Essays in International Economics

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ESSAYS IN INTERNATIONAL ECONOMICS

a dissertation

by

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Abstract

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This thesis includes two essays that analyze some features of the past financial crises. In the first chapter I study the possible reasons why investors reduced their holdings of foreign equities, and, at the same time, they increased their holdings of short-term government bonds, during the 2007 financial crisis that first hit the U.S. economy and soon became a world crisis. More precisely I analyze how the increases in uncertainty during the crisis affected capital flows. I use a two country DSGE model and I assume that there is trade in both goods and financial assets. I assume that each country is allowed to issue equities and government bonds, and I assume that each economy is hit by three types of shocks: Preference, productivity and government spending shocks. I proxy the increase in uncertainty with the introduction of uncertainty shocks, i.e. I allow the variances of the shocks to be time-varying. My findings show that uncertainty is a source of portfolio-dynamics that can contribute to explain, together with the other sources already identified in the literature, deviations of the portfolio from its steady-state. Investors choose their portfolio with the goal to smooth consumption. Therefore they want to hold assets with returns that display a negative covariance with consumption. When uncertainty shocks hit, the way in which the real variables of the model covary with asset returns changes. As a consequence, agents need to re-adjust their portfolios until when the shock has disappeared. I also show under which conditions it is rational for investors to increase their holdings of foreign government bonds and, at the same time, reduce their holdings of
foreign equity, in response to an increase in global uncertainty. My findings show that the response of the portfolio to an increase in uncertainty crucially depends on the source of uncertainty. If uncertainty comes from aggregate demand, it is optimal for agents to increase their holdings of foreign bonds and reduce their holdings of foreign equity. If instead the source of uncertainty is aggregate supply, agents find it optimal to increase their holdings of foreign equity and reduce their holdings of foreign bonds. This finding suggests that the movements of capital that took place during the crisis are compatible with an increase in uncertainty coming from aggregate demand. This result is supported by those theories that identify the collapse in demand as the main cause of the slump experienced by the U.S. and by many other economies during the crisis.

In the second chapter I study the currency denomination of the debt in emerging countries. Empirical studies have shown that emerging countries are often characterized by the presence of a high share of foreign currency denominated debt. As the debt crises of the 1990s show, the presence of foreign currency debt can be risky because, beyond creating a mismatch in the domestic firms balance sheets, it also constraints the traditional domestic policy instruments in dealing with home and foreign economic shocks. The reasons why such risky forms of international finance arise in the first place remain an open question. If foreign debt is so dangerous-as it is-it may be worth trying to give a micro-foundation to its emergence. Such a high share of foreign currency debt should be at least in part justified by the presence of some private benefits for the agents that choose this form of finance. The goal of this chapter is to rationalize the choice to borrow in dollars rather than in domestic currency on the international markets. In order to do so, I study how informational asymmetries and heterogeneous expectations can affect the choice of a borrower to expose herself to a currency risk. Furthermore I look at the policy implications of my findings to understand which policies could reduce the incentive of agents to dollarize. My model is a portfolio choice model with asymmetric information that analyzes how agents choose the currency denomination of their debt. The main findings of my model show that when domestic agents have a high informational
advantage and/or there is a low level of transparency on international markets, an increase in the
degree of dollarization might be observed, if the fundamentals are relatively strong. Alternatively,
if there is endogeneity between the exchange rate policy implemented by the monetary authority
and domestic agents decisions, a certain degree of complementarity in borrowers choices may arise,
thus creating a phenomenon of moral hazard. If domestic agents know that a high share of dollar
debt in the economy makes the exchange rate more rigid, they may want to coordinate on the
equilibrium where all the corporate debt in the economy is denominated in the same currency, even
when the fundamentals of the economy are relatively weak. These results have interesting policy
implications. A benevolent central bank that strongly bases her policy on the degree of dollarization
in the economy, can generate a coordination mechanism among the domestic borrowers that results
in a risky degree of dollarization. The solution would be to ex-ante choose a central banker with
a strong preference for a flexible exchange rate. My findings also show the importance of trans-
parency. Transparency does not necessarily coincide with public information. My model actually
shows that the precision of private sources of information determines the degree of dollarization.
If international markets could have access to some sources of private information, they would be
more willing to lend in pesos, when the fundamentals are relatively strong. As a consequence the
economy would not experience high levels of dollarization and would be better protected against
future negative shocks.
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This thesis is dedicated to my parents.
Chapter 1

Portfolio Choice and Flight to Safety

1.1 Introduction

The features of the financial crisis that started in the U.S. in August 2007 and soon spread to the rest of the world, have been extensively analyzed. Economists have found analogies between the current crisis and different crises that occurred in the past. Reinhart and Rogoff (2008a, 2008b) empirically document how several variables have behaved similarly during past financial crises and this one. Eichengreen (2008) also emphasizes how this crisis does not differ from the 1997-98 Asian crises, which were also characterized by lack of transparency, lax bank regulation, and connected lending that allowed some large institutions to enjoy privileged access to borrowed funds. However, this episode differs from the previous ones because the largest economy in the world was at the epicenter of the crisis. The U.S. financial crisis soon became a world crisis and the panic spread across economies. As a result, in this occasion, capital did not massively flow out of the crisis country generating the “sudden stops” and “capital reversals” that have instead characterized previous episodes of turmoil. Foreign investors reduced their holdings of U.S. equities indeed, but, at the same time, they increased their holdings of U.S. Treasuries, to such an extent that the U.S.
dollar started appreciating, and the yields on U.S. Treasury bills reached historical lows (Figures 1 and 2). The sudden change in the direction of the different types of capital, including Treasuries, U.S. and foreign corporate stocks and bonds, has been followed by a stabilization and a return to previous trends in 2009.

The decision of private markets to invest so heavily in the country from which the crisis originated has been explained as a flight to safety. The behavior of investors has been described as the result of “the insatiable need to accumulate safe debt instruments”. As international investors realized that U.S. corporate assets could not satisfy their demand, they decided to run to U.S. Treasuries. The reallocation of funds took place across asset classes, rather than across countries. But, what did U.S. investors do during the crisis? They dramatically reduced their holdings of foreign equity and, in some countries, they increased their holdings of short-term bonds. So, the question is: Under which conditions are foreign bonds a better hedging instrument than foreign equity? The goal of this paper is to identify the factors that can rationally justify the choice of agents to increase their holdings in foreign bonds and run away from foreign equity, when there is an increase in world uncertainty.

Earlier literature often abstracts from endogenous portfolio choices because of technical difficulties. The standard approach to solve DSGE models in fact relies on first-order approximations around the steady state that prove inadequate to capture second moments that determine portfolio choices. In a stochastic world, financial assets are differentiated by their degree of risk, and optimal portfolio choices are determined by correlations and variances of stochastic variables.

This paper makes use of the solution method recently developed by Devereux and Sutherland (2006). Their novel procedure consists in the combination of second-order approximations of the portfolio equations of the model with first-order approximations of non-portfolio equations. This

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1Caballero (2009), DeLong (2010) and Cochrane (2010), among many others.
2Caballero (2009).
allows to determine how the stochastic structure of the model affects the portfolio allocation and it becomes possible to characterize economy’s first-order response to stochastic shocks under the optimal portfolio. First-order dynamics of steady-state portfolio can be obtained by combining a third-order approximation of the portfolio equation and a second-order approximation of the rest of the model. Time variations in portfolios become relevant for macroeconomic dynamics at the second level of approximation.

The contribution of the paper is twofold: First, I show that uncertainty is a source of portfolio-dynamics that can contribute to explain, together with the other sources already identified in the literature, deviations of the portfolio from its steady-state.

Second, I show under which conditions it is rational for investors to increase their holdings of foreign government bonds and, at the same time, reduce their holdings of foreign equity, in response to an increase in global uncertainty. I use a two country DSGE model and I assume that there is trade in both goods and financial assets. Each country specializes in the production of one good, and trade is justified by the features of final consumption that is a bundle of domestic and foreign goods. The international asset portfolio includes two types of securities: stocks and bonds. Each country issues one government bond and one equity, denominated in local goods. There are three sources of shocks: one preference shock, productivity shocks and public spending shocks. Furthermore I proxy the increase in uncertainty with the introduction of uncertainty shocks, i.e. I allow the variances of the shocks to be time-varying. The number of shocks is larger than the number of assets and therefore the international financial markets are incomplete.

Investors choose their portfolio with the goal to smooth consumption. Therefore they want to hold assets with returns that display a negative covariance with consumption. The resulting steady-state portfolio features home bias in both bond and stock holdings. When uncertainty shocks hit, the way in which real variables covary with asset returns changes. As a consequence, agents need to re-adjust their portfolios until when the shock has disappeared.
My main findings are that the response of the portfolio to an increase in uncertainty crucially depends on the source of uncertainty. If uncertainty comes from aggregate demand, it is optimal for agents to increase their holdings of foreign bonds and reduce their holdings of foreign equity. If instead the source of uncertainty is aggregate supply, agents find it optimal to increase their holdings of foreign equity and reduce their holdings of foreign bonds.

This finding seems to suggest that the movements of capital that took place during the crisis are compatible with an increase in uncertainty coming from aggregate demand. This result is supported by those theories that identify the collapse in demand as the main cause of the slump experienced by the U.S. and by many other economies during the crisis\(^3\). On the one hand, the explosion of the housing bubble and the dramatic reduction in consumer spending in housing and durables can be described as an increase in the volatility of consumers’ preferences. On the other hand, the inability of governments to realize effective fiscal policies during the crisis can be considered a big source of uncertainty that did not help the recovery and that might have affected investors’ portfolio choice.

The model is also able to give reason of other episodes of capital movements observed in the past: A combination of aggregate demand and aggregate supply shocks is also able to replicate the massive capital outflows from the crisis country that have been observed in the late 90’s.

The rest of the paper is structured as follows. Section 2 gives a more detailed description of capital movements during the crisis. Section 3 presents the model set-up. In section 4 I briefly show the solution method, calibrate the model and discuss the results. Section 5 concludes.

### 1.2 A Look at the Crisis

In this section I show some more details on the movement of capital that took place at the beginning of the crisis. As already mentioned in the introduction, 2007 can be considered a turning point: \(^3\)Hall, (2011).
After 2007 foreign holdings of U.S. equities registered a dramatic reduction, while foreign holdings of U.S. Treasury bonds increased. The purpose of the model is to describe private agents’ investment decisions. It is well known that a large component of foreign holdings of U.S. Treasury bonds is represented by official holdings. Figures 3, 4 and 5 show that private and official investors’ holdings of Treasuries followed different paths after the panic: Private investors increased their holdings of U.S. Treasuries after 2007, but after 2008 they started reducing them. Official investors, instead, went on investing in U.S. Treasury bonds. Furthermore, it has been documented (Warnock, 2010) that private agents mainly hold short-term U.S. Treasuries, while governments mainly invest in long-term U.S. Treasuries. Private holdings of short-term Treasuries could have been considered a hedge against the risk, and, when the panic was over, things went back to normality and investors gradually started increasing U.S. equity holdings.

But what happened to U.S. holdings of foreign assets? As Figure (6) shows, there was a strong reduction in equity holdings and a mild reduction in bond holdings, while foreign direct investment kept increasing. But if we carefully look at U.S. holdings of foreign bonds, we can observe a difference in the behavior of short and long-term bond holdings. Figure 7 shows a classification of the countries that experienced an increase in U.S. holdings of their short-term debt, in millions of U.S. dollars.

In some cases, U.S. investors reduced their holdings of both long-term and short-term bonds, but, in some other cases, they only reduced their holdings of long-term foreign bonds, while they increased their holdings of short-term foreign bonds. For example, U.K., Ireland and Greece, registered a reduction in the U.S. holdings of their debt. However in the case of other countries, like Germany, Canada or Norway, there was a reduction in U.S. holdings of long-term debt, but, at the same time, an increase in U.S. holdings of short-term debt. The only exception is Switzerland that instead registered an increase in U.S. holdings of long-term debt and a reduction in U.S. holdings of short-term debt. Also developing countries like Brazil and Thailand registered an increase in
U.S. holdings of their short-term debt. Some developing countries had in fact been implementing rigorous fiscal and monetary policies in order to recover from their own crises. When the crisis hit the U.S., they were therefore characterized by relatively strong fundamentals that might have increased the attractiveness of their assets.

The choice of U.S. and foreign investors to increase their holdings of foreign short-term debt might be important in the description of the dynamics of capital flows during the crisis. There could have been an important symmetry in the behavior of investors during the crisis: As uncertainty increased, both U.S. and foreign investors used foreign short-term debt to protect themselves. This justifies my choice to use a two-country symmetric model in the rest of the paper.\footnote{In this version of the model I assume that the two countries have the same size. Size effects could be important in quantitatively determining steady-state portfolios, but they should not affect the qualitative predictions of the symmetric model, as shown in Ghironi et al. (2009).}

In what follows I assume that one country is the U.S. and the other one is the rest of the world.

