{C}^T_{t-1} = \frac{2(1 - \beta(1 - \xi))}{\beta \left[ (G - AC) - 2\bar{\alpha}(1 - \omega)(1 - \beta)C \right]} \frac{\dot{\mathbf{W}}_t}{1 - \beta(1 - \xi)}
\]

\[
\left[ 2\bar{\alpha}(1 - \beta)(1 - \omega) \left( (B - 1) + \frac{I\beta(1 - \rho^T)}{1 - \beta(1 - \xi)} \right) + (AB - E) - \frac{I\beta(1 - \rho^T)}{1 - \beta(1 - \xi)} (G - AC) \right]
\]

\[- \left[ 2\bar{\alpha}(1 - \beta)(1 - \omega) \left( D + \frac{K\beta(1 - \rho^N)}{1 - \beta(1 - \xi)} \right) + (AD - F) - \frac{K\beta(1 - \rho^N)}{1 - \beta(1 - \xi)} (G - AC) \right]
\]

\[
\left( \hat{z}^T_t - \hat{z}^T_{t-1} \right)
\]

\[
\left( \hat{z}^N_t - \hat{z}^N_{t-1} \right)
\]

(10)

\[\text{\textsuperscript{3}}\text{Detailed derivations are given in Appendix A2 and A3.}\]

\[\text{\textsuperscript{4}}\text{Detailed derivations are given in Appendix A4.}\]
With the solution for \((C^T_t - C^T_{t+1})\), and hence the dynamics of \((C^T_{t+i} - C^T_{t+i-1})\) according to equation (9), we can solve for the on-impact return differential:\(^5\)

\[
\hat{r}_{xt} = \frac{(1 - \beta)(1 - \omega)}{[(G - AC) - 2\delta(1 - \omega)(1 - \beta)]\epsilon_t^\beta} \left\{ [(B - 1)(G - AC) + C(AB - E)] \frac{\epsilon_t^\beta}{1 - \beta}\right\} - [D(G - AC) + C(AD - F)] \frac{\epsilon_t^\beta}{1 - \beta}\right\}.
\]

(11)

Without the non-tradable sector, \(a = 1\) and the solution for the return differential coincides with the results in Ghironi, Lee and Rebucci (2009):\(^6\)

\[
\hat{r}_{xt} = \frac{\sigma(1 + \varphi)(1 - \beta)(\omega - 1)}{(1 - \beta \rho) [\varphi(\omega - 1) + \sigma(\varphi + \omega) - 2\delta(1 - \omega)(1 - \beta)] \epsilon_t^\beta}.
\]

When \(a = 1\) and \(\varphi = 0\), or labor is inelastically supplied, the return differential is:

\[
\hat{r}_{xt} = \frac{1 - \beta}{1 - \beta \rho} \left[\frac{(\omega - 1)\epsilon_t^\beta}{\omega}\right],
\]

which is similar to the solution found in Devereux and Sutherland (2010):\(^7\) Combining equations (6), (10) and (11) gives the solution for \(\alpha_1\) :

\[
\alpha_1 = \frac{\beta py}{2(1 - \omega)(1 - \beta)} \left\{ \frac{\Psi_1 \Omega_1 \epsilon_t^\beta}{(1 - \beta \rho)^2} + \frac{\Psi_2 \Omega_2}{(1 - \beta \rho)^2} \right\} + \left\{ \frac{(\Psi_1 \Omega_2 + \Psi_2 \Omega_1) \epsilon_t^\beta N \sqrt{1}}{(1 - \beta \rho)^2} + \frac{(\Psi_1 \Omega_2 + \Psi_2 \Omega_1) \epsilon_t^\beta N \sqrt{1}}{(1 - \beta \rho)^2} \right\},
\]

where \(p\) and \(y\) are steady state relative price and output of home tradable sector in units of tradable consumption basket, and:

\[
\Phi_1 = B - 1 - IC \frac{1 - \beta}{1 - \beta(1 - \xi)}, \quad \Phi_2 = D - KC \frac{1 - \beta}{1 - \beta(1 - \xi)},
\]

\[
\Psi_1 = (AB - E) + I(G - AC) \frac{1 - \beta}{1 - \beta(1 - \xi)}, \quad \Psi_2 = (AD - F) + K(G - AC) \frac{1 - \beta}{1 - \beta(1 - \xi)},
\]

\[
\Omega_1 = (B - 1)(G - AC) + C(AB - E), \quad \Omega_2 = D(G - AC) + C(AD - F).
\]

The total value of home equity in the tradable sector is \(\beta py/((1 - \beta)\epsilon).\) Therefore, the proportion of home equity in the tradable sector held by home households, \(\delta^T,\) is given

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\(^5\)Detailed derivations are given in Appendix A5.

\(^6\)The solution coincides with the case in Ghironi, Lee and Rebucci (2009) when the government expenditure is zero and countries are symmetric.

\(^7\)In my model, innovation to the dividend differential at time \(t\) when \(a = 1\) and \(\varphi = 0\) is \((\omega - 1)\epsilon_t^\beta / \omega\).
by:
\[
\delta^T = \frac{\beta_{py}}{(1-\beta)} + \alpha_1 = 1 + \left[ \frac{1}{2(1 - \omega)} - \frac{\Psi_1 \Omega_1}{(1-\beta)\rho^T} + \frac{\Psi_2 \Omega_2}{(1-\beta)^2 \rho^T} \right] \epsilon.
\]

\section{2.14 Return to human capital}

In order to calculate the return to human capital, I suppose that each agent in each country can trade the claim on the human capital to other agents of the same country. The human capital is defined as:\footnote{Since the elasticity of the discount factor with respect to consumption is extremely small, \(\eta = 0.001\), the result is not much different from defining human capital as the summation of the stream of wage income discounted by \(\gamma_t\).}

\[
H_t = \sum_{i=0}^{\infty} \beta^i w_{t+i+1}.
\]

The return on such claim is thus:

\[
r_{ht} = \frac{H_t + w_t H_{t-1}}{H_{t-1}}.
\]

In equilibrium, every agent in one country will hold the same amount of human capital:

\[
H_i^t = H_j^t, \forall i, j \text{ in the same country. Therefore, } H_i^t = H_j^t = 0.
\]

The innovation to the human capital return differential can be expressed as:\footnote{detailed derivations are given in Appendix A6}

\[
\hat{r}_{ht} - E_{t-1}[\hat{r}_{ht}] = (1 - \beta) \left[ (G - AC) - 2\bar{\alpha}(1 - \omega)(1 - \beta)C \right] - \left[ B(G - AC) + C2\bar{\alpha}(1 - \beta)(1 - \omega) + C(AB - E) \right] \frac{e_T}{1 - \beta \rho^T} - \left[ D(G - AC) + C(AD - F) \right] \frac{e_N}{1 - \beta N}
\]

where \(\hat{r}_{ht} = \hat{r}_t - \hat{r}^*\).

