Fractional Monetary Dynamics

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Abstract

We test for fractional dynamics in U.S. monetary series, their various formulations and components, and velocity series. Using the spectral regression method, we find evidence of a fractional exponent in the differencing process of the monetary series (both simple-sum and Divisia indices), in their components (with the exception of demand deposits, savings deposits, overnight repurchase agreements, and term repurchase agreements), and the monetary base and money multipliers. No evidence of fractional behavior is found in the velocity series. Granger's (1980) aggregation hypothesis is evaluated and implications of the presence of fractional monetary dynamics are drawn.

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1. Introduction

This study investigates the presence of fractional dynamics in a variety of U.S. monetary series. Any dynamic macroeconomic model, whether an IS-LM based structure or a much more elaborate framework, will contain a number of economic variables which have been empirically identified as possessing fractional dynamics, or elements of strong persistence, in their time series representation. The model of fractionally integrated timeseries developed by Granger and Joyeux (1980) and Hosking (1981) allows for a fractional, as opposed to an integer, exponent in the differencing process of the time series. This avoids the 'knife-edge' unit root distinction while permitting a modelled series to exhibit the persistence, or 'long memory,' which characterizes many macroeconomic timeseries. For instance, fractionally integrated output series have been identified by Diebold and Rudebusch (1989) and Sowell (1992a). Such persistence is also evident in consumption (Diebold and Rudebusch (1991)), interest rates (Shea (1991)), and inflation rates (Baillie, Chung, and Tieslau (1996), Hassler and Wolters (1995), Baum, Barkoulas, and Caglayan (1997)).

The importance of measures of money in a macroeconomic modelling framework led Porter-Hudak (1990) to examine M1, M2 and M3 aggregates for fractional integration. The latter study provides the motivation for this paper, in which we extend Porter-Hudak's study of fractional integration in the monetary aggregates in several important ways in order to provide comprehensive evidence on the nature of fractional dynamic behavior in these series. More specifically, we test for fractional integration, using the spectral regression method developed by Geweke and Porter-Hudak (1983), in both simple-sum and Divisia monetary aggregates, monetary base, money multipliers, and velocity series. Given clear evidence of fractional integration in the aggregates, we subsequently try to identify which components of the monetary aggregates might be responsible for fractional integration and therefore evaluate Granger's (1980) aggregation hypothesis. The findings suggest that despite often-cited elements of instability, the monetary aggregates may contain predictable components due to their long-memory properties. The fractional dynamics which appear to characterize both monetary series and real measures may be exploitable in models of their comovements which go beyond the standard VAR framework.

The rest of the paper is constructed as follows. Section 2 presents the theory of fractionally integrated timeseries and the estimation method employed. Data and empirical results are discussed in Section 3. We conclude in Section 4 with a summary of our results.

2. The Fractionally Integrated Timeseries Model

The model of an autoregressive fractionally integrated moving average process of order (p,d,q), denoted by ARFIMA(p,d,q), with mean μ, may be written using operator notation as

$$\Phi(L)(1-L)^d\left(y_t-\mu\right)=\Theta(L)u_t, \qquad u_t \sim \text{i.i.d.}(0,\ \sigma_u^2) \tag{1}$$

where L is the backward-shift operator, $\Phi(L) = 1 - \phi_1 L - ... - \phi_p L^p$, $\Theta(L) = 1 + \vartheta_1 L + ... + \vartheta_q L^q$, and $(1-L)^d$ is the fractional differencing operator defined by

