Reason and dialectic in the argument against Protagoras in the Theaetetus

Author: Marina McCoy

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“Reason and Dialectic in the Argument against Protagoras in the *Theaetetus*”

I.

This paper examines a peculiar claim made in the dialogue between Socrates and Theaetetus in the *Theaetetus*: while it appears that Socrates has given a thorough logical refutation of the Protagorean view of knowledge as presented here (151e-186e), Socrates also claims that Protagoras himself would *not* accept the arguments if he were present. After the main argument is complete, Socrates adds this strange remark:

Hence it is likely that Protagoras, being older than we are, really is wiser as well; and if he were to stick up his head from below as far as the neck just here where we are, he would in all likelihood convict me twenty times over of talking nonsense, and show you up too for agreeing with me, before he ducked down to rush off again (171c-d).  

However, Socrates adds that it is more important that he and Theaetetus *do* agree that the argument is decisive than that Protagoras would (171d). This presents the interpreter of the dialogue with an interesting problem: how can the argument against Protagoras be understood as a refutation if it is admitted even after the argument is complete that not everyone would accept the argument? What sort of view of reason does this imply?

Most commentators read the refutation of the definition of knowledge as perception in one of two ways, either: (1) as an argument for why perception cannot ultimately yield anything worthy of the name knowledge, since knowledge must be of what is unchanging and unerring but perception is not (cf. Cornford 1934); or, more commonly, (2) as showing an internal inconsistency within the Protagorean position (cf. Bostock 1988; Burnyeat 1976; Fine 1998; McDowell 1973; Passmore 1961; Sayre 1969; and Vlastos 1956). The main dispute between commentators who take the second sort of
approach has revolved around whether Protagoras is a relativist or an infallibilist, and whether Plato is successful in refuting the position so presented. I propose that Socrates is not showing that the Protagorean position is internally inconsistent, impossible, or contradictory. Cornford is closer to the mark when he argues that the heart of the argument is whether other ordinary criteria that we have about knowledge must be abandoned if we adhere to the Protagorean position. But I wish to show that the argument here is decisive because Theaetetus the individual cannot accept the Protagorean argument because of his own activity and commitments as a mathematician, and not simply because it is in conflict with ordinary doxai more generally. For, Socrates acknowledges that a different person—Protagoras—in the end might still reject the arguments made here. Plato does not intend us as readers to understand the argument against Protagoras as a purely abstract one about inconsistent premises or ideas. Instead, Socrates' dialectical argument against Protagoras depends upon the beliefs and the character of Theaetetus for its success.

Understanding the refutation of the definition of knowledge as perception as dependent upon who Theaetetus is as an individual, and what he already believes, also makes more sense of the famous passage on Socratic midwifery. For, in that passage, Socrates claims that he helps others to give birth to ideas already within them (150d2-d9). If the argument against Protagoras is merely intended to be an argument about the internal consistency of someone else’s position, then it is hard to see how anything Theaetetus has within him is relevant to that argument. I suggest instead that understanding the argument against the Protagorean position as arising from Theaetetus’ own beliefs also
makes better sense of how Socrates understands the elenchic process to work, as expressed in the metaphor of a midwife.

Below I shall first examine how Protagoras’ position here is internally consistent and yet rejected by Theaetetus because of his own beliefs about knowledge and expertise. I then offer a few reflections on the dialectical nature of the argument in the *Theaetetus*. In particular, I examine how early sections of the dialogue (prior to the initial definition of knowledge) are central to the rejection of the Protagorean standpoint. The *Theaetetus* dialectically affirms the value of the philosophical while acknowledging the existence of a non-philosophical competing intellectual position.

II.

Theaetetus first proposes a definition of knowledge as “simply perception (αἰσθάνεσθαι)” (151e2). However, it is Socrates who gives this further content by equating this definition with Protagoras’ understanding of knowledge. Socrates quotes the famous, “Man is the measure of all things” dictum: “Man (ἄνθρωπος) is the measure of all things: of the things that are, that they are, and of the things that are not, that they are not,” and then interprets it such that ἄνθρωπος is given an individual interpretation. The individual human being’s perceptions are the final authority on what is, and what is not, such that individual perception is knowledge. If the wind feels cold to me, then it *is* cold to me. If it feels warm to you, then it *is* warm, to you. Socrates sums up the view, “[T]hings are for the individual as he perceives them” (152c2). The appearance or perception that I have is what a “thing” is. Socrates suggests for Protagoras that there is no distinction between how I perceive a thing, and what it is (for me).
Socrates implies that this rejection of the difference between a thing and an appearance has certain appeals: for example, it seems to avoid some of the problems with characterizing the relationship between an appearance or representation and the thing itself, through eliminating the distinction altogether. Such a definition of knowledge at least gives knowledge the characteristic of infallibility. Socrates hints at this advantage to his theory when he says perception seems to be a worthy candidate for knowledge, because it is “unerring,” as is appropriate for knowledge (152c4-5). We can bypass the difficulty of determining who is right or wrong about the temperature of the wind itself by suggesting that knowledge is not about comparing two or more perceptions with one another, or comparing perception and some external reality; rather, the question was wrongly posed to begin with. Whatever I perceive is already knowledge for me.

There has been some dispute over what precise position Protagoras takes here: relativism has been the most common account. While many have held that Protagoras is a relativist (see e.g., Bostock 1988; McDowell 1973; Sayre 1969; and Vlastos 1956), Burnyeat has given one of the most precise accounts of that position and what he sees as its inconsistency (Burnyeat 1976). Burnyeat argues that Protagoras is a relativist, that is, he claims that every judgment is true for the person whose judgment it is. That is, if it seems to x that p, then it is true for x that p. If I feel that the wind is cold, then for me it is cold; but if you feel that it is hot, it really is hot to you. Similarly, judgments about more complex matters are also relative: if x judges that p is true, then p is true for x. I generally agree with Burnyeat’s assessment of Protagoras as a relativist at the beginning of the discussion between Socrates and Theaetetus. However, the Protagorean position develops beyond a simple relativism much later in the discussion, when the relativistic
theory of truth is put into the larger context of a theory about wisdom and persuasion. This later section will prove to be significant for understanding how and why Socrates thinks that Protagoras' theory can withstand some of the arguments against him. (I delay discussion of these differences from Burnyeat's presentation of Protagoras until later in the paper when more of the Protagoras' position has been laid out.)