\section{2.15 The optimal portfolios}

\subsection{2.15.1 Benchmark calibration}

I pick \(\epsilon = 2.8\), which implies that the labor income share is 64% of the total output. I pick the elasticity of substitution between home and foreign tradables \(\omega = 1.8\). Backus,
Kehoe and Kydland (1994) estimate this parameter in the neighborhood of 1.5. Lai and Trefler (2002) estimate it to be 12 from disaggregated data. Similar to Tesar (1993), the elasticity of substitution between tradable and non-tradable goods is $\theta = 0.44$. I assume the coefficient of relative risk aversion (CRRA) $1/\sigma = 0.2$. The usual value of CRRA used in the business cycle literature is 1 or 2. However, there are empirical papers that estimate much lower values. Mankiw, Rotemberg and Summers (1985) estimate $1/\sigma$ to be in the range of 0.09 and 0.51. Amano and Wirjanto (1994) estimate $1/\sigma$ can be as low as 0.124. Pesenti and van Wincoop (2002) find it to be 0.02. Thus, the value of $1/\sigma = 0.2$ is still within the range found in the empirical literature. I pick $\alpha$ to be 0.3, which approximately corresponds to the trade volume of the U.S in 2011.\footnote{Trade data from Bureau of Economic Analysis. GDP data from IMF} The discount factor is set to 0.95, corresponding to the annual return of 5%. Following King and Rebelo (1999) and Ghironi, Lee and Rebucci (2009), the Frisch elasticity of labor supply is $\varphi = 4$. The autocorrelation coefficient of shocks is $\rho^T = \rho^N = 0.99$. Ireland (2001) estimates technology shock autocorrelation coefficient and find values as high as 0.9983 for quarterly data, corresponding to the value of 0.993 for annual data. The variance ratio of tradable and non-tradable sector relative technology shocks is 1.4 in Stockman and Tesar (2003), 2.5 in Corsetti, Dedola and Leduc (2008) and 7.2 in Benigno and Thoenissen (2008). I set the variance ratio $\iota = 4$, which is within the range of the estimated. The corresponding correlation of shocks are 0.35, 0.01 and 0.34 in these papers respectively. I set the correlation of shocks to be 0.25.

2.15.2 Complete market

There are two assets in the model: the home and foreign equities of the tradable sector. The financial market is complete when there are only two shocks: home and foreign tradable sector technology shocks. This is the case when either the size of the non-tradable sector is 0, or the non-tradable sector relative technology shock variance is zeros.

Complete market without the non-tradable sector:

When the size of the non-tradable sector is 0, $\alpha = 1$ and the proportion of home
equity held by home households becomes: $\delta^T = 1 - \epsilon / 2$. The solution coincides with Ghi-roni, Lee and Rebuffi (2009) and Devereux and Sutherland (2011). When $\epsilon = 2.8$, 64% of output is distributed toward labor income, the optimal portfolio is $\delta^T = \delta^T_{CM1} = -0.4$, and home agents should short sell home equity. This solution also coincides with Baxter and Jermann (1997). A foreign biased portfolio is optimal to hedge non-traded labor income risk.

**Complete market with the non-tradable sector and non-tradable sector relative technology shock variance is zero:**

When $\sigma^N = 0$, $\nu = \infty$, the optimal portfolio is $\delta^T = \delta^T_{CM2} = -0.59$ given the benchmark calibration for the rest of the parameters. The optimal portfolio in this case consists of slightly less home equity compared to the case of complete market without non-tradable goods. The intuition is in figure 6, which shows the impulse responses when the tradable sector relative technology shock hits. When the non-tradable sector is present, favorable home tradable sector relative technology shock raises home productivity in the tradable sector. High home wage in the home tradable sector draws a fraction of home labor in the non-tradable sector toward the tradable sector, decreasing home non-tradable output and consumption. Home tradable consumption decreases on impact relative to foreign to equalize to marginal utility of tradable consumption across countries. Thus, home agents should hold less home equity because the on-impact consumption in this case is smaller, relative to the case of complete market without non-tradable goods. The total consumption differential is highly correlated with the real exchange rate, which is consistent with the prediction of Backus and Smith (1993).

**The optimal portfolio as a function of labor share:**

The blue and green lines in figure 7 show the relationship between $\delta^T$ and labor income share. When the financial market is complete, the optimal portfolios are highly negatively correlated with labor income share. When the labor share increases, home

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11 The solution without the non-tradable sector coincides with the special case of Devereux and Sutherland (2011) when capital is perfectly correlated with labor income.
agents need more foreign equities to hedge the home non-traded labor income risk. Thus, the optimal portfolio $\delta^T$ consists of less home equity.

2.15.3 Incomplete Market

With the presence of the non-tradable sector and non-tradable sector relative technology shocks, the financial market is incomplete. The optimal portfolio, given the benchmark parameters, consists of 94% home equity, despite that labor income accounts for 64% of output and domestic human capital return is highly correlated with domestic equity return. In my model, the unconditional correlation of $(\hat{r}_{xt+1} - E_t[\hat{r}_{xt+1}])$ and $(\hat{r}_{ht}^{x} - E_t[r_{ht}^{x}])$ generated by the simulation is 0.77.

The optimal portfolio as a function of labor share:

The red line in figure 7 show the relationship between $\delta^T$ and the labor income share for the case of incomplete market. The change of $\delta^T$ is small when the labor income share changes.

What generates home bias and its insensitivity to the change in the labor income share?

When the labor income share = 0, my model generates home biased equity portfolios, which are the vertical intercept of the red line in figure 7. When the labor income share = 0, the result is intuitive, given the incomplete financial market and the complimentary relationship between tradable and non-tradable goods, as also observed in Pesenti and van Wincoop (2002) and Hnatkovska (2010). When the financial market is incomplete, home agents cannot fully insure against non-tradable sector relative technology shocks. When the home non-tradable sector relative technology shock hits and home non-tradable output and consumption increases, home marginal utility of tradable consumption is high since tradable and non-tradable goods are complements. The mobile labor market generates output co-movement across sectors, increasing the home tradable sector equity return. Home tradable sector equity is therefore a desirable asset to hedge home non-tradable sector relative technology shocks.
When the labor income share increases, on one hand, home agents would like to hold more foreign equity of the tradable sector to hedge the positive correlation between domestic equity return and labor income generated by tradable sector relative technology shocks. On the other hand, as capital share decreases, more tradable sector equity is needed to provide the same claim of a fraction of tradable output. As a result, home agents have incentive to hold more home equity to hedge non-tradable sector relative technology shocks. The incentives to hedge tradable and non-tradable sector relative technology shocks pull home bias in opposite directions. Consequently, the change in home bias is small when the labor income share changes.