### 1.3 The Model

This is an infinite horizon, two-country open economy model. There are two ex-ante symmetric countries, “Home” (H) and “Foreign” (F), each one populated by a representative household who consumes, works and trades a portfolio of financial assets. Each country specializes in the production of one good and issues stocks and government bonds, which are traded internationally. The international portfolio therefore consists of four assets: two equities and two government bonds. I allow for three types of shocks in each country: preference shocks, total factor productivity shocks, and government shocks. The number of shocks is larger than the number of assets available: This implies that financial markets are incomplete.
1.3.1 Preferences

The household in country $i = H, F$ chooses consumption $C^i$ and labor $l^i$ to maximize the intertemporal utility function:

$$\max E_t \sum_{s=0}^{\infty} \beta^s \left( \frac{(C^i_{t+s})^{1-\sigma}}{1-\sigma} - \frac{\chi (l^i_{t+s})^{1+\varphi}}{1+\varphi} \right)$$

(1.1)

with coefficient of relative risk aversion $\sigma > 0$ and a Frisch elasticity of labor supply $1/\varphi$. The Home final consumption basket combines home and foreign goods:

$$C^H_t = \left[ \kappa_t^{1/\theta} (c_{t}^{HH})^{(\theta-1)/\theta} + (1-\kappa_t)^{1/\theta} (c_{t}^{FH})^{(\theta-1)/\theta} \right]^{\theta/(\theta-1)}$$

(1.2)

where $c^{ij}$ is the amount of consumption good produced by country $i$ and consumed by country $j$. $\kappa$ is a time varying preference parameter equal to $\kappa_t = \kappa \exp(v_t)$, where $v_t$ is the preference shock that follows an AR(1) process: $v_t = \phi v_{t-1} + \epsilon_t^\nu$. The parameter $\kappa \in (0, 1)$ measures the share of local spending in consumption. If $\kappa > 0.5$, the agent is said to have a “home bias” preference in consumption.

Similarly, the consumption bundle for the foreign country depends on home and foreign goods, and the foreign preference parameter is $\kappa^*_t = \kappa \exp(-v_t)$. The Foreign consumption basket is:

$$C^F_t = \left[ (\kappa^*_t)^{1/\theta} (c_{t}^{FF})^{(\theta-1)/\theta} + (1-\kappa^*_t)^{1/\theta} (c_{t}^{HF})^{(\theta-1)/\theta} \right]^{\theta/(\theta-1)}.$$ 

(1.3)

A positive preference shock at the same time increases the consumption of the Home produced good and reduces the consumption of the Foreign produced good in both countries. The aggregate consumer price index for Home, $P^H_t$, is defined as:

$$P^H_t = \left[ \kappa_t (p^H_t)^{1-\theta} + (1-\kappa_t) (p^F_t)^{1-\theta} \right]^{1/\theta},$$

(1.4)

\footnote{The introduction of preference shocks generates incomplete markets, since the number of assets is smaller than the number of shocks. Furthermore the assumption of preference shocks during a crisis is not unrealistic: During a crisis agents for example prefer to postpone purchases of some goods, like consumer durables, while there are some other types of goods, like food, to which they cannot renounce.}
where \( p_H^t \) and \( p_F^t \) are the nominal prices of final home and foreign goods, denominated in the Home currency. Similarly, Foreign price index in Home currency is defined as:

\[
P_F^t = [\kappa^*_t (p_F^t)^{1-\theta} + (1 - \kappa^*_t)(p_H^t)^{1-\theta}]^{\frac{1}{1-\theta}}. \tag{1.5}
\]

In this economy the law of one price holds. Purchasing Power Parity (PPP) does not hold, and the real exchange rate is defined as the ratio between the Foreign price index over the Home price index: \( Q_t = \frac{p_F^t}{p_H^t} \).

### 1.3.2 Technology and Firms

In period \( t \), country \( i \) produces \( y_i^t \) units of good \( i \) according to the production function

\[
y_i^t = A_i^t l_i^\alpha, \tag{1.6}
\]

with \( i = H, F \) The stochastic productivity, \( A_i^t \), is an exogenous random variable that follows an AR(1) process:

\[
A_{i+1}^t = \phi^A A_i^t + \varepsilon_{i+1}^A. \tag{1.7}
\]

The Cobb-Douglas technology implies that a constant share of output \( \alpha \) is paid to workers. Therefore, the total cost of labor for country \( i \) is:

\[
w_i^t l_i^t = \alpha p_i^t y_i^t, \tag{1.8}
\]

where \( p_i^t \) is the price if the good produced in country \( i \) at time \( t \), and \( w_i^t \) is the wage rate of country \( i \) at time \( t \). Dividends of country \( i \) are defined as the share of output that is not paid to labor:

\[
d_i^t = (1 - \alpha)p_i^t y_i^t, \tag{1.9}
\]

\(^6\)In alternative, the assumption of a production function linear in labor and monopoly would have generated the same results.
1.3.3 The Government

The public sector of country $i$ issues at time $t$ government bonds $B_{t+1}^i$ denominated in the domestic good. The real price of the bond at time $t$ is $z_i^t$ and its return is one unit of good $i$ at time $t+1$. The government collects lump sum taxes $T_i^t$ in order to finance public spending. The budget constraint of the government is:

$$z_i^t B_{t+1}^i = \frac{p_i^t}{P_i^t} B_i^t + \frac{p_i^t}{P_i^t} G_i^t - T_i^t, \quad (1.10)$$

where public spending in country $i$, $G_i^t$, is assumed to fall entirely on the good produced by country $i$. Public spending follows an AR(1) process:

$$G_{t+1}^i = \phi G_t^i + \varepsilon_{t+1}^i. \quad (1.11)$$

1.3.4 Financial Markets

Each country $i$ issues two types of assets: government bonds and stocks. There is a bond denominated in the Home good and one denominated in the Foreign good: Buying one unit of the Home (Foreign) bond at time $t$ delivers one unit of Home (Foreign) good in the following period. Each country also issues a stock that represents a claim to country $i$ firm’s streams of cash-flows, $d^t$. The supply of each share is normalized to unity. Let $S_{t+1}^{ij}$ denote the number of shares of stock $i$ held by country $j$ household at the end of period $t$, while $B_{t+1}^{ij}$ represents holdings of bonds issued by country $i$ and held by country $j$. The budget constraint for the home economy at time $t$ is:

$$nfa_{t+1}^H + \frac{\phi}{2}(nfa_{t+1}^H)^2 = (Y_t^H - G_t^H) \frac{P_i^H}{P_i^H} - C_t^H + \gamma_t^F nfa_t^H + (\gamma_t^F - \gamma_t^H) q_{t-1}^H S_{t+1}^{HF}$$

$$+ (R_t^F - \gamma_t^F) z_{t-1}^F B_{t+1}^{HF} - (R_t^H - \gamma_t^F) z_t^H B_{t+1}^{HH}, \quad (1.12)$$

where $nfa^H$ are net foreign assets, and they are defined as:

$$nfa_{t+1}^H = z_t^F B_{t+1}^{HF} + q_t^F S_{t+1}^{HF} - q_t^H S_{t+1}^{HF} + z_t^H B_{t+1}^{HH}. \quad (1.13)$$
\( \gamma^H \) and \( \gamma^F \) are the returns on stocks issued by Home and Foreign and defined in terms of Home consumption:

\[
\gamma^H_t = \frac{(P_H^t q^H_t + d^H_t)}{P_H^t q^H_{t-1}},
\]

(1.14)

and

\[
\gamma^F_t = \frac{(P_H^t q^F_t + d^F_t)}{P_H^t q^F_{t-1}}.
\]

(1.15)

\( R^H_t \) and \( R^F_t \) are the returns on Home and Foreign bonds in terms of Home consumption, and they are defined as:

\[
R^H_t = \frac{p^H_t}{P_H^t z^H_{t-1}},
\]

(1.16)

and

\[
R^F_t = \frac{p^F_t}{P_H^t z^F_{t-1}}.
\]

(1.17)

The term \( \frac{\varphi}{2} (nfa^H_{t+1})^2 \) describes a convex cost of adjusting net foreign assets. It is a stationarity-inducing device that allows to pin down a unique, deterministic steady-state level of net foreign assets.

Foreign agents solve a similar portfolio allocation problem and their budget constraint is:

\[
nfa^F_{t+1} + \frac{\varphi}{2} (nfa^F_{t+1})^2 = (Y^F_t - G^F_t) \frac{P^F_t}{P^F_{t-1}} - C^F_t + \gamma^H_t nfa^H_t + (\gamma^H_t - \gamma^F_t) q^F_t S^F_{t+1} + (R^H_t - \gamma^H_t) z^H_{t-1} B^H_{t+1} - (R^F_t - \gamma^H_t) z^F_{t-1} B^F_{t+1},
\]

(1.18)

where Foreign net foreign assets \( nfa^F_{t+1} \), are defined as:

\[
nfa^F_{t+1} = z^H_{t+1} B_{t+1}^{HF} + q^H_t s^H_{t+1} - q^F_t s^F_{t+1} - z^F_t B_{t+1}^{FF}.
\]

(1.19)

### 1.3.5 Market Clearing Conditions

The resource constraints are

\[
c^H + c^F + G^H = Y^H_t
\]

(1.20)
\[ c_t^{FF} + c_t^{FH} + G_t^F = Y_t^F \]  

(1.21)

If we define the value of Home equity held by the two countries in terms of Home consumption, \( q_t^H S_{t}^{HH} \) and \( q_t^H S_{t}^{HF} \), as \( a_t^{HH} \) and \( a_t^{HF} \) the market clearing condition for the asset issued by Home economy is:

\[ a_t^{HH} + a_t^{HF} = q_t^H, \]

(1.22)

since the total number of equities is normalized to unity, \( S_{t}^{HH} + S_{t}^{HF} = 1 \). Similarly, by defining holdings of Foreign equity in terms of Home consumption, the market clearing condition for Foreign equity is:

\[ a_t^{FF} + a_t^{FH} = q_t^F. \]

(1.23)

In the case of bonds, we have:

\[ B_t^H = b_t^{HH} + b_t^{HF}, \]

(1.24)

\[ B_t^F = b_t^{FF} + b_t^{FH}, \]

(1.25)

where \( B_t^i \) is the total number of bonds issued by country \( i \) at time \( t \).

### 1.3.6 Uncertainty Shocks

Recently there has been an increasing interest in the analysis of the effects of uncertainty on macroeconomic variables\(^7\). Differently from the previous portfolio literature, I look at the effect of uncertainty shocks on the steady-state portfolio and on its dynamics. I introduce uncertainty through the assumption that the variances of the shocks that hit the economy are time-varying and can

\(^7\)For example see Benigno et al.(2010), Bloom (2009), Bloom, Floetotto and Jaimovich (2009), Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez and Uribe (2009).
deviate from their steady-state value according to the following stochastic processes:

\[
\begin{align*}
\sigma^2_{t+1}^A &= \rho^A \sigma^2_A + (1 - \rho^A)\sigma^2_{t+1}^Y + \eta^A u^A_{t+1}, \\
\sigma^2_{t+1}^G &= \rho^G \sigma^2_G + (1 - \rho^G)\sigma^2_{t+1}^G + \eta^G u^G_{t+1}, \\
\sigma^2_{t+1}^v &= \rho^v \sigma^2_v + (1 - \rho^v)\sigma^2_{t+1}^v + \eta^v u^v_{t+1},
\end{align*}
\]

with \(u^i_{t+1}\) is identically and independently distributed process with mean zero and unitary variance. \(\sigma^{2i}\) are the steady state values of the variances, with \(i = A, G, v\).

### 1.3.7 Optimality Conditions

The first-order conditions for country \(i\) with respect to equities, bonds and labor are:

\[
\begin{align*}
C_t^{i-\sigma} (1 + \varphi S_{t+1}^{H}) &= \beta E_t(C_{t+1}^{i-\sigma} \gamma_{t+1}^H), \\
C_t^{i-\sigma} (1 + \varphi S_{t+1}^{F}) &= \beta E_t(C_{t+1}^{i-\sigma} \gamma_{t+1}^F), \\
C_t^{i-\sigma} (1 + \varphi B_{t+1}^{H}) &= \beta E_t(C_{t+1}^{i-\sigma} R_{t+1}^H), \\
C_t^{i-\sigma} (1 + \varphi B_{t+1}^{F}) &= \beta E_t(C_{t+1}^{i-\sigma} R_{t+1}^F), \\
\chi^i_l &= C_t^{i-\sigma} \alpha A_t l_t^{\alpha-1}.
\end{align*}
\]

### 1.4 Model Solution

It is well known that in open economy macro literature, the optimal portfolio is indeterminate. Standard approximation methods that use only first-order approximations imply that certainty equivalence holds, and, as a result, all assets are perfect substitutes. In order to overcome this

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8As in Benigno et al. (2010), I assume that the exogenous state variables follow conditionally-linear stochastic processes, where the variances of the primitive shocks follow stochastic linear processes.
indeterminacy problem, Devereux and Sutherland (2006, 2010), provide a solution method that allows to determine steady-state portfolios. Their solution method is characterized by a two part solution: a second-order approximation of portfolio equations, in combination with a first-order approximation of the rest of the model. In this way it is possible to determine how the stochastic structure of the model affects the portfolio allocation, and it is possible to characterize the economy’s first-order response to stochastic shocks under the optimal portfolio.

Equations (6), (12)-(17), (20), (29)-(33), along with their foreign equivalent, may be solved in order to determine the path of quantities \( \{ C_t^H, C_t^F, l_t^H, l_t^F, y_t^H, y_t^F \} \), prices \( \{ w_t^H, w_t^F, q_t^H, q_t^F, z_t^H, z_t^F, T o t_t, Q_t \} \), rates of returns \( \{ \gamma_t^H, \gamma_t^F, R_t^H, R_t^F \} \), and the vector of steady-state asset holdings \( \{ a_t^{HH}, a_t^{HF}, a_t^{FF}, a_t^{FH}, b_t^{HH}, b_t^{HF}, b_t^{FF}, b_t^{FH} \} \).

Following the procedure that is described in Appendix A, it is possible to find the steady-state value of foreign equity holdings and foreign bond holdings:

\[
a_t^{HF} = f(\sigma_t^{2A}, \sigma_t^{2G}, \sigma_t^{2v}, \text{parameters}),
\]

\[
b_t^{HF} = f(\sigma_t^{2A}, \sigma_t^{2G}, \sigma_t^{2v}, \text{parameters}).
\]

Steady-state foreign equity and foreign bond holdings are a function of the parameters of the model and of the steady-state variances of the shocks.