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)\, L^k}{\Gamma(-d)\Gamma(k+1)} \tag{2}$$

with $\Gamma(\cdot)$ denoting the gamma function. The parameter d is allowed to assume any real value. The arbitrary restriction of d to integer values gives rise to the standard autoregressive integrated moving average (ARIMA) model. The stochastic process y_t is both stationary and invertible if all roots of $\Phi(L)$ and $\Theta(L)$ lie outside the unit circle and $|d| < 0.5$. The process is nonstationary for $d \geq 0.5$, as it possesses infinite variance, i.e. see Granger and Joyeux (1980). Assuming that $d \in (0, 0.5)$ and $d \neq 0$, Hosking (1981) showed that the correlation function, $\rho(\cdot)$, of an ARFIMA process is proportional to k^{2d-1} as $k \to \infty$. Consequently, the autocorrelations of the ARFIMA process decay hyperbolically to zero as $k \to \infty$ which is contrary to the faster, geometric decay of a stationary ARMA process. For $d \in (0, 0.5)$, $\sum_{j=-n}^{n} |\rho(j)|$ diverges as $n \to \infty$, and the ARFIMA process is said to exhibit long memory, or long-range positive dependence. The process is said to exhibit intermediate memory (anti-persistence), or long-range negative dependence, for $d \in (-0.5, 0)$. The process exhibits short memory for $d = 0$, corresponding to stationary and invertible ARMA modeling. For $d \in [0.5, 1)$ the process is mean reverting, even though it is not covariance stationary, as there is no long-run impact of an innovation on future values of the process.

Geweke and Porter-Hudak (1983) suggest a semiparametric procedure to obtain an estimate of the fractional differencing parameter d based on the slope of the spectral density function around the angular frequency $\xi = 0$.

The spectral regression is defined by

$$\ln\left\{I(\xi_{\lambda})\right\} = \beta_0 + \beta_1 \ln\left\{4\sin^2\left(\frac{\xi_{\lambda}}{2}\right)\right\} + \eta_{\lambda}, \qquad \lambda = 1,\ldots,\nu \tag{3}$$

where $I(\xi_{\lambda})$ is the periodogram of the time series at the Fourier frequencies of the sample $\xi_{\lambda} = \frac{2\pi\lambda}{T}\left(\lambda = 1,\ldots,\frac{T-1}{2}\right)$, T is the number of observations, and $\nu = g(T) << T$ is the number of Fourier frequencies included in the spectral regression.

Assuming that $\lim_{T\to\infty} g(T) = \infty$, $\lim_{T\to\infty}\left\{g(T)/T\right\} = 0$, and $\lim_{T\to\infty} \ln(T)^2/g(T) = 0$, the negative of the OLS estimate of the slope coefficient in (3) provides an estimate of d. Geweke and Porter-Hudak (1983) prove consistency and asymptotic normality for $d < 0$, while Robinson (1995) and Hassler (1993) prove consistency and asymptotic normality for $d \in (0, 0.5)$ in the case of Gaussian ARMA innovations in (1).

Other authors have used maximum likelihood methods (i.e., the exact maximum likelihood method proposed by Sowell (1992b)) or the approximate frequency domain maximum likelihood method proposed by Fox and Taqqu (1986)), which simultaneously estimate both the short-memory and long-memory parameters of the model. These estimation methods are computationally burdensome, rely on the correct specification of the high-frequency (ARMA) structure to obtain consistent parameter estimates, the final ARFIMA specification chosen generally varies across different selection criteria, and, in some cases, the maximum likelihood estimates of the fractional-differencing parameter appear to be sensitive to the parameterization of the high-frequency components of the series. As we are primarily interested in estimating the degree of long-term dependence in the monetary series without specification of a complete time series model, we favor the usage of the spectral regression method to estimate the fractional-differencing parameter.

3. Data and Empirical Estimates

We perform the analysis on a comprehensive set of U.S. monetary aggregates and their components. All data series are seasonally adjusted, monthly observations covering the period 1959:1 to 1995:10 unless otherwise indicated. The Appendix contains further details of the data set. All subsequent analysis is applied to the growth rates (first logarithmic differences) of the monetary series.