One interesting alternative to relativism has been offered by Fine, who argues that Protagoras is an infallibilist (Fine 1998). According to Fine, Protagoras holds that all beliefs are true simpliciter, i.e., just because they are believed. If A believes p, then p is true (not just true for A). Fine suggests that the infallibilist position is more likely because, among other reasons, the qualifiers (“for you” and the like) are at times omitted in the course of the discussion, a problem which has often been noted (cf. Vlastos 1956; Sayre 1969). If the qualifiers are sometimes dropped, then perhaps Protagoras is not really a relativist, but only an infallibilist. However, Socrates includes the qualifiers both in his initial description of the position and at nearly all of the major restatements of it. For example, consider the strength of: “ὥστε εἴτε τις εἶναι τι ὁνομάζει, τινὶ εἶναι ἢ τινὸς πρός τι ῥητέον αὐτῷ, εἴτε γίγνεσθαι (“so if someone says that anything ‘is,’ he must mention that it is to or of or relative to something, and also if he says it ‘becomes’”) (160b7-9).vi Fine also suggests that since the argument against Protagoras is effective against the infallibilist position, but not the relativistic one, infallibilism is a more likely position. But as I will argue below, Socrates’ comments suggest that he himself does not think that the arguments he and Theaetetus have given would be decisive for Protagoras (see 171c-d); this is all the more reason to suppose that infallibilism is not the position being argued against.
Socrates responds that Protagoras' theory is “not a common” theory. To a certain extent, the remainder of his argument against Protagoras might be summed up in this phrase: “not common” (ὦ φαυλον; found twice, at 151e9 and 152d2). Initially, it appears to be a compliment to Protagoras that his theory is sophisticated and uncommon; however, as the argument progresses, Socrates suggests that this out-of-the-ordinary nature of the theory is problematic. While the theory ensures that knowledge is infallible, it results in other consequences that conflict with ordinary ideas of knowledge. These consequences are initially discussed in metaphysical terms. Socrates proposes that a certain type of metaphysics underlies this theory of perception. If what we perceive is what is, then nothing "is" (εἶναι) in itself, but everything is “coming to be” (γίγνεσθαι) as a result of motion (157a1-2). It is a mistake to talk about the reality of an apple; rather, we ought to talk about the reality of the perception, which arises from the combined motion of the perceiving power and the motion of the perceived. Something in the perceiver’s perceptive powers combines with the motion of the perceived entity, to produce what has reality for us. This perception is constantly arising and therefore never truly static. In fact, it is a misnomer to talk about the perceiver’s powers, as if they were things, or the thing that is going to be perceived—we ought to talk about two motions that combine to produce another motion. The perceived apple is not an entity, but a process; neither are the forces which give rise to this perception things, but instead processes.

This metaphysical position is not in itself absurd. On the contrary, Socrates suggests that many of the great thinkers of the past—Empedocles, Herclitus, past poets and dramatists—held a similar view of the universe as a place of flux and change.

Moreover, many natural phenomena would seem to arise out of processes in motion, such
as fire, heat, and perhaps even the animating life force of the soul itself. This is all additional evidence in favor of the Protagorean metaphysical view. What is absurd, on this view, is the present state of language, which mistakenly talks about things instead of processes, and typically uses the word εἶναι instead of γίγνομαι. But we could just as easily say that our language ought to be reformed, so that it better reflects the true nature of reality, as reject the Protagorean theory on this basis alone.

As the first section of refutation begins, Socrates asks Theaetetus whether he finds this theory appealing. Theaetetus replies that he does not know whether Socrates himself believes in the position that Socrates has just expressed, or is just laying it out for the sake of argument. Socrates emphasizes that the whole point is whether Theaetetus believes it: “But you must have courage and patience; answer like a man whatever appears to you (φαίνεταί σοι) to be true about the things I ask you” (157d3-4). In this refutation of knowledge as reducible to appearances, Socrates suggests that the only way in which a reliable judgment about the fruitfulness of the idea is to be found in how the idea appears to Theaetetus.

The key points of objection that ensue are as follows. First, knowledge as perception cannot account for what seem to be clear-cut cases of misperception: dreaming and insanity, for example. However, Theaetetus points out that it is difficult to find any clear criteria by which to distinguish dreams from reality, presumably even if one were to set aside the Protagorean view of reality. And Socrates suggests that the Protagorean view can give an account of such experiences: whatever a person thinks at a given time is true for him, even in cases such as dreaming and madness. Even the self is not a stable entity, but changes and perceives things differently over time. We need not say that anything is
true universally; rather, we exclude the concept of universal truth altogether, and claim that truth is always relative to the individual percipient: "Hence, whether you apply the term ‘being’ to a thing or the term ‘becoming,’ you must always use the words ‘for somebody’ or ‘of something’ or ‘relatively to something’” (160b7-9). So this first objection seems not to be decisive after all.

In a second set of objections, Socrates argues that the Protagorean view reduces human knowledge to the level of animal sense perception. We cannot say that human beings are "knowers” any more than we can say that baboons or tadpoles are knowers. It seems to leave out altogether any uniquely human capacities to know. But we often do think of human beings as having a greater capacity to know the world than, say, houseflies. Thus Socrates jests, why didn’t Protagoras say, “Pig is the measure of all things” (161c4)? Moreover, if knowledge is individual perception, then it would seem that no person may be called any wiser than any other, including Protagoras himself. According to Protagoras’ theory, we cannot say that one species or even one individual is wiser than another.

Both of these initial objections focus on whether Protagoras' understanding of knowledge is sufficiently strong. However, Socrates soon afterwards suggests that if Protagoras were present, he would say that Socrates and Theaetetus were merely depending upon the ideas of the “mob,” arguing for what is merely plausible (πιθανολογίᾳ) or likely (εἰκότι) (162e5-163a1). The position that knowledge is perception has not in itself been shown to be untenable—rather, it simply leads to some uncommon consequences. So these objections, too, would seem to fall away.
A third set of objections comes with the introduction of a counterexample in which a person can perceive something, and yet not know it, as when we can hear people speaking a foreign language and yet do not understand it (163b). In such cases, we perceive but do not know. In addition, because the knowledge of the words’ meaning is separable from the perception of the sounds, this also suggests that when we do know a language, the knowledge is not to be found in the perceptions of the sounds themselves. That is, the ordinary use of the term “know” when I say that I “know” what someone else is saying, has content beyond perceiving that someone else is saying something.