2.15.4 The optimal portfolio as a function of the size of the tradable sector

Figure 8 shows the optimal portfolio $\delta^T$ as a function of the size of the tradable sector $a$. The horizontal asymptote is at $\delta^T = \delta_{CM1}^T = -0.4$. When $a = 1$, we have the optimal portfolio of the complete market case without the non-tradable sector. To gain an intuition on why $\delta^T$ decreases when $a$ increases, log linearize equation (1):

$$\hat{C}_T^T \left[ \frac{a(\sigma - \theta) - \sigma}{\theta \sigma} \right] + \hat{C}_N^N \left[ \frac{(1 - a)(\sigma - \theta)}{\theta \sigma} \right] = \lambda_t.$$

The left hand side is the log-linear of home agents’ marginal utility of tradable consumption. The higher $a$, the larger $a(\sigma - \theta) - \sigma)/(\theta \sigma)$ and the more impact a given deviation of tradable consumption, $\hat{C}_T^T$, has on marginal utility of tradable consumption. Thus, the tradable consumption risk increases. Similarly, the higher $a$, the smaller $(1 - a)(\sigma - \theta))/(\theta \sigma)$ and the less impact a given deviation of non-tradable consumption, $\hat{C}_N^N$, has on marginal utility of tradable consumption. Thus, the non-tradable consumption risk decreases. Consequently, home agents hold more foreign equity because the incentive to hedge tradable sector relative technology shocks is dominant.

2.15.5 The optimal portfolio as a function of the variance ratio

Figure 9 shows the optimal portfolio $\delta^T$ as a function of the variance ratio $\varrho$. The optimal portfolio decreases as $\varrho$ increases. When $\varrho$ increases, the non-tradable sector relative technology shock variance becomes smaller, relative to that of the tradable sector.
Home agents then tilt their portfolios toward foreign equity to hedge the tradable sector relative technology shocks. The horizontal asymptote is at $\delta_T = \delta_{CM2} = -0.59$. When $t$ approaches $\infty$, the non-tradable sector relative technology shock variance becomes infinitesimally small relative to that of the tradable sector, and $\delta_T$ approaches the optimal portfolio of the case of the complete market with the non-tradable sector.

2.15.6 The optimal portfolio as a function of $\sigma$ and $\omega$

Figure 10 shows the optimal portfolio $\delta^T$ as a function of $\sigma$ and $\omega$. Keeping $\sigma$ constant, $\delta^T$ increases as $\omega$ decreases. The closer $\omega$ is to 1, the more volatile the term of trade is and the more risk-sharing it provides when tradable sector relative productivity shocks hit. Thus, the tradable sector risk becomes smaller and home agents hold a more home biased portfolio. This result is similar to Cole and Obtsfeld (1991). Keeping $\omega$ constant, $\delta^T$ increases as $\sigma$ increases. The intuition also comes from the log-linear version of the marginal utility of tradable consumption:

$$C^T_t \left[ a(\sigma - \theta) - \sigma \right] + C^N_t \left[ (1 - a)(\sigma - \theta) \right] = \lambda_t,$$

or

$$C^T_t \left[ a(\sigma - \theta) - \sigma \right] + C^N_t (1 - a) \left[ \frac{1}{\theta} - \frac{1}{\sigma} \right] = \lambda_t.$$

The smaller $\sigma$ is, the more impact a given deviation $C^N_t$ has on the marginal utility of tradable consumption. Thus, the smaller $\sigma$, the "riskier" the non-tradable sector relative productivity shocks are, and a more home biased portfolio is needed to hedge these shocks.

2.16 Macroeconomics dynamics and consumption differential-real exchange rate correlation

2.16.1 Tradable sector relative technology shock

Figure 11 shows the impulse responses when the tradable sector relative technology shock hits. Higher home technology increases the the on-impact equity return differential. Subsequent equity return differentials are 0, as indicated by equation (7). Higher home productivity in the tradable sector increases the home wage above foreign. Home
agents, enjoying higher labor income and portfolio income, increase their consumptions and asset position. The relative price of non-tradable goods, which depends on relative wage, also jumps. Since the tradable consumption baskets are the same in both countries, the real exchange rate only depends on the relative price of non-tradables. Thus, the real exchange rate drops on impact. The total consumption differential and real exchange rate move in opposite directions following tradable sector relative technology shocks.

2.16.2 Non-tradable sector relative technology shock

Figure 12 shows the impulse responses when the non-tradable sector relative technology shock hits. Due to higher home productivity in the non-tradable sector, home non-tradable output and consumptions increase above foreign. Thus, home tradable and total consumptions also increase above foreign on impact. However, since the optimal portfolio does not fully insure against the non-tradable sector relative technology shocks, home agents have to spend their permanent wealth to support the higher home consumptions on-impact and few periods thereafter. The home net foreign asset position deteriorates. With less wealth, home agents in the long-run have to consume less and supply more labor at a lower wage, keeping the relative price of non-tradables at a lower level. Therefore, the real exchange rate is higher in the long-run. The total consumption differential and the real exchange rate move in opposite directions in the long run.

2.16.3 Simulation and consumption differential-Real exchange rate correlation

I generates two series of shocks $e^T_i$ and $e^N_i$ for 100 periods. The shocks are drawn from normal distribution with variance covariance matrix described in section 3. I then feed the shocks to the model and generate the time series for variables of the model. I HP-filter these series with smoothing parameter $\lambda = 6.25$, and calculate the correlation of the cyclical components of the consumption differential and the real exchange rate. I repeat the process 1000 times and take the average correlation. The unconditional correlation generates by my model is 0.
Figure 13 shows the unconditional correlation of consumption differential-Real exchange rate for different values of $\sigma$ and $\omega$. Keeping $\sigma$ constant, the correlation changes little when $\omega$ changes. When $\omega$ increases, the term of trade is less volatile and provides less risk-sharing. Thus, for a given optimal portfolio, the volatility generated by tradable sector relative technology shocks is higher, which tends to decrease the unconditional correlation. However, as $\omega$ increases, the optimal portfolio becomes less home biased, the jump in consumption differential due to higher home wealth generated by favorable tradable productivity shocks becomes smaller. Thus, the volatility generated by the non-tradable sector dominates, which increases the unconditional correlation. As a result, the correlation changes little when $\omega$ changes. Keeping $\omega$ constant, when $\sigma$ increases, the correlation becomes more negative. As $\sigma$ increases, the portfolio becomes more home biased and the volatilities generated by tradable sector relative technology shocks are larger, which decreases the correlation.

Figure 14 shows the unconditional correlation of consumption differential-Real exchange rate for different values of $\delta^T$ and $\omega$. $\delta^T$ in this case is not calculated from the model but rather, given exogenously. As one can see, for higher values of $\omega$, keeping $\omega$ fixed, the unconditional correlation greatly differs with different values of the steady state portfolio. Previous literature only consider the implied unconditional correlation of consumption differential-real exchange rate for given shock processes and conclude that such model and shock processes can generate a correlation that matches data. However, it is possible that the steady state portfolio implied by such shock processes will change the correlation. Thus, by jointly incorporating the home equity bias puzzle and the Backus Smith puzzle, not only do I generate a home equity biased portfolio, I also convincingly generate a low consumption differential-real exchange rate correlation.