We report fractional differencing estimates for estimation sample sizes of $\nu = T^{0.50}, T^{0.525}, T^{0.55}, T^{0.575}, \text{and}\, T^{0.60}$ for the spectral regression and impose the known theoretical variance of the spectral regression error $\frac{\pi^2}{6}$ in the construction of the $t-$statistic for d. Table 1 reports the spectral regression estimates of d for the simple-sum monetary aggregates, monetary base, and money multipliers.[1] The fractional integration estimates are fairly stable and fluctuate rather moderately across the sizes of the spectral regression considered. As panel (A) of Table 1 indicates, strong evidence of a fractional integration order is found in all simple-sum (M1, M2, M3, and L) monetary aggregates. The unit-root hypothesis in the growth rates of the simple-sum monetary indices is decidedly rejected and evidence of fractional dynamics with long memory features is established. If we compare the range of these estimates to those estimated by Porter-Hudak (1990) over the 1959-1986 period, we find values that are broadly comparable.[2]

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[1] We also applied the Phillips-Perron (PP, 1988) and Kwiatkowski, Phillips, Schmidt, and Shin (KPSS, 1992) unit-root tests to the growth rates of the monetary series. The combined use of these unit-root tests offers contradictory inference regarding the low-frequency behavior of most monetary series, which provides motivation for testing for fractional roots in these series. The long-memory evidence to follow reconciles the conflicting inference derived from the PP and KPSS tests. These results are not reported here but are available upon request from the authors.

[2] Our estimates of the fractional exponent are lower than her M1 estimate of 0.661, our M2 estimates are consistent with her corresponding estimate of 0.461, while our M3 estimates are higher than her estimate of 0.402.

Panel B of Table 1 reports the fractional exponents estimated for the adjusted monetary base and M1, M2, M3, and L multipliers. Significant evidence of long memory is obtained for the adjusted monetary base. The M1 multiplier series does not appear to contain long-memory features (the series in levels contains a single unit root). Fractional dynamics are observed for the rest of the money multipliers. The growth rates of the M3 and L multipliers may possibly contain a unit root, suggesting that the corresponding series in levels are $I(2)$ processes.

It is possible that the financial innovations of the 1970's (introduction of money market mutual funds, negotiable order of withdrawal (NOW) accounts, share drafts, and automatic transfer to savings (ATS) accounts) and the deregulation of the early 1980's (the Depository Institutions' Deregulation and Monetary Control Act of 1980 (DIDMCA) and the Garn-St. Germain Act of 1982) might have affected the low-frequency properties of the monetary series. In order to investigate whether these events affected the low-frequency properties of the monetary series, we reestimated the fractional differencing parameter for the monetary aggregates, monetary base, and money multipliers over two sample subperiods. The first starts in 1975:1 (roughly coinciding with the occurrence of financial innovations) while the second starts in 1982:1 (roughly coinciding with the aforementioned legislative changes); both subperiods end in 1995:10. The evidence over the two subperiods remains qualitatively unchanged as compared to that over the full sample thus suggesting robustness of the long-memory evidence with respect to the choice of the sample period.[3]

[3] Subsample estimates are not reported here, but are available upon request from the authors.

Analysis of Components of the Monetary Aggregates

Given the presence of a fractional exponent in the differencing process for the monetary aggregates, we now attempt to determine the sources of fractional dynamics. One explanation, attributed to Granger (1980), is that a persistent process can arise from the aggregation of constituent processes each of which has short memory. Granger (1980) showed that if a time series y_t is the sum of an infinite number of independent first-order Markov processes which have equal variances and whose autoregressive parameters are drawn independently from a beta distribution with support $(0,1)$, then the aggregated series is asymptotically fractionally integrated with $d < 0.5$. Granger and Ding (1996) extended the aggregation argument to mixtures of $I(d_j)$ processes for a range of distributions for d_j; they also showed that other data generating mechanisms, like time-varying coefficient models and possibly nonlinear models, can have the long-memory property. If long memory is artificially induced by aggregation, then it is not a genuine feature of the dependence structure of the series and therefore not economically interesting. To obtain further insight into Granger's aggregation hypothesis for the monetary series, we investigate whether selected components of the monetary indices exhibit fractional behavior.

Table 2 reports the fractional differencing estimates for several important components of the simple-sum M1, M2, and M3 monetary aggregates. The M1 components we consider include currency, demand deposits, and total checkable deposits. Robust evidence of long-term cycles with eventual positive dependence is obtained for all currency and total checkable deposits but not for the demand deposits.