Similarly, I can say that I know my friend when I shut my eyes, even though I cannot perceive him. We ordinarily think of perception and knowledge differently, in our use of those terms in everyday life. However, Socrates also objects to these arguments, on the grounds that they have only been “getting words to agree consistently,” like professional debators (164d). Socrates does not state explicitly which words they have been trying to get to agree. But in light of the later discussion of the terms knowledge, wisdom, and perception, perhaps Socrates’ point is that these examples rely upon ordinary senses of the words “know” and “perceive” which could in principle be used differently. For example, Protagoras could say that remembering a friend is a kind of a perception, too, and so would of course be included as an instance of knowledge. Or, Protagoras might very well say that when we perceive sounds in a foreign language that we do not understand, we have not really perceived them fully or adequately; we still do not need to claim that knowledge is distinct from perception. Again, the out-of-the-ordinary nature of the theory is not in itself a decisive objection to Protagoras’ theory. ix
It is at this point that Socrates provides his “spirited rescue” of the orphaned argument. Theaetetus seems ready to give up the Protagorean view, and yet Socrates is insistent that a further defense is needed before a full evaluation of knowledge as perception can be made. Socrates himself is not convinced that Protagoras' theory is being rejected for the right reasons. The difficulties posed in the first section of the dispute do not get to the heart of the differences between the Protagorean view of knowledge and that of Theaetetus.

Socrates-as-Protagoras asks that the position of knowledge as perception itself be refuted—he specifically asks his objectors to prove either that a person’s perceptions are not private events, or that a private perception can “be” for more than just the individual percipient. That is, “Protagoras” places the burden of refuting his theory upon directly refuting the fundamental claims made about perception, and not the various consequences that one would have to reluctantly accept if it were true. “Protagoras” is not interested in the implausibility of his theory, but rather in whether anyone can overcome the claim that perceptions are ultimately private and individual, such that they alone can claim to be knowledge.

To complicate matters further, Socrates-as-Protagoras develops an alternative theory of wisdom as distinct from knowledge. The wise person is not the person who knows more than others. Rather, he or she “can change the appearances”; the wise person “in any case where bad things both appear (φαίνεται) and are for one of us, works a change and makes good things appear and be (φαίνεσθαί τε καὶ εἶναι) for him” (166d7-9). Every appearance or perception is equally true. However, some positions are admittedly better or worse than others, and the wise person can effect a change in others’
appearances such that they have better perceptions than they did before. To say that the things that appear to the wise person are “true” or truer than the perceptions of others is a misnomer, for they are simply better and not truer. A wise doctor can make his patients perceive that eating healthy foods is good, while eating unhealthy foods is bad. The doctor does not possess a truth that the patient lacks, but rather a capacity to effect the change in the patient’s perceptions in a way that will benefit the patient.

Socrates-as-Protagoras develops a theory that is now closer to pragmatism than to pure relativism, although he maintains relativism with respect to the domain of truth. We can still say that wise people exist; however, wisdom is not the possession of knowledge but the capacity to change worse appearances to better ones. “Knowledge” and “truth” are terms to be applied only to the domain of perception, while “better” and “worse” apply to comparisons between perceptions. X Truth then becomes a concept subordinate to the category of the "beneficial"; knowledge is less important than the ability to persuade. Protagorean relativism as presented in the Theaetetus weakens, rather than strengthening, the centrality of truth and knowledge in living well. This is a sensible stance for a rhetorician to take, for then rhetoric, and not philosophy, is then the most powerful form of wisdom.

The heart of Socrates’ objection to the Protagorean redefinition of wisdom is that ordinarily, we do call people wise because they are knowledgeable. This is clearest in the case of technical skills: when we are in distress, or on the battlefield, or in a storm, we turn to those people whom we think possess knowledge that we lack. Socrates says, “In emergencies—if at no other time—you see this belief. When they are in distress, on the battlefield, in sickness or in a storm at sea, all men turn to their leaders in each sphere as
to God, and look to them for salvation because they are superior in precisely this one thing—knowledge (εἰδέναι)” (170a8-b2). Socrates says that we can conclude from the existence of experts that there are both true and false beliefs, since not everyone has knowledge.

Theaetetus finds this line of argument persuasive, but in fact this argument is open to the very same objection that Socrates-as-Protagoras made in response to the earlier arguments against Protagoras: why believe in what the many say? There is no reason to rely upon ordinary opinion as a judge of whether an intellectual position is a good one or not, Protagoras might say. Socrates reiterates the question in a more pointed way—does not Protagoras’ own wisdom as a teacher come from a certain type of knowledge that he possesses and others do not? But there is no reason that Protagoras would have to claim to that his wisdom lies in knowledge rather than in persuasiveness. In the end, Protagoras can meaningfully reiterate his own claim that wisdom is an ability to change the appearances, to give them "better" perceptions rather than worse ones, for some specific purpose (e.g., in order to pass a law that will benefit the city, or to get someone to take a dose of medicine that will make her healthier). Thus far there is no inconsistency in the Protagorean position; nor is it completely unreasonable. All that Socrates has shown in the course of his argumentation is once again that the Protagorean position is quite out of the ordinary.

III.

The key argument against Protagoras takes place from 170d-171c. This argument attempts to show that relativism is self-defeating at some level. However, while the
argument is sufficient for Theaeteteus to reject Protagoras’ position, Socrates also makes clear that Protagoras himself need not concede defeat to Socrates.

The steps of the argument proceed as follows:

1. According to Protagoras, a judgment a person makes is true for that person.

2. Others may disagree with your judgment; in such cases, for those other people, your belief is false and not true. This can be applied to Protagoras’ own beliefs such that:

3. Protagoras’ belief is true for him, but false for the majority of men.

4. “[T]he more those to whom it does not seem to be the truth outnumber those to whom it does, so much the more it isn’t than it is” (171a2-4). Because the judgment is not for more people than it is, the judgment considered in itself “is not” more than it “is.”

5. Protagoras must admit that the contrary of his opinion is true, since others believe that his opinion is false. That is, his own theory requires him to admit to its falsity.