2.17 Conclusion

My paper has been written to explain two features of the international equity home bias puzzle. First, the equity home bias exists in every country world-wide, despite the non-traded labor income that implies optimal foreign biased portfolio. Second, the equity home bias is not negatively correlated with the fraction of labor income, which is the implication in a standard model when the labor income and equity return
are positively correlated. My model generates a large home equity biased portfolio, despite the presence of non-traded human capital and the strongly positive correlation of its return with domestic equity return.

My model also generates a zero unconditional correlation of consumption differential-real exchange rate. I find that the correlation depends on the coefficient of relative risk aversion, $1/\sigma$, and elasticity of substitution between home and foreign tradable goods, $\omega$. Keeping $\sigma$ constant, the correlation change little when $\omega$ changes. Keeping $\omega$ constant, the correlation decreases when $1/\sigma$ decreases. Previous literature consider the Backus-Smith puzzle in an incomplete market setting without jointly solve for the optimal portfolio. I show that the steady state portfolio can greatly change the result of the correlation. It is possible that a given model and shock processes that can generate the consumption differential-real exchange rate correlation that matches data, but once consider the optimal portfolio implied by such shocks, the correlation can be greatly different. By jointly consider the two puzzles together, I can convincingly prove that with my model, shock processes, and the optimal portfolio implied by the model can generate a low correlation.
A1: The model in steady state

In the steady state, \( p^T = p^{T*} = p^N = p^{N*} = p = 1 \), \( y^T = y^{T*} = y = C^T = C^{T*} = L^T = L^{T*} = a, C^H = C^F = C^{H*} = C^{F*} = \frac{a}{2}, y^N = C^N = y^{N*} = C^{N*} = L^N = L^{N*} = 1 - a, w = w^* = \frac{c - 1}{\epsilon}, d^T = d^{T*} = \frac{a}{\epsilon}, r_1 = r_2 = \frac{1}{\beta}, r_x = 0, W = W^* = 0. \)

A2: Derivation for equation (8)

From the definition of \( W_t \), we have \( W_t = -W_t^* \) and \( \hat{W}_t = -\hat{W}_t^* \). Combining the log-linear version of the home and foreign budget constraints gives:

\[
2\hat{W}_t = 2\bar{\alpha} \hat{r}_{x,t} + \frac{2}{\beta} \hat{W}_{t-1} + \frac{1}{\epsilon} \left( \hat{d}_t^T - \hat{d}_t^{T*} \right) + \frac{Lw}{py} \left( \hat{w}_t - \hat{w}_t^* \right) - \frac{C^T}{py} \left( \hat{C}_t^T - \hat{C}_t^{T*} \right) - \frac{\epsilon - 1}{\epsilon} \frac{p^N C^N}{py} \left[ (\hat{C}_t^N - \hat{C}_t^{N*}) + (\hat{p}_t^N - \hat{p}_t^{N*}) \right].
\]

We will express variable differentials as functions of technology, tradable consumption and wage differentials. From the consumer first-order conditions for consumption, equation (1) and (2), and the firm optimal pricing equation in the non-tradable sector, we have: \( \frac{1}{\eta}(\hat{C}_t^T - \hat{C}_t^{N*}) = \hat{p}_t^N = \hat{w}_t - \hat{z}_t^N \). Thus, non-tradable consumption and price differentials can be written as:

\[
\hat{C}_t^N - \hat{C}_t^{N*} = \left( \hat{C}_t^T - \hat{C}_t^{T*} \right) - \theta \left[ (\hat{w}_t - \hat{w}_t^*) - (\hat{z}_t^N - \hat{z}_t^{N*}) \right],
\]

\[
\hat{p}_t^N - \hat{p}_t^{N*} = (\hat{w}_t - \hat{w}_t^*) - (\hat{z}_t^N - \hat{z}_t^{N*}).
\]

Log linearizing the firm optimal pricing equation in the tradable sector gives the equation for the term of trade (TOT): \( \hat{T}_t = \hat{p}_t^T - \hat{p}_t^{T*} = (\hat{w}_t - \hat{w}_t^*) - (\hat{z}_t^T - \hat{z}_t^{T*}) \). Demands for home tradable goods from home and foreign households are: \( C_i^H = \frac{1}{2}(p_i^T)^{-\omega}C_i^T \) and \( C_i^{H*} = \frac{1}{2}(p_i^T)^{-\omega}C_i^{T*} \) respectively. Market clearing condition for home tradable goods ensures: \( y_i^T = C_i^H + C_i^{H*} \). Combining the three equations above with equations for firms’ dividends and prices, one can express the tradable sector dividend and output
differentials as functions of tradable sector technology and wage differentials:

\[ \ddot{d}_t^T - \ddot{d}_t^{T*} = (1 - \omega) \left[ (\dot{\omega}_t - \dot{\omega}_t^*) - (\dot{z}_t^T - \dot{z}_t^{T*}) \right], \]

\[ \ddot{y}_t^T - \ddot{y}_t^{T*} = -\omega \left[ (\dot{\omega}_t - \dot{\omega}_t^*) - (\dot{z}_t^T - \dot{z}_t^{T*}) \right]. \]

Combining first-order conditions for tradable consumption and leisure, one can derive labor supply differential as a function of consumption, wage and non-tradable sector technology differentials:

\[ \hat{L}_t - \hat{L}_t^* = -\frac{\varphi}{\sigma} (\hat{C}_t^T - \hat{C}_t^{T*}) + \varphi \left( 1 - \frac{(\sigma - \theta)(1 - a)}{\sigma} \right) (\dot{\omega}_t - \dot{\omega}_t^*) + \frac{\varphi(\sigma - \theta)(1 - a)}{\sigma} (\dot{z}_t^N - \dot{z}_t^{N*}). \]

We can now express \( \hat{W}_t \) as function of technology, wage and tradable consumption differentials:

\[ 2\hat{W}_t = 2\hat{W}_{t-1} + 2\alpha \dot{r}_{x,t} + \left[ \frac{(1 - \omega)}{\epsilon} + \left( \frac{Lw}{\epsilon} \frac{\varphi(\sigma - \theta)(1 - a)}{\sigma} \right) - \frac{(\epsilon - 1)}{\epsilon} \frac{(1 - \theta)p^N C^N}{py} \frac{(\dot{\omega}_t - \dot{\omega}_t^*)}{\epsilon} \right] (\dot{\omega}_t - \dot{\omega}_t^*) \]

\[ - \frac{1}{\epsilon} (1 - \omega)(\dot{z}_t^T - \dot{z}_t^{T*}) + \left( \frac{Lw \varphi(\sigma - \theta)(1 - a)}{\sigma} + \frac{(\epsilon - 1)}{\epsilon} \frac{p^N C^N}{py}(1 - \theta) \right) (\dot{z}_t^N - \dot{z}_t^{N*}) \]

\[ - \left( \frac{Lw \varphi}{py} \frac{C^T + p^N S^{T-1}}{\sigma} \right) (\hat{C}_t^T - \hat{C}_t^{T*}). \]