For the M2 monetary aggregate, we analyze the following components: small time deposits (in commercial banks and thrift institutions), savings deposits (in commercial banks and thrift institutions), money market mutual funds, overnight repurchase agreements, and M2 minus M1. Evidence of a fractionally differenced process is found for the small time deposits series, the money market mutual funds, and M2 minus M1 series. No evidence of a fractional integration order is found in the savings deposit series and overnight repurchase agreements.

With respect to components of the M3 monetary index, we consider large time deposits (in commercial banks and thrift institutions), term Eurodollars, term repurchase agreements, and M3 minus M2. Evidence of a fractionally differenced process with long-memory features is obtained for the large time deposits in thrift institutions and possibly for the large time deposits in commercial banks, term Eurodollars, and the M3 minus M2 series. Long memory is not present in the term repurchase agreements series. The liquid assets (L) minus M3 series is also characterized by fractional behavior.

Overall, the following observations can be made from the analysis of the components of the simple-sum monetary aggregates. First, not all simple-sum component series exhibit fractional behavior. Therefore, long memory is not a universal feature of monetary series at a disaggregated level. The degree of persistence for each component series indicates which underlying series "cause" the long-memory property at a given level of disaggregation. And second, it does not appear that higher orders of integration are obtained for the more aggregated series, which is inconsistent with Granger's aggregation hypothesis. However, a more disaggregated data set may be needed in order to fully address the aggregation argument.

Analysis of Divisia Indices and Velocity Series

We subsequently test for a fractional integration order in an alternative set of monetary aggregates: the Divisia indices (Thornton and Yue (1992)). The Divisia monetary aggregates were proposed by Barnett et al. (1984) as superior to simple-sum aggregates which "implicitly view distant substitutes for money as perfect substitutes for currency." (1984, p.1051) Barnett et al. found that the Divisia aggregates performed considerably better in terms of causality tests, tests of the structural stability of money demand functions, and forecasting. They also noted that "the divergence between the time paths of the Divisia and the sum aggregates increases as the level of aggregation increases" (1984, pp.1075-76), so that we might expect the higher-level Divisia aggregates to possess different dynamic properties than their simple-sum counterparts. The spectral regression estimates reported in Table 3 suggest that a fractionally differenced model is an appropriate representation of the low-frequency behavior of the Divisia indices. Evidence of long-term persistence is unstable in the Divisia M1 measure but it is very strong in the more aggregated Divisia measures. The degree of persistence is similar in magnitude in the Divisia M2, M3, and L series. Although the Divisia measures are demonstrably different from their simple-sum counterparts, our qualitative conclusions for the simple-sum aggregates are not shaken by consideration of the Divisia measures.

Finally, the possibility of long-memory behavior in U.S. money velocity series is examined. The time series properties behavior of the velocity of money in the U.S. has attracted a great deal of attention in the literature given its implications for the monetarist position. Gould and Nelson (1974), Nelson and Plosser (1982), and Haraf (1986) conclude that money velocity contains a unit root. A similar conclusion is reached by Serletis (1995), even after allowing for the possibility of a one-time break in the intercept and the slope of the trend function at an unknown point in time.

Table 4 reports the fractional-exponent estimates for the growth rates of both simple-sum and Divisia velocities. A significantly negative d estimate for the growth-rate series would imply mean-reverting behavior for the series in levels, thus contradicting random-walk behavior. With the possible exception of the simple-sum M3 velocity series, there is no evidence of mean reversion in any of the velocity series. The unit-root null hypothesis is robust against fractional alternatives for the velocity series.