6. As a result, “the Truth of Protagoras is not true for anyone at all, not even for himself” (171c5-6).

We ought to more closely examine this argument. For, this argument would appear more than the others to be a purely logical argument, one which shows that Protagoras’ system is internally inconsistent, and therefore not acceptable even to him. However, here is where Socrates makes his strange remark that Protagoras himself probably would not accept the conclusion of this argument:

Hence it is likely that Protagoras, being older than we are, really is wiser as well; and if he were to stick up his head from below as far as the neck just here where we are, he would in all likelihood convict me twenty times over of talking nonsense, and show you up too for agreeing with me, before he ducked down to rush off again. But we have got to take
ourselves as we are, I suppose, and go on saying the things which seem to us to be (171c9-d7).

Why does Socrates think he will reject it? It is simply that Protagoras will say anything to win an argument, even if he knows his own position to be indefensible?

Burnyeat (1976) argues that Socrates does show that the Protagorean position is self-refuting. On Burnyeat’s reading, Plato is not just showing that Protagoras’ opinion is only true for himself, but false for others, and therefore an inadequate theory. Rather, Protagoras claims that whatever a person believes is true (in some strong sense) in that person’s world. As Burnyeat describes the Protagorean view, "To speak of how things appear to someone is to describe his state of mind, but to say that things are for him as they appear is to point beyond his state of mind to the way things actually are, not indeed in the world tout court (for Protagoras there is no such thing), but in the world as it is for him, in his world” (Burnyeat 1976, 181). But for one who does not believe in relativism (e.g., Socrates), relativism really is not true. If no one believes in Protagoras’ theory, then it is not true for anyone. On Burnyeat’s understanding of Protagorean relativism, when an opponent does not believe that Protagoras’ relativism is true, then it really is not true (in his world). So, according to Burnyeat, the above argument has some force if Protagoras’ opponent does not believe in relativism.

However, Socrates has not shown that relativism is self-refuting, for Protagoras would not accept the presuppositions behind the Socratic line of thinking here. All that Socrates has really shown is that Protagoras' theory is not true for the person who does not believe it. But this is not to say that it is true in some "world." That is, step four above claims that Protagoras’ theory requires him to infer from the difference between
the beliefs of one group of people (Protagoras' opponents) and the beliefs of another individual (Protagoras) something about the reality of what is actually the case. But, as McDowell has argued, Protagoras does not want to make any claims about the relationship between perceptual knowledge and reality, or to use Socrates’ language, “what is” (cf. McDowell 1973; Fine 1998). The whole point of the Protagorean theory is that it altogether avoids questions about truth as a relationship between perception and outside reality. Truth is to be applied only to the realm of each individual perception. We are not to speak of a public truth, or disagreements about what is, for this is an illegitimate application of the term truth, which by definition concerns itself only with individual perceptions, and not groups of perceptions, or comparisons of different individuals’ perceptions (cf. Kerferd 1981).

Burnyeat argues that Protagoras must mean that truth concerns more than individual perceptions. As Burnyeat puts it, "Protagoras' theory is, after all, a theory of truth and a theory of truth must link judgments to something else—the world, as philosophers often put it, though for a relativist the world has to be relativized to each individual" (Burnyeat 1976). However, one must ask whether this is really so obvious: is the Protagorean theory primarily a theory about truth, or a position that claims that demotes the importance of truth in relation to other concepts instead? While Socrates-as-Protagoras does claim that truth is relative to the individual, the speech from 166a-168c suggests that truth is a concept that is less important to Protagoras’ thought than the wise person’s ability to change the appearances. Protagoras says that truth is a term that only tells us something about a person’s beliefs; truth is only—one could even say, “merely”—the truth for someone. Protagoras seems to want to weaken, not strengthen, the idea of the
truth in order to make room for persuasion. In other words, Protagoras’ discussion of the wise as the beneficial suggests that “truth” is somewhat overblown as a concept; what is really important is there is belief and appearance, and the benefits of taking a certain point of view, benefits which are highly dependent upon context (166e-167d). There, his models of wisdom are the doctor, the politician, and the person who can persuade, i.e., the rhetorician (167b-c). In this sense, Protagoras is not a simple relativist, as he had at first appeared to be.

Socrates' argument presupposes that if people disagree about what they believe, there is some need to come to a resolution of the difference. This is because we do not just have our own perceptual experiences; it is because we as human beings tend to make truth claims about others' beliefs as well. For Socrates disagreements about the truth are important; they require some attention. Socrates states that other people believe that Protagoras’ belief is false; for them, it really is false. Protagoras might agree; however, it should be of little concern to Protagoras that others believe his theory to be false, because truth is not as important a concept for Protagoras as it is for Socrates. Protagoras says that disagreements can be resolved through changing the appearances, that is, changing the particular beliefs that a person has. Wisdom is this ability to change others' beliefs or appearances; so the task of resolving differences between perceptions (if resolution is required at all) properly would belong to rhetoric, and not to philosophy. Protagoras has removed the idea of truth from the public "world" altogether.

Other arguments that presuppose a public, intersubjective notion of truth as the foundation for an attack on Protagoras are going to fall prey to similar problems. For example, one might reasonably ask, when Protagoras says that the wise person can
produce a better rather than a worse perception in another person, isn’t he saying that the
better perception is *really* better, or *truly* better than the worse perception? Isn’t he
claiming that the wise person *knows* what is better or worse (as the not-so-wise person
does not)? This is another way of articulating Socrates’ arguments about experts: experts
are not just those who can produce what is better: they can produce what is better *because*
of their knowledge. But Protagoras can always say that the term truth need not be applied
to the intersubjective world: we can avoid using the term truth in reference to
comparisons of different people’s perceptions, but still maintain that some actions or
beliefs are better than others for a particular purpose.