To further solve for the dynamics, we need to solve for the wage differential. The wage differential is determined from the labor supply and demand equations. The total labor demand is the sum of labor demands in the tradable and non-tradable sectors. The labor demand equation is:

\[ \hat{L}_t - \hat{L}_t^* = \frac{L^T}{L} \left\{ -\omega \left[ (\dot{\omega}_t - \dot{\omega}_t^*) - (\dot{z}_t^T - \dot{z}_t^{T*}) \right] - (\dot{z}_t^T - \dot{z}_t^{T*}) \right\} \]

\[ + \frac{L^N}{L} \left\{ \left( \hat{C}_t^T - \hat{C}_t^{T*} \right) - \theta \left[ (\dot{\omega}_t - \dot{\omega}_t^*) - (\dot{z}_t^N - \dot{z}_t^{N*}) \right] - (\dot{z}_t^N - \dot{z}_t^{N*}) \right\}. \]

Combining labor demand and labor supply equations, we can solve for wage differential
as a function of tradable consumption and technology differentials:

\[
\hat{w}_t - \hat{w}_t^* = \frac{\varphi L + \sigma L^N}{\varphi L (\sigma - (\sigma - \theta)(1-a)) + \sigma (\omega L^T + \theta L^N)} (\hat{C}_t^T - \hat{C}_t^{T*}) \\
+ \frac{\sigma L^T (\omega - 1)}{\varphi L (\sigma - (\sigma - \theta)(1-a)) + \sigma (\omega L^T + \theta L^N)} (\hat{z}_t^T - \hat{z}_t^{T*}) \\
+ \frac{\sigma L^N (\theta - 1) - \varphi (\sigma - \theta)(1-a) L}{\varphi L (\sigma - (\sigma - \theta)(1-a)) + \sigma (\omega L^T + \theta L^N)} (\hat{z}_t^N - \hat{z}_t^{N*}).
\]

Plug the equation for wage differential into the equation for \( \hat{W}_t \), one can express the dynamics of net foreign assets as a function of tradable consumption and technology differentials:

\[
2\hat{W}_t = \frac{2}{\beta} \hat{W}_{t-1} + 2\hat{r}_{x,t} + \left[ \frac{(1 - \omega)}{\epsilon} + \frac{Lw}{py} \left( 1 + \frac{\varphi (\sigma - \theta)(1-a)}{\sigma} \right) - \frac{(\epsilon - 1) (1 - \theta) p^N C^N}{py} \right] \\
\left[ \frac{\varphi L + \sigma L^N}{\varphi L (\sigma - (\sigma - \theta)(1-a)) + \sigma (\omega L^T + \theta L^N)} (\hat{C}_t^T - \hat{C}_t^{T*}) \\
+ \frac{\sigma L^T (\omega - 1)}{\varphi L (\sigma - (\sigma - \theta)(1-a)) + \sigma (\omega L^T + \theta L^N)} (\hat{z}_t^T - \hat{z}_t^{T*}) \\
- \frac{\sigma L^N (1-\theta)}{\varphi L (\sigma - (\sigma - \theta)(1-a)) + \sigma (\omega L^T + \theta L^N)} (\hat{z}_t^N - \hat{z}_t^{N*}) \right] \\
- \frac{1}{\epsilon} (1 - \omega) (\hat{z}_t^T - \hat{z}_t^{T*}) + \left( \frac{Lw \varphi (\sigma - \theta)(1-a)}{py \sigma} + \frac{\epsilon - 1}{\epsilon} p^N C^N (\sigma - \theta)(1-a) \right) (\hat{z}_t^N - \hat{z}_t^{N*}) \\
- \frac{(Lw \varphi C^T + p^N S^{\epsilon-1})}{py} (\hat{C}_t^T - \hat{C}_t^{T*}).
\]
Let:

\[
A = \frac{(1 - \omega)}{\epsilon} + \frac{Lw \varphi}{py} \left[ 1 + \varphi - \frac{\varphi(\sigma - \theta)(1 - a)}{\sigma} \right] - \frac{(\epsilon - 1)(1 - \theta)p^N S}{\epsilon},
\]

\[
B = \sigma L^T (\omega - 1)
\]

\[
C = \varphi L + \sigma L^N
\]

\[
D = \sigma L^N (1 - \theta) + \varphi(\sigma - \theta)(1 - a) L
\]

\[
E = \frac{1}{\epsilon} (1 - \omega)
\]

\[
F = \frac{Lw \varphi}{py} \left[ \frac{(\sigma - \theta)(1 - a)}{\sigma} \right] + \frac{\epsilon - 1}{\epsilon} \frac{p^N S}{py} (1 - \theta)
\]

\[
G = \frac{Lw \varphi}{py} \frac{C^T + \rho S \varphi^T}{\varphi} \left[ \frac{1 - a - \epsilon - 1}{\epsilon} \right].
\]

A, B, C, D, F, G are simply constants which depend on parameters. One can rewrite \( \hat{W}_t \) as:

\[
2\hat{W}_t = \frac{2}{\beta} \hat{W}_{t-1} + 2\hat{\delta}_x t + A \left[ B(\hat{z}_t^T - \hat{z}_t^*T) + C(\hat{C}_t^T - \hat{C}_t^*T) - D(\hat{z}_t^N - \hat{z}_t^N*) \right]
\]

\[
- E(\hat{z}_t^T - \hat{z}_t^*T) + F(\hat{z}_t^N - \hat{z}_t^N*) - G(\hat{C}_t^T - \hat{C}_t^*T).
\]

Dividing both sides by 2, we can get equation (8).