4. Conclusions

We tested for fractional dynamics in a comprehensive set of U.S. monetary series containing simple-sum and Divisia aggregates, monetary base and money multipliers, selected components of simple-sum aggregates, and velocity series. Evidence of a fractional exponent is generally found in the differencing process of the monetary aggregates, the monetary base, and the money multipliers. Although not every component of the simple-sum aggregates exhibits long memory, the overall evidence is substantial and robust in support of fractional monetary dynamics with long-memory features. Our findings of fractional integration orders between one and two (and statistically distinguishable from one and two) is contrary to the conclusion reached by King et al. (1991) and Friedman and Kuttner (1992) that nominal money balances are $I(2)$ processes. A shock to the growth rate of the monetary series displays significant persistence, but it eventually dissipates. The money velocity series are best characterized as unit-root processes.

A number of implications and extensions for future research may be drawn from our empirical results. First, an ARFIMA model is established as an appropriate representation of the stochastic behavior of U.S. monetary indices and several of their key components. Since long memory represents nonlinear dependence in the first moment of the distribution and hence a potentially predictable component in the series dynamics, the possibility of improved forecasting via the estimation of an ARFIMA model arises. Porter-Hudak (1990) found superior out-of-sample forecasting performance of an ARFIMA model for the M1 aggregate versus a benchmark ARIMA model. Given the substantial fractional exponent in the differencing process in our series, similar improvements in forecasting accuracy may be expected to result from the estimation of an appropriate ARFIMA model for our data series. However, the development of complete ARFIMA models for our sample series and performance of forecasting experiments are beyond the scope of this paper and warrants future research. The extension of our approach to other industrial economies' monetary series may also be productive.

Second, these findings support the development of analytical models featuring monetary policies and rules that may give rise to long-range dependence in the monetary series. The nature and implications of the transmission of long-memory dynamics to other variables in a macroeconomic model should be investigated. We hypothesize that the dynamics of macroeconomic variables such as the price level, real output, and nominal interest rates may reflect fractional monetary dynamics by exhibiting some degree of persistence, as has been observed in a number of empirical studies of these series. Estimated models must take account of the observed long-memory properties of these macroeconomic variables, as their joint process will no longer be adequately characterized by a linear vector autoregression (VAR). The nonlinear relations arising in this context deserve further scrutiny. Finally, care must be exercised in interpreting results from regressions involving the growth rates of monetary series. Tsay and Chung (1995) have shown the existence of spurious effects in regressions involving two independent long memory fractionally integrated processes whose orders of integration sum up to a value greater than 0.5. Given our findings, many monetary series would be likely to trigger these effects.

Appendix

All data series are seasonally adjusted, monthly observations obtained from the Federal Reserve Bank of St Louis' FRED database, which contains series originally published by the Board of Governors of the Federal Reserve System. The Divisia aggregates series were originally published in Thornton and Yue (1992). For the Divisia M1, M2, M3, and L series, the sample period is 1960:1 to 1992:12. The sample period is 1959:1 to 1995:10 for the following series: simple-sum M1, M2, M3, L, currency in circulation, demand deposits, total checkable deposits, small time deposits at commercial banks, small time deposits at thrift institutions, savings deposits at commercial banks, savings deposits at thrift institutions, M2 minus M1, term Eurodollars, large time deposits at commercial banks, M3 minus M2, L minus M3, adjusted monetary base, M1 multiplier, M2 multiplier, M3 multiplier, and L multiplier. The sample period is 1973:11 to 1995:10 for money market mutual funds, 1969:1 to 1995:2 for overnight repurchase (RP) agreements, 1970:2 to 1995:10 for large-time deposits at thrift institutions, and 1969:10 to 1995:10 for term repurchase agreements. The sample periods for the simple-sum and Divisia-based velocity series are 1959:1 to 1995:10 and 1960:1 to 1992:12, respectively.