### 1.5 Portfolio Dynamics

DS (2006) show that a combination of third-order approximations of the portfolio equations and second-order approximations of the rest of the model delivers the path followed by the portfolio, when the economy is hit by a shock. DS (2006) showed that deviations of exogenous state variables from their steady-state are a source of portfolio changes. My contribution is to point out that

---

9From now on DS (2006).
uncertainty shocks can be an additional source of portfolio dynamics. Not only the shocks of the model, but also their time-varying variances can contribute to explain deviations of the portfolio from its steady-state.

Traditionally DSGE models rely on the assumption of homoscedastic shocks, but recently the attention has been focused on the effects of time-varying variances on the business cycle. As in Benigno et al. (2010) I rely on the assumption that the exogenous state variables follow a conditionally-linear stochastic process in which the variance of the primitive shocks are modelled through a stochastic linear process. As shown in Benigno et al. (2010), in the presence of uncertainty shocks the second-order approximation of the model is sufficient to capture the effects of uncertainty. It follows that when this principle is applied to the portfolio problem, first-order portfolio dynamics can be then generated by a combination of second and third-order approximations of the model.

The third-order approximations of the portfolio equations, as obtained in DS (2006), are:

\[ E_t[-\sigma \hat{C}_{t+1} \hat{D}_t + \frac{\sigma^2}{2} (\hat{C}_{t+1} \hat{D}_{t+1}^{(2)})] = 0, \tag{1.36} \]

and

\[ E_t[-\sigma \hat{C}_{t+1} \hat{R}_t + \frac{\sigma^2}{2} (\hat{C}_{t+1} \hat{R}_{t+1}^{(2)})] = 0, \tag{1.37} \]

where \( \hat{C}_{t+1}^{(2)} \) is the second-order approximation of consumption and \( \hat{D}_t^{(2)} \) and \( \hat{R}_t^{(2)} \) are the second-order approximations of the difference in the returns on equities and bonds. The second-order approximation of the differential in consumption is:

\[ \hat{C}_t^{(2)} = \eta_{C^{ap}a} n f a_t H + \eta_{C^D A^P} \hat{A}_t^D + \eta_{C^D G^D} \hat{G}_t^D + \eta_{C^D \xi^D} a \hat{r}_t^D + \eta_{C^D D^D} n f a_t \hat{\gamma}_t^D + \eta_{C^D A^D} \hat{A}_t^D + \eta_{C^D G^D} \hat{G}_t^D + \eta_{C^D \xi^D} \hat{\xi}_t + + \eta_{C^D \theta D} \hat{\theta}_t \tag{1.38} \]

where \( \hat{\xi}_t \) is now \( \hat{a}_t \hat{\gamma}_t^D \) and \( \hat{\theta}_t \) is now \( \hat{b}_t \hat{R}_t^D \). The second-order approximation of equity and bond return differential is instead:

\[ \hat{\gamma}_t^{(2)} = \eta_{\gamma^A \xi^A} \hat{\xi}_t^A + \eta_{\gamma^G \xi^G} \hat{\xi}_t^G + \eta_{\gamma^A(2)} \sigma_t^{2A} + \eta_{\gamma^G(2)} \sigma_t^{2G} \tag{1.39} \]
\[
\hat{R}_t^{D(2)} = \eta_{ReA} \varepsilon_t^A + \eta_{ReG} \varepsilon_t^G + \eta_{ReA(2)} \sigma_t^2 A + \eta_{ReG(2)} \sigma_t^2 G
\]  

(1.40)

Plugging (38), (39) and (40) in the equilibrium conditions (36) and (37) you find the equations that describe foreign holdings of Home assets dynamics, \( \hat{a}_t \) and \( \hat{b}_t \).

In Appendix B I solve a simplified portfolio model that allows me to get an analytical solution of the problem\(^{10}\). Time-varying volatilities are, together with state variables, a source of portfolio dynamics. The only exception is the log utility case, in which uncertainty does not contribute to explain portfolio departures from its steady state. An interesting property of the first-order dynamics generated by uncertainty is that actually portfolio dynamics coincide (except for a constant) with the sequence of steady-state portfolios that the problem would generate if we considered each individual realization of the stochastic variance as a steady-state variance. By looking at (B.32) and (B.36) from Appendix B you can observe that the first line of (B.36) is the time-indexed equivalent of (B.32), adjusted by a constant.

\[
\tilde{a} = -\frac{\eta_{CY} \eta_{\gamma Y} \sigma^{2Y} + \eta_{CG} \eta_{\gamma G} \sigma^{2G}}{\eta_{C}\xi (\eta_{\gamma Y} \sigma^{2Y} + \eta_{\gamma G} \sigma^{2G})}, \quad (B.32)
\]

\[
\hat{a}_t = \frac{1 - \sigma}{\sigma \eta_{C} \xi^{(2)} (\eta_{\gamma Y} \sigma_{t+1}^{2Y} + \eta_{\gamma G} \sigma_{t+1}^{2G})} \left( \eta_{CY} \eta_{\gamma Y} \sigma_{t}^{2Y} + \eta_{CG} \eta_{\gamma G} \sigma_{t}^{2G} \right) + \frac{1}{\eta_{C} \xi^{(2)}} \left( \frac{1 - \sigma}{\sigma} \eta_{C} \tilde{a} - \eta_{C} a T a_{t+1} + \frac{1}{\sigma} (\phi_{G} G_{t+1} \eta_{CG} + \phi_{A} A_{t+1} \eta_{CY}) \right) \; (B.36)
\]

This means that in this specific problem comparative statics and impulse responses provide the same piece of information, as long as only uncertainty is involved. This property implies that, in principle, a first-order approximation of non-portfolio equations and a second-order approximation of portfolio equations should be sufficient to analyze the effects of uncertainty on portfolio dynamics. The analytical solution shows that an increase in the volatility of Home endowment increases Foreign

\(^{10}\)The simplified model describes a two-country endowment economy. Each country issues one asset and the two economies are hit by government spending and endowment shocks.
asset’s holdings proportionally to the product of the elasticity of consumption to endowment $\eta_{CY}$ and the elasticity of the return to the endowment shock $\eta_{\gamma\varepsilon}$. Home endowment shocks have a positive effect on Home consumption and on Home returns. The increase in Home endowment volatility increases the covariance between Home consumption and Home asset returns and makes the Foreign asset a better hedging instrument against the endowment risk. Similarly, an increase in Home government spending uncertainty modifies Home holdings of the Foreign asset proportionally to the elasticity of government spending to consumption $\eta_{CG}$ and the elasticity of government spending to the Home asset return. $\eta_{\gamma\sigma}$. For reasonable parameter values, the sign of $\eta_{CG}$ is negative, as the intuition would suggest. An increase in government spending crowds out private spending. The fact that returns positively react to government spending shocks implies that their covariance with consumption is negative: An increase in the variance of Home government spending further reduces the covariance between consumption and Home returns. Home assets are therefore a good hedge against government spending volatility.

### 1.6 Numerical Results

#### 1.6.1 Calibration

The portfolio solution of the full model is a highly complicated expression that can only be described numerically. The parameter values are presented in Table 1. The discount factor, $\beta$, is set equal to 0.96, in order to have a steady state real interest rate of 4%. Agents in both countries maximize the same utility function. The constant risk aversion parameter, $\sigma$ is set to 3, since empirical evidence suggests that its value is between 2 and 3. The macro literature typically attributes to the Frisch elasticity a value between 2 and 4, while the findings of the micro literature suggest a much lower value for this elasticity. Consistently with the macro literature I set the parameter $\varphi$, equal to 0.5.
Following Devereux and Sutherland (2008), I set the persistence parameter of the preference shock to 0.95, while the persistence of government spending is 0.7 and the one of productivity is assumed to be 0.9. The standard deviation of all the shocks is equal to 0.001%, and all the shocks are \textit{i.i.d.}. The share of labor income, $\alpha$, equals 0.7, as suggested by empirical evidence. The home bias parameter $\kappa$ equals 0.8. The total number of bonds is normalized to 60% GDP, and the compatible ratio of taxes over GDP is 0.046.

1.6.2 Results

In this economy there are four assets and five shocks. If we reduce the number of shocks to four, we have locally complete markets\textsuperscript{11}, i.e., it is possible to build a portfolio that is able to replicate complete markets.Compatibly with the findings of Devereux and Sutherland (2010), the choice to invest in both assets depends on the presence of the preference shock. In the case without preference shock it is possible to replicate the complete asset market solution, characterized by complete home bias in the equity portfolio. \textsuperscript{12}.

Once the preference shock is introduced, the number of assets available is smaller than the number of shocks that hit the economy, and, therefore, agents need to hold also foreign equity in order to achieve the highest degree of risk sharing.

Given the parameter values, the optimal portfolio implies that each country holds 20% of GDP in

\textsuperscript{11}As showed in Coeurdacier and Gourinchas (2009), when the number of shocks equals the number of assets, rank and spanning conditions are satisfied, and one can replicate the efficient risk-sharing allocation up-to the first order.

\textsuperscript{12}As shown by DS (2010), this optimal portfolio is represented by zero holdings of foreign equity and a long or short position in foreign bonds, depending on parameter values. This portfolio achieves full cross country risk sharing and supports complete asset markets. Full risk sharing in fact requires consumption differentials adjusted for real exchange rate movements to be equalized across countries. Bonds allow a claim on the terms of trade and the deviation from full risk sharing across countries is proportional to the terms of trade too. Hence, a bond portfolio can ensure full cross-country risk sharing.
foreign equity, and 4% of GDP in foreign government bonds. The steady-state portfolio is therefore characterized by a long position in both foreign equity and foreign bond.

Figures 8, 9, and 10 show that the model matches the percentages of holdings observed in reality fairly accurately: In 2005 foreign holdings of Treasury bonds were 5% of U.S. GDP and foreign equity holdings were 16.6% of U.S. GDP. In 2007, foreign equity holdings were 23% of GDP and foreign bond holdings were 4.6% of GDP. As the coefficient of risk aversion increases equity home bias increases, while the home bias in bonds is reduced. When \( \sigma = 4 \) agents hold an amount of foreign equity equal to 19% of GDP, and foreign bond holdings are 5% of GDP.

In what follows, I analyze portfolio dynamics change during a crisis, when uncertainty increases in the economy. I use the volatilities of productivity, government spending and preference shocks as a proxy of uncertainty.

When the uncertainty shock hits the volatility of productivity, on impact Home holdings of foreign equity increase. As the uncertainty shock hits, holdings of Foreign equity jump to 30% of GDP. As time passes, they smoothly decrease until they go back to their steady-state (Figure 9). Foreign bond holdings are instead strongly reduced after an increase in the volatility of the productivity shock. When the shock hits, Home agents short-sell Foreign bonds and their foreign bond holdings equal \(-2\%\) of GDP. As variance goes back to its steady-state, foreign bond holdings go back to their initial value of 6% of GDP (Figure 10). Home equity returns increase when Home productivity is hit by a positive shock, while Home bond returns decrease when productivity is hit by the same shock. This happens because a positive productivity shock reduces the price of the Home good, but, at the same time it increases its production. This means that Home will experience an increase in total revenue and also in dividends. On the other hand, Home bonds still pay only one unit of Home good whose price has gone down: This results in a negative effect of productivity shocks on Home bond returns. Furthermore the elasticity of Home consumption to endowment shocks is positive. All this implies that an increase in the volatility of Home productivity shocks increases the covariance
between Home consumption and Home equity returns, while it reduces the covariance between Home consumption and Home bond returns. Agents therefore use Home bonds to hedge themselves against the high volatility in productivity shocks. They also increase their holdings of Foreign equity whose returns are not affected by Home uncertainty shocks. An increase in the volatility of government spending shocks induces agents to increase Home equity holdings. Foreign equity is reduced to 14% of GDP (Figure 11) and Foreign bond holdings are increased to 9.5% of GDP, as shown in figure 12. Government spending shocks have a negative but small impact on private consumption. Through their effect on relative Home prices, they increase bond returns and reduce equity returns. Furthermore, their effect on output also increases labor income. Agents, in order to smooth consumption, hedge the government spending risk through holdings of Foreign bonds and Home stocks, whose returns have a negative covariance with government spending shocks. An increase in the volatility of government spending shocks, makes these covariances even more negative and therefore agents readjust their portfolios increasing Foreign bond holdings and Home equity holdings. Finally an increase in the volatility of preference shocks results in an increase of both Foreign equity and bond holdings: Foreign equity holdings reach 31% of GDP and Foreign bond holdings reach 12% of GDP (Figure 13 and Figure 14). Preference shocks have a positive effect on consumption and on labor income. They also increase equity and bond returns. A positive preference shock for Home good increases the world demand for that good. It has a positive effect on relative prices and relative production. As a consequence labor income, Home equity and Home bond returns increase. An increase in the volatility of the Home preference shock increases the positive covariance between Home consumption and Home asset returns. Agents therefore diversify their portfolio by increasing their holdings of Foreign bonds and equity.
1.7 Conclusion

During the financial crisis that first hit the U.S. economy and soon became a world crisis, investors reduced their holdings of foreign equities, and, at the same time, they increased their holdings of short-term U.S. government bonds. The paper analyzes, within the context of a DSGE model, the hedging properties of foreign bond and foreign equity holdings during a crisis, when the degree of uncertainty is high. Uncertainty is here described by uncertainty shocks that make the variances of the model time-varying. Uncertainty shocks are a source of portfolio dynamics, together with deviations of the state variables from their steady-state. The portfolio dynamics generated by uncertainty and calculated through a combination of second and third-order approximations of the model display a very particular property: Portfolio dynamics are, de facto, an adjusted measure of the sequence of steady-state portfolios that would be observed if we considered in each instant any realization of the stochastic variance as a steady-state variance. This finding greatly simplifies the calculation of first-order portfolio dynamics.