To elaborate: in his speech about wisdom and the beneficial, Protagoras’
examples are from gardening and doctoring. There his language in speaking of what is
"better" relies upon the language of health: what is healthy, wholesome, or beneficial, is
what is good. For example, a wise gardener makes a plant have better rather than worse
perceptions (167b). Presumably, whether they are good or bad perceptions can be
measured by the way the plant grows. We can see whether a doctor’s actions are good or
bad by considering whether the patient changes his perceptions about what sorts of foods
are healthy, and is healed. But it is not strictly speaking necessary to bring in the
question of what the doctor or gardener believes, knows, or thinks. We care about the
effect of the doctor’s action and not whether the treatment is based on a theory of four
humors, germ theory, or homeopathy. Whatever any doctor believes is true for him, but
he is only wise if he effects a change in the patient’s appearances from worse to better.xvi

The person who wants to claim that when we say, “x is good” we already mean “x is truly
good,” is already presupposing that “truth” as a term applies to some part of the world
outside of perception. But this is exactly what is at issue. Protagoras can always say that he is not using “truth” in the way that others ordinarily do. He does not apply it to judgments about the value of perceptions—that is where the term “good” is applicable. Protagoras emphasizes that this is in part a terminological dispute when he says that some people who are “inexperienced” (ἀπειρίας) call what is a good perception a true perception (167b2); we can only compare perceptions against one another by attending to whether they are good, wholesome, beneficial, and so on.

Protagoras’ speech from 166a-168b, then, embeds the problem of truth in a much wider context of what sort of value should be given to persuasion and practice over theoretical concerns about truth. This speech reveals that Protagoras puts a much greater emphasis on praxis than on more theoretical approaches to truth: it was for this reason that Schiller found in Protagoras a precursor to his own version of contemporary humanism, as did later pragmatists (cf. Schiller 1908; Oehler 2002). Even more significantly, Protagoras considers those who have the power to persuade others to change their appearances (and so also their understanding of the truth) as the truly wise. But the concept that some external or transcendent standard needs to be part of good persuasion is excluded in Protagoras’ thought; truth is an almost superfluous concept for him, because what is good or beneficial varies so widely according to context. What matters is what a person believes, and how this motivates that person to, for example, take his medicine or submit to a painful medical procedure. If this is so, then on Protagoras’ view the rhetorician’s skill of persuading is far more valuable than the philosopher’s (untenable) claim to know. Plato does not present Protagoras as a fool bent on preserving a strange or inconsistent system, but rather as an intellectual who rejects a
strong notion of truth but then supplants the concept of truth with alternative conceptions of goodness and wisdom.

Given that Socrates acknowledges that Protagoras would come up from Hades if he could to tell us that he found the argument unsatisfying, Socrates himself must agree that Protagoras’ view is not logically self-refuting. Those who want to say that the position is self-refuting must explain why Socrates implies that it is not. Socrates recognizes that there is the possibility of maintaining a consistent Protagorean theory, saying after he has concluded his objections: “But so long as we keep within the limits of that immediate present experience of the individual which gives rise to perceptions and to perceptual judgments, it is more difficult to convict these latter of being untrue—but perhaps I’m talking nonsense. Perhaps it is not possible to convict them at all; perhaps those who profess that they are perfectly evident and are always knowledge may be saying what really is” (179c4-d1). He suggests that Protagoras’ theory is internally consistent, and holds out the possibility that Protagoras could be right.

We have then at the end of the argument between "Protagoras” and Socrates/Theaetetus two seemingly incommensurable positions: one that claims that a conception of knowledge must include the truth about judgments and what is, and one that favors persuasion and the ability to effect action as marks of wisdom while leaving truth to the realm of individual perception. However, there seems to be no obvious higher order perspective from which to judge the two positions, for the very criteria that could be used to make judgments about them are what is at issue in the first place. In addition, neither Socrates nor Protagoras finds the opinions of the majority to be a sufficient guide to what
is best. Instead, Socrates claims that we must continue our argument only with regard to the things that would seem to be true to Theaetetus and to Socrates (171d).

IV.

To sum up the previous section, Socrates’ arguments against Protagoras above rely upon certain assumptions: 1. There are real experts, and we call them experts because of their knowledge. 2. Wisdom and knowledge are equivalent terms: we call people wise on account of their knowledge. 3. The truth or falsity of another person’s beliefs is meaningful and important. Theories of knowledge should be in harmony with these basic ideas according to Socrates’ line of argument; but it is unclear why one might begin with such presuppositions in the first place.

Plato does not offer an abstract or universal argument on behalf of these assumptions. Rather, the answer is given in terms of why Theaetetus the mathematician believes in them. Early in the dialogue Socrates asks Theaetetus several questions that address issues of expertise, knowledge, and wisdom, even before any formal definitions are sought (145c-146a). First, the concept of expertise arises in Socrates’ initial question to Theaetetus. Theodorus has compared Theaetetus to Socrates. Socrates asks, if they were both holding lyres, and Theodorus said they were similarly tuned, should they just take Theodorus’ word for it? Shouldn’t they inquire into whether Theodorus was truly an expert in music? Theaetetus agrees that they ought to inquire into the matter. That is, Theaetetus affirms that simple acceptance of the authority of an expert on the basis of the opinions of others is insufficient. While Socrates runs through a number of different disciplines in which Theodorus could be an expert, it is when he comes to geometry that
Theaetetus exclaims that there is no doubt that Theodorus is an expert geometer. Theodorus seems to be a master of astronomy, arithmetic, and music, but Theaetetus seems less certain of these claims. We know that Theaetetus has already engaged in geometrical studies with Theodorus, but has not yet taken up these other areas, although he is anxious to do so (145c-d). Plato points out this feature of Theaetetus' relationship to Theodorus, suggesting that Theaetetus' experience of learning mathematics from Theodorus leads to his belief in Theodorus' mathematical expertise. That is, Theaetetus can identify the mathematical expertise of Theodorus because of his own experience of learning geometry from him. While this belief in Theodorus' expertise also implies a more general belief in the existence of knowledgeable experts, the particular experience is prior to the more abstract general principle.

Second, Socrates himself brings up the relationship between knowledge and wisdom. He wonders aloud whether one learns in order to become wise. Theaetetus agrees. Then an interesting passage ensues:

Socrates: And what makes men wise, I take it, is wisdom (σοφία)?
Thea: Yes.
Soc: And is this in any way different from knowledge (ἐπιστήμης)?
Thea: What?
Soc: Wisdom. Isn’t it the things which they know that men are wise about?
Thea: Well, yes.
Soc: So, knowledge and wisdom will be the same thing?
Thea: Yes. (145d6-e6).