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Table 1: Empirical Estimates of the Fractional-Differencing Parameter d for the Simple-Sum Monetary Aggregates, Monetary Base, and Money Multipliers

Monetary Series	$d(0.50)$	$d(0.525)$	$d(0.55)$	$d(0.575)$	$d(0.60)$
			(A)		
Simple-sum M1	0.552 (3.151) ***	0.573 (3.562) ***	0.457 (3.129) ***	0.421 (3.189) ***	0.483 (3.980) ***
Simple-sum M2	0.536 (3.056) ***	0.498 (3.093) ***	0.453 (3.104) ***	0.439 (3.327) ***	0.530 (4.367) ***
Simple-sum M3	0.668 (3.812) ***	0.658 (4.090) ***	0.607 (4.156) ***	0.665 (5.039) ***	0.737 (6.077) ***
Simple-sum L	0.639 (3.642) ***	0.721 (4.476) ***	0.645 (4.417) ***	0.679 (5.142) ***	0.754 (6.220) ***
			(B)		
Adj. Monetary Base	0.698 (3.984) ***	0.806 (5.005) ***	0.720 (4.924) ***	0.665 (5.035) ***	0.658 (5.428) ***
M1 Multiplier	0.104 (0.135)	0.135 (0.841)	0.186 (1.274)	0.228 (1.732) *	0.335 (2.761) ***
M2 Multiplier	0.729 (4.157) ***	0.708 (4.399) ***	0.817 (5.594) ***	0.748 (5.662) ***	0.695 (5.733) ***
M3 Multiplier	0.790 (4.506) ***	0.762 (4.735) ***	0.832 (5.696) ***	0.870 (6.586) ***	0.842 (6.944) ***
L Multiplier	0.815 (4.647) ***	0.896 (5.568) ***	0.891 (6.099) ***	0.958 (7.255) ***	0.924 (7.619) ***

Notes: The series are the growth rates (first logarithmic differences) of the monetary series. $d(0.50)$, $d(0.525)$, $d(0.50)$, $d(0.575)$, and $d(0.60)$ give the d estimates for spectral regression estimation sample sizes $\nu = T^{0.50}$, $\nu = T^{0.525}$, $\nu = T^{0.50}$, $\nu = T^{0.575}$, and $\nu = T^{0.60}$, respectively. The known theoretical error variance of $\frac{\pi^2}{6}$ is imposed in the construction of the $t-$statistics for the fractional differencing parameter d, which are given in parentheses. The superscripts ***, **, * indicate statistical significance for the null hypothesis $d = 0$ (corresponding to the presence of a unit root in the log levels of the series) against the alternative $d \neq 0$ at the 1, 5, and 10 per cent levels, respectively.

Table 2: Empirical Estimates of the Fractional-Differencing Parameter d for Selected Components of the Monetary Aggregates

Monetary Series	$d(0.50)$	$d(0.525)$	$d(0.55)$	$d(0.575)$	$d(0.60)$
M1 Components					
Currency	0.720 (4.104) ***	0.690 (4.284) ***	0.692 (4.736) ***	0.678 (5.131) ***	0.759 (6.259) ***
Demand Deposits	0.279 (1.595)	0.282 (1.756) *	0.294 (2.014) **	0.214 (1.624)	0.165 (1.360)
Total Checkable Deposits	0.443 (2.528) **	0.467 (2.903) ***	0.393 (2.689) ***	0.382 (2.895) ***	0.460 (3.798) ***
M2 Components					
Small Time Deposits (Commercial Banks)	0.551 (3.145) ***	0.573 (3.558) ***	0.485 (3.317) ***	0.413 (3.127) ***	0.325 (2.681) ***
Small Time Deposits (Thrift Institutions)	0.608 (3.468) ***	0.672 (4.177) ***	0.648 (4.436) ***	0.607 (4.594) ***	0.688 (5.676) ***
Savings Deposits (Commercial Banks)	-0.106 (-0.608)	-0.016 (-0.103)	0.039 (0.272)	0.006 (0.048)	0.071 (0.585)
Savings Deposits (Thrift Institutions)	-0.260 (-1.483)	-0.090 (-0.559)	-0.032 (-0.220)	-0.062 (-0.469)	0.034 (0.287)
Money Market Mutual Funds	0.309 (1.473)	0.370 (1.910) *	0.416 (2.375) **	0.481 (2.984) ***	0.553 (3.780) ***
Overnight RP Agreements	0.321 (1.595)	0.168 (0.927)	0.070 (0.426)	-0.044 (-0.298)	-0.063 (-0.464)
M2-M1	0.745 (4.250) ***	0.651 (4.042) ***	0.687 (4.705) ***	0.662 (5.012) ***	0.682 (5.623) ***
M3 Components					
Large Time Deposits (Commercial Banks)	0.277 (1.583)	0.247 (1.535)	0.276 (1.889) *	0.416 (3.155) ***	0.536 (4.425) ***
Large Time Deposits (Thrift Institutions)	0.496 (2.463) **	0.510 (2.816) ***	0.469 (2.837) ***	0.489 (3.196) ***	0.495 (3.603) ***
Term Eurodollars	0.380 (2.166) **	0.321 (1.998) **	0.345 (2.359) **	0.373 (2.825) ***	0.289 (2.382) **
Term RP Agreements	-0.095 (-0.475)	-0.152 (-0.842)	-0.183 (-1.107)	-0.125 (-0.840)	-0.113 (-0.827)
M3-M2	0.361 (2.057) **	0.349 (2.168) ***	0.400 (2.739) ***	0.512 (3.880) ***	0.553 (4.562) ***