The model is able to qualitatively replicate the change in portfolios observed during the crisis, under certain conditions: As world government spending volatility increases, it is optimal to increase the share of foreign bond holdings and reduce the share of foreign equity holdings. An increase in the variance of the government spending shock increases the covariance between bond returns and labor income and reduces the covariance between relative equity returns and labor income. Agents therefore stabilize their total income increasing their holdings of foreign bonds and reducing the ones of foreign equity. As world productivity volatility increases, it is optimal instead to increase the share of foreign equity holdings and reduce the share of foreign bond holdings.

The model suggests that the movement of capital observed during the crisis is compatible with shocks coming from the demand side of the market. These results find an important support in the theories that have been recently developed and that identify the collapse in demand as the main
cause of the slump experienced by the U.S. and by many other economies during the crisis.
Bibliography


Appendix A

A.0.1 Steady-State Portfolio

As shown by Devereux and Sutherland (2006), a second-order approximation of the portfolio problem is sufficient to capture the different features of assets and tie down a solution for steady-state holdings of foreign assets. The symmetric non stochastic steady state of the model is used as the approximation point for non-portfolio variables. In steady state, \( n \hat{f}a^H = n \hat{f}a^F = 0 \), \( \hat{\gamma}^H = \hat{\gamma}^F \), \( \bar{Y} = \bar{G} + \bar{C} \) and \( \beta = \frac{1}{\gamma} \). From the second-order approximation of the home country portfolio first order conditions, we get:

\[
E_t[\hat{\gamma}_{t+1}^D + \frac{1}{2}(\hat{\gamma}_{t+1}^H - \hat{\gamma}_{t+1}^F) - \rho \hat{C}_{H,t+1} \hat{\gamma}_{t+1}^D] = 0 \tag{A.1}
\]

with \( \hat{\gamma}^D = \hat{\gamma}^H - \hat{\gamma}^F \).

Similarly, for the foreign country:

\[
E_t[\hat{\gamma}_{t+1}^D + \frac{1}{2}(\hat{\gamma}_{t+1}^H - \hat{\gamma}_{t+1}^F) - \rho \hat{C}_{F,t+1} \hat{\gamma}_{t+1}^D - \hat{Q}_{t+1} \hat{\gamma}_{t+1}^D] = 0 \tag{A.2}
\]

Subtracting (A.2) from (A.1), we find one of the two equations that have to hold in equilibrium:

\[
E_t[\hat{\gamma}_{t+1}^D (\hat{C}_{H,t+1} - \hat{C}_{F,t+1} - \frac{\hat{Q}_{t+1}}{\sigma})] = 0. \tag{A.3}
\]
\[ E_t(\hat{\gamma}_{t+1}^D) = -\frac{1}{2} E_t(\hat{\gamma}_{H,t+1}^2 - \hat{\gamma}_{F,t+1}^2) + \rho \frac{1}{2} E_t[(\hat{C}_{H,t+1} + \hat{C}_{F,t+1} + \hat{Q}_{t+1})\gamma_{t+1}^D] \tag{A.4} \]

We can follow the same procedure for bonds too:

\[ E_t(\hat{R}_{t+1}^D(\hat{C}_{H,t+1} - \hat{C}_{F,t+1})) = 0, \tag{A.5} \]

\[ E_t(\hat{R}_{t+1}^D) = -\frac{1}{2} E_t(\hat{R}_{H,t+1}^2 - \hat{R}_{F,t+1}^2) + \rho \frac{1}{2} E_t[(\hat{C}_{H,t+1} + \hat{C}_{F,t+1})\hat{R}_{t+1}^D]. \tag{A.6} \]

The optimal value of portfolio holdings can be found by solving the first order accurate behavior of \((\hat{C}_{H,t+1} - \hat{C}_{F,t+1}), \hat{\gamma}_{t+1}^D,\) and \(\hat{R}_{t+1}^D\). This requires a first-order accurate solution of the non-portfolio equations of the model, as shown by Devereux and Sutherland (2006). The non-portfolio parts of the model are represented by the law of motion for net foreign assets and by the first order conditions with respect to equities and labor supply. The first-order approximation of the home budget constraint, around a symmetric steady state with zero net foreign assets, is:

\[ n\hat{f}_{a_{t+1}^H} + \varphi n\hat{f}_{a_{t+1}^H} + \hat{C}_t^H = y\hat{Y}_t^H - g\hat{G}_t^H + \hat{p}_t^H - \hat{\gamma}_t^H + \frac{1}{\beta} n\hat{f}_{a_t^H} + \frac{a_{HF}}{\beta C} (\hat{\gamma}_t^H - \hat{\gamma}_t^H) + \frac{b_{HF}}{\beta C} (\hat{R}_t^F - \hat{R}_t^H) \tag{A.7} \]

where \(g = \frac{G}{C}\) and similarly \(y = \frac{Y}{C}\). \(a_{HF}\) and \(b_{HF}\) represent the steady-state values of foreign holdings of home shares and bonds. Taking differences between the domestic and foreign budget constraint and using the condition \(n\hat{f}_{a_t^H} + n\hat{f}_{a_t^F} - \hat{Q}_t = 0\), we get:

\[ n\hat{f}_{a_{t+1}^H} = \frac{y}{2} \hat{Y}_t^D + \frac{1}{\beta} n\hat{f}_{a_t^H} - \frac{1}{2} \hat{C}_t^D + \frac{g}{2} \hat{G}_t^D + \hat{\gamma}_t^D + T_{ot_t} + \hat{Q}_t^D + \bar{b}\hat{R}_t^D \tag{A.8} \]

where \(\bar{a} = -\frac{a_{HF}}{\beta C}\), \(\bar{b} = -\frac{b_{HF}}{\beta C}\) and \(\hat{X}_t^D = \hat{X}_t^H - \hat{X}_t^F\). Terms of trade, \(Tot\) are defined as \(\hat{p}_t^H - \hat{p}_t^F\), while the real exchange rate \(Q\) is \(\hat{P}_t^F - \hat{P}_t^H\). The first-order approximations of the first order conditions in differential terms are:

\[ \hat{l}_t^D = \frac{1}{\varphi + \alpha + 1}(\hat{A}_t^D - \sigma \hat{C}_t^D), \tag{A.9} \]

\[ \hat{C}_t^D = E_t(\hat{C}_{t+1}^D), \tag{A.10} \]
\[
E_t(\hat{\gamma}^D_{t+1}) = 0, \tag{A.11}
\]
\[
E(\hat{R}^D_{t+1}) = 0, \tag{A.12}
\]

The state-space solution is characterized as follows:
\[
n\hat{f}a^H_{t+1} = \eta_{aa}\hat{f}a^H_t + \eta_{aAD}\hat{A}^D_t + \eta_{aGD}\hat{G}^D_t + \eta_{a\gamma}\hat{\xi}_t + \eta_{a\theta}\hat{\vartheta}_t + \eta_{a\upsilon}\hat{\upsilon}_t \tag{A.13}
\]
\[
\hat{C}^D_t = \eta_{CDa}\hat{f}a^H_t + \eta_{CDAD}\hat{A}^D_t + \eta_{CDDG}\hat{G}^D_t + \eta_{CD\xi}\hat{\xi}_t + \eta_{CD\theta}\hat{\vartheta}_t + \eta_{CD\upsilon}\hat{\upsilon}_t \tag{A.14}
\]
where \(\hat{\xi}_t = \tilde{a}\hat{\gamma}^D_t\) and \(\hat{\vartheta}_t = \tilde{b}\hat{R}^D_t\).

\[
\hat{\gamma}^D_t = \eta_{\gamma D\varepsilon}\varepsilon^A_t + \eta_{\gamma D\theta}\varepsilon^G_t + \eta_{\gamma D\upsilon}\varepsilon^v_t \tag{A.15}
\]
\[
\hat{R}^D_t = \eta_{RD\varepsilon}\varepsilon^A_t + \eta_{RD\theta}\varepsilon^G_t + \eta_{RD\upsilon}\varepsilon^v_t \tag{A.16}
\]

Substituting (A.13), (A.14), (A.15) and (A.16) into (A.3) and (A.5), we find the system of two equations that have to be solved in order to find the optimal portfolio \((a^*_1, b^*_1)\)
\[
E_t[(\eta_{CDa}\hat{f}a^H_{t+1} + \eta_{CDAD}\hat{A}^D_{t+1} + \eta_{CDDG}\hat{G}^D_{t+1} + \eta_{CD\xi}\hat{\xi}_{t+1} + \eta_{CD\theta}\hat{\vartheta}_{t+1} + \eta_{CD\upsilon}\hat{\upsilon}_{t+1})]
\]
\[
(\eta_{\gamma D\varepsilon}\varepsilon^A_{t+1} + \eta_{\gamma D\theta}\varepsilon^G_{t+1} + \eta_{\gamma D\upsilon}\varepsilon^v_{t+1}) = 0 \tag{A.17}
\]
\[
E_t[(\eta_{CDa}\hat{f}a^H_{t+1} + \eta_{CDAD}\hat{A}^D_{t+1} + \eta_{CDDG}\hat{G}^D_{t+1} + \eta_{CD\xi}\hat{\xi}_{t+1} + \eta_{CD\theta}\hat{\vartheta}_{t+1} + \eta_{CD\upsilon}\hat{\upsilon}_{t+1})]
\]
\[
(\eta_{RD\varepsilon}\varepsilon^A_{t+1} + \eta_{RD\theta}\varepsilon^G_{t+1} + \eta_{RD\upsilon}\varepsilon^v_{t+1}) = 0 \tag{A.18}
\]

The two following equations that will determine the steady state value of bond and equity holdings:
\[
\eta_{CDAD}\eta_{\gamma D\varepsilon}\sigma^2_A + \eta_{CDDG}\eta_{\gamma D\theta}\sigma^2_G + \eta_{CDD\xi}\hat{\alpha}(\eta_{\gamma D\varepsilon}\sigma^2_A + \eta_{\gamma D\theta}\sigma^2_G + \eta_{\gamma D\upsilon}\sigma^2_v) + \eta_{CD\gamma}\tilde{b}(\eta_{\gamma D\varepsilon}\eta_{ReA}\sigma^2_A + \eta_{\gamma D\theta}\eta_{ReG}\sigma^2_G + \eta_{\gamma D\upsilon}\eta_{Rev}\sigma^2_v) = 0 \tag{A.19}
\]
\[
\eta_{CDAD}\eta_{ReA}\sigma^2_A + \eta_{CDDG}\eta_{ReG}\sigma^2_G + \eta_{CD\xi}\hat{\alpha}(\eta_{\gamma D\varepsilon}\eta_{ReA}\sigma^2_A + \eta_{\gamma D\theta}\eta_{ReG}\sigma^2_G + \eta_{\gamma D\upsilon}\eta_{Rev}\sigma^2_v) + \eta_{CD\gamma}\tilde{b}(\eta_{ReA}\sigma^2_A^2 + \eta_{ReG}\sigma^2_G^2 + \eta_{Rev}\sigma^2_v^2) = 0 \tag{A.20}
\]

The two equations (A.19) and (A.20) are functions of the parameters of the model and of the variances of the shocks.
Appendix B

In what follows I show the analytical solution for the optimal portfolio and its dynamics, when there are uncertainty shocks. This is an infinite horizon, two-country open economy model. There are two ex-ante symmetric countries, “Home” (H) and “Foreign” (F), each one populated by a representative household who consumes and trades a portfolio of financial assets. There are two different goods: H good and F good. Final consumption is a CES aggregate of the two goods. The international portfolio consists of two assets: Home equity and Foreign equity. I allow for two types of shocks in each country: endowment shocks and government spending shocks. I also allow for uncertainty shocks: The variances of both endowment and government spending shocks are indexed by time. They stochastically move period by period according to their autoregressive processes. The number of shocks is larger than the number of assets available: This implies that financial markets are incomplete.

B.0.2 Preferences

The household in country $i = H, F$ chooses consumption $C^i_t$ to maximize the intertemporal utility function:

$$\max E_t \sum_{s=0}^{\infty} \beta^s \left( \frac{(C_{t+s}^i)^{1-\sigma}}{1-\sigma} \right)$$

(B.1)
with coefficient of relative risk aversion $\sigma > 0$. The Home final consumption basket combines home
and foreign goods:

$$C_t^H = \left[ 1/2^{1/\theta} (c_t^{HH})^{(\theta-1)/\theta} + 1/2^{1/\theta} (c_t^{FH})^{(\theta-1)/\theta} \right]^{\theta/(\theta-1)}$$  \hspace{1cm} (B.2)

where $c^{ij}$ is the amount of consumption good produced by country $i$ and consumed by country $j$.

The Foreign consumption basket is:

$$C_t^F = \left[ 1/2^{1/\theta} (c_t^{FF})^{(\theta-1)/\theta} + 1/2^{1/\theta} (c_t^{HF})^{(\theta-1)/\theta} \right]^{\theta/(\theta-1)}.$$  \hspace{1cm} (B.3)

The aggregate consumer price index for Home, $P_t^H$, is defined as:

$$P_t^H = \left[ 1/2(p_t^H)^{1-\theta} + 1/2(p_t^F)^{1-\theta} \right]^{1/\theta},$$  \hspace{1cm} (B.4)

where $p_t^H$ and $p_t^F$ are the nominal prices of final home and foreign goods, denominated in the Home
currency. Similarly, Foreign price index in Home currency is defined as:

$$P_t^F = \left[ 1/2(p_t^F)^{1-\theta} + 1/2(p_t^H)^{1-\theta} \right]^{1/\theta}.$$  \hspace{1cm} (B.5)

In this economy the law of one price holds. Purchasing Power Parity (PPP) also holds, since I
assume no home bias.