Although this passage is often overlooked, it seems important, for it is this equation between σοφία and ἐπιστήμη that Socrates-as-Protagoras denies at 166a-167b. Theaetetus at this point hardly seems even to understand Socrates’ question: perhaps it
seems too general to be interesting. However, Socrates finds it important to ask whether Theaetetus equates wisdom and knowledge. Theaetetus as a mathematician has a commitment to and interest in understanding more abstract truths, and already considers acquisition of this sort of knowledge to be a mark of true wisdom. In geometry, proof of a mathematical claim takes place apart from the usefulness or applicability of the idea in the practical realm. Theaetetus already values universal knowledge over practical benefit before the first definition of knowledge is ever offered.\textsuperscript{xviii}

The third assumption outlined above—that we can and do meaningfully speak about the truth or falsity of others' beliefs—is ultimately grounded in Theaetetus' commitment to the ideals of theoretical expertise and wisdom. In describing judgment, Socrates uses examples of experts in distinction from non-experts. The expert doctor can predict whether his patient will continue to have a fever or not better than the patient. An expert chef can predict whether her dish will turn out delicious better than the guests at dinner (178c-e). Mathematics requires an even stronger commitment to truth than medicine or cooking, however. Theaetetus' work as a geometer is not to show the practical value of a theorem, but to give a proof for why it must be true, a proof that can be demonstrated to others. Theaetetus already regards the truth as having intersubjective character before the more formal discussion of knowledge as perception ever begins.

Socrates is not here giving an argument that shows the internal inconsistency or the illogicality of the Protagorean view. Neither does he show that the Protagorean position is obviously unreasonable, for Protagoras develops a cohesive and meaningful theory. Rather, Socrates is concerned with whether his particular interlocutor, Theaetetus, can accept it. That is, Socrates is asking Theaetetus to make a judgment about the
"reasonableness" of the Protagorean position, by examining the implications of that position and whether they are consistent with his own experiences as a knower. If knowledge is perception, then everything is in flux; our language does not accurately reflect the nature of reality; there are no experts in knowledge; and the mark of wisdom is merely persuasiveness rather than knowledge. Theaetetus cannot accept such conclusions because he enters into the conversation with beliefs that are at odds with such conclusions, beliefs that are grounded in his experience as a student of mathematics. But while Theaetetus rejects the Protagorean theory, Plato goes out of his way to say that Protagoras would not if he were present. Socrates cares most about whether Theaetetus can accept these conclusions, because (somewhat paradoxically) the truth is to be found in Theaetetus himself.

Thus, when Socrates describes his own method in the Theaetetus in the well-known analogy of the midwife, he says that Theaetetus is pregnant, and Socrates’ only job is “to determine whether the young mind is being delivered of a phantom, that is, an error, or a fertile truth” (150c1-2). He describes his conversations with other interlocutors:

At first some of them may give the impression of being ignorant and stupid, but as time goes on and our association continues, all whom God permits are seen to make progress—a progress which is amazing both to other people and to themselves. And yet it is clear that this is not due to anything that they have learnt from me; it is that they discover within themselves a multitude of beautiful things (πολλὰ καὶ καλὰ̀) which they bring forth into the light (150d2-d9).

There are three important features to Socrates’ description of himself as midwife for the purposes of understanding the later argument against Protagoras. First, Socrates claims that he himself is past the age of “child-bearing” (149c4-5). Socrates says that he
is not the one giving birth here, but is aiding Theaetetus to give birth to his own ideas. If we take the arguments against Protagoras to be a rejection of the inadequacy of the sophist’s theory in abstraction from Theaetetus’ beliefs, it would be hard to understand Socrates’ claim of being “barren.” If all along Socrates knew the problems with Protagoras’ theory and could teach them, Theaetetus as a specific individual would be unimportant: anyone could answer these questions for him. In that case, Socrates would be no midwife but an expert. However, if the midwifery passage if taken seriously, it suggests that the answers to philosophical problems are inevitably sought in the soul of the individual. Socrates’ questions really are designed to draw out Theaetetus’ answers. He wants to show the young mathematician why he, Theaetetus, does not really believe that knowledge is perception even if this initial definition at first appears right.

Second, Socrates claims that the process of philosophical childbirth is by its nature difficult and painful; he suggests that it is even worse than the pain experienced by women in childbirth, for while their physical labor ends within a relatively short time, those undergoing philosophical labor are up “day and night” (151a7; cf Hemmenway 1990). Socrates says that his art both helps to bring on these pains and to soothe them (151a 8). Socrates is clear that Theaeteus is one who is pregnant, and possibly already in labor. Just before the midwife analogy is offered, Theaetetus exclaims that he cannot stop worrying about the nature of knowledge when he has heard others speak on such topics (148e1-5). Neither his own nor others’ answers seem sufficient, and yet he cannot stop wondering about the questions. As Roochnik has argued, Plato continually portrays Theaetetus as someone who is driven by a sense of wonder (θαυμάζειν) about the world (cf. Roochnik 2002 and Polanksky 1992). The term θαυμάζειν occurs frequently in the
Theaetetus (see, for example, 142a2; 142c5; 144a3; 144b6; 150d5; 161c3; 162c3; 162c8; cf. Roochnik 2002). Theaetetus is described as being willing to put his “heart” (προθυμία) into seeking to understanding whatever he is curious about (148d1-4). At 162c8, Theaetetus describes himself as being in a state of amazement or wonder. Socrates specifically connects this experience of wonder about the world to what characterizes the philosopher: "For this is an experience which is characteristic of a philosopher, this wondering: this is where philosophy begins and nowhere else" (155d1-3). Plato presents Theaetetus as someone who has this love of wisdom.