Liquid Assets-M3	0.225 (1.283)	0.208 (1.297)	0.277 (1.894) *	0.307 (2.329) ***	0.433 (3.573) ***

Notes: All series are in the form of growth rates. See notes in Table 1 for explanation of Table.

Table 3: Empirical Estimates of the Fractional-Differencing Parameter d for the Divisia Monetary Aggregates

Monetary Series	$d(0.50)$	$d(0.525)$	$d(0.55)$	$d(0.575)$	$d(0.60)$
Divisia M1	0.262	0.267	0.261	0.365	0.423
	(1.400)	(1.618)	(1.704) *	(2.660) ***	(3.379) ***
Divisia M2	0.629	0.494	0.433	0.404	0.413
	(3.363) ***	(2.989) ***	(2.833) ***	(2.944) ***	(3.300) ***
Divisia M3	0.553	0.498	0.459	0.454	0.465
	(2.954) ***	(3.012) ***	(2.996) ***	(3.307) ***	(3.711) ***
Divisia L	0.558	0.533	0.482	0.511	0.489
	(2.983) ***	(3.223) ***	(3.148) ***	(3.726) ***	(3.906) ***

Notes: All series are in the form of growth rates. See notes in Table 1 for explanation of Table.

Table 4: Empirical Estimates of the Fractional-Differencing Parameter d for the Velocity Series

Velocity Series	$d(0.50)$	$d(0.525)$	$d(0.55)$	$d(0.575)$	$d(0.60)$
Simple-sum M1	0.227	0.217	0.181	0.247	0.308
	(1.214)	(1.314)	(1.186)	(1.800) *	(2.462) ***
Simple-sum M2	-0.328	-0.268	-0.270	-0.102	-0.046
	(-1.755) *	(-1.623)	(-1.763) *	(-0.744)	(-0.371)
Simple-sum M3	-0.408	-0.342	-0.319	-0.132	-0.069
	(-2.179) **	(-2.066) **	(-2.086) **	(-0.963)	(-0.553)
Simple-sum L	-0.180	-0.172	-0.184	-0.041	0.003
	(-0.962)	(-1.040)	(-1.207)	(-0.301)	(0.031)
Divisia M1	0.225	0.201	0.142	0.212	0.215
	(1.206)	(1.219)	(0.929)	(1.547)	(1.717) *
Divisia M2	0.019	0.036	0.012	0.123	0.168
	(0.104)	(0.218)	(0.083)	(0.901)	(1.345)
Divisia M3	-0.043	-0.017	-0.027	0.123	0.163
	(-0.230)	(-0.103)	(-0.178)	(0.902)	(1.305)
Divisia L	-0.012	-0.006	-0.018	0.101	0.136
	(-0.068)	(-0.038)	(-0.121)	(0.735)	(1.090)

Notes: All series are in the form of growth rates. See notes in Table 1 for explanation of Table.