\textbf{B.0.3 Stochastic Processes}

The endowment and government spending shocks follow an AR(1) process:

$$\hat{Y}_{t+1}^D = \phi^Y \hat{Y}_t^D + \varepsilon_{t+1}^Y$$  \hspace{1cm} (B.6)

$$\hat{G}_{t+1}^D = \phi^G \hat{G}_t^D + \varepsilon_{t+1}^G$$  \hspace{1cm} (B.7)

where $Y^D$ and $G^D$ denote the log of the productivity and the government spending shocks. $\hat{Y}^D$ is
the difference between the Home endowment of Home good and the Foreign endowment of Foreign
good. The innovations to the log processes (B.6) and (B.7), $\varepsilon_{t+1}^Y$ and $\varepsilon_{t+1}^G$ are identically and
independently distributed with mean zero and variance $\sigma^2_{t+1}$ and $\sigma^2_{t+1}$, respectively.

Furthermore, I assume that the variance of the shocks is time-varying. When an uncertainty shock hits the economy at time $t+1$, the variances of $\varepsilon^Y_{t+1}$ and $\varepsilon^G_{t+1}$ depart from their steady-state and follow a mean reverting process:

$$\sigma^2_{t+1} = \rho^Y \sigma^2_Y + (1 - \rho^Y) \sigma^2_t + \eta^Y u^Y_{t+1} \tag{B.8}$$

$$\sigma^2_{t+1} = \rho^G \sigma^2_G + (1 - \rho^G) \sigma^2_t + \eta^G u^G_{t+1} \tag{B.9}$$

where $u^Y_{t+1}$ and $u^G_{t+1}$ are identically and independently distributed process with mean zero and unitary variance. Since $\rho^Y$ and $\rho^G$ are smaller than 1, as time passes, the variances converge to their steady-states $\sigma^2Y$ and $\sigma^2G$.

The budget constraint for the home economy at time $t$ is:

$$nf a^H_{t+1} = (Y^H_t - G^H_t) \frac{P^H_t}{P^H_t} - C^H_t + \gamma^F_t nfa^H_t + (\gamma^H_t - \gamma^H_{t-1}) q^H_{t-1} S^{HF}_t \tag{B.10}$$

where $nfa^H$ are net foreign assets, and they are defined as:

$$nfa^H_{t+1} = q^F_t S^{FH}_{t+1} - q^H_t S^{HF}_{t+1}. \tag{B.11}$$

$S^{ij}$ stands for the stock issued by country $i$ and held by country $j$, while $q^i$ is the price of the stock issued by country $i$. $\gamma^H$ and $\gamma^F$ are the returns on the stocks issued by Home and Foreign and they are defined in terms of the consumption good:

$$\gamma^H_t = \frac{(P^H_t q^H_t + p^H_t Y^H_t)}{P^H_t q^H_{t-1}}, \tag{B.12}$$

and

$$\gamma^F_t = \frac{(P^F_t q^F_t + p^F_t Y^F_t)}{P^H_t q^F_{t-1}}. \tag{B.13}$$

\footnote{As in Benigno et al. (2010), I assume that the exogenous state variables follow conditionally-linear stochastic processes, where the variances of the primitive shocks follow stochastic linear processes}
where the dividend paid by country’s $i$ equity depends on the endowment of that country in that period. Foreign agents solve a similar portfolio problem and their budget constraint is:

$$nfa_{t+1}^F = (Y_t^F - G_t^F)\frac{P_t^F}{P_t} - C_t^F + \gamma_t^H nfa_t^F + (\gamma_t^H - \gamma_t^F)q_{t-1}^H S_{t+1}^{HF}, \quad (B.14)$$

where $nfa^F$ are net foreign assets for the Foreign country, and they are defined as:

$$nfa_{t+1}^F = q_t^H S_{t+1}^{HF} - q_t^F S_{t+1}^{FH}. \quad (B.15)$$

As in Devereux and Sutherland (2006), I define the Foreign holdings of Home equity:

$$a_t = q_{t-1}^H S_t^{HF} \quad (B.16)$$

### B.0.4 Market Clearing Conditions

The resource constraints are

$$C_t^{HH} + C_t^{HF} + G_t^H = Y_t^H \quad (B.17)$$
$$C_t^{FH} + C_t^{FF} + G_t^F = Y_t^F \quad (B.18)$$

The market clearing condition for Home stocks is:

$$S_t^{HH} + S_t^{HF} = 1. \quad (B.19)$$

The market clearing condition for Foreign equity is:

$$S_t^{FF} + S_t^{FH} = 1. \quad (B.20)$$

### B.0.5 Optimality Conditions

The first-order conditions for country $i$ with respect to equities are:

$$C_t^{i-\sigma} = \beta E_t(C_{t+1}^{i-\sigma} \gamma_{t+1}^H), \quad (B.21)$$
\[ C_t^{\pi-\sigma} = \beta E_t(C_{t+1}^{\pi-\sigma} \gamma_{t+1}^F), \quad (B.22) \]

As shown by DS (2006), a second-order approximation of the portfolio problem is sufficient to capture the different features of assets and tie down a solution for steady-state holdings of foreign assets. The symmetric non stochastic steady state of the model is used as the approximation point for non-portfolio variables. In steady state, \( nfa^H = nfa^F = 0, \gamma^H = \gamma^F, Y = G + C \) and \( \beta = \frac{1}{\gamma} \).

From the second-order approximation of the home country portfolio first-order conditions, we get:

\[ E_t[\hat{\gamma}_{t+1}^D + \frac{1}{2}(\hat{\gamma}_{H,t+1}^2 - \hat{\gamma}_{F,t+1}^2) - \sigma \hat{C}_{H,t+1} \gamma_{D,t+1}] = 0 \quad (B.23) \]

with \( \hat{\gamma}_D = \hat{\gamma}_H - \hat{\gamma}_F \). Similarly, for the foreign country:

\[ E_t[\hat{\gamma}_{t+1}^D + \frac{1}{2}(\hat{\gamma}_{H,t+1}^2 - \hat{\gamma}_{F,t+1}^2) - \sigma \hat{C}_{F,t+1} \gamma_{D,t+1}] = 0. \quad (B.24) \]

Subtracting (B.24) from (B.23), we find the equations that have to hold in equilibrium:

\[ E_t[\hat{\gamma}_{t+1}^D (\hat{C}_{H,t+1} - \hat{C}_{F,t+1})] = 0 \quad (B.25) \]

\[ E_t(\hat{\gamma}_{t+1}^D) = -\frac{1}{2}E_t(\hat{\gamma}_{H,t+1}^2 - \hat{\gamma}_{F,t+1}^2) + \frac{1}{2}E_t[(\hat{C}_{H,t+1} + \hat{C}_{F,t+1}) \gamma_{D,t+1}] \quad (B.26) \]

Taking a first order approximation of Home and Foreign budget constraints and subtracting the Foreign budget constraint from the Home one, yields the following law of motion for Home net foreign assets:

\[ n \hat{f}a_{t+1}^H = \frac{y}{2} \hat{Y}_t^D + \frac{1}{\beta} n \hat{f}a_t^H - \frac{1}{2} \hat{C}_t^D - \frac{g}{2} \hat{G}_t^D + \tilde{a} \gamma_t^D + \frac{y - g}{2} \hat{p}_t^D \quad (B.27) \]

where

\[ \tilde{a} = -\frac{a}{\beta C} \quad (B.28) \]

\( a \) being the steady-state value of \( a_t \) as defined in (B.16). The state-space solution is characterized as follows:

\[ n \hat{f}a_{t+1}^H = n \hat{f}a_t^H + \eta_{aY} \hat{Y}_t^D + \eta_{aG} \hat{G}_t^D + \eta_{a\xi} \hat{\xi}_t \quad (B.29) \]
\[
\hat{C}_t^D = \eta_{CDa}n\hat{f}_t^H + \eta_{CDY^D}\hat{Y}_t^D + \eta_{CDG^D}\hat{G}_t^D + \eta_{CD\xi}\hat{\xi}_t \tag{B.30}
\]

\[
\hat{\gamma}_t^D = \eta_{\gamma\varepsilon\varepsilon_t^Y} + \eta_{\gamma\varepsilon\varepsilon_t^G} \tag{B.31}
\]

where \(\hat{\xi}_t = \tilde{a}\hat{\gamma}_t^D\). The DS solution for \(\tilde{a}\) is:

\[
\tilde{a} = -\frac{\eta_{CY^D}\sigma_{2Y} + \eta_{CG^D}\sigma_{2G}}{\eta_{C\xi}(\eta_{\gamma\varepsilon\sigma_{2Y}} + \eta_{\gamma\varepsilon\sigma_{2G}})}.
\]  

### B.0.6 Portfolio Dynamics

In this section I discuss the role of time-varying variances in portfolio dynamics. DS (2006) show that a combination of third-order approximations of the portfolio equations and second-order approximations of the rest of the model delivers the path followed by the portfolio when the economy is hit by a shock. It is first of all necessary to notice that the dynamics generated by the shocks are different from the ones generated by their volatility. As shown in Benigno et al. (2010), in the presence of uncertainty shocks the second-order approximation is sufficient to capture the effects of uncertainty. First-order portfolio dynamics can be then generated by a combination of second and third-order approximations of the model. The third-order approximation of the portfolio equations, as obtained in DS (2006), is:

\[
E_t[-\sigma\hat{C}_{t+1}\hat{\gamma}_{t+1}^D + \frac{\sigma^2}{2}(\hat{C}_{t+1}\hat{\gamma}_{t+1}^D) - \frac{\sigma}{2}(\hat{C}_{t+1}\hat{\gamma}_{t+1}^{D(2)})] = 0, \tag{B.33}
\]

where \(\hat{C}_{D(2)}^t\) and \(\hat{\gamma}_{D(2)}^t\) are the second-order approximations of consumption and return differentials.

The second-order approximation of the differential in consumption is:

\[
\hat{C}_{D(2)}^t = \eta_{CDa}n\hat{f}_t^H + \eta_{CDA}\hat{A}_t^D + \eta_{CDG}\hat{G}_t^D + \eta_{CD\xi}\hat{\gamma}_t +
\eta_{CD\eta_D}n\hat{f}_t^H + \eta_{CD\eta_D}\hat{A}_t^D + \eta_{CDG}\hat{G}_t^D + \eta_{CD\xi}\hat{\xi}_t, \tag{B.34}
\]

where \(\hat{\xi}_t\) is now the product between the time-varying portfolio and the first-order approximation of the returns, \(\hat{a}_t\hat{\gamma}_t^D\). The second-order approximation of the return differential is instead:

\[
\hat{\gamma}_{D(2)}^t = \eta_{\gamma\varepsilon\varepsilon_t^A} + \eta_{\gamma\varepsilon\varepsilon_t^G} + \eta_{\gamma\varepsilon\varepsilon_t^{2A}} + \eta_{\gamma\varepsilon\varepsilon_t^{2G}} \tag{B.35}
\]
Plugging (B.34) and (B.35) in (B.33) and solving for \( \hat{a}_t \) leads to the following expression:

\[
\hat{a}_t = \frac{(1 - \sigma)(\eta_{CY}\gamma_q^y[\rho^Y\sigma^2Y + (1 - \rho^Y)\sigma_i^2] + \eta_{CG}\gamma_q^G[\rho^G\sigma^2G + (1 - \rho^G)\sigma_i^2])}{\sigma}\eta_{CD}\xi(2)(\eta_{CY}^2[\rho^Y\sigma^2Y + (1 - \rho^Y)\sigma_i^2] + \eta_{CG}^2[\rho^G\sigma^2G + (1 - \rho^G)\sigma_i^2]) + \frac{1}{\eta_{CD}\xi(2)}\left(1 - \frac{\sigma}{\sigma}\eta_C\bar{a} - \eta_C a_t + 1 + \frac{\phi^G\hat{G}_t^D\eta_{CG} + \phi^A\hat{Y}_t^D\eta_{CY}}{\sigma}\right)
\]

(B.36)

The resulting expression for the portfolio dynamics \( \hat{a}_t \) shows that besides the sources of dynamics already identified by the literature (i.e., the state variables of the model, \( nfa, G \) and \( A \)) also the time-varying variances of the model can generate portfolio dynamics around the steady-state. An interesting finding is that this new source of dynamics only works when the risk-aversion parameter \( \sigma \) is different from one. When agents maximize a log-utility function, time-varying volatilities do not contribute to explain portfolio dynamics.
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Table B.1: Parameter Values

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Chapter 2

Foreign Currency Debt and Expectations

2.1 Introduction

Empirical studies \(^1\) have shown that emerging countries are often characterized by the presence of a high share of foreign currency denominated debt. This stylized fact, often referred to as the “Original Sin of the International Finance”, has been described as the inability of developing countries to borrow in their own currency on the international markets. Since the “Original Sin” seems to affect also emerging countries characterized by strong fundamentals and sound policies, its main determinants have been identified in the way international markets operate, rather than in the specific features of each country.

As the debt crises of the 1990s show, the presence of foreign currency debt can be risky because, beyond creating a mismatch in the domestic firms’ balance sheets, it also constraints the traditional domestic policy instruments in dealing with home and foreign economic shocks. The reasons why such risky forms of international finance arise in the first place remain an open question. If foreign

debt is so dangerous—as it is—it may be worth trying to give a micro-foundation to its emergence. Such a high share of foreign currency debt should be at least in part justified by the presence of some private benefits for the agents that choose this form of finance.

The goal of this paper is to rationalize the choice to borrow in dollars rather than in domestic currency on the international markets. In order to do so, I study how informational asymmetries and heterogeneous expectations can affect the choice of a borrower to expose herself to a currency risk. Furthermore I look at the policy implications of my findings to understand which policies could reduce the incentive of agents to dollarize.