**A3: Derivation for equation (9)**

The total consumption differential can be written as a function of tradable and non-tradable consumption differentials. Substitute the non-tradable consumption differential with the function of wage and technology differentials, we can get the following
equation for the total consumption differential:

\[
\hat{C}_t - \hat{C}^*_t = \left( \hat{C}^T_t - \hat{C}^T_t^* \right) - (1 - a) \theta \left[ (\hat{w}_t - \hat{w}^*_t) - (\hat{z}^N_t - \hat{z}^{N*}_t) \right].
\]

From Appendix A2, the wage differential can be written as:

\[
\hat{w}_t - \hat{w}^*_t = C \left( \hat{C}^T_t - \hat{C}^T_t^* \right) + B \left( \hat{z}^T_t - \hat{z}^{T*}_t \right) - D \left( \hat{z}^N_t - \hat{z}^{N*}_t \right).
\]

Combining the above two equations with the log-linear version of equation (1) and equation (4), shock processes and the discount factor process, one can get:

\[
(1 - \xi)(\hat{C}^T_t - \hat{C}^T_t^*) + (1 - \rho^T)I (\hat{z}^T_t - \hat{z}^{T*}_t) - (1 - \rho^N)K (\hat{z}^N_t - \hat{z}^{N*}_t) = E_t \left[ (\hat{C}^T_{t+1} - \hat{C}^T_{t+1}^*) \right],
\]

where:

\[
I = \frac{(\sigma - \theta)(\omega - 1)a(1 - a)}{\varphi + \omega a + \theta(1 - a) + (1 - a) (\sigma - \theta)(1 - a)},
\]

\[
K = \frac{(\sigma - \theta)(1 - a)[\sigma(1 - a) + \sigma \omega a + \varphi [\sigma - (\sigma - \theta)(1 - a)]]}{\sigma [\varphi + \omega a + \theta(1 - a) + (1 - a)(\sigma - \theta)(1 - a)]},
\]

\[
\xi = \frac{\sigma \eta}{1 + C(1 - a)(\sigma - \theta)}.
\]

The general formula is

\[
(1 - \xi)^i(\hat{C}^T_t - \hat{C}^T_t^*) + \frac{1 - \rho^T}{1 - \xi - \rho^T} \left[ (1 - \xi)^i - (\rho^T)^i \right] I (\hat{z}^T_t - \hat{z}^{T*}_t)
\]

\[
- \frac{1 - \rho^N}{1 - \xi - \rho^N} \left[ (1 - \xi)^i - (\rho^N)^i \right] K (\hat{z}^N_t - \hat{z}^{N*}_t) = E_t \left[ (\hat{C}^T_{t+i} - \hat{C}^T_{t+i}^*) \right].
\]
We prove it by induction. The statement is true for $i = 1$, since the general formula simply becomes:

$$(1 - \xi) (\hat{C}_t^T - \hat{C}_t^{T*}) + (1 - \rho^T) I (\hat{z}_t^T - \hat{z}_t^{T*}) - (1 - \rho^N) K (\hat{z}_t^N - \hat{z}_t^{N*})$$

$$= E_t \left[ (\hat{C}_{t+1} - \hat{C}_{t+1}^{T*}) \right],$$

which is shown above. Suppose the statement is true for $i = n$, and we have:

$$(1 - \xi)^n (\hat{C}_t^T - \hat{C}_t^{T*}) + \frac{1 - \rho^T}{1 - \xi - \rho^T} \left[(1 - \xi)^n - (\rho^T)^n\right] I (\hat{z}_t^n - \hat{z}_t^{T*})$$

$$- \frac{1 - \rho^N}{1 - \xi - \rho^N} \left[(1 - \xi)^n - (\rho^N)^n\right] K (\hat{z}_t^n - \hat{z}_t^{N*}) = E_t \left[ (\hat{C}_{t+n} - \hat{C}_{t+n}^{T*}) \right].$$

We will show that the statement is true for $i = n + 1$. The first order condition for period $t + n$ yields:

$$E_t \left[ (1 - \xi) (\hat{C}_{t+n}^T - \hat{C}_{t+n}^{T*}) + (1 - \rho^T) I (\hat{z}_{t+n}^T - \hat{z}_{t+n}^{T*}) - (1 - \rho^N) K (\hat{z}_{t+n}^N - \hat{z}_{t+n}^{N*}) \right]$$

$$= E_t \left[ \hat{C}_{t+n+1} - \hat{C}_{t+n+1}^{T*} \right]$$

We use the assumption that the statement is true for $i = n$ and substitute $E_t \left[ (\hat{C}_{t+n}^T - \hat{C}_{t+n}^{T*}) \right]$ with:

$$(1 - \xi)^n (\hat{C}_t^T - \hat{C}_t^{T*}) + \frac{1 - \rho^T}{1 - \xi - \rho^T} \left[(1 - \xi)^n - (\rho^T)^n\right] I (\hat{z}_t^n - \hat{z}_t^{T*})$$

$$- \frac{1 - \rho^N}{1 - \xi - \rho^N} \left[(1 - \xi)^n - (\rho^N)^n\right] K (\hat{z}_t^n - \hat{z}_t^{N*}).$$
We also use the fact that $E_t[\hat{z}_t - \hat{z}_t^*] = (\rho^j) (\hat{z}_t - \hat{z}_t^*)$, $j = T, N$:

$$(1 - \xi) \left[ (1 - \xi)^n (\hat{C}_t - \hat{C}_t^*) + \frac{1 - \rho^T}{1 - \xi - \rho^T} [(1 - \xi)^n - (\rho^T)^n] I (\hat{z}_t - \hat{z}_t^*) \right]$$

$$(1 - \xi) \left[ (1 - \xi)^n - (\rho^N)^n \right] K (\hat{z}_t^N - \hat{z}_t^{N^*}) + (1 - \rho^T)(\rho^T)^n I (\hat{z}_t^T - \hat{z}_t^{T^*})$$

Thus, the statement is true for $i = n + 1$ and hence, our proof for the general formula for consumption dynamics.

**A4: Derivation for equation (10)**

In order to solve for the on-impact tradable consumption differential, we need to solve for the on-impact return differential as a function of tradable consumption and technology differentials. We start from the basic equation for the return of home assets:

$$r_{1t} = \frac{d_{1t}^T + Z_{1t}}{Z_{1t-1}},$$

where $Z_{1t}$ is the price of the home assets in period $t$. The log-linear version of the above equation is:

$$\hat{r}_{1t} = (1 - \beta) d_{1t}^T + \beta Z_{1t} - \hat{Z}_{1t-1}.$$
The same equations holds for subsequent period returns:

\[ \beta \hat{r}_{1t+1} = \beta(1 - \beta)\tilde{d}_{t+1} + \beta^2 \hat{Z}_{1t+1} - \beta \hat{Z}_{1t}, \]

...etc.