Considerable space at the beginning of the Theaetetus is also devoted to Theaetetus' person: the first six pages of the dialogue center around his character and his work in mathematics (142a-148d). There, Theaetetus is described as intellectually gifted, mild in temper, and courageous (ἀνδρεία) (144a); his courage on the battlefield is also emphasized in the prologue (142b). Later in the dialogue, Socrates says that courage is necessary if Theaetetus is to continue to pursue the argument in which they are engaged (157d). That is, Socrates says that courage is closely connected to philosophical inquiry. Theaetetus' sense of wonder, courage, and intelligence all contribute to his drive to go beyond the internally coherent but ultimately unsatisfying claims of Protagoras that there are no universal truths to be sought. Theaetetus is not satisfied with Protagoras’ practical theory because of these intellectual labor pains provoked by his philosophical curiosity, his desire to know. xix

A third interesting feature of the midwife analogy is Socrates’ claim to be a “matchmaker: just as midwives have a little-known ability to make matches that will produce the best physical children, Socrates says that he also knows what sorts of ideas
will be the most fruitful for those whom he is assisting (150a1-b2; cf. Hemmenway 1990). In other words, Socrates chooses the direction in which his questions will go in response to the particular person with whom he is working. The implication is that a different partner in conversation would require a different set of questions. Socrates’ main interest here is not to evaluate the ideas of the historical Protagoras, but rather to examine how using this theory can aid Theaetetus in assessing his initial belief that knowledge is perception. This explains why Socrates seems to move unexpectedly between offering complex defenses of Protagoras and criticisms of the theory rather than being immediately one-sided. His ultimate objective is to allow Theaetetus to refine, revise, or reject his own views, and to bring to birth a better definition later in the conversation (which is itself open to revision and rejection). Socrates uses Protagorean theory as a tool to get at Theaetetus’ beliefs: Theaetetus’ ideas are the “beautiful things” that Socrates is interested in bringing to light.

Such reliance upon Theaetetus’ beliefs about knowledge raises questions as to whether Plato thought that these beliefs about knowledge and expertise are rational or objective in any universal sense. That is, does Plato view the assumptions of Theaetetus as rational and justifiable in some non-circular way so that anyone must reject Protagoras’ starting position? One might argue that Theaetetus has special or privileged knowledge as a mathematician that allows him to understand abstract knowledge in a way that non-mathematicians cannot. Perhaps mathematics orients a person towards the concept of universal truth, and provides the student with the experience of understanding universal, abstract truths that will inform his understanding of knowledge more generally. (Consider
the Republic’s recommendations that students study various forms of mathematics and harmony as preparation for philosophy at 523a-532a.)

However, when one examines the content of Theaetetus’ work as a mathematician, the picture becomes somewhat more complicated. For Theaetetus’ major contribution to mathematics focuses on the concept of what would today be called irrational numbers (now represented as $\sqrt{3}$, $\sqrt{5}$ and the like). His work in this area is explicitly described in the dialogue (147d-148b). We learn that Theodorus has been working on the problem of incommensurable areas. Theodorus was able to demonstrate 3 square feet, 5 square feet, and numbers up to 17 square feet are incommensurable with a unit of 1, through a series of individual proofs. However, Theaetetus has just made a major contribution, even surpassing his own teacher's discovery: Theaetetus has discovered how to classify numbers with irrational roots and those with rational roots into separate categories. (Theaetetus describes this in terms of those numbers that fit into the category of equilateral or oblong, as the problem was addressed in geometrical terms in ancient mathematics rather than in the language of rational or irrational; see Brown 1969 for an extensive and detailed discussion of this work in Theaetetus’ mathematics.) That is, Theaetetus has been able to separate incommensurable from commensurable quantities in a more universal way.\textsuperscript{xxi}

Theaetetus’ work in mathematics perfectly mirrors the situation with respect to the discussion of knowledge as perception. Plato was well aware of the concept of the incommensurable, as he uses the term οὐ σύμμετροι to describe the mathematical problem at 147d; that is, he understood that certain lengths cannot be “grasped” or measured by a common term. Similarly, Plato sets side by side two approaches to
understanding knowledge that also seem to be incommensurable. No common term can evaluate both the approaches of Theaetetus and of Protagoras adequately, for their concepts of truth, wisdom, and knowledge are entirely different. The main body of Theaetetus’ work was to grapple with what is real and yet does not seem to be capable of expression with a single measure; here, too, Plato seems to be pointing out the possibility of real conflict that cannot be resolved with an obvious, indisputable concept of the rational.

In addition, part of what is at issue in any discussion of knowledge is what the ground rules are for how one goes about arguing, and what the accepted mode of argumentation is going to be. Plato does not have a theory of formal logic, but even if one were available to him, it would not suffice for the purposes of this discussion in the *Theaetetus*. For part of what is in question in searching for a definition of knowledge is not only the content of the definitions, but also the method by which one ought to argue in the first place. As Griswold has argued, the numerous breakdowns in conversation between Socrates and Protagoras in the *Protagoras* stem from more basic conflicts over values such as personal responsibility, self-sufficiency, and accountability to reason (cf. Griswold 1999). In the *Protagoras*, Protagoras and Socrates continually fail to talk to one another because they cannot even agree upon what an acceptable mode of argumentation is, or what the ultimate purpose of a conversation would be. Perhaps part of the reason that the conversation between Theaetetus and “Protagoras” can take place in a coherent way at all is that Socrates the philosopher takes on the role of Protagoras. xxii

Nonetheless, Theaetetus’ project as a mathematician is to make a kind of rational sense of the irrational. Theaetetus’ work to discover a way to analyze and categorize the
irrational and rational numbers is itself a rational activity. He works to impose a sort of order on what is seemingly chaotic, but an order that acknowledges and maintains the concept of the irrational within this larger rational framework. Similarly, Plato seeks to capture in the dialogue form a conversation that preserves the incommensurability of the standpoints about knowledge held by Protagoras and Theaetetus, but also in its very mode of conversation embodies the rational as Socrates understands it. That is, in the *Theaetetus*, Plato seems to recognize the existence of a competing intellectual position that from its own standpoint is not fully “captured” by the Socratic position, while still affirming the Socratic/philosophical standpoint. A key question, then, is how that philosophical standpoint is affirmed.

While Plato does not address the question directly, Socrates’ claim that the conversation must always proceed from Theaetetus’ beliefs alone suggests that no simple answer can be given. That is, nowhere in the conversation does Plato provide a universal refutation of Protagoras that can withstand all potential objections. Instead, Plato seems to affirm a notion of the “rational” as inevitably embedded in the experiences of the particular interlocutors with whom Socrates speaks. Philosophy is, in this sense, a personal activity. That is, Theaetetus already has an understanding of the rational that guides how he answers Socrates’ questions, but his understanding of the rational goes far beyond what is internally consistent or logical to include broader ideas about expertise, wisdom, and learning that stem from his experience of mathematics and even from his character. Plato acknowledges that our characters and experiences also guide and inform our judgments of what is rational, and whether the philosophical understanding of rationality is valuable to begin with. While there is no non-circular way to begin the
process of reasoning about reasoning, this is not necessarily a cause for despair; it is rather an affirmation of the dialectical and personal nature of philosophy.