My model is a portfolio choice model that analyzes how agents choose in which currency they want to borrow. In my model I have three types of agents: domestic borrowers, international investors and a central bank in the domestic country. I analyze two different settings of the model: In one case I assume that domestic agents receive private signals on the economic fundamentals, while international investors only observe a public signal that is common knowledge. The presence of private signals generates a coordination game among domestic agents that make their choices on the basis of their expectations and on their expectations on the expectations of the market (higher order beliefs). In the second setting, instead, I assume that domestic agents have an informational advantage on the international investors, but among them have access to the same piece of information. The degree of dollarization is therefore the result of a game between two large players: A domestic borrower and an international lender that observe different pieces of information. The international agent observes a public signal, while the domestic one observes the public signal plus a “private” signal.

The main findings of my model show that when domestic agents have a high informational advantage and/or there is a low level of transparency on international markets, an increase in the degree of dollarization might be observed, if the fundamentals are relatively strong. Alternatively, if there is endogeneity between the exchange rate policy implemented by the monetary authority and do-
mestic agents’ decisions, a certain degree of complementarity in borrowers’ choices may arise, thus creating a phenomenon of moral hazard. If domestic agents know that a high share of dollar debt in the economy makes the exchange rate more rigid, they may want to coordinate on the equilibrium where all the corporate debt in the economy is denominated in the same currency, even when the fundamentals of the economy are relatively weak.

These results have interesting policy implications. First of all they show that the “type” of the central bank can create moral hazard. A benevolent central bank that strongly bases her policy on the degree of dollarization in the economy, can generate a coordination mechanism among the domestic borrowers that results in a risky degree of dollarization. The solution would be to ex-ante choose a central banker with a strong preference for a flexible exchange rate. My findings also show the importance of transparency. Transparency does not necessarily coincides with public information. My model actually shows that the precision of private sources of information determines the degree of dollarization. If international markets could have access to some sources of private information, they would be more willing to lend in pesos, when the fundamentals are relatively strong. As a consequence the economy would not experience high levels of dollarization and would be better protected against future negative shocks.

The logic of my model resembles the one used in the literature of Global Games, first introduced by Carlsson and van Damme (1993), and then applied by Morris and Shin (1998, 1999, 2001, 2004) to different economic contexts, like currency crises and debt rollovers, where the complementarity of agents’ actions plays a crucial role. Through the introduction of private information in the economy, they show how it is possible to interpret certain phenomena as the result of higher order beliefs.

The theoretical literature on foreign currency debt in emerging economies has investigated this phe-

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2Similarly to the solution proposed in the monetary literature to fight inflation bias, Rogoff (1985).
nomenon often emphasizing more its consequences than its causes\textsuperscript{3}. However the positive issue of why emerging market economies do in fact rely on foreign currency denominated debt has instead received limited attention so far. The papers that address this issue emphasize the role of the expectations and the interaction between domestic borrowers and the Central Bank. Chamon and Hausman (2002), Jeanne (2003), Cowan and Do (2003) and Velasco and Chang (2004) show how the interaction between domestic agents and their central bank can result in full dollarization of the economy. One common problem is the difficulty to determine the optimal degree of dollarization in the economy: All the mentioned papers, in fact, find either corner solutions or indeterminate solutions. In my paper, instead, I manage to solve the indeterminacy problem. Through the introduction of informational asymmetries I in fact show that the degree of dollarization is an interior solution and is proportional to the difference in agents expectations.

The reminder of this paper is organized as follows. Section 2 presents the basic model. Section 3 sketches two possible settings of the game: a coordination game among the borrowers, considering the international lenders as a unique player, and a simple game between domestic borrowers and international lenders, considered as two big players. Section 4 briefly considers some policy implications. Section 5 concludes.

### 2.2 The Model

Consider a single-period small open economy populated by a continuum of domestic agents distributed over the unit interval. Agents are at the same time producers and consumers. There is a domestic currency, called \textit{peso}, which is issued by the domestic Central Bank. There is also a foreign currency, called \textit{dollar}. To finance their production, at the beginning of the period the domestic

firms borrow on the international market, here represented by a continuum of risk neutral lenders. Each firm can borrow in pesos or dollars. As a consequence the firm’s optimal borrowing policy determines the final degree of dollarization in the economy. The choice concerning the currency denomination of the debt will be influenced by the difference between lenders’ and borrowers’ expectations about the future exchange rate changes. The difference in agents’ expectations comes directly from the different information sets they dispose of. The final exchange rate depreciation (appreciation) will be determined by the Central Bank at the end of the period, after observing the true state of the economy and the degree of dollarization.

2.2.1 International Investors

International lenders are risk neutral. Their opportunity cost is given by the international interest rate $i^*$. The following zero-profit condition must hold for every value of $m$, the share of domestic currency debt chosen by the generic domestic agent:

$$ (1 + i) = (1 + i^*) E_L (1 + \Delta e) $$

where $i$ is the domestic interest rate and $E_L (1 + \Delta e)$ is the international lender’s expectation about the future devaluation. The supply of funds is infinitely elastic and the international banks are willing to lend both in domestic and foreign currency to the domestic firms, if and only if the returns given by the combination of peso and dollar debt are at least equal to the international rate of return $i^*$. In other words, the Uncovered Interest Parity condition holds for the international lenders.

2.2.2 Domestic Borrowers

Each domestic firm produces a domestic good $q$, and to finance its production it borrows funds on the international markets. Domestic agents maximize a CARA utility function whose argument is
net output, i.e. their consumption. The choice of the currency denomination of their debt results from the maximization of their \textit{ex-ante} utility function:

\[
E_i U [q - (1 - m_i) (1 + i^*) (1 + \Delta e) - m_i (1 + i)]
\]

\[
= E_i [\exp[-A[q - (1 - m_i) (1 + i^*) (1 + \Delta e) - m_i (1 + i)]]]
\]

\[
= -E_i[\exp(-A[E(q - (1 - m_i) (1 + i^*) (1 + \Delta e) - m_i (1 + i) \mid I_i) +
\n- \frac{A}{2} \text{Var}(q - (1 - m_i) (1 + i^*) (1 + \Delta e) - m_i (1 + i)) \mid I_i)]
\]

Let

\[
\Xi_i = [(Aq - A(1 - m_i)(1 + i^*) \, E_i (1 + \Delta e) +
- Am_i (1 + i) \mid I_i) - \frac{1}{2} A^2 (1 - m_i)^2 (1 + i^*)^2 \text{VAR}(1 + \Delta e) \mid I_i]
\]

where \(A\) is the coefficient of risk-aversion that is the same for all the domestic agents. Agent’s optimization problem reduces to choosing the optimal share of peso-debt, \(m_i\) to maximize \(\Xi_i\), given her information set \(I_i\).

\subsection*{2.2.3 The Central Bank}

In this economy there is a benevolent Central Bank whose policy instrument is the exchange rate. The Central Bank chooses the exchange rate policy in order to maximize an exogenous objective function, whose arguments are the \textit{ex-post} utility function of the domestic economy (calculated by aggregating domestic agents’ utility functions) and a term that measures the difference between the exchange rate devaluation and an adverse shock \(y\) that hits the economy. The inclusion in the
utility function of this term can be interpreted as the attempt of the central bank to implement an exchange rate policy that mirrors changes in the economic fundamentals. If the economy is hit by an adverse shock (for example to the shadow exchange rate) the Central Bank should proportionally depreciate the exchange rate.

\[
\max_{\Delta e} W_{CB} = \lambda \left[ \int (Aq - A(1 - m_i)(1 + i^*)(1 + \Delta e) - Am_i(1 + i) - \frac{1}{2} A^2(1 - m_i)^2(1 + i)^2 VAR(\Delta e)I_i)di \right] - \frac{1}{2} [\Delta e - y]^2
\] (2.2)

that becomes when we integrate over \( i \):

\[
\max_{\Delta e} \lambda [Aq - A(1 - M)(1 + i^*)(1 + \Delta e) - AM(1 + i) - \frac{1}{2} A^2(1 - M)^2(1 + i)^2 VAR(\Delta e)I_i] - \frac{1}{2} [\Delta e - y]^2
\] (2.3)

where \( M = \int m_i di \) describes the degree of domestic currency debt in the economy. From the he first order conditions the optimal policy rule followed by the Central Bank under discretion is:

\[
\Delta e = -\lambda A (1 - M)(1 + i^*) + y
\] (2.4)

**Proposition 1** The opposite biases that influence the Central Bank monetary policy when there is the contemporaneous presence of a high level of dollarization in the economy and an adverse shock, result in the phenomenon usually known as “Fear of Floating”.

“Fear of floating” is the expression commonly used to describe the exchange rate policy implemented in developing countries. It has been in fact observed that these countries had a tendency to keep their exchange rate fixed, even if they had formally adopted an flexible exchange rate regime. This behavior introduced many distortions in the financial markets that became evident during the crises in the late 90’s. The monetary policy rule is linear in the degree of dollarization of the domestic
economy \((1 - M)\) and in the shock to the fundamentals. Given \(y\), a higher share of dollar debt creates a bias towards an appreciation of the exchange rate. On the other hand, a higher \(y\) leads to an exchange rate devaluation. This rule shows how the exchange rate devaluation is influenced by the degree of dollarization in the economy: a higher presence of dollar debt reduces the incentives of the Central Bank to depreciate and instead creates a bias towards an appreciation. This justifies the stylized fact that emerging economies adopt at least \(de facto\) fixed exchange rate regimes.

### 2.2.4 Equilibrium Share of Peso Debt

The optimal share of peso debt for the borrower \(i\) is given by the constrained maximization of her ex-ante utility function, as follows:

\[
\max_m [Aq - A(1 - m_i)(1 + i)(E_i(1 + \Delta e)|I_i) - Am_i(1 + i) - \frac{1}{2}A^2(1 - m_i)^2(1 + i)^2VAR(\Delta e)|I_i]
\]

\[\text{s.t.}\]
\[\begin{align*}
(1 + i) &= (1 + i)E_L(1 + \Delta e), \\
E_i(\Delta e|I_i) &= -\lambda AE_i[(1 - M)](1 + i) + E_i(y|I_i), \\
E_L(\Delta e|I_L) &= -\lambda AE_i[(1 - M)](1 + i) + E_L(y|I_L).
\end{align*}\]

The resulting optimal share of peso-debt is

\[
m_i = 1 + \frac{(E_i(1 + \Delta e) | I_i)(1 + i)}{AVAR(\Delta e | I_i)(1 + i)^2} - \frac{(E_L(1 + \Delta e) | I_L)(1 + i)}{A(1 + i)^2VAR(\Delta e | I_i)}
\]

where \((E_i(1 + \Delta e)|I_i)\) and \((E_L(1 + \Delta e)|I_L)\) are the expectations about the exchange rate devaluation formulated, respectively, by the atomistic domestic borrower and the international lenders, given their information set. Expectations are formulated on the basis of the policy rule (2.4) that is
common knowledge in this economy. The difference in lenders’ and borrowers’ expectations is the main reason that justifies the decision to dollarize. Assuming that \( i^* = 0 \) the share of peso debt can be written simply as a function of international lenders’ and the borrowers’ expectations about \( \Delta e \):

\[
m_i = 1 + \frac{E_i(\Delta e | I_i)}{\operatorname{AVAR}(E_i(\Delta e | I_i))} - \frac{E_L(\Delta e | I_L)}{\operatorname{AVAR}(E_l(\Delta e | I_i))}
\]  

(2.10)

If \( E_i(\Delta e | I_i) = E_L(\Delta e | I_L) \) the domestic borrower chooses to borrow only in domestic currency. This choice is the optimal one as it allows the agent to hedge herself against any shock to the exchange rate. The main reason that justifies the choice to borrow in dollars is that the expectations about the stability of the exchange rate are different for the international lenders and the borrowers with \( E_i(\Delta e) < E_L(\Delta e) \). In that case the share of peso debt \( m_i \) is less than 1. There is a continuum of equilibria that crucially depend on the inequality between \( E_i(\Delta e | I_i) \) and \( E_L(\Delta e | I_L) \).

### 2.3 The Game

It has been shown that the optimal share of peso debt (2.9) depends upon the difference between lenders’ and borrowers’ expectations about the devaluation of the exchange rate. The choice to borrow in foreign currency can signal a difference in the information available to the two sets of agents considered. If lenders expect a higher depreciation rate than borrowers, the domestic interest rate will increase making the decision to borrow in pesos more expensive. As a consequence, domestic agents will have to compare the currency risk associated with dollar debt and the cost of borrowing in domestic currency, given lenders’ pessimism, and choose accordingly. The policy rule (2.4) followed by the Central Bank shows that agents’ strategies are characterized by a certain degree of complementarity: The higher the share of dollar debt in the economy is, the higher the bias of the Central Bank to fix or appreciate the exchange rate. The degree of dollarization in the
domestic economy is described as the result of a coordination game. Two settings of the game are analyzed:

- A coordination game among a continuum of atomistic domestic agents that have to choose the optimal share of peso debt, given the presence of an international investor, represented by the international market as a whole.

- A game between two large players, a domestic borrower and an international investor.