Summing up all of the equations for returns of home asset gives:

\[ \hat{r}_{1t} = (1 - \beta) \left[ \tilde{d}^T_t + \beta \tilde{d}^T_{t+1} + ... \right] - \hat{Z}_{1t-1}. \]

Since the same equation applies for the returns of foreign assets, the return differential at time \( t \) is given by:

\[ \hat{r}_{xt} = (1 - \beta) E_t \left[ \left( \hat{C}_t^T - \hat{C}_t^{T*} \right) + \beta (\hat{C}_{t+1}^T - \hat{C}_{t+1}^{T*}) + ... \right] - \left( \hat{Z}_{1t-1} - \hat{Z}_{2t-1} \right). \]

Replacing dividend differential with wage and technology differentials into the above equation gives:

\[ \hat{r}_{xt} = (1 - \beta) E_t \left[ \left( \hat{w}_t - \hat{w}_t^{*} \right) + \beta (\hat{w}_{t+1} - \hat{w}_{t+1}^{*}) + \beta^2 (\hat{w}_{t+2} - \hat{w}_{t+2}^{*}) + ... \right] - \left( \hat{Z}_{1t-1} - \hat{Z}_{2t-1} \right). \]

From equation (8):

\[ (G - AC)(\hat{C}_t^T - \hat{C}_t^{T*}) = \frac{2}{\beta} \hat{W}_{t-1} - 2 \hat{W}_t + 2\alpha \hat{r}_{xt} + (AB - E)(\hat{z}_t^T - \hat{z}_t^{T*}) - (AD - F)(\hat{z}_t^N - \hat{z}_t^{N*}). \]

Similar equations hold for subsequent periods. Therefore:

\[ \sum_{i=0}^{\infty} (G - AC) \beta^i E_t (\hat{C}_{t+i}^T - \hat{C}_{t+i}^{T*}) = \frac{2}{\beta} \hat{W}_{t-1} + 2\alpha \hat{r}_{xt} + \frac{AB - E}{1 - \beta \rho^T}(\hat{z}_t^T - \hat{z}_t^{T*}) - \frac{AD - F}{1 - \beta \rho^N}(\hat{z}_t^N - \hat{z}_t^{N*}). \]
We use the no-Ponzi condition in the above summation. Plugging the equation for the on-impact return differential into the equation above gives:

\[
\sum_{i=0}^{\infty} (G - AC) \beta^i E_t(\hat{C}_{t+i}^T - \hat{C}_{t+i}^{T*}) = \frac{2}{\beta} \hat{W}_{t-1} \\
+ 2\bar{\alpha}\left\{ (1 - \beta)(1 - \omega) \left\{ \frac{B - 1}{1 - \beta \rho_T} (\hat{z}_t^T - \hat{z}_t^{T*}) + C \sum_{i=0}^{\infty} \beta^i E_t(\hat{C}_{t+i}^T - \hat{C}_{t+i}^{T*}) - \frac{D}{1 - \beta \rho_N} (\hat{z}_t^N - \hat{z}_t^{N*}) \right\} \right\} \\
+ \frac{AB - E}{1 - \beta \rho_T} (\hat{z}_t^T - \hat{z}_t^{T*}) - \frac{AD - F}{1 - \beta \rho_N} (\hat{z}_t^N - \hat{z}_t^{N*}).
\]

From equation (9) that we prove in Appendix A3, we can express \( \sum_{i=0}^{\infty} \beta^i E_t(\hat{C}_{t+i}^T - \hat{C}_{t+i}^{T*}) \) as function of \( (\hat{z}_t^T - \hat{z}_t^{T*}) \):

\[
E_t \sum_{i=0}^{\infty} \beta^i (\hat{C}_{t+i}^T - \hat{C}_{t+i}^{T*}) = \frac{\hat{C}_t^T - \hat{C}_t^{T*}}{1 - \beta(1 - \xi)} + \frac{\beta(1 - \rho_T)}{[1 - \beta(1 - \xi)][1 - \beta \rho_T]} I(\hat{z}_t^T - \hat{z}_t^{T*}) \\
- \frac{\beta(1 - \rho_N)}{[1 - \beta(1 - \xi)][1 - \beta \rho_N]} K(\hat{z}_t^N - \hat{z}_t^{N*}).
\]

Combining the two equations above, we can get equation (10).

**A5: Derivation for equation (11)**

From the equation for return differential proved in Appendix A3, we have:

\[
\hat{r}_{xt} = (1 - \beta)(1 - \omega) \left\{ \frac{B - 1}{1 - \beta \rho_T} (\hat{z}_t^T - \hat{z}_t^{T*}) + C \sum_{i=0}^{\infty} \beta^i E_t(\hat{C}_{t+i}^T - \hat{C}_{t+i}^{T*}) \\
- \frac{D}{1 - \beta \rho_N} (\hat{z}_t^N - \hat{z}_t^{N*}) \right\} - \left( \hat{Z}_{t-1} - \hat{Z}_{2t-1} \right),
\]

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Similar to the step used in Appendix A4, substitute \( \sum_{i=0}^{\infty} \beta^i E_t(\hat{C}^T_{t+i} - \hat{C}^{T\ast}_{t+i}) \) with time \( t \) tradable consumption and technology differentials:

\[
\hat{r}_{xt} = (1 - \beta)(1 - \omega) \left\{ \frac{B - 1}{1 - \beta \rho T} (\hat{z}^T_t - \hat{z}^{T\ast}_t) + C \sum_{i=0}^{\infty} \beta^i E_t(\hat{C}^T_{t+i} - \hat{C}^{T\ast}_{t+i}) - \frac{D}{1 - \beta \rho N} (\hat{z}_t^N - \hat{z}^{N\ast}_t) \right\} - (\hat{Z}_{1t-1} - \hat{Z}_{2t-1}),
\]

\[
= (1 - \beta)(1 - \omega) \left\{ \frac{B - 1}{1 - \beta \rho T} (\hat{z}^T_t - \hat{z}^{T\ast}_t) + \frac{C}{1 - \beta(1 - \xi)} \left\{ (\hat{C}^T_t - \hat{C}^{T\ast}_t) + \frac{\beta(1 - \rho T)}{(1 - \beta \rho T)} I_t - \frac{\beta(1 - \rho N)}{(1 - \beta \rho N)} K_t \right\} - \frac{D}{1 - \beta \rho N} (\hat{z}_t^N - \hat{z}^{N\ast}_t) \right\} - (\hat{Z}_{1t-1} - \hat{Z}_{2t-1}),
\]

\[
= (1 - \beta)(1 - \omega) \left\{ \frac{B - 1}{1 - \beta \rho T} (\hat{z}^T_t - \hat{z}^{T\ast}_t) + \frac{C}{[1 - \beta(1 - \xi)] \beta [(G - AC) - 2\bar{a}(1 - \omega)(1 - \beta)C]} \left\{ 2\bar{a}(1 - \beta)(1 - \omega) \frac{B - 1}{1 - \beta \rho T} + \frac{AB - E}{1 - \beta \rho T} \right\} (\hat{z}^T_t - \hat{z}^{T\ast}_t) \right\} - \left\{ 2\bar{a}(1 - \beta)(1 - \omega) \frac{D}{1 - \beta \rho N} + \frac{AD - F}{1 - \beta \rho N} \right\} (\hat{z}_t^N - \hat{z}^{N\ast}_t) - (\hat{Z}_{1t-1} - \hat{Z}_{2t-1}).
\]
We use the trick $\dot{r}_x = \dot{r}_x - E_{t-1}[\dot{r}_x]$ to get rid of terms containing $W_{t-1}$ and $(\tilde{Z}_{t-1} - \tilde{Z}_{2t-1})$ and $(\tilde{z}_t^i - \tilde{z}_t^{i*})$, $i = T, N$