The *Theaetetus* is a deeply dialectical work: it begins with the beliefs, assumptions, and histories of the particular individuals involved in conversation, but also ever asks those individuals to put to the test those ideas. The dialectical process is open-ended in its method: the direction in which the conversation proceeds is not predetermined by a set of scientific rules, but is ever reliant upon Theaetetus’ current perceptions and beliefs about the subject matter at hand. Like a good midwife who uses practical judgment to discern how to aid a woman at different moments of her labor, Socrates needs to adapt his own questioning to Theaetetus’ current state of laboring. This dialectical process is also presented as incomplete by its nature. Theaetetus begins with a far more superficial understanding of his own ideas about knowledge than he ends up with by the end of the conversation; however, he arrives at no final and complete definition. Plato thereby affirms to some extent Protagoras’ contention that γίγνεσθαι is as important as εἶναι with respect to human knowledge: Theaetetus as a knower is in a state of coming to be and never in a state of completion with respect to his own knowledge of knowledge. Socrates presents the philosopher as always in a middling state, in between the simple affirmation of beliefs informed by character and experience, and the achievement of a universal and objective truth. Both the particular and the universal constitute the human world; but Plato presents their opposition as a creative tension that gives birth to many “beautiful things.”

Marina McCoy

Boston College
Bibliography


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Bemelmans agrees that there is something incommensurable between Protagoras’ and Socrates’ approaches. However, Bemelmans does not examine the relevance of Theaetetus’ beliefs to the final argument, beliefs I argue below are crucial to the resolution of the conflict for Socrates and Theaetetus.

For the infallibilist interpretation, see Gail Fine, “Plato’s Refutation of Protagoras in the Theaetetus” Apeiron 31 (1998) 201-234. Burnyeat provides a thorough summary and examination of these main avenues of interpretation in his commentary on the Theaetetus See M. Burnyeat, ed, Theaetetus of Plato, pp. 7-65.

I abstain here entirely from making claims about the historical Protagoras and what elements might or might not be genuinely “Protagorean,” as this poses numerous interpretive difficulties.

For extensive discussions of the mathematics of Theaetetus and their philosophical relevance, see Malcolm Brown, “Theaetetus: Knowledge as Continued Learning.” Journal of the History of Philosophy, 7 (1969) 359-379, and Martin Andic and Malcolm

vi My translation. Burnyeat suggests that the qualifiers are legitimately dropped once Socrates has established that the Protagorean theory is not true for those who do not believe it (e.g., at 171a). See M. Burnyeat, “Protagoras and Self-Refutation in Plato’s *Theaetetus*,” *Philosophical Review* 85 (1976) 172-195. However, I find his understanding of the nature of the self-refutation argument problematic for reasons that I describe below in the course of explaining the later arguments. Perhaps he drops the qualifiers at 171a because Theaetetus’ view of knowledge is what is operative at that point in the argument.


viii It was this line of thinking that led Cornford to suggest that Plato thought that Protagoras gave an adequate account of perception, but not of knowledge. Knowledge must be of what is, while Protagoras' ontology relies upon becoming rather than being, and so falls short. See Francis M. Cornford, *Plato’s Theaetetus*, trans. with a running commentary (Indianapolis: Bobbs-Merrill Press, 1934).

ix McDowell also shows that Theaetetus' reply is inadequate for another reason: showing that people know what they perceive, and that they do not perceive what they do not know, does not also show that the knowledge that they lack is identical to the perception that they lack (see McDowell, p. 160).
Working from her claim that Protagoras is an infallibilist, Fine argues that Plato successfully refutes Protagoras by showing that not all beliefs can be true, i.e., in particular second-order beliefs. See Fine, 201-234. However, this section suggests that Protagoras would have a response: truth is not a term to be applied to our judgments either; we can still distinguish between better and worse judgments without claiming that one is truer than the next.

Plato's *Gorgias*, of course, takes up the question as to whether the ability to persuade justly requires knowledge of justice, but this is an issue not addressed directly here.

While this section of the argument takes place between Socrates and Theodorus, it is Theaetetus who is asked to use his own beliefs to evaluate the two competing standpoints.

As McDowell puts it, "it depends on any given person's judgment whether any proposition is the truth for him or not; not that every person's judgment must be given weight in deciding, by a count of heads, to what extent any proposition is true simpliciter" (McDowell, p. 170).


See Burnyeat, “Protagoras and Self-Refutation in Plato’s *Theaetetus,*”, p. 181.

As Polansky says, the medical example can be persuasive since we can say both that all perceptions are true, and that all who are sick want to become healthy; so health can be a good without bringing in some "true" concept of health (see Ronald M. Polansky,

xvii That is, while pragmatists have correctly identified an anti-transcendental and practical aspect to Protagoras’ thought, he is not really a pragmatist. Pragmatism views judgments which have been tested, or which are useful, as “true” in a larger social context. See Klaus Oehler, "Protagoras from the Perspective of Modern Pragmatism," *Transactions of the Charles S. Peirce Society*. 38 (2002) 207-214. However, Protagoras demotes the value of the term truth and instead emphasizes the power to change appearances as central to wisdom.

xviii This is not to exclude Polansky's interesting point that Theaetetus may not recognize a form of wisdom that does not entail knowledge of particular arts and sciences (i.e., dialectical knowledge; see Polansky pp. 45-46); but Theaetetus at least excludes the mere ability to persuade as true wisdom, and so his strong association of knowledge and wisdom here has a positive effect.

xix See David Roochnik, *Beautiful City: The Dialectical Character of Plato’s Republic* (Ithaka: Cornell U. Press, 2003) for an extensive discussion of the nuanced role that mathematics plays in the *Republic*, including the similarities between math and philosophy in their attempts to grasp the universal (and important differences).

xx Of course, this does not mean that Plato has no more general interest in the theory; here I am focusing on the nature of Socratic method within the world of the dialogue.

xxi Most scholars understand Theaetetus to have been the major contributor to the material that later forms Book Ten of Euclid's geometry. See Brown, 359-379.
See Charles Griswold, "Relying on Your Own Voice: An Unsettled Rivalry of Moral
examination of their conflict. Consider also the Gorgias 487b in which Callicles
describes Socrates’ method as trivial and pointless.

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