2.3.1 The Coordination Game among the Domestic Borrowers

Domestic and international agents at time 0 have a common prior about the adverse shock \( y \) that hits the economy. The shock is normally distributed with mean \( \mu \) and variance \( \sigma_y^2 \). At time 1 the shock is realized, but it is not directly observed. At time 2 domestic borrowers receive a private signal \( x_i = y + \epsilon_i \) that describes the shock and that they use to update their prior. The error term \( \epsilon_i \) is normally distributed over the population of borrowers with mean 0 and finite variance \( \epsilon_i \sim N(0, \sigma_e^2) \). At time 3 Borrowers and lenders formulate their expectations on \( \Delta e \) and \( M \), and the final share of peso debt in the economy is determined. The last stage of the game takes place when the Central Bank, after observing the actual values of \( M \) and \( y \) chooses how to move the exchange rate. The borrowers follow the policy rule (4) to update their expectations on \( \Delta e \), as follows:

\[
E_i(\Delta e \mid x_i, \mu) = -\lambda A E_i[(1 - M) \mid x_i, \mu](1 + i^*) + E_i(y \mid x_i, \mu) \tag{2.11}
\]

The updated expectation of \( y \) after observing the private signal is:

\[
E_i(y \mid x_i, \mu) = \frac{\sigma_e^2}{\sigma_e^2 + \sigma_y^2} \mu + \frac{\sigma_y^2}{\sigma_e^2 + \sigma_y^2} x_i. \tag{2.12}
\]
The precisions of the two signals are $\alpha = \frac{1}{\sigma^2_y}$ and $\beta = \frac{1}{\sigma^2_\epsilon}$, and the updated expectation of $y$ can be written as:

$$E_i(y \mid x_i, \mu) = \frac{\alpha \mu + \beta x_i}{\alpha + \beta}.$$  \hspace{1cm} (2.13)

The conditional expectation of the exchange rate devaluation is:

$$E_i(\Delta e \mid x_i, \mu) = -\lambda A (E_i(1 - M) \mid x_i, \mu) + E_i(y \mid x_i, \mu) = -\lambda A + \lambda AE_i(M) + \frac{\alpha \mu + \beta x_i}{\alpha + \beta}.$$  \hspace{1cm} (2.14)

Plugging the expression for $E_i(\Delta e \mid x_i, \mu)$ into (10), $m_i$ becomes:

$$m_i = 1 - \frac{E_L(\Delta e \mid \mu) + \lambda A}{AVAR(\Delta e)} + \frac{\lambda AE_i(M \mid x_i, \mu) + E_i(y \mid x_i, \mu)}{AVAR(\Delta e)}.$$  \hspace{1cm} (2.15)

It depends on borrower’s expectations on the fundamentals and on the total share of peso debt in the economy, and on lenders’ expectations on the future devaluation rate. Each agent cares not only about her own beliefs on the state of the economy, but also about other agents’ beliefs. To find the equilibrium value of $m_i$ I use the method of the undetermined coefficients. To find $E_i(M \mid x_i, \mu)$ I guess the generic rule followed by the domestic borrower $j$ in the economy. The strategy followed by the generic agent $j$ depends on both the public (i.e. the common prior) and the private signals she observes, plus a constant.

$$m_j = K_x x_j + K_\mu \mu + \gamma$$  \hspace{1cm} (2.16)

The total share of peso debt in the economy is found by integrating the individual share over the continuum of domestic agents as follows:

$$M = K_x y + K_\mu \mu + \gamma$$  \hspace{1cm} (2.16)
And finally the expectation of the domestic agent $i$ under study is formulated in the following way:

$$E_i (M \mid x_i, \mu) = E_i (K_x y + K_\mu \mu + \gamma) = K_x \left( \frac{\alpha \mu + \beta x_i}{\alpha + \beta} \right) + K_\mu \mu + \gamma$$  \hspace{1cm} (2.17)

After finding agent $i$’s expectation on the strategy followed by the rest of the market, I use the optimal rule (2.4) to get the equilibrium share of peso debt for each domestic agent:

$$m_i = 1 - \frac{[E_L (\Delta e \mid \mu) + \lambda A] (\alpha + \beta) - \lambda A (\alpha + \beta) \gamma}{AVAR (\Delta e) (\alpha + \beta)} + \frac{\lambda A K_x \alpha + \lambda A K_\mu (\alpha + \beta) + \alpha}{AVAR (\Delta e) (\alpha + \beta)} \mu + \frac{\lambda A K_x \beta + \beta}{AVAR (\Delta e) (\alpha + \beta)} x_i$$

The expression for $m_i$ now depends only on the public and the private signals, and on lenders’ expectations about the exchange rate devaluation $E_L (\Delta e \mid \mu)$. International lenders formulate their expectations only on the basis of the common prior on $y$. The expression for $E_L (\Delta e \mid \mu)$ is as follows:

$$E_L (\Delta e \mid \mu) = -\lambda A + \lambda A E_L (M \mid \mu) + E_L (y \mid \mu)$$  \hspace{1cm} (2.18)

Since $M = K_x y + K_\mu \mu + \gamma$ is a function of parameters that are common knowledge in the economy, lenders’ expectations are defined as:

$$E_L (M \mid \mu) = (K_x + K_\mu) \mu + \gamma.$$  \hspace{1cm} (2.19)

$E_L (\Delta e \mid \mu)$ can then be expressed as follows:

$$E_L (\Delta e \mid \mu) = -\lambda A + \lambda A [(K_x + K_\mu) \mu + \gamma] + E_L (y \mid \mu).$$  \hspace{1cm} (2.20)

The expression for $m_i$ becomes:
Equating the coefficients that appear in the guess and those that characterize the optimal share of peso debt, the final expression for the individual optimal share of peso debt becomes:

\[
m_i = 1 - \frac{\lambda AK_x \beta + \beta}{AVAR(\Delta e \mid x_i, \mu) (\alpha + \beta) \mu} + \frac{\lambda AK_x \beta + \beta}{AVAR(\Delta e \mid x_i, \mu) (\alpha + \beta)} x_i.
\]  

(2.21)

Finally the conditional variance of the exchange rate is:

\[
Var(\Delta e \mid x_i, \mu) = \frac{(\lambda AK_x + 1)^2}{ \alpha + \beta}.
\]  

(2.23)

Under the condition that \( \alpha \neq -\beta \), the only acceptable solution is

\[
Var(\Delta e \mid x_i \mu) = \frac{1}{2(\alpha + \beta)} [(1 + 2\lambda \beta) + \sqrt{4\beta \lambda + 1}].
\]  

(2.24)

Substituting the value of \( VAR(\Delta e \mid x_i, \mu) \) into the expression for \( m_i \), the optimal share of peso debt chosen by the domestic agent \( i \) depends exclusively on the precision of the private signal, on the coefficient of risk aversion \( A \), on the parameter \( \lambda \) that characterizes the policy rule and on the value of the two signals:

\[
m_i = \frac{\beta}{A \left[ \frac{1}{2} \left( (1 + 2\lambda \beta) + \sqrt{4\beta \lambda + 1} \right) - \lambda \beta \right]} (x_i - \mu) + 1.
\]  

(2.25)

The share of peso debt depends on the difference between the private and the public signal and on the precision of the former. A highly precise private signal describing pretty strong fundamentals induces the agent who observes it to choose dollar debt that seems relatively cheaper and not too risky.

It is worth noting that the precision of the public signal does not enter the expression for the equilibrium share of peso debt. This result is quite different from the ones usually found in global
game models where the coordination motive results in a weight given to the public signal that is increasing in its precision. Here instead there is a positive term in $\beta$ involving the private signal and a corresponding negative term involving the public signal. The share of peso debt overreacts to the private signal, while the information content of the public signal is suppressed. This seems to confirm the intuition that the choice concerning the currency denomination of the debt mainly depends on the differences in the availability of information. Within the context of an ordinary coordination game the public signal influences the final outcome not only because of the information it conveys, but also because of its ability to re-create a certain degree of common knowledge among agents. The role of public information becomes here secondary instead, because it is contemporaneously used by two different categories of agents. The currency denomination of the debt can be interpreted as a signal of strong fundamentals that domestic agents send to the international markets. The ability to send such a signal comes directly from the informational advantage they have with respect to international agents. The signal is stronger when the private information is more precise. By integrating $m_i$ over the continuum of domestic borrowers, the optimal share of peso debt $M$ is:

$$M = \frac{2}{A \left[ 1 + \sqrt{4\beta \lambda + 1} \right]} \left[ \beta (y - \mu) \right] + 1$$  \hspace{1cm} (2.26)

The optimal share of peso debt in the economy resulting from the coordination game among the domestic borrowers depends on the spread between the actual value of the fundamentals and their common prior and on the parameters describing the precision of the private signal, the risk aversion and the weight given to the social utility in the objective function of the Central Bank. When the realization of the shock equals its prior all agents’ expectations are the same and the share of dollar debt in the economy is zero ($M = 1$).
Comparative Statics

It is possible to infer the following propositions from equation (2.26):

**Proposition 2** When the fundamentals are strong, i.e. \( y < \mu \), an increase in the precision of the private signal observed by agents results in an increase of the degree of dollarization in the economy.

The partial derivative of \( M \) with respect to \( \beta \) is:

\[
\frac{\partial M}{\partial \beta} = \frac{(y - \mu)}{A \left[ 1 + \sqrt{4\beta \lambda + 1} \right]^2} \left[ \frac{\sqrt{4\beta \lambda + 1} + 2\lambda \beta + 1}{\sqrt{4\beta \lambda + 1}} \right]
\]

If the precision of the private source of information available to the domestic borrowers increases, we observe an increase in the degree of dollarization. If the fundamentals are strong in fact it is less risky in the borrowers’ eyes to borrow in foreign currency and, as a result, the average share of peso debt decreases. On the other hand, if the realization of the shock \( y \) is higher than its prior, an increase in \( \beta \) increases the degree of peso debt in the economy. In other terms, a high precision of the private signal leads the currency denomination choices in the direction that is more consistent with the actual state of the economy.

**Proposition 3** When the fundamentals are weak, i.e. \( y > \mu \), an increase in the parameter \( \lambda \) increases the degree of dollarization in the economy.

The partial derivative of \( M \) with respect to \( \lambda \) is:

\[
\frac{\partial M}{\partial \lambda} = - \frac{4\beta^2 (y - \mu)}{A \left[ 1 + \sqrt{4\beta \lambda + 1} \right]^2} \sqrt{4\beta \lambda + 1}
\]

In correspondence of relatively weaker conditions of the fundamentals (\( y > \mu \)), a higher protection of the Central Bank towards those that borrow in dollars increases the degree of dollarization. Even though the fundamentals are not very strong, the stronger commitment of the monetary authority
to keep the exchange rate appreciated increases the degree of dollarization observed. Agents expose themselves to a risk that otherwise they would not have run. This is what I define “dollarization due to moral hazard”.

2.3.2 Simple Game between Two Large Players

The optimal share of peso debt for the atomistic domestic borrower is given as before by (2.10). In order to analyze other factors that influence the degree of dollarization in the economy, I will slightly change the set-up of the game and show how things go under different assumptions. Both domestic and international investors have a common prior about the shock, that is normally distributed with finite mean and variance: $y \sim N(\mu, \sigma_y^2)$. In order to emphasize the supply side factors that can be involved in the process of dollarization leaving aside the informational asymmetries among domestic agents, I assume in this new version of the game that after the shock takes place, all the domestic borrowers observe the same private signal about the fundamentals $x_i = y + \epsilon_i$, where $\epsilon_i = \epsilon_j$ with $\epsilon \sim N(0, \sigma_\epsilon^2)$. In this case the signal is private in the sense that it can be observed only by the domestic agents and not by the international markets. The direct consequence of this assumption is that all the borrowers choose the same composition of debt, observing exactly the same information. In this case the problem of Higher Order Beliefs is not taken into consideration anymore, and $m_i = m = M$. International investors do not precisely see the average share of peso debt in the economy. They observe a public signal $z$, instead. The degree of dollarization in the economy is given by the combination of borrowers’ and lenders’ expectation about the movements in the exchange rate. The Central Bank in the last stage observes $M$ and $y$ and implements the optimal monetary policy, as before. Since all the borrowers observe the same “private” signal $x$ I will not use the subscript $i$. The share peso debt of the single borrower coincides with the degree of domestic currency debt in the economy. Equation (2.10) becomes:
\[ M = 1 + \frac{E(\Delta e \mid x, \mu)}{AVAR(\Delta e \mid x, \mu)} - \frac{E_L(\Delta e \mid z, \mu)}{AVAR(\Delta e \mid x, \mu)}. \tag{2.27} \]

Employing the policy rule (4), it is possible to see that the expectations about the movements in the exchange rate formulated by the domestic market are as follows:

\[ E(\Delta e \mid x, \mu) = -\lambda A + \lambda AM + E(y \mid x, \mu) \tag{2.28} \]

The variance of the depreciation rate coincides now with the variance of the shock:

\[ Var(\Delta e \mid x, \mu) = \frac{\sigma_y^2 \sigma_{\epsilon}^2}{\sigma_y^2 + \sigma_{\epsilon}^2} \tag{2.29} \]

If we call \( \alpha = \frac{1}{\sigma_y^2} \) and \( \beta = \frac{1}{\sigma_{\epsilon}^2} \)

\[ Var(\Delta e \mid x, \mu) = \frac{1}{\alpha + \beta}. \tag{2.30} \]

The international borrowers observe a public signal describing the share of peso debt in the economy \( z = M + \eta \) where \( \eta \sim N(0, \sigma_{\eta}^2) \). Their expectations are still formulated on the basis of the optimal policy rule (4) as follows:

\[ E_L(\Delta e \mid \mu, z) = -\lambda A + \lambda AM + \lambda A\eta + \mu. \tag{2.31} \]

Plugging the expectations into (2.27), the final expression for the total share of peso debt is a function of the precision of the public and the private signals, the two signals themselves, the noise of the signal observed by the lenders and the parameter \( \lambda \) that appears in the utility function of the Central Bank.

\[ M = 1 + \frac{(\alpha + \beta)}{A} \left( \frac{\beta(x - \mu)}{\alpha + \beta} - \lambda A\eta \right). \tag{2.32} \]
The total share of domestic currency denominated debt in the economy is lower when the private signal observed within the domestic economy is lower than the common prior shared by all the agents, and gives rise to more optimistic expectations about the state of the fundamentals. Another important element is the noise $\eta$. If the low level of transparency on the international markets induces the lenders to overestimate the share of peso debt in the economy, the expectations of devaluations of the lenders will be higher and therefore the supply of domestic currency denominated funds will be lower.