$$
\dot{r}_x = (1 - \beta)(1 - \omega) \left\{ (B - 1) \frac{\hat{e}_t^T}{1 - \beta \rho^T} + \frac{C}{(G - AC) - 2\alpha(1 - \omega)(1 - \beta)C} \left[ \{2\alpha(1 - \beta)(1 - \omega)(B - 1) + (AB - E)\} \frac{\hat{e}_t^T}{1 - \beta \rho^T} - \{2\alpha(1 - \beta)(1 - \omega)D + (AD - F)\} \frac{\hat{e}_t^N}{1 - \beta \rho^N} \right] - D \frac{\hat{e}_t^N}{1 - \beta \rho^N} \right\}
$$

$$
= \frac{(1 - \beta)(1 - \omega)}{|(G - AC) - 2\alpha(1 - \omega)(1 - \beta)C|} \left\{ (B - 1) \frac{\hat{e}_t^T}{1 - \beta \rho^T} + C \left[ \{2\alpha(1 - \beta)(1 - \omega)(B - 1) + (AB - E)\} \frac{\hat{e}_t^T}{1 - \beta \rho^T} - \{2\alpha(1 - \beta)(1 - \omega)D + (AD - F)\} \frac{\hat{e}_t^N}{1 - \beta \rho^N} \right] - D \frac{\hat{e}_t^N}{1 - \beta \rho^N} \right\}
$$

This is equation (11).

**A4: Derivation for equation (12)**

The steady state values are $\bar{r}_h = \frac{1}{\beta}$ and $\bar{H} = \frac{\beta \omega}{1 - \beta}$. One can log-linear the definition of human capital and its return:

$$(E_t - E_{t-1})\dot{H}_t = \frac{(1 - \beta)}{\beta} \sum_{i=0}^{\infty} \beta^i (E_t - E_{t-1}) \bar{w}_{t+1+i}$$

$$(E_t - E_{t-1})\dot{r}_t = (1 - \beta)(E_t - E_{t-1}) \bar{w}_t + \beta(E_t - E_{t-1})\dot{H}_t$$

Innovation to return to human capital can then be expressed as innovation to wages:

$$(E_t - E_{t-1})\dot{r}_t = (1 - \beta) \sum_{i=0}^{\infty} \beta^i (E_t - E_{t-1})[\bar{w}_{t+i}]$$

$$(E_t - E_{t-1})(\dot{r}_t - \dot{r}_t^*) = (1 - \beta) \sum_{i=0}^{\infty} \beta^i (E_t - E_{t-1}) (\bar{w}_{t+i} - \bar{w}_{t+i}^*)$$

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We previously showed that 
\[ \hat{\omega}_{t+i} - \hat{\omega}^{*}_{t+i} = B*(\hat{z}_{t+i}^T - \hat{z}^{*T}_{t+i}) + C(\hat{\theta}_{t+i}^T - \hat{\theta}^{*T}_{t+i}) - D*(\hat{z}_{t+i}^N - \hat{z}^{*N}_{t+i}) \].

Substitute in to find the formula for \( \hat{r}_{ht} - \hat{r}^{*}_{ht} \):

\[
(E_t - E_{t-1})e^{T}_{xt} = (1 - \beta)(E_t - E_{t-1}) \sum_{i=0}^{\infty} \beta^i \left\{ B(\hat{z}^T_{t+i} - \hat{z}^{*T}_{t+i}) + C(\hat{\theta}^T_{t+i} - \hat{\theta}^{*T}_{t+i}) - D(\hat{z}^N_{t+i} - \hat{z}^{*N}_{t+i}) \right\}
\]

\[
= (1 - \beta) \left\{ \frac{B}{1 - \beta \rho^T} e^T_t + (E_t - E_{t-1})C \sum_{i=0}^{\infty} \beta^i (\hat{\theta}^T_{t+i} - \hat{\theta}^{*T}_{t+i}) - \frac{D}{1 - \beta \rho^N} e^N_t \right\}
\]

\[
= (1 - \beta) \left\{ \frac{B}{1 - \beta \rho^T} e^T_t + C \left\{ \frac{(E_t - E_{t-1})[\hat{\theta}^T_{t} - \hat{\theta}^{*T}_{t}]}{1 - \beta} + I e^T_t \frac{\beta(1 - \rho^T)}{(1 - \beta)(1 - \beta \rho^T)} \right. \right. \\
\left. \left. - Ke^N_t \frac{\beta(1 - \rho^N)}{(1 - \beta)(1 - \beta \rho^N)} \right) \right\} - \frac{D}{1 - \beta \rho^N} e^N_t \}
\]

Innovation to human capital return differential becomes:

\[
\hat{r}^h_{xt} - E_{t-1}[\hat{r}^h_{xt}] = (1 - \beta) \left\{ \frac{1}{[(G - AC) - 2\bar{\alpha}(1 - \omega)(1 - \beta)C]} \left\{ \frac{2\bar{\alpha}(1 - \beta)(1 - \omega)}{1 - \beta \rho^T} \frac{B - 1}{1 - \beta \rho^T} + \frac{AB - E}{1 - \beta \rho^T} \right\} e^T_t \right. \\
\left. - \left\{ 2\bar{\alpha}(1 - \beta)(1 - \omega) \frac{D}{1 - \beta \rho^N} + \frac{AD - F}{1 - \beta \rho^N} \right\} e^N_t \right) \}
\]

which is equation (12) in the model.
2.19 Figures

Figure 1: Labor Income Share and Equity Home Bias 2007

Data from OECD and Seren and Vanpee (2007)

Figure 2: Labor Income Share Over Time

Data from OECD
Figure 3: Home Equity Bias Over Time

Home bias calculated from data from CPIS and World Federation of Exchange

Figure 4: Real Exchange Rate and Relative Consumption for USA and GBR

Annual data from World Bank. Both series are logged and HP-filtered using the smoothing parameter $\lambda = 6.25$
Figure 5: Real Exchange Rate and Relative Consumption for USA and JPN

Annual data from World Bank. Both series are logged and HP-filtered using the smoothing parameter $\lambda = 6.25$

Figure 6: Complete market impulse responses, tradable sector relative technology shock
Figure 7: Optimal portfolio and labor share

Figure 8: Optimal portfolio and the size of the tradable sector
Figure 9: Optimal portfolio and variance ratio of relative technology shocks

Figure 10: Optimal portfolio as function of $\sigma$ and $\omega$
Figure 11: Incomplete market, tradable sector relative technology shock

Figure 12: Incomplete market, non-tradable sector relative technology shock
Figure 13: Unconditional correlation of consumption differential—Real exchange rate as function of $\sigma$ and $\omega$

Figure 14: Unconditional correlation of consumption differential—Real exchange rate as function of $\delta^T$ and $\omega$
Bibliography