**Comparative Statics**

From equation (2.32) it is possible to infer the following propositions:

**Proposition 4** An increase in the size of the noise that affects the signal $z$ observed by the international lenders reduces the share of peso debt.

$$\frac{\partial M}{\partial \eta} = -\lambda (\alpha + \beta)$$

A low level of transparency on the international markets increases the level of dollarization. If international lenders overstate the share of peso debt in the economy, they will increase their devaluation expectations and with them also the price of domestic currency denominated funds. Such an increase in the share of dollar debt is greater in correspondence of high values of $\lambda$ and highly precise signals about the fundamentals.

**Proposition 5** An increase in the value of the parameter $\lambda$ reduces the share of peso debt.

$$\frac{\partial M}{\partial \lambda} = -\eta (\alpha + \beta)$$
An increase in the parameter that weights the social welfare in the utility function of the Central Bank results in an increase in the degree of dollarization, when the low level of transparency on the international markets induces the lenders to overestimate the share of peso debt in the domestic economy. A paternalist central bank increases agents’ incentives to contract dollar debt. If international investors overestimate the presence of domestic currency denominated debt ($\eta > 0$), it will become more expensive to borrow in pesos and the degree of dollarization will increase.

**Proposition 6** *An increase in the precision $\beta$ of the private signal that domestic agents observe, decreases the share of peso debt, when international investors overestimate the degree of dollarization. This effect is even stronger when the domestic market has optimistic expectations about the fundamentals.*

\[
\frac{\partial M}{\partial \beta} = \frac{(x - \mu)}{A} - \lambda \eta
\]

A higher precision of the private signal that the domestic economy observes increases the level of dollarization, when the fundamentals described by the private signal are stronger than the ones implied by the public signal. The degree of dollarization is also influenced by the policy parameter $\lambda$ and by the degree of transparency on the international markets. If international investors overestimate the share of peso debt in the economy ($\eta > 0$), their expectations of devaluation will be high and the supply of domestic currency denominated debt will be reduced.

**Proposition 7** *An increase in the precision of the public signal increases the degree of dollarization, when the international markets overestimate the presence of peso debt in the economy.*

\[
\frac{\partial M}{\partial \alpha} = -\lambda \eta
\]
An increase in the precision of the public signal increases the degree of dollarization, when international markets have high expectations of devaluation. In this case the value of the two signals doesn’t appear, while only $\lambda$ and $\eta$ influence the degree of dollarization.

2.4 Policy Implications

The simple exercise of comparative statics conducted in both the frameworks analyzed gives rise to very similar results. Even if the expression of the average peso debt in the economy $M$ comes from different contexts, its reaction to a marginal increase of some key parameters like $\lambda$ (the weight of the social utility in the objective function of the Central Bank), and $\beta$ (the precision of the private signal observed only by the domestic borrowers), goes in the same direction. An increase in $\lambda$ produces the same effect of a decrease in the borrowers’ risk aversion. If there is an increase in the weight given to the social utility in the objective function of the Central Bank, people are more willing to borrow in dollars because they know the central bank will protect them, avoiding any depreciation of the exchange rate. This creates a moral hazard problem in the domestic economy because agents will be willing to borrow in foreign currency even when the economy is relatively weak.

An increase in the precision of the private signal about the fundamentals, $\beta$, lowers the peso debt in the economy, when the fundamentals are relatively strong. It is in fact reasonable to think that if the precision of the private signal increases, the weight given by domestic borrowers to the private information increases as well, and the choice to borrow in dollars ends up revealing this additional information they dispose of.

Another important parameter able to influence the degree of dollarization is the size of the error term $\eta$. A marginal increase in $\eta$ results always in an increase in the share of dollar debt, and the presence of this parameter in the other partial derivatives always makes the increase in the
degree of dollarization stronger. Higher transparency on international markets about the true level of dollarization would, in some cases, help reduce the share of dollar debt, in economies already characterized by a high level of dollarization.

It would be interesting to analyze the effect on $M$ produced by an increase in the sources of information available on the international markets. The presence of private information on the international markets could reduce the necessity of the domestic economy to borrow in dollars. This would happen because international investors, having access to pieces of information that are not public, could be more willing to lend in pesos. This would reduce the degree of dollarization even when the fundamentals are relatively strong and would protect the economy from future negative shocks.

2.5 Conclusion

The paper analyzes the factors that determine the high share of foreign currency debt that we observe in many emerging markets, even if characterized by strong fundamentals and sound policies. Through a simple model I show how the combination of demand and supply side factors can explain the emergence of the phenomenon known as ”Original Sin”. The main result is that the different information sets available to the agents in the economy can justify the formation of different expectations about the soundness of the domestic market and, therefore, the choice to dollarize. The most important factors that help explain the share of dollar debt in the economy are the informational advantage about the state of the economy that domestic agents have over international markets, the exchange rate policy and the degree of transparency on the international markets. The relative incidence of supply and demand side factors on the determination of the final degree of dollarization in the economy can change depending on the time and on the markets considered. In general the message of the model is that a high dollar debt even in strong economies, can be due to a
willingness of such countries to signal their good shape on the one hand, and to the low ability of the international markets to catch this signal on the other. It is worth noting that a crucial role is played by the Central Bank that with its exchange rate policy can deeply influence agents’ behavior and, eventually, create dangerous distortions in the economy.
Bibliography


Appendix C

In order to analyze whether the optimal share of peso debt chosen by each agent in the economy depends on the nature of the information available (public or private), I slightly modify some assumptions on the signals observed by the agents in the economy. If a certain degree of public information is shared only by domestic agents, the usual result concerning the typical over-reaction to public information is reestablished, even though the private information goes on playing a crucial role. This exercise helps clarify the main intuition according to which the problem analyzed in the model displays important complementarities in domestic agents’ actions.

There is a common prior about the adverse shock that will hit the domestic market shared by the whole economy. The shock $y$ is normally distributed with mean $\phi$ and precision $\theta$, $y \sim N\left(\phi, \frac{1}{\theta}\right)$. After the shock has taken place, domestic agents update their expectations by observing a public signal $\mu$, such that $\mu = y + \eta$, with $\eta \sim N\left(0, \frac{1}{\alpha}\right)$ and a private signal $x_i$, such that $x_i = y + \epsilon_i$, with $\epsilon_{i\text{iid}} \sim N\left(0, \frac{1}{\beta}\right)$ and $\epsilon_i \neq \epsilon_j$. The difference in the availability of information is now given by the presence of a public and a private signal that only domestic agents can observe.

Moving from equation (2.10) that describes the optimal share of peso debt chosen, the guess concerning the generic domestic agent $j$’s actions depends now on the prior, the public signal and the private signal:
\[ m_j = k_x x_j + k_\phi \phi + k_\mu + \gamma \]  

(C.1)

The total amount of peso debt can be found by integrating over the continuum of domestic agents

\[ M = k_x y + k_\phi \phi + k_\mu + \gamma \]  

(C.2)

\[ E_i (M \mid \phi, \mu, x_i) = k_x E_i (y \mid \phi, \mu, x_i) + k_\phi \phi + k_\mu + \gamma \]  

(C.3)

International investors’ expectations about the exchange rate devaluation depend, as before, only on the common prior

\[ E_L (\Delta e \mid \phi) = -\lambda A + \lambda A E_L (M \mid \phi) + E_L (y \mid \phi) \]  

(C.4)

\[ E_L (M \mid \phi) = (k_x + k_\phi + k_\mu) \phi + \gamma \]  

(C.5)

\[ E_L (\Delta e \mid \phi) = -\lambda A + \lambda A ((k_x + k_\phi + k_\mu) \phi + \gamma) + \phi \]  

(C.6)

Plugging (C.3) and (C.6) into (2.10) I get

\[ m_i = \frac{1 + -\lambda A \left((k_x (\alpha + \beta) + k_\mu (\vartheta + \alpha + \beta)) \phi \right) - \phi (\alpha + \beta)}{AVAR(\Delta e) (\vartheta + \alpha + \beta)} + \frac{\lambda A [k_x \alpha \mu + k_\mu \mu (\vartheta + \alpha + \beta)] + \alpha \mu}{AVAR(\Delta e) (\vartheta + \alpha + \beta)} + \frac{\lambda A k_x \beta x_i + \beta x_i}{AVAR(\Delta e) (\vartheta + \alpha + \beta)} \]  

(C.7)

The average degree of domestic currency denominated debt in the economy becomes:
\[ M = 1 + \frac{2\beta}{A [(1 + 2\lambda \beta) + \sqrt{1 + 4\beta\lambda}]} (y - \phi) + \]
\[ + \frac{\alpha}{A [(1 + 2\lambda \beta) + \sqrt{1 + 4\beta\lambda}]} (\mu - \phi) + \]
\[ + \frac{\alpha (2\lambda \beta + \sqrt{1 + 4\beta\lambda})}{A [(1 + 2\lambda \beta) + \sqrt{1 + 4\beta\lambda}]} \mu - \frac{\alpha (2\lambda + 1)}{A [(1 + 2\lambda \beta) + \sqrt{1 + 4\beta\lambda}]} \phi \]

The introduction of a public signal \( \mu \) observable only by the domestic borrowers reestablishes, to a certain extent, the usual properties of the public information in the context of a coordination game. The final degree of peso debt in the economy depends (positively) on the public signal and on the realized value of the shock, while it depends negatively on the common prior. More precisely, the degree of peso debt depends on the difference between the actual realization of the shock and its prior (as before) and now—with the introduction of a public signal—also on the difference between the public signal itself and the common prior, weighted by the precision of the former. From the first two elements of (C.8) we can infer that the share of peso debt “over-reacts” to the actual realization of the shock and to the public signal, while the information content of the common prior is to some extent suppressed. In other terms, the final degree of dollarization depends crucially on the additional information domestic agents dispose of. The public signal enters the expression for \( M \) with a higher weight, but still the private signal matters for the final degree of peso debt. This finding is due to the presence of actions that are at the same time substitutes and complements. Since the game takes place on two levels, as a coordination game within the domestic economy and as a game between the domestic agents and the international markets, this result combines the different natures of agents’ actions.
Appendix D

It has been shown that the monetary policy crucially influences agents’ borrowing decision. A large value of $\lambda$ in the policy function means that agents’ utility enters the welfare function with a high weight. A higher degree of “paternalism” of the monetary authority seems to increase the incentive of domestic agents to borrow in foreign currency. That is to say, we observe a phenomenon of moral hazard due to the strong protection from exchange rate depreciations given by the central bank to those that dollarize. In the paper I analyzed the role of monetary policy under discretion, but it is worth studying how things change under commitment. Under a credible commitment the monetary authority that wants to rule out any coordination motive among agents and reduce as much as possible the moral hazard, announces an exchange rate rule that does not depend on the average degree of dollarization in the economy. I assume that the central bank credibly commits to a rule such that the movements in the exchange rate are only justified by the occurrence of supply shocks denoted as before by $y$. 

$$\Delta e = \gamma(y)$$

(D.1)

The optimal share of domestic currency denominated debt, $m_i$, depends now on the expectations formulated on the basis of the new rule (D.1). Foreign investors formulate their expectations on the basis of their information set given simply by the common prior on $y$, while domestic agents use
both the sources of information they have, given by the prior and the private signal they observe.

Equation (2.10) now becomes

\[ m_i = 1 + \frac{\gamma \left( \frac{\alpha \mu + \beta x_i}{\alpha + \beta} \right) - \gamma \mu}{AVAR(\Delta e | x_i, \mu)} \]

The conditional variance of \( \Delta e \) is

\[ VAR(\Delta e | x_i, \mu) = \frac{\gamma^2}{\alpha + \beta} \]

\[ m_i = 1 + \frac{\beta(x_i - \mu)}{A\gamma} \quad (D.2) \]

The total share of peso debt in the economy therefore is

\[ M = 1 + \frac{\beta(y - \mu)}{A\gamma} \quad (D.3) \]

In order to find the optimal degree of dollarization under commitment it is necessary to maximize the social welfare function with respect to \( \gamma \) :

\[
\max_{\gamma} W_{CB}^C = \lambda[Aq - A(1 - M)(1 + i^*)(1 + \Delta e) - AM(1 + i)] - \frac{1}{2}A^2(1 - M)^2VAR(\Delta e)] - \frac{1}{2}[\Delta e - y]^2 \quad (D.4)
\]

\[
\max_{\gamma} \frac{\lambda[Aq + \frac{\beta(y - \mu)}{\gamma}]}{\gamma} \left( 1 + \gamma y \right) - A[1 + \frac{\beta(y - \mu)}{A\gamma}] \left( 1 + \gamma \mu \right) -
\]

\[
\frac{1}{2}A^2 \left( \frac{\beta(y - \mu)}{A\gamma} \right)^2 \frac{\gamma^2}{\alpha + \beta} - \frac{1}{2} \left[ \gamma y - y \right]^2 \quad (D.5)
\]

The optimal value of \( \gamma \) is:
\[ \gamma = \frac{y^2 - \mu A \lambda}{y^2} \]  \hspace{1cm} (D.6)

It is now possible to compare the optimal share of peso debt in the economy under commitment \((M^c)\) with the one that emerges under discretion \((M^d)\):

\[ M^c = 1 + \frac{\beta (y - \mu) y^2}{A (y^2 - \mu A \lambda)} \]  \hspace{1cm} (D.7)

\[ M^d = \frac{2}{A \left[ 1 + \sqrt{4 \beta \lambda + 1} \right]} \left[ \beta (y - \mu) \right] + 1 \]  \hspace{1cm} (D.8)